EDITORIAL PREFACE

The International Journal of Scientific and Statistical Computing (IJSSC) is an effective medium for interchange of high quality theoretical and applied research in Scientific and Statistical Computing from theoretical research to application development. This is the first issue of volume first of IJSSC. International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas.

Submissions reflect the important developments, extensions, and applications in statistical modeling. IJSSC goal is to be multidisciplinary in nature, promoting the cross-fertilization of ideas between scientific computation and statistical computation. IJSSC is refereed journal and invites researchers, practitioners to submit their research work that reflect new methodology on new computational and statistical modeling ideas, practical applications on interesting problems which are addressed using an existing or a novel adaptation of an computational and statistical modeling techniques and tutorials & reviews with papers on recent and cutting edge topics in computational and statistical concepts.

IJSSC editors understand that how much it is important for authors and researchers to have their work published with a minimum delay after submission of their papers. They also strongly believe that the direct communication between the editors and authors are important for the welfare, quality and wellbeing of the Journal and its readers. Therefore, all activities from paper submission to paper publication are controlled through electronic systems that include electronic submission, editorial panel and review system that ensures rapid decision with least delays in the publication processes.

To build international reputation of IJSSC, we are disseminating the publication information through Google Books, Google Scholar, Directory of Open Access Journals (DOAJ), Open J Gate, ScientificCommons, Docstoc, Scribd, CiteSeerX and many more. Our International Editors are working on establishing ISI listing and a good impact factor for IJSSC. I would like to remind you that the success of the journal depends directly on the number of quality articles submitted for review. Accordingly, I would like to request your participation by submitting quality manuscripts for review and encouraging your colleagues to submit quality manuscripts for review.
One of the great benefits that IJSSC editors provide to the prospective authors is the mentoring nature of the review process. IJSSC provides authors with high quality, helpful reviews that are shaped to assist authors in improving their manuscripts.

Editorial Board Member
International Journal of Scientific and Statistical Computing (IJSSC)
Editorial Board Members (EBMs)

Dr. De Ting Wu
Morehouse College (United States of America)
Table of Content

Volume 1, Issue 2, December 2011

Pages

7-19 A Fuzzy Arithmetic Approach for Perishable Items in Discounted Entropic Order Quantity Model
Monalisha Pattnaik, P.K. Tripathy

20-26 A Customizable Model of Head-Related Transfer Functions Based on Pinna Measurements
Navarun Gupta, Armando Barreto

27-39 A Retail Category Inventory Management Model Integrating Entropic Order Quantity and Trade Credit Financing
Pradip Kumar Tripathy, S. Pradhan
A Fuzzy Arithmetic Approach for Perishable Items in Discounted Entropic Order Quantity Model

P.K. Tripathy
P.G. Dept. of Statistics, Utkal University,
Bhubaneswar-751004, India.
msccompsc@gmail.com

M. Pattnaik
Dept. of Business Administration, Utkal University,
Bhubaneswar-751004, India.
omalisha_1977@yahoo.com

Abstract

This paper uses fuzzy arithmetic approach to the system cost for perishable items with instant deterioration for the discounted entropic order quantity model. Traditional crisp system cost observes that some costs may belong to the uncertain factors. It is necessary to extend the system cost to treat also the vague costs. We introduce a new concept which we call entropy and show that the total payoff satisfies the optimization property. We show how special case of this problem reduce to perfect results, and how post deteriorated discounted entropic order quantity model is a generalization of optimization. It has been imperative to demonstrate this model by analysis, which reveals important characteristics of discounted structure. Further numerical experiments are conducted to evaluate the relative performance between the fuzzy and crisp cases in EnOQ and EOQ separately.

Key Words: Discounted Selling Price, Fuzzy, Instant Deterioration, Inventory.

1. INTRODUCTION

In this paper we consider a continuous review, using fuzzy arithmetic approach to the system cost for perishable items. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in [5] and [13]. From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. For this reason, we consider the same since no researcher have discussed EnOQ model by introducing the holding cost and disposal cost as the fuzzy number. The model provides an approach for quantifying these benefits which can be substantial, and should be reflected in fuzzy arithmetic system cost. Our objective is to find optimal values of the policy variables when the criterion is to optimize the expected total payoff over a finite horizon.

In addition, the product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs, bakery items etc. suffer from depletion by decent spoilage. Lot of articles are available in inventory literature considering deterioration. Interested readers may consult the survey paper of [10], [18], [16], [15], [4] and [9] classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Products which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles are available in inventory literature considering deterioration. If this product starts to deteriorate as soon as it is received in the stock, then there is no option to provide pre-deterioration discount. Only we may give post deterioration discount on selling price.

Every organisation dealing with inventory faces a numbers of fundamental problems. Pricing decision is one of them. In the development of an EOQ system, we usually omit the case of discounting on
serving price. But in real world, it exists and is quite flexible in nature. On the other hand, in order to motivate customers to order more quantities for instant deterioration model usually supplier offers discount on selling prices. [2] developed an inventory model under continuous discount pricing, [11] studied an inventory problem under the condition that multiple discounts can be used to sell excess inventory. [14] mentioned that discount is considered temporarily for exponentially decaying inventory model. However, most of the studies except few, do not attempt to unify the two research streams: temporary price reductions and instant deterioration. This paper outlines the issue in details.

The awareness of the importance of including entropy cost is increasing everyday. Indeed, entropy is frequently defined as the amount of disorder in a system. The above consideration leads us to some important points. First, the use of entropy must be carefully planned, taking into account the multiplicity of objectives inherent in this kind of decision problem. Second, these are several economic strategies with conflicting objectives in this kind of decision making process. [12] proposed an analogy between the behaviour of production system and the behaviour of physical system. The main purpose of this research is to introduce the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement process.

In last two decades the variability of inventory level dependent demand rate on the analysis of inventory system was described by researchers like [17], [1] and [3]. They described the demand rate as the power function of on hand inventory. There is a vast literature on stock development inventory and its outline can be found in the review article by [19] where he unified two types of inventory level dependent demand by considering a periodic review model. Researchers such as [1], [16], [17], [4], [6] and [8] discussed the EOQ model assuming time value of money, demand rate, deterioration rate, shortages and so on a constant or probabilistic number or an exponential function. In this paper we consider demand as a constant function for instant deterioration model.

The paper tackles to investigate the effect of the approximation made by using the average payoff when determining the optimal values of the policy variables. The problem consists of the simultaneous optimization of fuzzy entropic EOQ and crisp entropic EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed with the help of [7] and optimum solution is obtained through LINGO software. In order to make the comparisons equitable a particular evaluation function based on discount is suggested. Numerical experiments are carried out to analyse the magnitude of the approximation error. However, a discount during post deterioration time, fuzzy system cost which might lead to a non-negligible approximation errors. The remainder of this paper is organised as follows. In section 2 assumptions and notations are provided for the development of the model. Section 3 describes the model formulation. Section 4 develops the fuzzy model. Section 5 provides mathematical analysis. In section 6, an illustrative numerical experiment is given to illustrate the procedure of solving the model. Finally section 7 concludes this article with a brief summary and provides some suggestions for future research.

\[
\text{TABLE-1: Major Characteristics of Inventory Models on Selected Researches.}
\]

<table>
<thead>
<tr>
<th>Author(s) and published Year</th>
<th>Structure of the Model</th>
<th>Deterioration</th>
<th>Inventory Model Based on</th>
<th>Discount allowed</th>
<th>Demand (constant)</th>
<th>Backlogging allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahata et al. (2006)</td>
<td>Fuzzy</td>
<td>Yes (constant)</td>
<td>EOQ</td>
<td>No</td>
<td>Constant</td>
<td>No</td>
</tr>
<tr>
<td>Panda et al. (2009)</td>
<td>Crisp</td>
<td>Yes (constant)</td>
<td>EOQ</td>
<td>Yes</td>
<td>Stock dependent</td>
<td>Yes (partial)</td>
</tr>
<tr>
<td>Jaber et al. (2008)</td>
<td>Crisp</td>
<td>Yes (on hand inventory)</td>
<td>EnOQ</td>
<td>No</td>
<td>Unit selling price</td>
<td>No</td>
</tr>
<tr>
<td>Vujosevic et al. (1996)</td>
<td>Fuzzy</td>
<td>No</td>
<td>EOQ</td>
<td>No</td>
<td>Constant</td>
<td>No</td>
</tr>
<tr>
<td>Skouri et al. (2007)</td>
<td>Crisp</td>
<td>Yes (Weibull)</td>
<td>EOQ</td>
<td>No</td>
<td>Ramp</td>
<td>Yes (partial)</td>
</tr>
<tr>
<td>Present paper (2010)</td>
<td>Fuzzy</td>
<td>Yes (constant)</td>
<td>EnOQ</td>
<td>Yes</td>
<td>Constant</td>
<td>No</td>
</tr>
</tbody>
</table>
2. NOTATIONS AND ASSUMPTIONS

Notations

- $C_0$ : set up cost
- $c$ : per unit purchase cost of the product
- $s$ : constant selling price of the product per unit ($s > c$)
- $h$ : holding cost per unit per unit time
- $d$ : disposal cost per unit.
- $r$ : discount offer per unit after deterioration.
- $Q_1$ : order level for post deterioration discount on selling price with instant deterioration.
- $Q_2$ : order level for no discount on selling price with instant deterioration.
- $T_1, T_2$ : cycle lengths for the above two respective cases.

Assumptions

- Replenishment rate is infinite.
- The deterioration rate $\theta$ is constant and ($0 < \theta < 1$)
- Demand is constant and defined as follows.
- $R(I(t)) = a$
- $r, (0 \leq r \leq 1)$ is the percentage discount offer on unit selling price during instant deterioration.
- $\alpha = (1 - r)^n$ ($n \in \mathbb{R}$) is the effect of discounting selling price on demand during deterioration. $\alpha$ is determined from priori knowledge of the seller with constant demand.
- 5. The entropy generation must satisfy $\sigma = \frac{d\sigma(t)}{dt}$ where, $\sigma(t)$ is the total entropy generated by time $t$ and $S$ is the rate at which entropy is generated. The entropy cost is computed by dividing the total commodity flow in a cycle of duration $T_i$. The total entropy generated over time $T_i$ as

$$\sigma(T_i) = \int_0^{T_i} S dt, \quad S = \frac{R(I(t))}{s} = \frac{a}{s}$$

Entropy cost per cycle is

$$EC(T_i) = \frac{Q_i}{\sigma(T_i)} (i=1, 2)$$

3. MATHEMATICAL MODEL

At the beginning of the replenishment cycle the inventory level raises to $Q_1$. As the time progresses it is decreased due to instantaneous stock with constant demand. Ultimately inventory reaches zero level at $T_1$. As instant deterioration starts from origin, r% discount on selling price is provided to enhance the demand of decreased quality items. This discount is continued for the rest of the replenishment cycle. Then the behaviour of inventory level is governed by the following system of linear differential equation.

$$\frac{dI(t)}{dt} = -\left[\alpha a + \theta I(t)\right] \quad 0 \leq t \leq T_1$$

with the initial boundary condition

$$I(0) = Q_1$$
and
$$I(T_1) = 0$$

Solving the equations,

$$I(t) = \frac{\alpha a}{\theta} \left[ e^{\theta(T_1-t)} - 1 \right] \quad 0 \leq t \leq T_1$$

$$Q_1 = \frac{\alpha a}{\theta} \left[ e^{\theta T_1} - 1 \right]$$

Holding cost and disposal cost of inventories in the cycle is,
\[ HC + DC = \left( h + \theta d \right) \int_0^{\frac{T}{a}} I(t) \, dt \]

Purchase cost in the cycle is given by \( PC = cQ_1 \).

