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Intuitionistic Fuzzy W- Closed Sets and Intuitionistic Fuzzy W -Continuity

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Abstract

The aim of this paper is to introduce and study the concepts of intuitionistic fuzzy w-closed sets, intuitionistic fuzzy w-continuity and intuitionistic fuzzy w-open & intuitionistic fuzzy w-closed mappings in intuitionistic fuzzy topological spaces.

Key words: Intuitionistic fuzzy w-closed sets, Intuitionistic fuzzy w-open sets, Intuitionistic fuzzy w-connectedness, Intuitionistic fuzzy w-compactness, intuitionistic fuzzy w-continuous mappings.

2000, Mathematics Subject Classification: 54A

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [23] in 1965 and fuzzy topology by Chang [4] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [7], fuzzy connectedness [21], fuzzy separation axioms [3], fuzzy continuity [8], fuzzy g-closed sets [15] and fuzzy g-continuity [16] have been generalized for intuitionistic fuzzy topological spaces. In the present paper we introduce the concepts of intuitionistic fuzzy w-closed sets; intuitionistic fuzzy w-open sets, intuitionistic fuzzy w-connectedness, intuitionistic fuzzy w-compactness and intuitionistic fuzzy w-continuity obtain some of their characterization and properties.

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2. PRELIMINARIES

Let $X$ be a nonempty fixed set. An intuitionistic fuzzy set $A[1]$ in $X$ is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X \}$, where the functions $\mu_A : X \rightarrow [0,1]$ and $\gamma_A : X \rightarrow [0,1]$ denote the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set $A$ respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. The intuitionistic fuzzy sets $\emptyset = \{<x, 0, 0> : x \in X \}$ and $1 = \{<x, 1, 0> : x \in X \}$ are respectively called empty and whole intuitionistic fuzzy set on $X$. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X \}$ is called a subset of another intuitionistic fuzzy set $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X \}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X \}$ is the intuitionistic fuzzy set $A^c = \{<x, \gamma_A(x), \mu_A(x)> : x \in X \}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $\mathcal{A} = \{<x, \mu_A(x), \gamma_A(x)> : x \in X \}$ (i.e., $\bigvee \mathcal{A}$) of $X$ be the intuitionistic fuzzy set $\cap \mathcal{A} = \{<x, \wedge \mu_A(x), \vee \gamma_A(x)> : x \in X \}$ (resp. $\cup \mathcal{A} = \{<x, \vee \mu_A(x), \wedge \gamma_A(x)> : x \in X \}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X \}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X \}$ are said to be q-coincident ($A \equiv B$) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\mathcal{F}$ of intuitionistic fuzzy sets on a non empty set $X$ is called an intuitionistic fuzzy topology [5] on $X$ if the intuitionistic fuzzy sets $\emptyset, 1 \in \mathcal{F}$, and $\mathcal{F}$ is closed under arbitrary union and finite intersection. The ordered pair $(X, \mathcal{F})$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\mathcal{F}$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in $X$ is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains $A$ is called the closure of $A$. It denoted $cl(A)$. The union of all intuitionistic fuzzy open subsets of $A$ is called the interior of $A$. It is denoted $int(A)$ [5].

**Lemma 2.1** [5]: Let $A$ and $B$ be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \mathcal{F})$. Then:

(a) $\bigvee (A \cup B) \subseteq A \subseteq B^c$.
(b) $A$ is an intuitionistic fuzzy closed set in $X$ if and only if $cl(A) = A$.
(c) $A$ is an intuitionistic fuzzy open set in $X$ if and only if $int(A) = A$.
(d) $cl(A^c) = (int(A))^c$.
(e) $int(A^c) = (cl(A))^c$.
(f) $A \subseteq B$ if and only if $int(A) \subseteq int(B)$.
(g) $A \subseteq B$ if and only if $cl(A) \subseteq cl(B)$.
(h) $cl(A \cup B) = cl(A) \cup cl(B)$.
(i) $int(A \cap B) = int(A) \cap int(B)$

**Definition 2.1** [6]: Let $X$ be a nonempty set and $c \in X$ a fixed element in $X$. If $\alpha \in (0,1]$ and $\beta \in [0,1)$ are two real numbers such that $\alpha + \beta \leq 1$ then:

(a) $c(\alpha, \beta) = < x, c_{\alpha}, c_{1-\beta}>$ is called an intuitionistic fuzzy point in $X$, where $\alpha$ denotes the degree of membership $c(\alpha, \beta)$, and $\beta$ denotes the degree of non membership of $c(\alpha, \beta)$.
(b) $c(\beta) = < x, 0, 1-c_{1-\beta}>$ is called a vanishing intuitionistic fuzzy point in $X$, where $\beta$ denotes the degree of non membership of $c(\beta)$.

**Definition 2.2** [7]: A family $\{G_i : i \in \Lambda \}$ of intuitionistic fuzzy sets in $X$ is called an intuitionistic fuzzy open cover of $X$ if $\cup\{G_i : i \in \Lambda \} = 1$ and a finite subfamily of an intuitionistic fuzzy open cover $\{G_i : i \in \Lambda \}$ of $X$ which also an intuitionistic fuzzy open cover of $X$ is called a finite sub cover of $\{G_i : i \in \Lambda \}$.

**Definition 2.3** [7]: An intuitionistic fuzzy topological space $(X, \mathcal{F})$ is called fuzzy compact if every intuitionistic fuzzy open cover of $X$ has a finite sub cover.
Definition 2.4[8]: An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \( (X,\mathcal{I}) \) is called intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) if there exists an intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) \( U \) such that \( U \subseteq A \subseteq \text{cl}(A) \) (resp. \( \text{int}(U) \subseteq A \subseteq U \)).

Definition 2.5[21]: An intuitionistic fuzzy topological space \( X \) is called intuitionistic fuzzy connected if there is no proper intuitionistic fuzzy set of \( X \) which is both intuitionistic fuzzy open and intuitionistic fuzzy closed.

Definition 2.6[15]: An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \( (X,\mathcal{I}) \) is called:
(a) Intuitionistic fuzzy g-closed if \( \text{cl}(A) \subseteq O \) whenever \( A \subseteq O \) and \( O \) is intuitionistic fuzzy open.
(b) Intuitionistic fuzzy g-open if its complement \( A^c \) is intuitionistic fuzzy g-closed.

Remark 2.1[15]: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true.

Definition 2.7[18]: An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \( (X,\mathcal{I}) \) is called:
(a) Intuitionistic fuzzy sg-closed if \( \text{scl}(A) \subseteq O \) whenever \( A \subseteq O \) and \( O \) is intuitionistic fuzzy semi open.
(b) Intuitionistic fuzzy sg-open if its complement \( A^c \) is intuitionistic fuzzy sg-closed.

Remark 2.2[18]: Every intuitionistic fuzzy semi-closed (resp. intuitionistic fuzzy semi-open) set is intuitionistic fuzzy sg-closed (intuitionistic fuzzy sg-open) but its converse may not be true.

