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EDITORIAL PREFACE

The International Journal of Scientific and Statistical Computing (IJSSC) is an effective medium for interchange of high quality theoretical and applied research in Scientific and Statistical Computing from theoretical research to application development. This is the first issue of volume first of IJSSC. International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas.

The initial efforts helped to shape the editorial policy and to sharpen the focus of the journal. Starting with volume 2, 2011, IJSSC appears in more focused issues. Besides normal publications, IJSSC intend to organized special issues on more focused topics. Each special issue will have a designated editor (editors) – either member of the editorial board or another recognized specialist in the respective field.

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IJSSC editors understand that how much it is important for authors and researchers to have their work published with a minimum delay after submission of their papers. They also strongly believe that the direct communication between the editors and authors are important for the welfare, quality and wellbeing of the Journal and its readers. Therefore, all activities from paper submission to paper publication are controlled through electronic systems that include electronic submission, editorial panel and review system that ensures rapid decision with least delays in the publication processes.

To build international reputation of IJSSC, we are disseminating the publication information through Google Books, Google Scholar, Directory of Open Access Journals (DOAJ), Open J Gate, ScientificCommons, Docstoc, Scribd, CiteSeerX and many more. Our International Editors are working on establishing ISI listing and a good impact factor for IJSSC. I would like to remind you that the success of the journal depends directly on the number of quality articles submitted for review. Accordingly, I would like to request your participation by submitting quality manuscripts for review and encouraging your colleagues to submit quality manuscripts for review. One of the great benefits that IJSSC editors provide to the prospective authors is the mentoring nature of the review process. IJSSC provides authors with high quality, helpful reviews that are shaped to assist authors in improving their manuscripts.

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Optimum Algorithm for Computing the Standardized Moments Using MATLAB 7.10(R2010a)

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Abstract

A fundamental task in many statistical analyses is to characterize the location and variability of a data set. A further characterization of the data includes skewness and kurtosis. This paper emphasizes the real time computational problem for generally the r^{th} standardized moments and specially for both skewness and kurtosis. It has therefore been important to derive an optimum computational technique for the standardized moments. A new algorithm has been designed for the evaluation of the standardized moments. The evaluation of error analysis has been discussed. The new algorithm saved computational energy by approximately 99.95%than that of the previously published algorithms.

Keywords: Statistical Toolbox**,** Mathematics, MATLAB Programming

1. INTRODUCTION

The formula used for Z –score appears in two virtually identical forms, recognizing the fact that we may be dealing with sample statistics or population parameters. These formulae are as follow:

$$z_i = \frac{x_i - \bar{x}}{s} \text{ Sample statistics} \qquad (1)$$

$$Z_i = \frac{x_i - \mu}{\sigma} \text{ Population statistics} \qquad (2)$$

Where:

x_i a row score to be standardized
n sample size
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ Sample mean
μ Population mean
s Sample standard deviation
σ Population standard deviation
z Sample z score
Z Populationz score.

Subtracting the mean centers the distribution and dividing by the standard normalizes the distribution. The interesting properties of Z score are that they have a zero mean (effect of centering) and a variance and standard of one (effect of normalizing). We can use Z score to compare samples coming from different distributions [1].

The most common and useful measure of dispersion is the standard deviation. The formula for sample standard deviation is as follow:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$ Sample standard deviation (3)

The population standard deviation is as follow:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$ Population standard deviation (4)

2. MOMENTS

In statistics, the moments are a method of estimation of population parameters such as mean, variance, skewness, and kurtosis from the sample moments.

a) Central Moments

Central moment is called moment about the mean. The central moments provide quantitative indices for deviations of empirical distributions. The $r^{\underline{th}}$central is given by:

$$m_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^r$$

$$or:$$

$$m_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^r \tag{5}$$

Where:

m_r $r^{\underline{th}}$ Sample and population central moments

b) Standardized Moment

The $r^{\underline{th}}$ standardized moment in statistics is the $r^{\underline{th}}$ central moment divided by σ^r (standard deviation raised to power r) as follow:

$$\alpha_r = \frac{m_r}{\sigma^r} \tag{6}$$

Where:

α_r $r^{\underline{th}}$standardized moment

From Eq.(4), Eq.(5), & Eq.(6), We have:

$$\alpha_r = \frac{1}{n}\frac{\sum (x-\mu)^r}{\sigma^r} = \frac{1}{n}\Sigma(Z)^r$$

$$= \frac{(1/n)\sum (x-\mu)^r}{\left(\sqrt{(1/n)\sum (x-\mu)^2}\right)^r} = \frac{m_r}{\left(\sqrt{m_2}\right)^r}$$

Therefore:

$$\alpha_r = \frac{1}{n}\Sigma(Z)^r = \frac{m_r}{\left(\sqrt{m_2}\right)^r} \tag{7}$$

Where:

m_2 Second central moments

c) Computing Population Standardized Moments From Sample z Score

In the real world, finding the standard deviation of an entire population is unrealistic except in certain cases such as standardized testing, where every element of a population is sampled. In most cases, the standard deviation is estimated by examining a random sample taken from the population as defined by eq.(3).