Entropy cost in the cycle is
\[ EC = \frac{Q_1 \text{ with deterioration}}{\sigma(T_1)} = \frac{Q_1}{\sigma(T_1)} \]

Total sales revenue in the order cycle can be found as
\[ SR = s \left[ \alpha (1-r) \int_0^{\frac{T}{a}} \frac{\alpha}{T_1} \, dt \right] \]

Thus total profit per unit time of the system is
\[ \frac{1}{T_1} [SR - PC - HC - DC - EC - OC] \]

On integration and simplification of the relevant costs, the total profit per unit time becomes
\[ \pi_1 = \frac{1}{T_1} \left[ \alpha \alpha (1-r) aT_1 - h \left( e^{\theta T_1} - 1 \right) - T_1 \left( a\alpha - \theta a\alpha \right) + \frac{sQ_1}{aT_1} - cQ_1 - C_0 \right] \]

If the product starts to deteriorate as soon as it is received in the stock, then there is only one option we may give post deterioration discount. The post deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product, i.e. \( s(1-r) - c > 0 \). Applying this constraint on unit total profit function we have the following maximization problem.

Maximize \( \pi_1(r,T_1) \)
Subject to \( r < 1 - \frac{c}{s} \)
\( \forall r, T_1 \geq 0 \)
\[ \pi_1 = F_1 + F_2h + F_3d \]
where
\[ F_1 = \frac{1}{T_1} \left[ \alpha \alpha (1-r) aT_1 - \frac{sQ_1}{aT_1} - cQ_1 - C_0 \right] \]
\[ F_2 = -\frac{1}{T_1} \left[ e^{\theta T_1} - 1 \right] - T_1 \left( a\alpha - \theta a\alpha \right) \]
\[ F_3 = -\frac{1}{T_1} \left[ a\alpha \left( e^{\theta T_1} - 1 \right) - T_1 \right] \]

4. FUZZY MODEL
We replace the holding cost and disposal cost by fuzzy numbers \( \tilde{h} \) and \( \tilde{d} \) respectively. By expressing \( \tilde{h} \) and \( \tilde{d} \) as the normal triangular fuzzy numbers \((h_1, h_0, h_2)\) and \((d_1, d_0, d_2)\), where \( h_1 = h - \Delta_1, \ h_0 = h, \ h_2 = h + \Delta_2, \ d_1 = d - \Delta_3, \ d_0 = d, \ d_2 = d + \Delta_4 \) such that
$0 < \Delta_1 < h, \ 0 < \Delta_2, \ 0 < \Delta_3 < d, \ 0 < \Delta_4; \Delta_1, \Delta_2, \Delta_3 \text{ and } \Delta_4$ are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy disposal cost are considered as:

\[
\mu_h(h) = \begin{cases} 
\frac{h - h_1}{h_0 - h_1}, & h_1 \leq h \leq h_0 \\
\frac{h_2 - h}{h_2 - h_0}, & h_0 \leq h \leq h_2 \\
0, & \text{otherwise}
\end{cases}
\]  
(9)

\[
\mu_d(d) = \begin{cases} 
\frac{d - d_1}{d_0 - d_1}, & d_1 \leq d \leq d_0 \\
\frac{d_2 - d}{d_2 - d_0}, & d_0 \leq d \leq d_2 \\
0, & \text{otherwise}
\end{cases}
\]  
(10)

Then the centroid for $\bar{h}$ and $\bar{d}$ are given by

\[
M_h = \frac{h_1 + h_2 + h}{3} = h + \frac{\Delta_2 - \Delta_1}{3} \quad \text{and} \quad M_d = \frac{d_1 + d_2 + d_2}{3} = d + \frac{\Delta_4 - \Delta_3}{3}
\]
respectively.

For fixed values of $r$ and $T_1$, let

\[
Z(h, d) = F_1(r, T_1) + F_2(r, T_1)h + F_3(r, T_1)d = y \\
h = \frac{y - F_1 - F_2d}{F_2} = \frac{\Delta_2 - \Delta_1}{3} \quad \text{and} \quad \frac{\Delta_4 - \Delta_3}{3} = \psi_2
\]

Let

\[
\mu_{\tilde{\mu}_z(h, d)}(\psi) = \sup_{(h, d) \in Z(h, d)} \{\mu_h(h) \lor \mu_d(d)\}
\]

By extension principle the membership function of the fuzzy profit function is given by

\[
\mu_{\tilde{\mu}_z(h, d)}(\psi) = \sup_{d_1 \leq d \leq d_2} \left\{ \mu_h \left( \frac{y - F_1 - F_2d}{F_2} \right) \lor \mu_d(d) \right\}
\]
(11)

Now,

\[
\mu_h \left( \frac{y - F_1 - F_2d}{F_2} \right) = \begin{cases} 
\frac{y - F_1 - F_2h_1 - F_3d}{F_2(h_0 - h_1)}, & u_2 \leq d \leq u_1 \\
\frac{F_1 + F_2h_2 + F_3d - y}{F_2(h_2 - h_0)}, & \frac{y - F_1 - F_2h_1 - F_3d}{F_2(h_0 - h_1)} \leq u_2 \leq u_1 \\
0, & \text{otherwise}
\end{cases}
\]  
(12)

where,

\[
u_1 = \frac{y - F_1 - F_2h_1}{F_3}, \quad u_2 = \frac{y - F_1 - F_2h_1}{F_3}, \quad u_3 = \frac{y - F_1 - F_2h_2}{F_3}
\]

When $u_2 \leq d$ and $d \leq u_1$ then $y \leq F_1 + F_2h_1 + F_3d_0$ and $y \geq F_1 + F_2h_1 + F_3d_1$. It is clear that for every $y \in [F_1 + F_2h_1 + F_3d_1, F_1 + F_2h_0 + F_3d_0]$, $\mu_z(y) = PP'$. From the equations (9) and (12) the value of $PP'$ may be found by solving the following equation:
\[
d - d_1 = \frac{y - F_i - F_2h_i - F_3d}{d_0 - d_1} = \frac{F_2(h_0 - h_i)}{d_0 - d_1} \\
d = \frac{(y - F_i - F_2h_i)(d_0 - d_1) + F_3d_1(h_0 - h_i)}{F_2(h_0 - h_i) + F_3(d_0 - d_1)}
\]
or
\[
PP' = \frac{d - d_1}{d_0 - d_1} = \frac{y - F_i - F_2h_i - F_3d}{F_2(h_0 - h_i) + F_3(d_0 - d_1)} = \mu_i(y)
\]

Therefore, \(PP'\), (say). \(\text{(13)}\)

When \(d \leq d_1\) and \(d \leq d_2\) then \(y \leq F_i + F_2h_i + F_3d_1\) and \(y \geq F_i + F_2h_0 + F_3d_0\). It is evident that for every \(y \in [F_i + F_2h_0 + F_3d_0, F_i + F_2h_2 + F_3d_2]\), \(\mu_i(y) = PP''\). From the equations (9) and (12), the value of \(PP''\) may be found by solving the following equation:
\[
\frac{d_2 - d}{d_0 - d_1} = \frac{F_i + F_2h_2 + F_3d - y}{F_2(h_2 - h_0)} \\
\]
or,
\[
PP'' = \frac{d_2 - d}{d_0 - d_1} = \frac{F_i + F_2h_2 + F_3d_2 - y}{F_2(h_2 - h_0) + F_3(d_2 - d_0)} = \mu_2(y)
\]

Therefore, \(PP''\), (say). \(\text{(14)}\)

Thus the membership function for fuzzy total profit is given by
\[
\mu_{\tilde{y_1}, \tilde{y_2}}(y) = \begin{cases} 
\mu_i(y) : & F_i + F_2h_i + F_3d_1 \leq y \leq F_i + F_2h_0 + F_3d_0 \\
\mu_2(y) : & F_i + F_2h_0 + F_3d_0 \leq y \leq F_i + F_2h_2 + F_3d_2 \\
0 : & \text{otherwise}
\end{cases}
\]

\(\text{(15)}\)

Now, let \(P_1 = \int_{-\infty}^{\infty} \mu_{\tilde{y_1}, \tilde{y_2}}(y) \, dy\) and \(R_1 = \int_{-\infty}^{\infty} y \mu_{\tilde{y_1}, \tilde{y_2}}(y) \, dy\)

Hence, the centroid for fuzzy total profit is given by
\[
\tilde{x}_1 = M_{\text{TP}}(r, T_1) = \frac{R_1}{P_1} \\
= F_1(r, T_1) + F_2(r, T_1)h + F_3(r, T_1)d + \psi_1F_2(r, T_1) + \psi_2F_3(r, T_1) \\
M_{\text{TP}}(r, T_1) = F_1 + (h + \psi_1)F_2 + (d + \psi_2)F_3 
\text{(16)}
\]

where, \(F_1(r, T_1), F_2(r, T_1)\) and \(F_3(r, T_1)\) are given by equations (6), (7) and (8).

The post-deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product, i.e. \(s(1-r)-c>0\).

Applying this constraint on the unit total profit function in equation (17) we have the following maximization problem.