Definition 2.8[12]: An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \( (X,\mathcal{I}) \) is called:
(a) Intuitionistic fuzzy gs-closed if \( \text{scl}(A) \subseteq O \) whenever \( A \subseteq O \) and \( O \) is intuitionistic fuzzy open.
(b) Intuitionistic fuzzy gs-open if its complement \( A^c \) is intuitionistic fuzzy gs-closed.

Remark 2.3[12]: Every intuitionistic fuzzy sg-closed (resp. intuitionistic fuzzy sg-open) set is intuitionistic fuzzy gs-closed (intuitionistic fuzzy gs-open) but its converse may not be true.

Definition 2.9: [5] Let \( X \) and \( Y \) are two nonempty sets and \( f: X \rightarrow Y \) is a function.
(a) If \( B = \{<y, \mu_B(y), \gamma_B(y)> : y \in Y\} \) is an intuitionistic fuzzy set in \( Y \), then the preimage of \( B \) under \( f \) denoted by \( f^{-1}(B) \), is the intuitionistic fuzzy set in \( X \) defined by
\[
f^{-1}(B) = \{<x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)> : x \in X\}.
\]
(b) If \( A = \{<x, \lambda_A(x), \nu_A(x)> : x \in X\} \) is an intuitionistic fuzzy set in \( X \), then the image of \( A \) under \( f \) denoted by \( f(A) \) is the intuitionistic fuzzy set in \( Y \) defined by
\[
f(A) = \{<y, f(\lambda_A)(y), f(\nu_A)(y)> : y \in Y\}
\]
where \( f(\nu_A) = 1 - f(1- \nu_A) \).

Definition 2.10[8]: Let \( (X,\mathcal{I}) \) and \( (Y,\sigma) \) be two intuitionistic fuzzy topological spaces and let \( f: X \rightarrow Y \) be a function. Then \( f \) is said to be
(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of \( Y \) is an intuitionistic fuzzy open set in \( X \).
(b) Intuitionistic fuzzy semi continuous if the pre image of each intuitionistic fuzzy open set of \( Y \) is an intuitionistic fuzzy semi open set in \( X \).
(c) Intuitionistic fuzzy closed if the image of each intuitionistic fuzzy closed set in \( X \) is an intuitionistic fuzzy closed set in \( Y \).
(d) Intuitionistic fuzzy open if the image of each intuitionistic fuzzy open set in \( X \) is an intuitionistic fuzzy open set in \( Y \).
Definition 2.6[12, 16, 17, 19]: Let (X, ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let \( f: X \rightarrow Y \) be a function. Then \( f \) is said to be

(a) Intuitionistic fuzzy g-continuous [16] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g-closed in X.
(b) Intuitionistic fuzzy gc-irresolute[17] if the pre image of every intuitionistic fuzzy g-closed in Y is intuitionistic fuzzy g-closed in X.
(c) Intuitionistic fuzzy sg-continuous [19] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg-closed in X.
(d) Intuitionistic fuzzy gs-continuous [12] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gs-closed in X.

Remark 2.4[12, 16, 19]:
(a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [16].
(b) Every intuitionistic fuzzy semi continuous mapping is intuitionistic fuzzy sg-continuous, but the converse may not be true [19].
(c) Every intuitionistic fuzzy sg-continuous mapping is intuitionistic fuzzy gs-continuous, but the converse may not be true [12].
(d) Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gs-continuous, but the converse may not be true [12].

3. INTUITIONISTIC FUZZY W-CLOSED SET

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, ℑ) is called an intuitionistic fuzzy w-closed if \( \text{cl}(A) \subseteq O \) whenever \( A \subseteq O \) and O is intuitionistic fuzzy semi open.

Remark 3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy w-closed but its converse may not be true.

Example 3.1: Let \( X = \{a, b\} \) and \( ℑ = \{ \emptyset, 1, U \} \) be an intuitionistic fuzzy topology on X, where U= \{ <a,0.5,0.5>,<b, 0.4, 0.6 >\}. Then the intuitionistic fuzzy set \( A = \{<a,0.5,0.5>,<b,0.5,0.5>\} \) is intuitionistic fuzzy w-closed but it is not intuitionistic fuzzy closed.

Remark 3.2: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed but its converse may not be true.

Example 3.2: Let \( X = \{a, b\} \) and \( ℑ = \{ \emptyset, 1, U \} \) be an intuitionistic fuzzy topology on X, where U= \{ <a,0.7,0.3>,<b, 0.6, 0.4 >\}. Then the intuitionistic fuzzy set \( A = \{<a,0.6,0.4>,<b,0.7,0.3>\} \) is intuitionistic fuzzy g-closed but it is not intuitionistic fuzzy w-closed.

Remark 3.3: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy sg-closed but its converse may not be true.

Example 3.3: Let \( X = \{a, b\} \) and \( ℑ = \{ \emptyset, 1, U \} \) be an intuitionistic fuzzy topology on X, where U= \{ <a,0.5,0.5>,<b, 0.4, 0.6 >\}. Then the intuitionistic fuzzy set \( A = \{<a,0.5,0.5>,<b,0.3,0.7>\} \) is intuitionistic fuzzy sg-closed but it is not intuitionistic fuzzy w-closed.
**Remark 3.4:** Remarks 2.1, 2.2, 2.3, 3.1, 3.2, 3.3 reveals the following diagram of implication.

\[
\begin{array}{ccc}
\text{Intuitionistic fuzzy} & \xrightarrow{\text{Closed}} & \text{Intuitionistic fuzzy} \\
\text{w-closed} & \xrightarrow{} & \text{g-closed} \\
\text{Intuitionistic fuzzy} & \xrightarrow{\text{Semi closed}} & \text{Intuitionistic fuzzy} \\
\text{sg-closed} & \xrightarrow{} & \text{gs-closed}
\end{array}
\]

**Theorem 3.1:** Let \((X, 3)\) be an intuitionistic fuzzy topological space and \(A\) is an intuitionistic fuzzy set of \(X\). Then \(A\) is intuitionistic fuzzy w-closed if and only if \(\overline{\text{cl}}(AqF) \Rightarrow \overline{\text{cl}}(\text{AqF})\) for every intuitionistic fuzzy semi closed set \(F\) of \(X\).

**Proof:** **Necessity:** Let \(F\) be an intuitionistic fuzzy semi closed set of \(X\) and \(\overline{\text{cl}}(AqF)\). Then by Lemma 2.1(a), \(A \subseteq F^c\) and \(F^c\) intuitionistic fuzzy semi open in \(X\). Therefore \(\text{cl}(A) \subseteq F^c\) by Def 3.1 because \(A\) is intuitionistic fuzzy w-closed. Hence by lemma 2.1(a), \(\overline{\text{cl}}(\text{AqF})\).

**Sufficiency:** Let \(O\) be an intuitionistic fuzzy semi open set of \(X\) such that \(A \subseteq O\) i.e. \(A \subseteq (O)^c\) Then by Lemma 2.1(a), \(\overline{\text{cl}}(A, O^c)\) and \(O^c\) is an intuitionistic fuzzy semi closed set in \(X\). Hence by hypothesis \(\overline{\text{cl}}((A, O^c))\). Therefore by Lemma 2.1(a), \(\text{cl}(A) \subseteq ((O)^c)\) i.e. \(\text{cl}(A) \subseteq O\) Hence \(A\) is intuitionistic fuzzy w-closed in \(X\).