From eq.(5) & eq.(7), We have:

$$\alpha_r = \frac{1}{n}\Sigma(Z)^r = \frac{m_r}{\left(\sqrt{m_2}\right)^r}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{\left(\sqrt{(1/n)\sum (x-\bar{x})^2}\right)^r}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{(1/n)^{r/2}(n-1)^{r/2}\left(\sum (x-\bar{x})^2/(n-1)\right)^{r/2}}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{((n-1)/n)^{r/2}\left(S^2\right)^{r/2}}$$

$$= \frac{1}{n}\left(\frac{n}{n-1}\right)^{r/2}\sum (x-\bar{x})^r / S^r$$

$$= \frac{1}{n}\left(\frac{n}{n-1}\right)^{r/2}\sum z^r$$

Therefore:

$$\alpha_r = \frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}\sum z^r \qquad (8)$$

Equation(8) represents the general equation for computing the r[th] standardized moments of sample z-score.

d) Simplified Standardized Moments

From eq.(8), the term $\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}$ can be simplified using binomial theorem, since it can obtain the binomial series which is valid for any real number k if $|x| < 1$ as follow:

$$(1+x)^k = 1 + k\,x + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots] \qquad (9)$$

The term $\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}$ can be rewritten in the following form:

$$\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2} = \frac{1}{n}\left(\frac{n-1}{n}\right)^{-rl2} = \frac{1}{n}\left(1-\frac{1}{n}\right)^{-rl2}$$

(10)

By replacing x by $\frac{-1}{n}$ and k by $\frac{r}{2}$ we have:

$$\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}} = \frac{1}{n}\left[1-\frac{1}{n}\right]^{-\frac{r}{2}}$$

$$= \frac{1}{n}\left[1 + \frac{\left(-\frac{r}{2}\right)}{1!}\left(-\frac{1}{n}\right) + \frac{\left(-\frac{r}{2}\right)\left(-\frac{r}{2}-1\right)}{2!}\left(-\frac{1}{n}\right)^2 + \cdots\right]$$

$$= \frac{1}{n}\left[1 + \frac{r}{2n} + \frac{r}{4n^2} + \frac{r^2}{8n^2} + \cdots\right]$$

$$= \left[\frac{1}{n} + \frac{r}{2n^2} + \frac{r}{4n^3} + \frac{r^2}{8n^3} + \cdots\right] \qquad (11)$$

For large values of n ,we get:

$$\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}} \cong \left[\frac{1}{n}+\frac{r}{2n^2}\right] \qquad (12)$$

Substituting Eq.(12) in eq.(8), we get:

$$\alpha_{sr} = \left[\frac{1}{n}+\frac{r}{2n^2}\right]\sum_{i=1}^{n} z_i^r$$

$$= \left[\frac{2n+r}{2n^2}\right]\sum_{i=1}^{n} z_i^r \qquad (13)$$

Where:

α_{sr} r^{th} simplified standardized moments.

e) Mathematical Formulae of Standardized and Simplified Moments

Using Eq.(8) & Eq.(13), we can get the following formulae:

Name	r^{th}	Standardized moments	Simplified moments
Mean	1	$\alpha_1 = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{1}{2}}\sum_{i=1}^{n} z_i = 0$	$\alpha_{1s} = \left[\frac{2n+1}{2n^2}\right]\sum_{i=1}^{n} z_i = 0$
Variance	2	$\alpha_2 = \left[\frac{1}{n-1}\right]\sum_{i=1}^{n} z_i^2 = 1$	$\alpha_{2s} = \left[\frac{n+1}{n^2}\right]\sum_{i=1}^{n} z_i^2 = 1$
Skewness	3	$\alpha_3 = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{3}{2}}\sum_{i=1}^{n} z_i^3$	$\alpha_{3s} = \left[\frac{2n+3}{2n^2}\right]\sum_{i=1}^{n} z_i^3$
Kurtosis	4	$\alpha_4 = \frac{n}{(n-1)^2}\sum_{i=1}^{n} z_i^4$	$\alpha_{4s} = \left[\frac{n+2}{n^2}\right]\sum_{i=1}^{n} z_i^4$

f) Ratio Between Population and Sample z-Score

From Eq.(7) & Eq.(8), we can get the exact and simplified ratio of population and sample z-score as follow:

Since:

$$\frac{1}{n}\sum_{i=1}^{n} Z_i^r = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r$$

We get:

$$\frac{\sum_{i=1}^{n} Z_i^r}{\sum_{i=1}^{n} z_i^r} = \left[\frac{n}{n-1}\right]^{\frac{r}{2}} \qquad \textit{exact ratio} \qquad (14)$$

And from Eq.(7) & Eq.(13), we can get:

$$\frac{1}{n}\sum_{i=1}^{n} Z_i^r = \left[\frac{2n+r}{2n^2}\right]\sum_{i=1}^{n} z_i^r$$

$$\frac{1}{n}\sum_{i-1}^{n} Z_i^r = \frac{1}{n}\left[\frac{2n+r}{2n}\right]\sum_{i-1}^{n} z_i^r$$

$$\frac{\sum_{i=1}^{n} Z_i^r}{\sum_{i=1}^{n} z_i^r} = \left[1+\frac{r}{2n}\right] \qquad \textit{simplified ratio} \qquad (15)$$