Maximize \(M_{\text{TP}}(r, T_1)\)
Subject to,
\[
r < 1 - \frac{c}{s} \\
\forall \quad r, T_1 \geq 0 \quad \text{(18)}
\]

Our objective here is to determine the optimal values of \(r\) and \(T_1\) to maximize the unit profit function. It is very difficult to derive the results analytically. Thus some numerical methods must be applied to derive the optimal values of \(r\) and \(T_1\), hence the unit profit function. There are several methods to
cope with constraint optimization problem numerically. But here we use penalty function method \[7\] and LINGO software to derive the optimal values of the decision variables.

a. Special Case

b. I Model for instant deterioration with no discount

In this case order level and unit profit function for model with constant deterioration and constant demand with no discount are obtained from (3) and (4) by substituting \( r=0 \) as

\[
Q_2 = \frac{a}{\theta} \left( e^{\alpha z} - 1 \right) \tag{19}
\]

From equation (4) total profit per unit time becomes

\[
\pi_2(T_2) = \frac{TP_2}{T_2} = \frac{1}{T_2} \left[ saT_2 - (h + \theta d)\frac{a}{\theta} \left( e^{\alpha z} - 1 \right) - \frac{sQ_2}{aT_2} - cQ_2 - C_0 \right]
\]

\[
= F_4 + F_5 h + F_6 d
\]

where,

\[
F_4 = \frac{1}{T_2} \left[ saT_2 - \frac{sQ_2}{aT_2} - cQ_2 - C_0 \right] \tag{21}
\]

\[
F_5 = -\frac{a}{\theta T_2^2} \left[ e^{\alpha z} - 1 \right] - T_2 \tag{22}
\]

\[
F_6 = -\frac{a}{\theta T_2^2} \left[ e^{\alpha z} - 1 \right] - T_2 \tag{23}
\]

Thus we have to determine \( T_2 \) from the fuzzy maximization problem

maximize \( M_{\tilde{T}_2} (T_2) \)

\[ \forall \ T_2 \geq 0 \]

where, \( M_{\tilde{T}_2} (T_2) = F_4 + (h+\psi_1)F_5 + (d+\psi_2)F_6 = \tilde{\pi}_2 \).

5. MODEL ANALYSIS THEOREM

For \( n \neq 1, \tilde{\pi}_1 > \tilde{\pi}_2 \) if \( r < \min \left\{ \frac{c}{s}, 1 - \frac{n}{s(n-1)} \right\} \).

Proof:

The values of \( \tilde{\pi}_1 \) for fixed \( r \) are always less than optimal value of \( r \). Thus it is sufficient to show that \( \tilde{\pi}_1 > \tilde{\pi}_2 \) for fixed \( r \). Here, \( T_1 \) is the cycle length when post deterioration discount is applied on unit selling price to enhance the demand of decreased quality items. For the enhancement of demand the inventory depletion rate will be higher and consequently the cycle time will reduce. \( T_2 \) is the cycle length when no discount is applied on selling price. Obviously \( T_2 \) is greater than \( T_1 \). Without loss of generality let both the profit function \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) are positive.

\[
\tilde{\pi}_1 - \tilde{\pi}_2 = \frac{TP_1}{T_1} - \frac{TP_2}{T_2} \geq \frac{TP_1 - TP_2}{T_2}
\]
It is sufficient to show that \( \frac{TP_1 - TP_2}{T_2} > 0 \). If it can be shown that \( \frac{TP_1 - TP_2}{T_2} \) is an increasing function of \( r \) then our purpose will be served. Now differentiating it with respect to \( r \) we have,

\[
\frac{\partial (\bar{\pi}_1 - \bar{\pi}_2)}{\partial r} = \frac{1}{T_2} \left[ \left( s(n-1)(1-r) + n \frac{h + \theta d}{\theta} \right) aT_2 \right] + \left[ -c - \frac{h + \theta d}{\theta} - \frac{s}{aT_2} \right] \left[ n(e^{\theta T_2} - 1) \right]
\]

Therefore, \( \frac{TP_1 - TP_2}{T_2} > 0 \Rightarrow \frac{\partial (\bar{\pi}_1 - \bar{\pi}_2)}{\partial r} > 0 \),

i.e. if

\[
\left( s(n-1)(1-r) + n \frac{h + \theta d}{\theta} \right) + \left[ -c - \frac{h + \theta d}{\theta} - \frac{s}{aT_2} \right] \left[ n(e^{\theta T_2} - 1) \right] > 0
\]

Now, \( \frac{\theta T_2}{\theta r} > 1 \).

we have,

\[
s(n-1)(1-r) + n \left[ -c - \frac{s}{aT_2} \right] > 0
\]

i.e.

\[
r < \frac{s(n-1)}{s(n-1)}
\]

We have the restriction \( r < 1 - \frac{c}{s} \).

Therefore, \( \bar{\pi}_1 > \bar{\pi}_2 \) if

\[
r < \min \left\{ 1 - \frac{c}{s}, 1 - \frac{\left( c + \frac{s}{aT_2} \right)}{s(n-1)} \right\}
\]

(27)

Theorem indicates that for \( n \neq 1 \) post instant deterioration discount on unit selling price produces higher profit than that of instant deterioration with no discount on unit selling price in fuzzy environment, if the percentage of post deterioration discount on unit selling price is less than \( \min \left\{ 1 - \frac{c}{s}, 1 - \frac{\left( c + \frac{s}{aT_2} \right)}{s(n-1)} \right\} \).

A simple managerial indication is that in pure inventory scenario if the product deteriorates after a certain time then it is always more profitable to apply only post deterioration discount on unit selling price and the amount of percentage discount must be less than the limit provided in equation (27) for the post deterioration discount.

6. NUMERICAL EXAMPLE
LINGO software is used to solve the aforesaid numerical example.
We redo the same example of [18] to see the optimal replenishment policy while considering the fuzzy holding cost, fuzzy disposal cost and entropy cost. The parameter values are a=80, b=0.3, h=0.6, d=2.0, s=10.0, C0=100.0, c=4.0, θ=0.03, n =2.0, Δ1=0.1, Δ2=0.2, Δ3=0.5, Δ4=0.8.

After 185 and 50 iterations in Table 2 we obtain the optimal replenishment policy for instant deterioration fuzzy entropic order quantity models with post deterioration discount and no discount respectively. The total profits for both the cases obtained here is at least 4.12% and 3.76% respectively less than that in [18] i.e. our CEOQ models. This is because we modified the model by introducing the hidden cost that is entropy cost where the optimal values for both the cases are 21.03623 and 20.28649 respectively. In Tables 3 and 4 we obtain the numerical results of different models like FEnOQ, FEOQ, CEnOQ and CEOQ for above two cases separately. The behaviour of the total profit to the lot size and the cycle length of post deterioration discounted model is shown in Figure 1.

**TABLE-2:** The Numerical Results of the Instant Deterioration Fuzzy Entropic Order Quantity (FEnOQ) Models 
(i=1,2)

<table>
<thead>
<tr>
<th>Model</th>
<th>Local optimal solution found at iteration</th>
<th>r</th>
<th>Ti</th>
<th>Qi</th>
<th>EC</th>
<th>πi</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEnOQ (Only post deterioration discount)</td>
<td>185</td>
<td>0.0350</td>
<td>1.8221</td>
<td>160.8798</td>
<td>21.0362</td>
<td>354.1393</td>
</tr>
<tr>
<td>FEnOQ (No discount)</td>
<td>50</td>
<td>-</td>
<td>1.8814</td>
<td>154.8204</td>
<td>20.28649</td>
<td>353.6979</td>
</tr>
<tr>
<td>% change</td>
<td>-</td>
<td>-</td>
<td>-3.1457</td>
<td>3.9136</td>
<td>3.6958</td>
<td>0.1248</td>
</tr>
</tbody>
</table>

**TABLE-3:** Comparison of Results for the different Post Deterioration Discount Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Local optimal solution found at iteration</th>
<th>r</th>
<th>Ti</th>
<th>Qi</th>
<th>EC</th>
<th>πi</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEnOQ</td>
<td>185</td>
<td>0.0350</td>
<td>1.8221</td>
<td>160.8798</td>
<td>21.0362</td>
<td>354.1393</td>
</tr>
<tr>
<td>FEOQ</td>
<td>193</td>
<td>0.0673</td>
<td>1.6063</td>
<td>151.3270</td>
<td>-</td>
<td>366.6226</td>
</tr>
<tr>
<td>CEnOQ</td>
<td>105</td>
<td>0.0392</td>
<td>1.8561</td>
<td>165.4009</td>
<td>21.1392</td>
<td>357.0641</td>
</tr>
<tr>
<td>CEOQ</td>
<td>196</td>
<td>0.0708</td>
<td>1.6367</td>
<td>155.4112</td>
<td>-</td>
<td>369.3739</td>
</tr>
</tbody>
</table>
7. COMPARATIVE EVALUATION

Table 2 shows that 3.4% discount on post deterioration model is provided on unit selling price to earn 0.12% more profit than that with no discounted instant deterioration model. From Table 5 it indicates that the uncertainty and entropy cost are provided on the post deterioration discount model to lose 3.4%, 0.81% and 4.12% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. Similarly it shows that the no discounted deterioration model to lose 3.08%, 0.78% and 3.76% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. This paper investigates a computing schema for the EOQ in fuzzy sense. From Tables 3 and 4 it shows that the fuzzy and crisp results are very approximate, i.e. it permits better use of EOQ as compared to crisp space arising with the little change in holding cost and in disposal cost respectively. It indicates the consistency of the crisp case from the fuzzy sense.

**TABLE-4:** Comparison of Results for the different No Discounted Instant Deterioration Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Local optimal solution found at iteration</th>
<th>$T_2$</th>
<th>$Q_2$</th>
<th>$EC$</th>
<th>$\pi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEnOQ</td>
<td>50</td>
<td>1.8814</td>
<td>154.8207</td>
<td>20.28649</td>
<td>353.6979</td>
</tr>
<tr>
<td>FEOQ</td>
<td>32</td>
<td>1.7203</td>
<td>141.2193</td>
<td>-</td>
<td>364.9558</td>
</tr>
<tr>
<td>CEnOQ</td>
<td>48</td>
<td>1.9239</td>
<td>158.4196</td>
<td>20.2931</td>
<td>356.5054</td>
</tr>
<tr>
<td>CEOQ</td>
<td>34</td>
<td>1.7592</td>
<td>144.4996</td>
<td>-</td>
<td>367.5178</td>
</tr>
</tbody>
</table>

**TABLE-5:** Relative Error (RE) of Post Deterioration Discount and No Discounted Deterioration FEnOQ Models with the different Models
8. CRITICAL DISCUSSION

When human originated data like holding cost and disposal cost which are not precisely known but subjectively estimated or linguistically expressed is examined in this paper. The mathematical model is developed allowing post deterioration discount on unit selling price in fuzzy environment. It is found that, if the amount of discount is restricted below the limit provided in the model analysis, then the unit profit is higher. It is derived analytically that the post deterioration discount on unit selling price is to earn more revenue than the revenue earned for no discount model. The numerical example is presented to justify the claim of model analysis. Temporary price discount for perishable products to enhance inventory depletion rate for profit maximization is an area of interesting research. This paper introduces the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement of the process. This paper examines the idea by extending the analysis of [18] by introducing fuzzy approach and entropy cost to provide a firm its optimum discount rate, replenishment schedule, replenishment order quantity simultaneously in order to achieve its maximum profit.