**Theorem 3.2:** Let \(A\) be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space \((X, 3)\) and \(c(\alpha, \beta)\) be an intuitionistic fuzzy point of \(X\) such that \(c(\alpha, \beta)\) is \(\text{cl}(A)\) then \(\overline{\text{cl}}(c(\alpha, \beta)A)\).

**Proof:** If \(\overline{\text{cl}}(c(\alpha, \beta))A\) then by Lemma 2.1(a), \(\overline{\text{cl}}(c(\alpha, \beta)) \subseteq A^c\) which implies that \(A \subseteq (\overline{\text{cl}}(c(\alpha, \beta)))^c\) and so \(\text{cl}(A) \subseteq (\text{cl}(c(\alpha, \beta)))^c \subseteq (c(\alpha, \beta))^c\), because \(A\) is intuitionistic fuzzy w-closed in \(X\). Hence by Lemma 2.1(a), \(\overline{\text{cl}}(c(\alpha, \beta)\text{cl}(A))\), a contradiction.

**Theorem 3.3:** Let \(A\) and \(B\) are two intuitionistic fuzzy w-closed sets in an intuitionistic fuzzy topological space \((X, 3)\), then \(A \cup B\) is intuitionistic fuzzy w-closed.

**Proof:** Let \(O\) be an intuitionistic fuzzy semi open set in \(X\), such that \(A \cup B \subseteq O\). Then \(A \subseteq O\) and \(B \subseteq O\). So, \(\text{cl}(A) \subseteq O\) and \(\text{cl}(B) \subseteq O\). Therefore \(\text{cl}(A) \cup \text{cl}(B) = \text{cl}(A \cup B) \subseteq O\). Hence \(A \cup B\) is intuitionistic fuzzy w-closed.

**Remark 3.2:** The intersection of two intuitionistic fuzzy w-closed sets in an intuitionistic fuzzy topological space \((X, 3)\) may not be intuitionistic fuzzy w-closed. For,

**Example 3.2:** Let \(X = \{a, b, c\}\) and \(U, A\) and \(B\) be the intuitionistic fuzzy sets of \(X\) defined as follows:

\[
\begin{align*}
U &= \{<a, 1, 0>, <b, 0, 1>, <c, 0, 1>\} \\
A &= \{<a, 1, 0>, <b, 1, 0>, <c, 0, 1>\} \\
B &= \{<a, 1, 0>, <b, 0, 1>, <c, 1, 0>\}
\end{align*}
\]
Let $\mathcal{I} = \{0, 1, U\}$ be intuitionistic fuzzy topology on $X$. Then $A$ and $B$ are intuitionistic fuzzy w-closed in $(X, \mathcal{I})$ but $A \cap B$ is not intuitionistic fuzzy w-closed.

**Theorem 3.4**: Let $A$ be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space $(X, \mathcal{I})$ and $A \subseteq B \subseteq \text{cl}(A)$. Then $B$ is intuitionistic fuzzy w-closed in $X$.

**Proof**: Let $O$ be an intuitionistic fuzzy semi open set such that $B \subseteq O$. Then $A \subseteq O$ and since $A$ is intuitionistic fuzzy w-closed, $\text{cl}(A) \subseteq O$. Now $B \subseteq \text{cl}(A) \Rightarrow \text{cl}(B) \subseteq \text{cl}(A) \subseteq O$. Consequently $B$ is intuitionistic fuzzy w-closed.

**Definition 3.2**: An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \mathcal{I})$ is called intuitionistic fuzzy w-open if and only if its complement $A^c$ is intuitionistic fuzzy w-closed.

**Remark 3.5**: Every intuitionistic fuzzy open set is intuitionistic fuzzy w-open. But the converse may not be true. For

**Example 3.4**: Let $X = \{a, b\}$ and $\mathcal{I} = \{0, 1, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.4, 0.6>\}$. Then intuitionistic fuzzy set $B = \{<a, 0.5, 0.5>, <b, 0.5, 0.5>\}$ is an intuitionistic fuzzy w-open in intuitionistic fuzzy topological space $(X, \mathcal{I})$ but it is not intuitionistic fuzzy open in $(X, \mathcal{I})$.

**Remark 3.6**: Every intuitionistic fuzzy w-open set is intuitionistic fuzzy g-open but its converse may not be true.

**Example 3.5**: Let $X = \{a, b\}$ and $\mathcal{I} = \{0, 1, U\}$ be an intuitionistic fuzzy topology on $X$, where $U = \{<a, 0.5, 0.5>, <b, 0.4, 0.6>\}$. Then the intuitionistic fuzzy set $A = \{<a, 0.4, 0.6>, <b, 0.3, 0.7>\}$ is intuitionistic fuzzy g-open in $(X, \mathcal{I})$ but it is not intuitionistic fuzzy w-open in $(X, \mathcal{I})$.

**Theorem 3.5**: An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \mathcal{I})$ is intuitionistic fuzzy w-open if $F \subseteq \text{int}(A)$ whenever $F$ is intuitionistic fuzzy semi closed and $F \subseteq A$.

**Proof**: Follows from definition 3.1 and Lemma 2.1

**Remark 3.4**: The union of two intuitionistic fuzzy w-open sets in an intuitionistic fuzzy topological space $(X, \mathcal{I})$ may not be intuitionistic fuzzy w-open. For the intuitionistic fuzzy set $C = \{<a, 0.4, 0.6>, <b, 0.7, 0.3>\}$ and $D = \{<a, 0.2, 0.8>, <b, 0.5, 0.5>\}$ in the intuitionistic fuzzy topological space $(X, \mathcal{I})$ in Example 3.2 are intuitionistic fuzzy w-open but their union is not intuitionistic fuzzy w-open.

**Theorem 3.6**: Let $A$ be an intuitionistic fuzzy w-open set of an intuitionistic fuzzy topological space $(X, \mathcal{I})$ and $\text{int}(A) \subseteq B \subseteq A$. Then $B$ is intuitionistic fuzzy w-open.

**Proof**: Suppose $A$ is an intuitionistic fuzzy w-open in $X$ and $\text{int}(A) \subseteq B \subseteq A$. Then $A^c \subseteq B^c \subseteq (\text{int}(A))^c \Rightarrow A^c \subseteq B^c \subseteq \text{cl}(A^c)$ by Lemma 2.1(d) and $A^c$ is intuitionistic fuzzy w-closed it follows from theorem 3.4 that $B^c$ is intuitionistic fuzzy w-closed. Hence $B$ is intuitionistic fuzzy w-open.

**Definition 3.3**: An intuitionistic fuzzy topological space $(X, \mathcal{I})$ is called intuitionistic fuzzy semi normal if for every pair of two intuitionistic fuzzy semi closed sets $F_1$ and $F_2$ such that $\text{cl}(F_1 \cap F_2)$, there exists two intuitionistic fuzzy semi open sets $U_1$ and $U_2$ in $X$ such that $F_1 \subseteq U_1$, $F_2 \subseteq U_2$ and $\text{cl}(U_1 \cap U_2)$. 