Though lower amount of percentage discount on unit selling price in the form of post deterioration discount for larger time results in lower per unit sales revenue, still it is more profitable. Because the inventory depletion rate is much higher than for discount with enhanced demand resulting in lower amount inventory holding cost and deteriorated items. Thus it can be conjectured that it is always profitable to apply post deterioration discount on unit selling price to earn more profit. Thus the firm in this case can order more to get earn more profit.

These models can be considered in a situation in which the discount can be adjusted and number of price changes can be controlled. Extension of the proposed model to unequal time price changes and other applications will be a focus of our future work.

9. CONCLUSION

This paper provides an approach to extend the conventional system cost including fuzzy arithmetic approach for perishable items with instant deterioration for the discounted entropic order quantity model in the adequacy domain. To compute the optimal values of the policy parameters a simple and quite efficient policy model was designed. Theorem determines effectively the optimal discount rate $r$ for post deterioration discount. Finally, in numerical experiments the solution from the instant deteriorated model evaluated and compared to the solutions of other different EnOQ and traditional EOQ policies.

However, we saw few performance differences among a set of different inventory policies in the existing literature. Although there are minor variations that do not appear significant in practical terms, at least when solving the single level, incapacitated version of the lot sizing problem. From our analysis it is demonstrated that the retailer’s profit is highly influenced by offering post discount on selling price. The results of this study give managerial insights to decision maker developing an optimal replenishment decision for instant deteriorating product. Compensation mechanism should
also be included to induce collaboration between retailer and dealer in a meaningful supply chain. We conclude this paper by summarizing some of the managerial insights resulting from our work.

In general, for normal parameter values the relative payoff differences seem to be fairly small. The optimal solution of the suggested post deterioration discounted model has a higher total payoff as compared with no discounted model. Conventional wisdom suggests that workflow collaboration in a fuzzy entropic model in a varying deteriorating product in market place are promising mechanism and achieving a cost effective replenishment policy. Theoretically such extensions would require analytical paradigms that are considerably different from the one discussed in this paper, as well as additional assumptions to maintain tractability.

The approach proposed in the paper based on EnOQ model seems to be a pragmatic way to approximate the optimum payoff of the unknown group of parameters in inventory management problems. The assumptions underlying the approach are not strong and the information obtained seems worthwhile. Investigating optimal policies when demand are generated by other process and designing models that allow for several orders outstanding at a time, would also be challenging tasks for further developments. Its use may restrict the model's applicability in the real world. Future direction may be aimed at considering more general deterioration rate or demand rate. Uses of other demand side revenue boosting variables such as promotional efforts are potential areas of future research. There are numerous ways in which one could consider extending our model to encompass a wider variety of operating environments. The proposed paper reveals itself as a pragmatic alternative to other approaches based on constant demand function with very sound theoretical underpinnings but with few possibilities of actually being put into practice. The results indicate that this can become a good model and can be replicated by researchers in neighbourhood of its possible extensions. As regards future research, one other line of development would be to allow shortage and partial backlogging in the discounted model.

10. REFERENCES


A Customizable Model of Head-Related Transfer Functions Based on Pinna Measurements

Navarun Gupta
Assistant Professor, Department of Electrical Engineering
University of Bridgeport
Bridgeport, CT 06604, USA

Armando Barreto
Associate Professor, Department of Electrical Engineering
Florida International University
Miami, FL 33174, USA

Abstract

This paper proposes a method to model Head-Related Transfer Functions (HRTFs) based on the shape and size of the outer ear. Using signal processing tools, such as Prony’s signal modeling method, a dynamic model of the pinna has been obtained, that completes the structural model of HRTFs used for digital audio spatialization.

Listening tests conducted on 10 subjects showed that HRTFs created using this pinna model were 5% more effective than generic HRTFs in the frontal plane. This model has been able to reduce the computational and storage demands of audio spatialization, while preserving a sufficient number of perceptually relevant spectral cues.

Keywords: HRTF, Binaural, HRIR, Pinna, Model

1. INTRODUCTION

HRTFs represent the transformation undergone by the sound signals, as they travel from their source to both of the listener’s eardrums. This transformation is due to the interaction of sound waves with the torso, shoulder, head and outer ear of a listener [9]. Therefore, the two components of these HRTF pairs (left and right) are typically different from each other, and pairs corresponding to sound sources at different locations around the listener are different. Furthermore, since the physical elements that determine the transformation of the sounds reaching the listener’s eardrums (i.e., the listener’s head, torso and pinnae), are somewhat different for different listeners, and so should be their HRTF sets [2].

Currently, some spatialization systems make use of HRTFs that are empirically measured for each prospective user. These “custom” HRTFs are anthropometrically correct for each user, but the equipment, facilities and expertise required to obtain these “measured HRTF pairs”, constrain their application to high-end, purpose-specific sound spatialization systems only [2]. For most consumer-grade applications, sound spatialization systems resort to the use of “generic” transfer functions, measured from a manikin with “average” physical characteristics [7], which, evidently is a fundamentally imperfect approach.

This paper reports on our work to advance an alternative approach to sound spatialization, based on the postulation of anthropometrically-related “structural models” [6] that will transform a single-channel audio signal into a left/right binaural spatialized pair, according to the sound source simulation. Specifically, the work reported here proposes linkages between the parameters of the HRTF model and key anthropometric features.
of the intended listener’s pinna, so that the model, and consequently the resulting HRTFs are easily “customizable” according to a small set of anthropometric measurements.

2. MEASUREMENTS AND IMPLEMENTATION

Current sound spatialization systems use HRTFs, represented by their corresponding impulse response sequences, the Head-Related Impulse Responses, (HRIRs) to process, by convolution, a single-channel digital audio signal, resulting in the two components (left and right) of a binaural spatialized sound. When these two channels are delivered to the listener through headphones, the sound will seem to emanate from the source location corresponding to the HRIR pair used for the spatialization process [4].

In our laboratory, we use the Ausim3D’s HeadZap HRTF Measurement System [1]. This system measures a 256-point impulse response for both the left and the right ear using a sampling frequency of 96 KHz. Golay codes are used to generate a broad-spectrum stimulus signal delivered through a Bose Acoustimass speaker. The response is measured using miniature blocked meatus microphones placed at the entrance to the ear canal on each side of the head. Under control of the system, the excitation sound is issued and both responses (left and right ear) are captured. Since the Golay code sequences played are meant to represent a broad-band excitation equivalent to an impulse, the sequences captured in each ear are the impulse responses corresponding to the HRTFs. The system provides these measured HRIRs as a pair of 256-point minimum-phase vectors, and an additional delay value that represents the Interaural Time Difference (ITD), i.e., the additional delay observed before the onset of the response collected from the ear that is farthest from the speaker position. In addition to the longer onset delay of the response from the “far” or “contralateral ear” (with respect to the sound source), this response will typically be smaller in amplitude than the response collected in the “near” or “ipsilateral ear”. The difference in amplitude between HRIRs in a pair is referred to as the Interaural Intensity Difference (IID).

Our protocol records HRIR pairs from source locations at the 72 possible combinations of $\phi = \{-36^\circ, -18^\circ, 0^\circ, 18^\circ, 36^\circ, 54^\circ\}$ and $\theta = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ, -150^\circ, -120^\circ, -90^\circ, -60^\circ, -30^\circ\}$. The left (L) and right (R) HRIRs collected for a source location at azimuth $\phi$ and elevation $\theta$ are symbolized by $h_{L, \theta, \phi}$ and $h_{R, \theta, \phi}$, respectively. The creation of a spatialized binaural sound (left and right channels) involves convolving the single-channel digital sound to be spatialized, $s(n)$, with the HRIR pair corresponding to the azimuth and elevation of the intended virtual source location:

$$y_{L, \theta, \phi}(n) = \sum_{k=-\infty}^{\infty} h_{L, \theta, \phi}(k) \cdot s(n-k)$$

and

$$y_{R, \theta, \phi}(n) = \sum_{k=-\infty}^{\infty} h_{R, \theta, \phi}(k) \cdot s(n-k)$$

(1)

3. STRUCTURAL MODEL

Structural models of HRTF are based on the premise that each anthropometric feature of the listener affects the HRTF in a way that can be described mathematically [11]. Because such a model has its origin in the physical characteristics of the entities involved in the phenomenon, it should be possible to derive the value of its parameters (for a given source location), from the sizes of those entities, i.e., the anthropometric features of the intended listener. Proper identification of such parameters and adequate association of their numerical values with the anthropometric features of the intended listener may provide a mechanism to interactively adjust a generic base model to the specific characteristics of an individual. One of the most practical models has been proposed by Brown and Duda [6]. Their model is illustrated in Figure 1:
the pinna model has remained an open question, so far, and it is the objective of our work. Carlile [7] divides its parameters to anthropometric features of the listener’s pinna. Taking into account the information emulated by a one-pole/one-zero model [5]. This model, incorporating the IID and ITD effects, can account for an approximate localization, particularly in the horizontal plane (elevation, $\phi = 0^\circ$). However, to produce elevation effects, a good pinna sub-model is required. The definition and anthropometric characterization of the pinna model has remained an open question, so far, and it is the objective of our work. Carlile [7] divides pinna models according to the main phenomenon that they address: Resonating, diffractive and reflective. From these, reflective models have attracted the most attention in the literature.

The intent of our work is to define a functional pinna sub-model that has anthropometric plausibility and then associate its parameters to anthropometric features of the listener’s pinna. Taking into account the information available about the existence of a resonant effect implemented by the ear’s concha [13] and according to the reflective pinna models discussed previously, we propose that the pinna may, in turn, be modeled as the series connection of an equivalent second-order resonator and a series of characteristic echoes, representing the delayed and attenuated secondary paths taken by the incoming sound, in addition to a “direct path”. A block diagram representation of this model is shown in Figure 2.

It should be noted that this pinna model allows for the “direct-path”, $F_0$, and each one of the “echoes”, $F_1$, $F_2$, and $F_3$, to be affected by a different equivalent resonance.

FIGURE 1: Right Channel half for Brown & Duda’s Structural HRTF Model. The model comprises a symmetric left half (not shown).

FIGURE 2: Proposed Pinna Model
Accordingly, HRIRs are envisioned as the impulse response of this model, which will be the superposition of four damped sinusoidals (the impulse responses of each of the 2nd order resonators), characterized by their frequency, $f$, and damping factor, $\sigma$. These damped sinusoidals are altered in their amplitude according to the $\rho_k$ parameters, and delayed according to the $\tau_k$ parameters.

Thus, the instantiation of this proposed model will require the identification of the $f_k$, $\sigma_k$, $\rho_k$, and $\tau_k$ values, to characterize the parameters of the model that successfully approximates an HRIR collected for a given azimuth and elevation, through the output provided by the pinna model. The main challenge in this operation is the fact that the several replicas of the damped oscillation are irreversibly mixed together, partially overlapping in time, in the measured HRIR. This problem was addressed by the sequential application of Prony’s modeling algorithm [10, 12] to partial segments of the response. Prony’s method approximates a given signal $\mu(t)$ as the superposition of $p$ damped sinusoidals:

$$\mu(t) = \sum_{j=1}^{p} \rho_j e^{(\sigma_j t)} \sin(2\pi f_j t + \xi_j)$$  \hspace{1cm} (2)

Figure 3 shows the four F components, obtained using this method, from a measured HRIR.

5. MEASUREMENT OF ANTHROPOMETRIC FEATURES

Key anthropometric features of the ears of the 15 experimental subjects in the study (same 15 subjects for whom the HRIRs were empirically measured with the Ausim 3D system) were captured by means of digital photography (including a distance reference), and laser 3-D scanning, using a Polhemus FastScan handheld scanner.
The features measured are: Ear length (EL), Ear width (EW), Concha width (CW), Concha height (CH), Helix length (HL), Concha area (CA), Concha volume (CV) and Concha depth (CD).

6. ASSOCIATION BETWEEN MODEL PARAMETERS AND ANTHROPOMETRIC MEASUREMENTS

Following the procedure described in the two preceding sections two independent sets of data were available for each pinna of each one of the 15 subjects in the study:

Estimated Model Parameters (for frontal plane sites):

\[ r_{0\phi}, \alpha_{0\phi}, r_{1\phi}, \tau_{1\phi}, r_{2\phi}, \tau_{2\phi}, r_{3\phi}, \tau_{3\phi}, r_{4\phi} \text{ and } \tau_{4\phi}, \text{ for } \phi = -36^\circ, -18^\circ, 0^\circ, 18^\circ, 36^\circ, \text{ and } 54^\circ \]

(Note, here \( r_0 \) and \( \alpha_0 \) are the magnitude and angle of the poles of the resonator, which define the resonator response \( F_0(n) \), in terms of its frequency \( f_0 \) and its damping factor \( \sigma_0 \))

Measured Anthropometric Features:
EL, EW, CH, CW, CA, CV, CD and HL

Under the assumption that the model parameters depend of the anthropometric features, a general dependency equation may be set, for each model parameter. For example, for the amplitude of the first replica in the pinna model, \( \rho_1 \), at \( \phi = 54^\circ \), the following equation may be set up:

\[ \rho_{0\phi=54} = K_{EL} \text{EL} + K_{EW} \text{EW} + K_{CH} \text{CH} + K_{CW} \text{CW} + K_{CA} \text{CA} + K_{CV} \text{CV} + K_{CD} \text{CD} + K_{HL} \text{HL} + B \]

Coalescing the data from both ears, at the same elevation, (under the assumption of symmetry), 30 equations like the one above can be set up, for each model parameter, at each elevation. Each group of 30 equations can then be analyzed through multiple regression to estimate the values of the constants (KEL, KEW, ..., KHL, B). The multiple regression analysis was carried out using the Statistical Package for the Social Sciences (SPSS).

7. MODEL EVALUATION AND RESULTS

Using the predictive equations found above, for each subject tested, a Model HRTF was created. Ultimately, the efficiency of the modeled sequences obtained by predicting the model parameters from the anthropometric measurements of the subjects was gauged in listening tests. In these tests, white noise bursts were spatialized using the modeled HRIR sequences, that had been obtained based on the ear measurements of the subject under test, for the six elevations under study. The order in which these elevations where used for the spatialization was randomized. Each elevation was simulated four times (i.e., there were 24 trials for each side of the head.) In each trial the subject would listen to each spatialized sound and then use a graphic user interface to indicate the perceived elevation. Since the spatialization was performed to emulate six specific locations, the absolute value of the angular difference between the perceived elevation and the emulated one would be considered as the elevation error for the trial. The subjects listened to the original, modeled and generic HRTFs. Figure 4 illustrates the average angular error (across all 10 subjects) experienced in the perception of the different emulated elevations for Original, Generic (B&K) and Model HRIRs. The global average error (across all subjects and all elevations) with the original HRTFs, was 23.7°. The corresponding global average error with modeled HRIRs was 29.9°. Finally, the global average error when the subjects used the generic HRIRs, collected from the B&K manikin, was 31.4°.

It should be noted, however, that near the horizontal plane (e.g., between \( \phi = -18^\circ \) and \( \phi = 36^\circ \)), the performance of the modeled HRIRs was close to or better than, that of the individually measured HRIRs.
8. CONCLUSIONS
This paper has presented a proposed functional model of the pinna, to be used as the output block in a structural HRTF model. Although this study resorted to the use of a relatively expensive 3-D laser scanner and specialized software to determine some of the anthropometric features of our subjects, which is a prerequisite to the use of the predictive equations developed in this research, it is likely that empirical relationships can be found to obtain these feature values from two-dimensional high-resolution photographs (commonly available) and a few direct physical measurements in the subject.

9. ACKNOWLEDGEMENT
This research was supported by NSF grants, HRD-0317692, IIS-0308155, and CNS-0426125.

10. REFERENCES


A Retail Category Inventory Management Model Integrating Entropic Order Quantity and Trade Credit Financing

P.K. Tripathy
P.G. Dept. of Statistics, Utkal University,
Bhubaneswar-751004, India.

S. Pradhan
Dept. of Mathematics, Orissa Engineering College,
Bhubaneswar-751007, India.

Abstract

A retail category inventory management model that considers the interplay of entropic product assortment and trade credit financing is presented. The proposed model takes into consideration of key factors like discounted-cash-flow. We establish a stylized model to determine the optimal strategy for an integrated supplier-retailer inventory system under the condition of trade credit financing and system entropy. This paper applies the concept of entropy cost estimated using the principles of thermodynamics. The classical thermodynamics reasoning is applied to modelling such systems. The present paper postulates that the behaviour of market systems very much resembles those of physical systems. Such an analogy suggests that improvements to market systems might be achievable by applying the first and second laws of thermodynamics to reduce system entropy (disorder). This paper synergises the above process of entropic order quantity and trade credit financing in an increasing competitive market where disorder and trade credit have become the prevailing characteristics of modern market system. Mathematical models are developed and numerical examples illustrating the solution procedure are provided.

Key words: Discounted Cash-Flow, Trade Credit, Entropy Cost.

1. INTRODUCTION

In the classical inventory economic order quantity (EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practice or when the economy turns sour, the supplier allows credit for some fixed time period in settling the payment for the product and does not charge any interest from the customer on the amount owned during this period. Goyal (1985) developed an EOQ model under the conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's (1985) model to consider the deteriorating items. Chung (1999) presented an EOQ model by considering trade credit with DCF approach. Chang (2004) considered the inventory model having deterioration under inflation when supplier credits linked to order quantity. Jaber et al. (2008) established an entropic order quantity (EnOQ) model for deteriorating items by applying the laws of thermodynamics. Chung and Liao (2009) investigated an EOQ model by using a discounted-cash-flows (DCF) approach and trade credit depending on the quantity ordered.

The specific purpose of this paper is to trace the development of entropy related thought from its thermodynamic origins through its organizational and economic application to its relationship to discounted-cash-flow approach.
2. Model Development

2.1 Basis of the Model:
The classical economic order quantity (EOQ) or lot sizing model chooses a batch size that minimizes the total cost calculated as the sum of two conflicting cost functions, the order/setup cost and the inventory holding costs. The entropic order quantity (EnOQ) is derived by determining a batch size that minimizes the sum of the above two cost and entropic cost.

Notations

- \( T \) : the inventory cycle time, which is a decision variable;
- \( C, A \) : the purchase cost and ordering cost respectively;
- \( h \) : the unit holding cost per year excluding interest change;
- \( D \) : the demand rate per unit time.
- \( D(T) \) : the demand rate per unit time where cycle length is \( T \).
- \( r \) : Discount rate (opportunity cost) per time unit.
- \( Q \) : Procurement quantity;
- \( M \) : the credit period;
- \( W \) : quantity at which the delay payment is permitted;
- \( \sigma(t) \) : total entropy generated by time \( t \).
- \( S \) : rate of change of entropy generated at time \( t \).
- \( E(t) \) : Entropy cost per cycle;
- \( PV1(T) \) : Present value of cash-out-flows for the basic EnOQ model;
- \( PV2(T) \) : present value of cash-out-flows for credit only on units in stock when \( T \leq M \).
- \( PV3(T) \) : Present value of cash-out-flows for credit only on units in stock when \( T \geq M \).
- \( P(t), P_0(t) \) : unit price and market equilibrium price at time \( t \) respectively.
- \( PV_\infty(T) \) : the present value of all future cash-flows.
- \( T^* \) : the optimal cycle time or \( PV_\infty(T) \) when \( T > 0 \).

ASSUMPTIONS

1. The demand is constant.
2. The ordering lead time is zero.
3. Shortages are not allowed.
4. Time period is infinite.
5. If \( Q < W \), the delay in payment is not permitted, otherwise, certain fixed trade credit period \( M \) is permitted. That is, \( Q < W \) holds if and only if \( T < W / D \).
6. During the credit period, the firm makes payment to the supplier immediately after use of the materials. On the last day of the credit period, the firm pays the remaining balance.

2.2 Commodity flow and the entropy cost:
The commodity flow or demand/unit time is of the form
\[
D = -k(P(t) - P_0(t))(1)
\]
The concept represented by equation (1) is analogous to energy flow (heat or work) between a thermodynamics system and its environment where \( k \) (analogous to a thermal capacity) represents the change in the flow for the change in the price of a commodity and is measured in additional units sold per year per change in unit price e.g. units/year/$.