Theorem 3.7: If $F$ is intuitionistic fuzzy semi closed and $A$ is intuitionistic fuzzy w–closed set of an intuitionistic fuzzy semi normal space $(X, \mathcal{I})$ and $\overline{\langle A, F \rangle}$. Then there exists intuitionistic fuzzy semi open sets $U$ and $V$ in $X$ such that $\text{cl} (A) \subseteq U, F \subseteq V$ and $\overline{\langle U \cup V \rangle}$.

Proof: Since $A$ is intuitionistic fuzzy w-closed set and $\overline{\langle A, F \rangle}$, by Theorem (3.1), $\overline{\text{cl} (A)} \subseteq F$ and $(X, \mathcal{I})$ is intuitionistic fuzzy semi normal. Therefore by Definition 3.3 there exists intuitionistic fuzzy semi open sets $U$ and $V$ in $X$ such that $\text{cl} (A) \subseteq U, F \subseteq V$ and $\overline{\langle U \cup V \rangle}$.

Theorem 3.8: Let $A$ be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space $(X, \mathcal{I})$ and $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I}^*)$ is an intuitionistic fuzzy irresolute and intuitionistic fuzzy closed mapping then $f(A)$ is an intuitionistic w-closed set in $Y$.

Proof: Let $A$ be an intuitionistic fuzzy w-closed set in $X$ and $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I}^*)$ is an intuitionistic fuzzy continuous and intuitionistic fuzzy closed mapping. Let $f(A) \subseteq G$ where $G$ is intuitionistic fuzzy semi open in $Y$ then $A \subseteq f^{-1}(G)$ and $f^{-1}(G)$ is intuitionistic fuzzy semi open in $X$ because $f$ is intuitionistic fuzzy irresolute. Now $A$ be an intuitionistic fuzzy w-closed set in $X$, by definition 3.1 $\text{cl}(A) \subseteq f^{-1}(G)$. Thus $f(\text{cl}(A)) \subseteq G$ and $f(\text{cl}(A))$ is an intuitionistic fuzzy closed set in $Y$ (since $\text{cl}(A)$ is intuitionistic fuzzy closed in $X$ and $f$ is intuitionistic fuzzy closed mapping). It follows that $\text{cl} (f(A)) \subseteq f(\text{cl}(A)) = f(\text{cl}(A)) \subseteq G$. Hence $\text{cl}(f(A)) \subseteq G$ whenever $f(A) \subseteq G$ and $G$ is intuitionistic fuzzy semi open in $Y$. Hence $f(A)$ is intuitionistic fuzzy w-closed set in $Y$.

Theorem 3.9: Let $(X, \mathcal{I})$ be an intuitionistic fuzzy topological space and $\text{IFSO}(X)$ (resp.$\text{IFC}(X)$) be the family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy closed) sets of $X$. Then $\text{IFSO}(X) = \text{IFC}(X)$ if and only if every intuitionistic fuzzy set of $X$ is intuitionistic fuzzy w-closed.

Proof: Necessity: Suppose that $\text{IFSO}(X) = \text{IFC}(X)$ and let $A$ is any intuitionistic fuzzy set of $X$ such that $A \subseteq U \in \text{IFSO}(X)$ i.e. $U$ is intuitionistic fuzzy semi open. Then $\text{cl}(A) \subseteq \text{cl}(U) = U$ because $U \in \text{IFSO}(X)$ = $\text{IFC}(X)$. Hence $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is intuitionistic fuzzy semi open. Hence $A$ is w- closed set.

Sufficiency: Suppose that every intuitionistic fuzzy set of $X$ is intuitionistic fuzzy w-closed. Let $U \in \text{IFSO}(X)$ then since $U \subseteq U$ and $U$ is intuitionistic fuzzy w- closed, $\text{cl}(U) \subseteq U$ then $U \in \text{IFC}(X)$. Thus $\text{IFSO}(X) \subseteq \text{IFC}(X)$. If $T \in \text{IFC}(X)$ then $T^c \in \text{IFO}(X) \subseteq \text{IFSO}(X)$ hence $T \in \text{IFO}(X)$ hence $\text{IFO}(X) \subseteq \text{IFSO}(X)$. Consequently $\text{IFC}(X) \subseteq \text{IFSO}(X)$ and $\text{IFSO}(X) = \text{IFC}(X)$.

4: INTUITIONISTIC FUZZY W-CONNECTEDNESS AND INTUITIONISTIC FUZZY W-COMPACTNESS

Definition 4.1: An intuitionistic fuzzy topological space $(X, \mathcal{I})$ is called intuitionistic fuzzy w – connected if there is no proper intuitionistic fuzzy set of $X$ which is both intuitionistic fuzzy w-open and intuitionistic fuzzy w- closed.

Theorem 4.1: Every intuitionistic fuzzy w-connected space is intuitionistic fuzzy connected.

Proof: Let $(X, \mathcal{I})$ be an intuitionistic fuzzy w –connected space and suppose that $(X, \mathcal{I})$ is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set $A (A \neq \emptyset, A \neq X)$ such that $A$ is both...
intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic w-open ((resp. intuitionistic fuzzy w-closed), X is not intuitionistic fuzzy w-connected, a contradiction.

**Remark 4.1:** Converse of theorem 4.1 may not be true for ,

**Example 4.1:** Let \( X = \{a, b\} \) and \( \mathcal{I} = \{\emptyset, \{a, b\}, U\} \) be an intuitionistic fuzzy topology on X, where \( U = \{\langle a, 0.5, 0.5\rangle, \langle b, 0.4, 0.6\rangle\} \). Then intuitionistic fuzzy topological space \((X, \mathcal{I})\) is intuitionistic fuzzy connected but not intuitionistic fuzzy w-connected because there exists a proper intuitionistic fuzzy set \( A = \{\langle a, 0.5, 0.5\rangle, \langle b, 0.5, 0.5\rangle\} \) which is both intuitionistic fuzzy w-open and intuitionistic fuzzy w-closed. But this is a contradiction to the fact that X is intuitionistic fuzzy w-connected.

**Theorem 4.2:** An intuitionistic fuzzy topological space \((X, \mathcal{I})\) is intuitionistic fuzzy w-connected if and only if there exists no non zero intuitionistic fuzzy open sets \( A \) and \( B \) in X such that \( B = A^c \).

**Proof:** Necessity: Suppose that \( A \) and \( B \) are intuitionistic fuzzy w-open sets such that \( A \neq \emptyset \neq B \) and \( A = B^c \). Since \( A = B^c \), \( B \) is an intuitionistic fuzzy w-open set which implies that \( B^c = A \) is intuitionistic fuzzy w-closed set and \( B \neq \emptyset \) this implies that \( B^c \neq 1 \) i.e. \( A \neq 1 \). Hence there exists a proper intuitionistic fuzzy set \( A \) (\( A \neq \emptyset \), \( A \neq 1 \)) such that \( A \) is both intuitionistic fuzzy w-open and intuitionistic fuzzy w-closed. But this is a contradiction to the fact that X is intuitionistic fuzzy w-connected.

**Sufficiency:** Let \((X, \mathcal{I})\) is an intuitionistic fuzzy topological space and \( A \) is both intuitionistic fuzzy w-open set and intuitionistic fuzzy w-closed set in X such that \( \emptyset \neq A \neq 1 \). Now take \( A = B^c \). In this case \( B \) is an intuitionistic fuzzy w-open set and \( A \neq 1 \). This implies that \( B = A^c \neq \emptyset \) which is a contradiction. Hence there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy w-open and intuitionistic fuzzy w-closed. Therefore intuitionistic fuzzy topological \((X, \mathcal{I})\) is intuitionistic fuzzy w-connected.

**Definition 4.2:** Let \((X, \mathcal{I})\) be an intuitionistic fuzzy topological space and \( A \) be an intuitionistic fuzzy set X. Then w-interior and w-closure of \( A \) are defined as follows.

- \( \text{wcl}(A) = \cap \{K: K \text{ is an intuitionistic fuzzy w-closed set in X and } A \subseteq K\} \)
- \( \text{wint}(A) = \cup \{G: G \text{ is an intuitionistic fuzzy w-open set in X and } G \subseteq A\} \)

**Theorem 4.3:** An intuitionistic fuzzy topological space \((X, \mathcal{I})\) is intuitionistic fuzzy w-connected if and only if there exists no non zero intuitionistic fuzzy w-open sets \( A \) and \( B \) in X such that \( B = (\text{wcl}(A))^c \), \( A = (\text{wcl}(B))^c \).

**Proof:** Necessity: Assume that there exists intuitionistic fuzzy sets \( A \) and \( B \) such that \( A \neq \emptyset \neq B \) in X such that \( B = A^c \), \( B = (\text{wcl}(A))^c \), \( A = (\text{wcl}(B))^c \). Since \( (\text{wcl}(A))^c \) and \( (\text{wcl}(B))^c \) are intuitionistic fuzzy w-open sets in X, which is a contradiction.

**Sufficiency:** Let \( A \) be both an intuitionistic fuzzy w-open set and intuitionistic fuzzy w-closed set such that \( \emptyset \neq A \neq 1 \). Taking \( B = A^c \), we obtain a contradiction.

**Definition 4.3:** An intuitionistic fuzzy topological space \((X, \mathcal{I})\) is said to be intuitionistic fuzzy \( T_{1/2} \) if every intuitionistic fuzzy w-closed set in X is intuitionistic fuzzy closed in X.

**Theorem 4.4:** Let \((X, \mathcal{I})\) be an intuitionistic fuzzy w-connected space, then the following conditions are equivalent:

- (a) \( X \) is intuitionistic fuzzy w-connected.
(b) X is intuitionistic fuzzy connected.

**Proof:** (a) ⇒ (b) follows from Theorem 4.1

(b) ⇒ (a): Assume that X is intuitionistic fuzzy w- $T_{1/2}$ and intuitionistic fuzzy w-connected space. If possible, let X be not intuitionistic fuzzy w-connected, then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy w-open and w-closed. Since X is intuitionistic fuzzy w-$T_{1/2}$, A is intuitionistic fuzzy open and intuitionistic fuzzy closed which implies that X is not intuitionistic fuzzy connected, a contradiction.

**Definition 4.4:** A collection \{ $A_i : i \in \Lambda$\} of intuitionistic fuzzy w-open sets in intuitionistic fuzzy topological space $(X, \mathcal{I})$ is called intuitionistic fuzzy w-open cover of intuitionistic fuzzy set $B$ of $X$ if $B \subseteq \bigcup \{ A_i : i \in \Lambda \}$

**Definition 4.5:** An intuitionistic fuzzy topological space $(X, \mathcal{I})$ is said to be intuitionistic fuzzy w-compact if every intuitionistic fuzzy w-open cover of $X$ has a finite sub cover.

**Definition 4.6:** An intuitionistic fuzzy set $B$ of intuitionistic fuzzy topological space $(X, \mathcal{I})$ is said to be intuitionistic fuzzy w-compact relative to $X$, if for every collection \{ $A_i : i \in \Lambda$\} of intuitionistic fuzzy w-open subset of $X$ such that $B \subseteq \bigcup \{ A_i : i \in \Lambda \}$ there exists finite subset $\Lambda_0$ of $\Lambda$ such that $B \subseteq \bigcup \{ A_i : i \in \Lambda_0 \}$.

**Definition 4.7:** A crisp subset $B$ of intuitionistic fuzzy topological space $(X, \mathcal{I})$ is said to be intuitionistic fuzzy w-compact as intuitionistic fuzzy subspace of $X$.

**Theorem 4.5:** A intuitionistic fuzzy w-closed crisp subset of intuitionistic fuzzy w-compact space is intuitionistic fuzzy w-compact relative to $X$.

**Proof:** Let $A$ be an intuitionistic fuzzy w-closed crisp subset of intuitionistic fuzzy w-compact space $(X, \mathcal{I})$. Then $A^c$ is intuitionistic fuzzy w-open in $X$. Let $M$ be a cover of $A$ by intuitionistic fuzzy w-open sets in $X$. Then the family \{ $M, A^c$\} is intuitionistic fuzzy w-open cover of $X$. Since $X$ is intuitionistic fuzzy w-compact, it has a finite sub cover say \{ $G_1, G_2, G_3, \ldots, G_n$\}. If this sub cover contains $A^c$, we discard it. Otherwise leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy w-open sub cover of $A$. Therefore $A$ is intuitionistic fuzzy w-compact relative to $X$.

## 5: INTUITIONISTIC FUZZY W-CONTINUOUS MAPPINGS

**Definition 5.1:** A mapping $f : (X, \mathcal{I}) \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous if inverse image of every intuitionistic fuzzy closed set of $Y$ is intuitionistic fuzzy w-closed set in $X$.

**Theorem 5.1:** A mapping $f : (X, \mathcal{I}) \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous if and only if the inverse image of every intuitionistic fuzzy open set of $Y$ is intuitionistic fuzzy w-open in $X$.

**Proof:** It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set $U$ of $Y$.

**Remark 5.1** Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy w-continuous, but converse may not be true. For,

**Example 5.1** Let $X = \{a, b\}$, $Y = \{x, y\}$ and intuitionistic fuzzy sets $U$ and $V$ are defined as follows:

$U = \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle$

$V = \langle x, 0.5, 0.5 \rangle, \langle y, 0.5, 0.5 \rangle$
Let $\mathcal{I} = \{0, 1, U\}$ and $\sigma = \{0, 1, V\}$ be intuitionistic fuzzy topologies on $X$ and $Y$ respectively. Then the mapping $f: (X, \mathcal{I}) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy w-continuous but not intuitionistic fuzzy continuous.

**Remark 5.2** Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy g-continuous, but converse may not be true. For,

**Example 5.2:** Let $X = \{a, b\}$, $Y = \{x, y\}$ and intuitionistic fuzzy sets $U$ and $V$ are defined as follows:

$U = \{(a, 0.7, 0.3), (b, 0.6, 0.4)\}$
$V = \{(x, 0.6, 0.4), (y, 0.7, 0.3)\}$

Let $\mathcal{I} = \{0, 1, U\}$ and $\sigma = \{0, 1, V\}$ be intuitionistic fuzzy topologies on $X$ and $Y$ respectively. Then the mapping $f: (X, \mathcal{I}) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy g-continuous but not intuitionistic fuzzy w-continuous.