Let \( P(t) \) be the unit price at time \( t \) and \( P_0(t) \) the market equilibrium price at time \( t \), where \( P(t) < P_0(t) \) for every \( t \in [0, T] \). At constant demand rate \( P(t)=P \) and \( P_0(t)=P_0 \) noting that when \( P < P_0 \), the direction of the commodity flow is from the system to the surroundings. The entropy generation rate must satisfy
\[
S = \frac{d\sigma(t)}{dt} = k\left(\frac{P}{P_0} + \frac{P_0}{P} - 2\right)
\]
To illustrate, assume that the price of a commodity decreases according to the following relationship.
\[ P(t) = P(0) - \frac{a}{T} t \]

where, \( a = P(0) - P(T) \) as linear form of price being time dependent.

\[ E(t) = \frac{D}{\sigma(T)} = \frac{TP_0^2}{a} \ln \left[ 1 - \frac{aT}{TP(0) - TP_0} \right] - P_0T = \]

Entropy cost per cycle.

Case-1: Instantaneous cash-flows (the case of the basic EnOQ model)

The components of total inventory cost of the system per cycle time are as follows:

(a) Ordering cost = \( A + Ae^{-rT} + Ae^{-2rT} + \ldots = \frac{A}{1 - e^{-rT}} \)

(b) Present value of the purchase cost can be shown as

\[ PV_1(T) = \frac{A}{1 - e^{-rT}} + \frac{cT^2}{1 - e^{-rT}} \left( \frac{a}{2} + p_0 - P(0) \right) + \frac{hck}{r^2} \left( \frac{kT}{r} (p_0 - P(0)) \right) \]

So, the present value of all future cash-flows in this case is

\[ PV_1(T) = \frac{A}{1 - e^{-rT}} + \frac{cT^2}{1 - e^{-rT}} \left( \frac{a}{2} + p_0 - P(0) \right) + \frac{hck}{r^2} \left( \frac{kT}{r} (p_0 - P(0)) \right) + \frac{TP_0^2}{a} \ln \left[ 1 - \frac{a}{p_0 - P_0} \right] - P_0T \]

(1)

Case-2: Credit only on units in stock when \( T \leq M \).

During the credit period \( M \), the firm makes payment to the supplier immediately after the use of the stock. On the last day of the credit period, the firm pays the remaining balance. Furthermore, the credit period is greater than the inventory cycle length. The present value of the purchase cost can be shown as

\[ PV_2(T) = \frac{A}{1 - e^{-rT}} - \frac{ck}{r} (P(0) - P_0) + \frac{cka}{Tr^2} \left( \frac{aT}{1 - e^{-rT}} - \frac{Tr}{r} \right) \]

The present value of all future cash-flows in this case is

\[ PV_2(T) = \frac{A}{1 - e^{-rT}} - \frac{ck}{r} (P(0) - P_0) + \frac{cka}{Tr^2} \left( \frac{aT}{1 - e^{-rT}} - \frac{Tr}{r} \right) + \frac{hck}{r^2} \left[ P(0) - P_0 - \frac{2a}{rT} \right] \]

(2)

Case-3: Credit only on units in stock when \( T \geq M \).

The present value only on units in stock can be shown as
Now our main aim is to minimize the present value of all future cash-flow cost $PV_\infty(T)$. That is

$$
\text{Minimize } PV_\infty(T) \\
\text{subject to } T > 0.
$$

We will discuss the situations of the two cases,

(A) Suppose $M > W/D$

In this case we have

$$
PV_\infty(T) = \begin{cases} 
PV_1(T) & \text{if } 0 < T < W/D \\
PV_2(T) & \text{if } W/D \leq T < M \\
PV_3(T) & \text{if } M \leq T.
\end{cases}
$$

It was found that

$$
PV_1(T) - PV_2(T) > 0 \quad \text{and} \quad PV_1(T) - PV_3(T) > 0
$$

for $T > 0$ and $T \geq M$ respectively.

which implies

$$
PV_1(T) > PV_2(T) \quad \text{and} \quad PV_1(T) > PV_3(T)
$$

for $T > 0$ and $T \geq M$ respectively.

Now we shall determine the optimal replenishment cycle time that minimizes present value of cash-out-flows. The first order necessary condition for $PV_1(T)$ in (1) to be minimized is expressed as

$$
\frac{\partial PV_1(T)}{\partial T} = 0
$$

which implies

$$
- \frac{re^{-rt}A}{(1-e^{-rt})^2} + \frac{2CT(1-e^{-rt})-re^{rt}CT^2}{(1-e^{-rt})^2} \left( \frac{ak}{2} + P_0 k - kP(0) \right) + \frac{hcka}{r^2} \left[ \frac{-re^{-rt}(1+e^{-rt})}{(1-e^{-rt})^3} \right]
$$
\[
+ \frac{hck}{r} (P_0 - P(0)) \left[ \frac{1 - e^{-\tau T}}{e^{-\tau T}} - e^{-\tau T} T \right] + \frac{2hck}{r^2} \ln \left[ 1 - \frac{a}{P(0) - P_0} \right] - P_0 = 0
\]

(4)

Similarly, \( PV_2(T) \) in equation(2) to be minimized is

\[
\frac{\partial PV_2(T)}{\partial T} = 0
\]

which implies

\[
- \frac{re^{-\tau A}}{(1 - e^{-\tau T})^2} - \frac{ck}{r} \left[ -\frac{e^{-\tau (1 - e^{-\tau T})}}{e^{-\tau T} (1 - e^{-\tau T})^2} \right] + \frac{hck}{r^2} \left[ -\frac{e^{-\tau (1 - e^{-\tau T}) - (1 + e^{-\tau T})e^{-\tau T}}}{(1 - e^{-\tau T})^2} \right]
\]

Likewise, the first order necessary condition for \( PV_3(T) \) in equation(3) to be minimized is

\[
\frac{\partial PV_3(T)}{\partial T} = 0
\]

which leads

\[
- \frac{re^{-\tau A}}{(1 - e^{-\tau T})^2} - \frac{hck e^{-\tau T} (1 - e^{-\tau T}) - (1 + e^{-\tau T})e^{-\tau T}}{r^2 (1 - e^{-\tau T})}
\]

Furthermore, we let

\[
\Delta_1 = \frac{\partial PV_1(T)}{\partial T} \bigg|_{T=W/D} \quad (7)
\]

\[
\Delta_2 = \frac{\partial PV_2(T)}{\partial T} \bigg|_{T=W/D} \quad (8)
\]

\[
\Delta_3 = \frac{\partial PV_3(T)}{\partial T} \bigg|_{T=M} \quad (9)
\]

**Lemma-1**

(a) If \( \Delta_1 \geq 0 \), then the total present value of \( PV_1(T) \) has the unique minimum value at the point \( T = T_1 \) where \( T_1 \in (0, W/D) \) and satisfies \( \frac{\partial PV_1}{\partial T} = 0 \).

(b) If \( \Delta_1 < 0 \), then the value of \( T_1 \in (0, W/D) \) which minimizes \( PV_1(T) \) does not exist.

Proof:

Now taking the second derivative of \( PV_1(T) \) with respect to \( T \), we have

\[
\frac{r^2 e^{-\tau T} (1 - e^{-\tau T})^2 + 2r^2 e^{-\tau T} (1 - e^{-\tau T})}{(1 - e^{-\tau T})^2}
\]
We obtain from the above expression, which implies is strictly increasing function of T in the interval (0,W/D).

Also we know that,

Thus for all $T \in (0,W/D)$ and also we find $\frac{\partial^2 PV_1(T)}{\partial T^2} < 0$ for all $T \in (0,W/D)$. Thus $PV_1(T)$ is a strictly decreasing function of T in the interval $(0,W/D)$. Therefore, we can not find a value of T in the open interval $(0,W/D)$ that minimizes $PV_1(T)$. This completes the proof.

Lemma-2

(a) If $\Delta_2 \leq 0 \leq \Delta_1$, then the total present value of PV2(T) has the unique minimum value at the point $T = T_2$ where $T_2 \in (W/D,M)$ and satisfies $\frac{\partial PV_2}{\partial T} = 0$.

(b) If $\Delta_2 > 0$, then the present value PV2(T) has a minimum value at the lower boundary point $T = W/D$.

(c) If $\Delta_2 < 0$, then the present value PV2(T) has a minimum value at the upper boundary point $T = M$.

Proof:

$\frac{\partial^2 PV_2(T)}{\partial T^2} = \frac{r^2 e^{-\gamma T} \left(1-e^{-\gamma T}\right)^2 + 2r^2 e^{-\gamma T} \left(1-e^{-\gamma T}\right)}{(1-e^{-\gamma T})^4}$
\[ -\frac{ck a e^{-rT}}{\left(1 - e^{-rT}\right)^{2}} + \frac{hck a e^{-2rT}}{r\left(1 + e^{-rT}\right)^{2}} + \frac{hck (P_0 - P(0))}{(1 + e^{-rT})^{2}} \left\{ e^{-rT} \left(1 + rT\right) - 2r \right\} \]
\[ + \frac{2hr^2 c k (P_0 - P(0)) e^{-rT}}{r^2 T^3} + \frac{2ck a}{r^2 T^3} - 4hck a \]

which is \(>0\) where \(T \in [W/D, M]\).

Which implies \(\frac{\partial PV_2}{\partial T}\) is strictly increasing function of \(T\) in the interval \((W/D, M)\).

Also we have

If \(\Delta_2 \leq 0\), then by applying the intermediate value theorem there exists a unique value

\[ T_2 \in [W/D, M] \]

so that

\[ \frac{\partial PV_2(T)}{\partial T} \bigg|_{T=T_2} = 0 \]

with respect to \(T\) at the point \(T_2\) we have

Thus \(T_2 \in [W/D, M]\) is the unique solution to \(PV_2(T)\).

Now if \(\Delta_2 > 0\), then the total present value of \(PV_3(T)\) has the unique minimum value at the point

\[ T_3 \in (M, \infty) \]

satisfies

\[ \frac{\partial PV_3}{\partial T} = 0 \]

Therefore, \(PV_2(T)\) is strictly increasing function of \(T\) in the interval \((W/D, M)\). Therefore has a minimum value at the lower boundary point \(T = W/D\) and similarly we can prove has a minimum value at upper boundary point \(T=M\).

**Lemma-3**

(a) If \(\Delta_3 \leq 0\), then the total present value of \(PV_3(T)\) has the unique minimum value at the point \(T=T_3\) on \(T_3 \in (M, \infty)\) and satisfies

\[ \frac{\partial PV_3}{\partial T} = 0 \]

(b) If \(\Delta_3 > 0\), then the present value of \(PV_3(T)\) has a minimum value at the boundary \(T=M\).

**Proof:**

The proof is same to the lemma-2.