**Remark 5.3** Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy sg-continuous, but converse may not be true. For,

**Example 5.1** Let $X = \{a, b\}$, $Y = \{x, y\}$ and intuitionistic fuzzy sets $U$ and $V$ are defined as follows:

$U = \{(a, 0.5, 0.5), (b, 0.4, 0.6)\}$
$V = \{(x, 0.5, 0.5), (y, 0.3, 0.7)\}$

Let $\mathcal{I} = \{0, 1, U\}$ and $\sigma = \{0, 1, V\}$ be intuitionistic fuzzy topologies on $X$ and $Y$ respectively. Then the mapping $f: (X, \mathcal{I}) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy sg-continuous but not intuitionistic fuzzy w-continuous.

**Remark 5.4:** Remarks 2.4, 5.1, 5.2, 5.3 reveals the following diagram of implication:

```
Intuitionistic fuzzy          Intuitionistic fuzzy          Intuitionistic fuzzy
Continuous                    w-continuous                        g-continuous
                          \downarrow                                 \downarrow
Intuitionistic fuzzy                 Intuitionistic fuzzy            Intuitionistic fuzzy
Semi continuous                           sg-continuous                     gs-continuous
                          \downarrow                               \downarrow
```

**Theorem 5.2:** If $f: (X, \mathcal{I}) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy w-continuous then for each intuitionistic fuzzy point $c(\alpha, \beta)$ of $X$ and each intuitionistic fuzzy open set $V$ of $Y$ such that $f(c(\alpha, \beta)) \subseteq V$ there exists a intuitionistic fuzzy w-open set $U$ of $X$ such that $c(\alpha, \beta) \subseteq U$ and $f(U) \subseteq V$.

**Proof:** Let $c(\alpha, \beta)$ be intuitionistic fuzzy point of $X$ and $V$ be intuitionistic fuzzy open set of $Y$ such that $f(c(\alpha, \beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis $U$ is intuitionistic fuzzy w-open set of $X$ such that $c(\alpha, \beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

**Theorem 5.3:** Let $f: (X, \mathcal{I}) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy w-continuous then for each intuitionistic fuzzy point $c(\alpha, \beta)$ of $X$ and each intuitionistic fuzzy open set $V$ of $Y$ such that $f(c(\alpha, \beta)) \subseteq V$, there exists a intuitionistic fuzzy w-open set $U$ of $X$ such that $c(\alpha, \beta) \subseteq U$ and $f(U) \subseteq V$.

**Proof:** Let $c(\alpha, \beta)$ be intuitionistic fuzzy point of $X$ and $V$ be intuitionistic fuzzy open set of $Y$ such that $f(c(\alpha, \beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis $U$ is intuitionistic fuzzy w-open set of $X$ such that $c(\alpha, \beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$. 

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**Theorem 5.4:** If $f : (X,3) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy w-continuous, then $f(wcl(A) \subseteq cl(f(A)))$ for every intuitionistic fuzzy set $A$ of $X$.

**Proof:** Let $A$ be an intuitionistic fuzzy set of $X$. Then $cl(f(A))$ is an intuitionistic fuzzy closed set of $Y$. Since $f$ is intuitionistic fuzzy w-continuous, $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy w-closed in $X$. Clearly $A \subseteq f^{-1}(cl((A))$. Therefore $wcl(A) \subseteq wcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Hence $f(wcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set $A$ of $X$.

**Theorem 5.5:** A mapping $f$ from an intuitionistic fuzzy $w$-$T_{1/2}$ space $(X,3)$ to an intuitionistic fuzzy topological space $(Y,\sigma)$ is intuitionistic fuzzy semi-continuous if and only if it is intuitionistic fuzzy w-continuous.

**Proof:** Obvious

**Remark 5.5:** The composition of two intuitionistic fuzzy w-continuous mapping may not be intuitionistic fuzzy w-continuous.

**Example 5-5:** Let $X = \{a, b\}$, $Y = \{x, y\}$ and $Z = \{p, q\}$ and intuitionistic fuzzy sets $U, V$ and $W$ defined as follows:

- $U = \{<a, 0.5, 0.5>, <b, 0.4, 0.6>\}$
- $V = \{<x, 0.5, 0.5>, <y, 0.3, 0.7>\}$
- $W = \{<p, 0.6, 0.4>, <q, 0.4, 0.6>\}$

Let $3 = \{\emptyset, 1, U\}$, $\sigma = \{\emptyset, 1, V\}$ and $\mu = \{\emptyset, 1, W\}$ be intuitionistic fuzzy topologies on $X$, $Y$ and $Z$ respectively. Let the mapping $f : (X,3) \rightarrow (Y,\sigma)$ defined by $f(a) = x$ and $f(b) = y$ and $g : (Y,\sigma) \rightarrow (Z,\mu)$ defined by $g(x) = p$ and $g(y) = q$. Then the mappings $f$ and $g$ are intuitionistic fuzzy w-continuous but the mapping $gof: (X,3) \rightarrow (Z,\mu)$ is not intuitionistic fuzzy w-continuous.

**Theorem 5.6:** If $f : (X,3) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy w-continuous and $g : (Y,\sigma) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-continuous. Then $gof : (X,3) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-continuous.

**Proof:** Let $A$ is an intuitionistic fuzzy closed set in $Z$. then $g^{-1}(A)$ is intuitionistic fuzzy closed in $Y$ because $g$ is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy w-closed in $X$. Hence $gof$ is intuitionistic fuzzy w-continuous.

**Theorem 5.7:** If $f : (X,3) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy w-continuous and $g : (Y,\sigma) \rightarrow (Z,\mu)$ is intuitionistic fuzzy g-continuous and $(Y,\sigma)$ is intuitionistic fuzzy $(T_{1/2})$ then $gof : (X,3) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-continuous.

**Proof:** Let $A$ is an intuitionistic fuzzy closed set in $Z$, then $g^{-1}(A)$ is intuitionistic fuzzy g-closed in $Y$. Since $Y$ is $(T_{1/2})$, then $g^{-1}(A)$ is intuitionistic fuzzy g-closed in $Y$. Hence $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy w-closed in $X$. Hence $gof$ is intuitionistic fuzzy w-continuous.

**Theorem 5.8:** If $f : (X,3) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy gc-irresolute and $g : (Y,\sigma) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-continuous. Then $gof : (X,3) \rightarrow (Z,\mu)$ is intuitionistic fuzzy g-continuous.

**Proof:** Let $A$ is an intuitionistic fuzzy gc-closed set in $Z$, then $g^{-1}(A)$ is intuitionistic fuzzy w-closed in $Y$, because $g$ is intuitionistic fuzzy w-continuous. Since every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed set, therefore $g^{-1}(A)$ is intuitionistic fuzzy g-closed in $Y$. Then $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g-closed in $X$, because $f$ is intuitionistic fuzzy gc-irresolute. Hence $gof : (X,3) \rightarrow (Z,\mu)$ is intuitionistic fuzzy g-continuous.

**Theorem 5.9:** An intuitionistic fuzzy $w$-continuous image of a intuitionistic fuzzy $w$-compact space is intuitionistic fuzzy compact.
Proof: Let $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I})$ is intuitionistic fuzzy $w$-continuous map from an intuitionistic fuzzy $w$-compact space $(X, \mathcal{I})$ onto an intuitionistic fuzzy topological space $(Y, \mathcal{I})$. Let $\{A_i: i \in \Lambda\}$ be an intuitionistic fuzzy open cover of $X$ then $\{f^{-1}(A_i): i \in \Lambda\}$ is an intuitionistic fuzzy $w$-open cover of $X$. Since $X$ is intuitionistic fuzzy $w$-compact, it has a finite intuitionistic fuzzy subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$. Since $f$ is onto $\{A_1, A_2, \ldots, A_n\}$ is an intuitionistic fuzzy open cover of $Y$ and so $(Y, \mathcal{I})$ is intuitionistic fuzzy compact.

Theorem 5.10: If $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I})$ is intuitionistic fuzzy $w$-continuous surjection and $X$ is intuitionistic fuzzy $w$-connected then $Y$ is intuitionistic fuzzy connected.

Proof: Suppose $Y$ is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set $G$ of $Y$ which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy open set of $X$, which is both intuitionistic fuzzy $w$-open and intuitionistic fuzzy $w$-closed, because $f$ is intuitionistic fuzzy $w$-continuous surjection. Hence $X$ is not intuitionistic fuzzy $w$-connected, which is a contradiction.

6. INTUITIONISTIC FUZZY $W$-OPEN MAPPINGS

Definition 6.1: A mapping $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I})$ is intuitionistic fuzzy $w$-open if the image of every intuitionistic fuzzy open set of $X$ is intuitionistic fuzzy $w$-open set in $Y$.

Remark 6.1: Every intuitionistic fuzzy open map is intuitionistic fuzzy $w$-open but converse may not be true. For,

Example 6.1: Let $X = \{a, b\}$, $Y = \{x, y\}$ and the intuitionistic fuzzy set $U$ and $V$ are defined as follows:

$U = \{< a, 0.5, 0.5 >, < b, 0.4, 0.6 >\}$
$V = \{< x, 0.5, 0.5 >, < y, 0.3, 0.7 >\}$

Then $\mathcal{I}_X = \{\emptyset, U, 1\}$ and $\mathcal{I}_Y = \{\emptyset, V, 1\}$ be intuitionistic fuzzy topologies on $X$ and $Y$ respectively. Then the mapping $f: (X, \mathcal{I}_X) \rightarrow (Y, \mathcal{I}_Y)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy $w$-open but it is not intuitionistic fuzzy open.

Theorem 6.1: A mapping $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I})$ is intuitionistic fuzzy $w$-open if and only if for every intuitionistic fuzzy set $U$ of $X$ $f(\text{int}(U)) \subseteq \text{wint}(f(U))$.

Proof: Necessity Let $f$ be an intuitionistic fuzzy $w$-open mapping and $U$ is an intuitionistic fuzzy open set in $X$. Now $\text{int}(U) \subseteq U$ which implies that $f(\text{int}(U)) \subseteq f(U)$. Since $f$ is an intuitionistic fuzzy $w$-open mapping, $f(\text{int}(U))$ is intuitionistic fuzzy $w$-open set in $Y$ such that $f(\text{int}(U)) \subseteq f(U)$ therefore $f(\text{int}(U)) \subseteq \text{wint}(f(U))$.

Sufficiency: For the converse suppose that $U$ is an intuitionistic fuzzy open set of $X$. Then $f(U) = f(\text{int}(U)) \subseteq \text{wint}(f(U))$. But $\text{int}(f(U)) \subseteq f(U)$. Consequently $f(U) = \text{wint}(U)$ which implies that $f(U)$ is an intuitionistic fuzzy $w$-open set of $Y$ and hence $f$ is an intuitionistic fuzzy $w$-open.

Theorem 6.2: If $f: (X, \mathcal{I}) \rightarrow (Y, \mathcal{I})$ is an intuitionistic fuzzy $w$-open map then $\text{int}(f^{-1}(G)) \subseteq f^{-1}(\text{wint}(G))$ for every intuitionistic fuzzy set $G$ of $Y$.

Proof: Let $G$ is an intuitionistic fuzzy set of $Y$. Then $\text{int}(f^{-1}(G))$ is an intuitionistic fuzzy open set in $X$. Since $f$ is intuitionistic fuzzy $w$-open $f(\text{int}(f^{-1}(G)))$ is intuitionistic fuzzy $w$-open in $Y$ and hence $f(\text{int}(f^{-1}(G))) \subseteq \text{wint}(f(f^{-1}(G))) \subseteq \text{wint}(G)$. Thus $\text{int}(f^{-1}(G)) \subseteq f^{-1}(\text{wint}(G))$. 

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Theorem 6.3: A mapping \( f : (X, \sigma) \rightarrow (Y, \alpha) \) is intuitionistic fuzzy w-open if and only if for each intuitionistic fuzzy set \( S \) of \( Y \) and for each intuitionistic fuzzy closed set \( U \) of \( X \) containing \( f^{-1}(S) \) there is a intuitionistic fuzzy w-closed \( V \) of \( Y \) such that \( S \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Proof: Necessity: Suppose that \( f \) is an intuitionistic fuzzy w-open map. Let \( S \) be the intuitionistic fuzzy closed set of \( Y \) and \( U \) is an intuitionistic fuzzy closed set of \( X \) such that \( f^{-1}(S) \subseteq U \). Then \( V = (f^{-1}(S))^c \) is intuitionistic fuzzy w-closed set of \( Y \) such that \( f^{-1}(V) \subseteq U \).

Sufficiency: For the converse suppose that \( F \) is an intuitionistic fuzzy open set of \( Y \). Then \( f^{-1}((f(F))^c) \subseteq F^c \) and \( F^c \) is intuitionistic fuzzy closed set in \( X \). By hypothesis there is an intuitionistic fuzzy w-closed set \( V \) of \( Y \) such that \( \{ f(F) \}^c \subseteq V \) and \( f^{-1}(V) \subseteq F \). Therefore \( F^c \subseteq (f^{-1}(V))^c \subseteq F \). Hence \( V^c \subseteq f((f^{-1}(V))^c) \subseteq V^c \) which implies \( f(F) = V^c \). Since \( V^c \) is intuitionistic fuzzy w-open set of \( Y \). Hence \( f(F) \) is intuitionistic fuzzy w-open set of \( Y \) and thus \( f \) is intuitionistic fuzzy w-open map.

Theorem 6.4: A mapping \( f : (X, \sigma) \rightarrow (Y, \alpha) \) is intuitionistic fuzzy w-open if and only if \( f^{-1}(wcl(B)) \subseteq cl f^{-1}(B) \) for every intuitionistic fuzzy set \( B \) of \( Y \).