**Proposition 1**

(i) \(2e^{rT} - 2 - rT > 0\)

(ii) \(e^{2rT} + 1 - 3e^{rT} > 0\)

**Proof:**

(i) \(2e^{-rT} - 2 - rT = 2(e^{-rT} - 1) - rT\)

\[ = 2 \left(1 + rt + \frac{(rT)^2}{2!} + \ldots \right) - rT \]

\[ = rt + 2 \left(\frac{(rT)^2}{2!} + \frac{(rT)^3}{3!} + \ldots \right) \]

which is always + ve as value of \(r\) and \(T\) are always positive.

(ii) \(e^{2rT} + 1 - 3e^{rT}\)
\[
\begin{align*}
&= 1 + 2rT + \frac{(2rT)^2}{2!} + \frac{(3rT)^3}{3!} + \ldots + 1 - 3 \left(1 + rT + \frac{(rT)^2}{2!} + \frac{(rT)^3}{3!} + \ldots\right) \\
&= -1 - rT + \frac{(rT)^2}{2} + 6\frac{(rT)^3}{6} + \ldots \\
&= -1 - rT + \frac{(rT)^2}{2} + (rT)^3 + 10.5(rT)^4
\end{align*}
\]
which is also give a positive value a r and T are always positive.

By this two position it is easy to say that \( \Delta_1 > \Delta_2 \).

Then the equations (7) – (9) yield
\[
\begin{align*}
\Delta_1 < 0 & \iff PV_1'(W / D) < 0 \iff T_1^* > W / D \quad \text{(10)} \\
\Delta_2 < 0 & \iff PV_2'(W / D) < 0 \iff T_2^* > W / D \quad \text{(11)} \\
\Delta_3 < 0 & \iff PV_3'(M) < 0 \iff T_3^* > M \quad \text{(12)} \\
\Delta_3 < 0 & \iff PV_3'(M) < 0 \iff T_3^* > M \quad \text{(13)}
\end{align*}
\]

From the above equations we have the following results.

Theorem-1

(1) If \(\Delta_1 > 0, \Delta_2 \geq 0\) and \(\Delta_3 > 0\), then \(PV_\omega(T^*) = \min\{PV_\omega(T_1^*), PV_\omega(W / D)\}\). Hence \(T^*\) is \(T_1^*\) or W/D associated with the least cost.

(2) If \(\Delta_1 > 0, \Delta_2 < 0\) and \(\Delta_3 > 0\), then \(PV_\omega(T^*) = PV_\omega(T_2^*)\). Hence \(T^*\) is \(T_2^*\).

(3) If \(\Delta_1 > 0, \Delta_2 < 0\) and \(\Delta_3 \leq 0\), then \(PV_\omega(T^*) = PV_\omega(T_3^*)\). Hence \(T^*\) is \(T_3^*\).

(4) If \(\Delta_1 \leq 0, \Delta_2 < 0\) and \(\Delta_3 > 0\), then \(PV_\omega(T^*) = PV_\omega(T_2^*)\). Hence \(T^*\) is \(T_2^*\).

(5) If \(\Delta_1 \leq 0, \Delta_2 < 0\) and \(\Delta_3 \leq 0\), then \(PV_\omega(T^*) = PV_\omega(T_3^*)\). Hence \(T^*\) is \(T_3^*\).

Proof:

(1) If \(\Delta_1 > 0, \Delta_2 \geq 0\) and \(\Delta_3 > 0\), which imply that \(PV_1'(W / D) > 0\), \(PV_2'(W / D) \geq 0\), \(PV_2'(M) > 0\) and \(PV_3'(M) > 0\). From the above lemma we implies that

(i) \(PV_3(T)\) is increasing on \([M, \infty)\)

(ii) \(PV_2(T)\) is increasing on \([W / D, M)\)

(iii) \(PV_1(T)\) is increasing on \([T_1^*, W / D)\) and decreasing on \((0, T_1^*)\)

Combining above three, we conclude that \(PV_\omega(T)\) has the minimum value at \(T = T_1^*\) on \((0, W / D)\) and \(PV_\omega(T)\) has the minimum value at \(T = W / D\). Hence, \(PV_\omega(T^*) = \min\{PV_\omega(T_1^*), PV_\omega(W / D)\}\). Consequently, \(T^*\) is \(T_1^*\) or W/D associated with the least cost.

(2) If \(\Delta_1 > 0, \Delta_2 < 0\) and \(\Delta_3 > 0\), which imply that \(PV_1'(W / D) > 0\), \(PV_1'(W / D) < 0\), \(PV_2'(M) > 0\) and \(PV_3'(M) > 0\) which implies that \(T_1^* = W / D\), \(T_2^* > W / D\), \(T_2^* < M\) and \(T_3^* < M\) respectively. Furthermore from the lemma

(i) \(PV_3(T)\) is increasing on \([M, \infty)\)
Since $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, which implies that $PV_1'(W/D) > 0$, $PV_2'(W/D) < 0$, $PV_2'(M) \leq 0$ and $PV_3'(M) \leq 0$, which imply that $T_1^* < W/D$, $T_2^* > W/D$, $T_2^* \geq M$ and $T_3^* > M$, respectively. Furthermore, from the lemma it implies that

1. $PV_3(T)$ is decreasing on $(M, T_3^*)$ and increasing on $(T_3^*, \infty)$
2. $PV_2(T)$ is decreasing on $(W/D, M)$
3. $PV_1(T)$ is decreasing on $(0, T_1^*)$ and increasing on $(T_1^*, W/D)$.

Combining all above, we conclude that $PV_\omega(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\omega(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since, $PV_2(T)$ is decreasing on $(0, T_2^*)$, $T_1^* < W/D$ and $T_2^* \geq M > W/D$ we have $PV_1(T_1^*) > PV_2(T_2^*)$, $PV_2(T_1^*) > PV_3(M)$ and $PV_3(M) > PV_3(T_3^*)$. Hence we conclude that $PV_\omega(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, $T^* = T_3^*$.

If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which implies that $PV_1'(W/D) \leq 0$, $PV_1'(W/D) \leq 0$, $PV_2'(M) > 0$ and $PV_3'(M) > 0$, which imply that $T_1^* \geq W/D$, $T_2^* > W/D$, $T_2^* < M$ and $T_3^* < M$. Furthermore, we have

1. $PV_3(T)$ is increasing on $[M, \infty)$
2. $PV_2(T)$ is decreasing on $[W/D, T_2^*)$ and increasing on $(T_2^*, M]$.
3. $PV_1(T)$ is decreasing on $(0, W/D)$.

Since $PV_1(W/D) > PV_2(W/D)$, and $PV_2(W/D) > PV_2(T^*)$.

So we conclude that $PV_\omega(T)$ has the minimum value at $T = T_2^*$ on $(0, \infty)$. Consequently, $T^* = T_2^*$.

If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, which gives that $PV_1'(W/D) \leq 0$, $PV_2'(W/D) < 0$, $PV_2'(M) \leq 0$ and $PV_3'(M) \leq 0$, and which imply that $T_1^* \geq W/D$, $T_2^* > W/D$, $T_2^* \geq M$ and $T_3^* \geq M$ respectively. Furthermore, from the lemma it implies that

1. $PV_3(T)$ is decreasing on $(M, T_3^*)$ and increasing on $[T_3^*, \infty)$
2. $PV_2(T)$ is decreasing on $[W/D, M]$. 

From the above we conclude that $PV_\omega(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\omega(T)$ has the minimum value at $T = T_2^*$ on $[W/D, \infty)$. Since $PV_1(T) > PV_2(T)$ and $T > 0$. Then $PV_\omega(T^*) = PV_\omega(T_1^*)$ and $T^*$ is $T_2^*$.
(iii) \( PV_1(T) \) is decreasing on \((0, W/D)\).

Since \( PV_1(W/D) > PV_2(W/D) \), combining the above we conclude that \( PV\_w(T) \) has the minimum value at \( T = T_3^* \) on \((0, \infty)\). Consequently, \( T^* \) is \( T_3^* \). This completes the proof.

(B) Suppose \( M \leq W/D \).

Here \( PV\_w(T) \) can be expressed as follows:

\[
PV\_w(T) = \begin{cases} 
PV_1(T) & \text{if } 0 < T < W/D \\
PV_2(T) & \text{if } W/D \leq T
\end{cases}
\]

\[
\left. \frac{\partial P_1'(T)}{\partial T} \right|_{T=W/D} = \Delta_4
\]

and let

\[
(14)
\]

By using proposition 1, we have

\( \Delta_4 - \Delta_4 > 0 \), which leads to \( \Delta_4 > \Delta_4 \).

From (14), we also find that

\[
\Delta_4 < 0 \text{ iff } PV_1(W/D) < 0 \text{ iff } T_3^* > W/D
\]

Lemma-4

(a) If \( \Delta_4 \leq 0 \), then the present value of \( PV_1(T) \) possesses the unique minimum value at the point \( T = T_3 \), where \( T_3 \in [W/D, \infty) \) and satisfies \( \frac{\partial P_1'(T)}{\partial T} = 0 \).

(b) If \( \Delta_4 > 0 \), then the present value of \( PV_1(T) \) possesses a minimum value at the boundary point \( T = W/D \).

Proof: The proof is similar to that of Lemma-2.

Theorem-2

(1) If \( \Delta_1 > 0 \) and \( \Delta_4 \geq 0 \), then \( PV\_w(T^*) = \min\{PV\_w(T_1^*), PV\_w(W/D)\} \). Hence \( T^* \) is \( T_1^* \) or W/D is associated with the least cost.

(2) If \( \Delta_1 > 0 \) and \( \Delta_4 < 0 \), then \( PV\_w(T^*) = \min\{PV\_w(T_1^*), PV\_w(T_3^*)\} \). Hence \( T^* \) is \( T_1^* \) or \( T_3^* \) is associated with the least cost.

(3) If \( \Delta_1 \leq 0 \) and \( \Delta_4 < 0 \), then \( PV\_w(T^*) = PV\_w(T_3^*) \). Hence \( T^* \) is \( T_3^* \).