Proof: Necessity: Suppose that \( f \) is an intuitionistic fuzzy w-open map. For any intuitionistic fuzzy set \( B \) of \( Y \) \( f^{-1}(B) \subseteq cl(f^{-1}(B)) \). Therefore by theorem 6.3 there exists an intuitionistic fuzzy w-closed set \( F \) in \( Y \) such that \( B \subseteq F \) and \( f^{-1}(F) \subseteq cl(f^{-1}(B)) \). Therefore we obtain that \( f^{-1}(wcl(B)) \subseteq f^{-1}(F) \subseteq cl f^{-1}(B) \).

Sufficiency: For the converse suppose that \( B \) is an intuitionistic fuzzy open set of \( Y \). and \( F \) is an intuitionistic fuzzy closed set of \( X \) containing \( f^{-1}(B) \). Put \( V = cl(B) \), then we have \( B \subseteq V \) and \( V \) is w-closed and \( f^{-1}(V) \subseteq cl(f^{-1}(B)) \subseteq F \). Then by theorem 6.3 \( f \) is intuitionistic fuzzy w-open.

Theorem 6.5: If \( f : (X, \sigma) \rightarrow (Y, \alpha) \) and \( g : (Y, \alpha) \rightarrow (Z, \mu) \) be two intuitionistic fuzzy map and \( gof : (X, \sigma) \rightarrow (Z, \mu) \) is intuitionistic fuzzy w-open. If \( g : (Y, \alpha) \rightarrow (Z, \mu) \) is intuitionistic fuzzy w-irresolute then \( f : (X, \sigma) \rightarrow (Y, \alpha) \) is intuitionistic fuzzy w-open map.

Proof: Let \( H \) be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space \((X, \sigma)\). Then \( (gof) (H) \) is intuitionistic fuzzy w-open set of \( Z \) because \( gof \) is intuitionistic fuzzy w-open map. Now since \( g : (Y, \alpha) \rightarrow (Z, \mu) \) is intuitionistic fuzzy w-irresolute and \( (gof) (H) \) is intuitionistic fuzzy w-open set of \( Z \) therefore \( g^{-1}((gof)(H)) = f(H) \) is intuitionistic fuzzy w-open set in intuitionistic fuzzy topological space \( Y \). Hence \( f \) is intuitionistic fuzzy w-open map.

7. INTUITIONISTIC FUZZY W-CLOSED MAPPINGS

Definition 7.1: A mapping \( f : (X, \sigma) \rightarrow (Y, \alpha) \) is intuitionistic fuzzy w-closed if image of every intuitionistic fuzzy closed set of \( X \) is intuitionistic fuzzy w-closed set in \( Y \).

Remark 7.1 Every intuitionistic fuzzy closed map is intuitionistic fuzzy w-closed but converse may not be true. For,

Example 7.1: Let \( X = \{a, b\}, Y = \{x, y\} \) then the mapping \( f : (X, \sigma) \rightarrow (Y, \alpha) \) defined in Example 6.1 is intuitionistic fuzzy w-closed but it is not intuitionistic fuzzy closed.

Theorem 7.1: A mapping \( f : (X, \sigma) \rightarrow (Y, \alpha) \) is intuitionistic fuzzy w-closed if and only if for each intuitionistic fuzzy set \( S \) of \( Y \) and for each intuitionistic fuzzy open set \( U \) of \( X \) containing \( f^{-1}(S) \) there is a intuitionistic fuzzy w-open set \( V \) of \( Y \) such that \( S \subseteq V \) and \( f^{-1}(V) \subseteq U \).
**Proof:** **Necessity:** Suppose that \( f \) is an intuitionistic fuzzy \( w \)-closed map. Let \( S \) be the intuitionistic fuzzy closed set of \( Y \) and \( U \) is an intuitionistic fuzzy open set of \( X \) such that \( f^{-1}(S) \subset U \). Then \( V = Y - f^{-1}(U) \) is intuitionistic fuzzy \( w \)-open set of \( Y \) such that \( f^{-1}(V) \subset U \).

**Sufficiency:** For the converse suppose that \( F \) is an intuitionistic fuzzy closed set of \( X \). Then \( (f(F))^{c} \) is an intuitionistic fuzzy \( w \)-open set in \( X \) such that \( f^{-1}(f(F))^{c} \subset F^{c} \). By hypothesis there is an intuitionistic fuzzy \( w \)-open set \( V \) of \( Y \) such that \( (f(F))^{c} \subset V \) and \( f^{-1}(V) \subset F^{c} \). Therefore \( F \subset f^{-1}(V) \). Hence \( V^{c} \subset f(F) \subset f(f^{-1}(V)) \subset V^{c} \) which implies \( f(F) = V^{c} \). Since \( V^{c} \) is intuitionistic fuzzy \( w \)-closed set of \( Y \). Hence \( f(F) \) is intuitionistic fuzzy \( w \)-closed in \( Y \) and thus \( f \) is intuitionistic fuzzy \( w \)-closed map.

**Theorem 7.2:** If \( f(X,3) \rightarrow (Y,\sigma) \) is intuitionistic fuzzy semi continuous and intuitionistic fuzzy \( w \)-closed map and \( A \) is an intuitionistic fuzzy \( w \)-closed set of \( X \), then \( f(A) \) intuitionistic fuzzy \( w \)-closed.

**Proof:** Let \( f(A) \subset O \) where \( O \) is an intuitionistic fuzzy semi open set of \( Y \). Since \( f \) is intuitionistic fuzzy semi continuous therefore \( f^{-1}(O) \) is an intuitionistic fuzzy semi open set of \( X \) such that \( A \subset f^{-1}(O) \). Since \( A \) is intuitionistic fuzzy \( w \)-closed of \( X \) which implies that \( cl(A) \subset f^{-1}(O) \) and hence \( f(cl(A)) \subset O \) which implies that \( cl(f(A)) \subset O \) therefore \( cl(f(A)) \subset O \) whenever \( f(A) \subset O \) where \( O \) is an intuitionistic fuzzy semi open set of \( Y \). Hence \( f(A) \) is an intuitionistic fuzzy \( w \)-closed set of \( Y \).

**Corollary 7.1:** If \( f(X,3) \rightarrow (Y,\sigma) \) is intuitionistic fuzzy \( w \)-continuous and intuitionistic fuzzy closed map and \( A \) is an intuitionistic fuzzy \( w \)-closed set of \( X \), then \( f(A) \) intuitionistic fuzzy \( w \)-closed.

**Theorem 7.3:** If \( f(X,3) \rightarrow (Y,\sigma) \) is intuitionistic fuzzy closed and \( g : (Y,\sigma) \rightarrow (Z,\mu) \) is intuitionistic fuzzy \( w \)-closed. Then \( gof : (X,3) \rightarrow (Z,\mu) \) is intuitionistic fuzzy \( w \)-closed.

**Proof:** Let \( H \) be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space \( (X,3) \). Then \( f(H) \) is intuitionistic fuzzy closed set of \( (Y,\sigma) \) because \( f \) is intuitionistic fuzzy \( w \)-closed map. Now \( gof \) \((H) = g(f(H)) \) is intuitionistic fuzzy \( w \)-closed set in intuitionistic fuzzy topological space \( Z \) because \( g \) is intuitionistic fuzzy \( w \)-closed map. Thus \( gof : (X,3) \rightarrow (Z,\mu) \) is intuitionistic fuzzy \( w \)-closed.

**REFERENCES**

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