Proof:

(1) If \( \Delta_1 > 0 \) and \( \Delta_4 > 0 \), which imply that \( PV_1'(W/D) > 0 \) and \( PV_3'(W/D) \geq 0 \), and also \( T_1^* \leq W/D \) and \( T_3^* \leq W/D \). Furthermore, we have

(i) \( PV_1(T) \) is increasing on \([W/D, \infty)\)

(ii) \( PV_1(T) \) is decreasing on \((0, T_1^*)\) and increasing on \([T_1^* < W/D)\). Combining the above we conclude that \( PV\_w(T) \) has the minimum value at \( T = T_1^* \) on \((0, W/D)\) and \( PV\_w(T) \) has the minimum value at \( T = W/D \) on \([W/D, \infty)\). Hence, \( PV\_w(T^*) = \min\{PV\_w(T_1^*), PV\_w(W/D)\} \). Consequently, \( T^* \) is \( T_1^* \) or W/D associated with the least cost.
(2) If \( 0 > \Delta_4 < 0 \), which imply that \( PV_1'(W/D) > 0 \) and \( PV_3'(M) < 0 \) which implies that \( T_1^* < W/D \) and \( T_3^* > W/D \) and also

(i) \( PV_3(T) \) is decreasing on \([W/D, T_3^*]\) and increasing on \([T_3^*, \infty)\).

(ii) \( PV_1(T) \) is decreasing on \((0, T_1^*)\) and increasing on \((T_1^*, W/D)\). Combining (i) and (ii) we conclude that \( PV_\infty(T) \) has the minimum value at \( T = T_1^* \) on \((0, W/D)\) and \( PV_\infty(T) \) has the minimum value at \( T = T_3^* \) on \([W/D, \infty)\). Hence, \( PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(T_3^*)\} \).

Consequently, \( T^* \) is \( T_1^* \) or \( T_3^* \) associated with the least cost.

(3) If \( 0 \leq \Delta_4 \leq 0 \), which implies that \( PV_1'(W/D) \leq 0 \) and \( PV_3'(M) < 0 \) and \( T_1^* \geq W/D \) and \( T_3^* > W/D \) and also

(i) \( PV_3(T) \) is decreasing on \([W/D, T_3^*]\) and increasing on \([T_3^*, \infty)\).

(ii) \( PV_1(T) \) is decreasing on \((0, W/D)\)

From which we conclude that \( PV_\infty(T) \) is decreasing on \((0, W/D)\) and \( PV_\infty(T) \) has the minimum value at \( T = T_3^* \) on \([W/D, \infty)\). Since, \( PV_1(W/D) > PV_3(W/D) \), and conclude that \( PV_\infty(T) \) has minimum value at \( T = T_3^* \) on \((0, \infty)\). Consequently \( T^* \) is \( T_3^* \).

This completes the proof.

**NUMERICAL EXAMPLES**

The followings are considered to be its base parameters \( A = $5/\text{order}, r = 0.3/\$, C = $1, a = 1, k = 2.4, \) \( P_0(\text{Market Price}) = $3, \) Price at the beginning of a cycle \( P(0) = $1, D = -k(P(0) - P_0) = -2.4(1-3) = 4.8\)

**Example-1**

If \( M = 2, W = 2, W/D < M \)

\( \Delta_1 = 31.47 > 0, \Delta_2 = -2.5020 < 0, \Delta_3 = -1.520559 < 0 \)

\( T^* = T_3 = 2.55, \quad PV_3(T^*) = 36.910259 \)

**Example-2**

If \( M = 2, W = 3, W/D < M \)

\( \Delta_1 = 48.416874 > 0, \Delta_2 = -0.112 < 0, \Delta_3 = -1.52 < 0 \)

\( T^* = T_3 = 2.55, \quad PV_3(T^*) = 36.910259 \)

**Example-3**

If \( M = 5, W = 3, W/D < M \)

\( \Delta_1 = 48.51 > 0, \Delta_2 = -0.112 < 0, \Delta_3 = 2.1715 > 0 \)

\( T^* = T_2 = 3.1, \quad PV_2(T^*) = 36.302795 \)

**Example-4**

If \( M = 20, W = 6, W/D < M \)

\( \Delta_1 = 83.5186 > 0, \Delta_2 = 1.47 > 0, \Delta_3 = 4.001 > 0 \)

\( T^* = T_1 = 0.7442, \quad PV_1(T^*) = 48.134529 \)

**Example-5**

If \( M = 5, W = 2, W/D < M \)

\( \Delta_1 = 36.5465 > 0, \Delta_2 = -2.5 < 0, \Delta_3 = 2.738 > 0 \)
Example-6
If \( M=2, W=1, \frac{W}{D}<M \)
\[
\Delta_1 = -1.546 < 0, \quad \Delta_2 = -15.079 < 0, \quad \Delta_3 = -1.52 < 0
\]
\[
T^* = T_2^* = 3.1, \quad PV_2(T) = 36.302795
\]

Example-7
If \( M=10, W=1, \frac{W}{D}<M \)
\[
\Delta_1 = -1.546 < 0, \quad \Delta_2 = -15.079 < 0, \quad \Delta_3 = 3.728554 > 0
\]
\[
T^* = T_2^* = 3.1, \quad PV_2(T) = 36.910259
\]

Example-8
If \( M=1, W=5, \frac{W}{D}<M \)
\[
\Delta_1 = 164.83 > 0, \quad \Delta_2 = 2.946 > 0
\]
\[
T^* = T_1^* = 0.7442, \quad PV_1(T) = 48.134529
\]

Example-9
If \( M=10, W=5, \frac{W}{D}<M \)
\[
\Delta_1 = 83.518661 > 0, \quad \Delta_2 = -1.837413 < 0
\]
\[
T^* = T_3^* = 2.66, \quad PV_3(T) = 34.98682
\]

Based on the above computational result of the numerical examples, the following managerial insights are obtained and following comparative evaluation are observed. If the supplier does not allow the delay payment, cash-out-flow is more but practically taking in view of real-world market, to attract the retailer/customer credit period should be given and it observed that it should be less than equal to the inventory cycle to achieve the better goal. Furthermore, it is preferable for the supplier to opt a credit period which is marginally small.

**CONCLUSION AND FUTURE RESEARCH**

This paper suggested that it might be possible to improve the performance of a market system by applying the laws of thermodynamics to reduce system entropy (or disorder). It postulates that the behaviours of market systems very much resembles that of physical system operating within surroundings, which include the market and supply system.

In this paper, the suggested demand is price dependent. Many researchers advocated that the proper estimation of input parameters in EOQ models which is essential to produce reliable results. However, some of those costs may be difficult to quantify. To address such a problem, we propose in this paper accounting for an additional cost (entropy cost) when analysing EOQ systems which allow a permissible delay payments if the retail orders more than or equal to a predetermined quantity. The results from this paper suggest that the optimal cycle time is more sensitive to the change in the quantity at which the fully delay payment is permitted.

An immediate extension is to investigate the proposed model to determine a retailer optimal cycle time and the optimal payment policy when the supplier offers partially or fully permissible delay in payment linking to payment time instead of order quantity.

**ACKNOWLEDGEMENTS**

The authors sincerely acknowledge the anonymous referees for their constructive suggestion.

**REFERENCES**


(7) Chung, K.J., Huang, C.K., 2000. The convexities of the present value functions of all future cash out flows in inventory and credit. Production planning and control, 11, 133-140.


CALL FOR PAPERS

Volume: 2  Issue: 1
ISSN: 2180-1339

About IJSSC
International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas. Submissions must reflect important developments, extensions, and applications in statistical modeling. IJSSC also encourages submissions that describe scientifically interesting, complex or novel statistical modeling aspects from a wide diversity of disciplines, and submissions that embrace the diversity of scientific and statistical modeling.

IJSSC goal is to be multidisciplinary in nature, promoting the cross-fertilization of ideas between scientific computation and statistical computation. IJSSC is refereed journal and invites researchers, practitioners to submit their research work that reflect new methodology on new computational and statistical modeling ideas, practical applications on interesting problems which are addressed using an existing or a novel adaptation of an computational and statistical modeling techniques and tutorials & reviews with papers on recent and cutting edge topics in computational and statistical concepts.

To build its International reputation, we are disseminating the publication information through Google Books, Google Scholar, Directory of Open Access Journals (DOAJ), Open J Gate, ScientificCommons, Docstoc and many more. Our International Editors are working on establishing ISI listing and a good impact factor for IJSSC.

IJSSC List of Topics
The realm of International Journal of Scientific and Statistical Computing (IJSSC) extends, but not limited, to the following:

- Annotated Bibliography of Articles for the Statist
- Bibliography for Computational Probability and Sta
- Current Index to Statistics
- Guide to Statistical Computing
- Solving Non-Linear Systems
- Statistics and Statistical Graphics
- Theory and Applications of Statistics and Probabil
- Annals of Statistics
- Computational Statistics
- Environment of Statistical Computing
- Mathematics of Scientific Computing
- Statistical Computation and Simulation
- Symbolic computation
**Important Dates**

**Volume:** 2  
**Issue:** 1  
**Paper Submission:** January 31, 2011  
**Author Notification:** March 01, 2011  
**Issue Publication:** March /April 2011
CALL FOR EDITORS/REVIEWERS

CSC Journals is in process of appointing Editorial Board Members for **INTERNATIONAL JOURNAL OF SCIENTIFIC AND STATISTICAL COMPUTING (IJSSC)**. CSC Journals would like to invite interested candidates to join **IJSSC** network of professionals/researchers for the positions of Editor-in-Chief, Associate Editor-in-Chief, Editorial Board Members and Reviewers.

The invitation encourages interested professionals to contribute into CSC research network by joining as a part of editorial board members and reviewers for scientific peer-reviewed journals. All journals use an online, electronic submission process. The Editor is responsible for the timely and substantive output of the journal, including the solicitation of manuscripts, supervision of the peer review process and the final selection of articles for publication. Responsibilities also include implementing the journal’s editorial policies, maintaining high professional standards for published content, ensuring the integrity of the journal, guiding manuscripts through the review process, overseeing revisions, and planning special issues along with the editorial team.


Please remember that it is through the effort of volunteers such as yourself that CSC Journals continues to grow and flourish. Your help with reviewing the issues written by prospective authors would be very much appreciated.

Feel free to contact us at coordinator@cscjournals.org if you have any queries.
Contact Information

Computer Science Journals Sdn Bhd
M-3-19, Plaza Damas Sri Hartamas
50480, Kuala Lumpur MALAYSIA

Phone: +603 6207 1607
        +603 2782 6991
Fax:     +603 6207 1697

BRANCH OFFICE 1
Suite 5.04 Level 5, 365 Little Collins Street,
MELBOURNE 3000, Victoria, AUSTRALIA

Fax: +613 8677 1132

BRANCH OFFICE 2
Office no. 8, Saad Arcad, DHA Main Bulevard
Lahore, PAKISTAN

EMAIL SUPPORT
Head CSC Press: coordinator@cscjournals.org
CSC Press: cscpress@cscjournals.org
Info: info@cscjournals.org