Eq.(14) and Eq.(15) appear to be very dependent on the sample size. Therefore the ratio between population and sample z-score(required for computing therth standardized moments) depends on the sample size as given in Table_1. This table shows the variation. Figure_1 shows that the sample z score gets closer to population Z score. Therefore, computing standardized moments using simplified technique is recommended for small sample size.

g) Formulae of Skewness and Kurtosis Applied in Statistical Packages

The usual estimators of the population skewness and kurtosis used in Minitab, SAS, SPSS, and Excel are defined as follow [2], [3],[4]:

$$G_1 = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s}\right)^3 \qquad (16)$$

$$G_2 = \left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)} \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{s^4}\right\} - 3\frac{(n-1)^2}{(n-2)(n-3)} + 3 \qquad (17)$$

Where:

$s$is the sample standard deviation.

$G_1$is the usual estimator of population skewness.

$G_2$is known as the excess kurtosis(without adding 3).

Sample size(n)	Exact ratio		Simplified ratio	
	r=3	r=4	r=3	r=4
20	1.07998	1.10803	1.07500	1.10000
30	1.05217	1.07015	1.05000	1.06667
50	1.03077	1.04123	1.03000	1.04000
100	1.01519	1.02030	1.01500	1.02000
200	1.00755	1.01008	1.00750	1.01000
400	1.00376	1.00502	1.00375	1.00500
600	1.00251	1.00334	1.00250	1.00333
1000	1.00150	1.00200	1.00150	1.00200
1400	1.00107	1.00143	1.00107	1.00143
2000	1.00075	1.00100	1.00075	1.00100
2600	1.00058	1.00077	1.00058	1.00077
3000	1.00050	1.00067	1.00050	1.00066
3600	1.00042	1.00056	1.00042	1.00055
4000	1.00038	1.00050	1.00038	1.00050
4500	1.00033	1.00044	1.00033	1.00044
5000	1.00030	1.00040	1.00030	1.00040
5500	1.00027	1.00036	1.00027	1.00036
6000	1.00025	1.00033	1.00025	1.00033
8000	1.00019	1.00025	1.00019	1.00025
10000	1.00015	1.00020	1.00015	1.00020

TABLE 1: Ratio between population and sample z-score

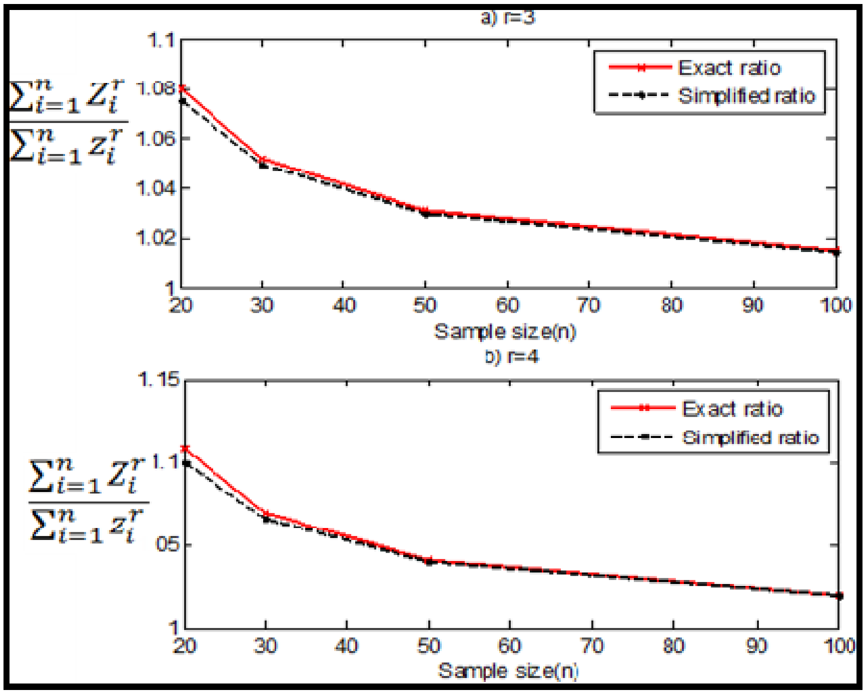

FIGURE 1: Ratio between population and sample z-score.

h) Error Analysis of Standardized Moments

The absolute relative error(ARE) between the standardized and simplified moments is given by:

$$ARE = \left|\frac{standardised - simplified}{standardised}\right| = \left|\frac{\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r - \frac{1}{n}\left[1+\frac{r}{2n}\right]\sum_{i=1}^{n} z_i^r}{\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r}\right|$$

$$= \left|\frac{\left[\frac{n}{n-1}\right]^{\frac{r}{2}} - \left[1+\frac{r}{2n}\right]}{\left[\frac{n}{n-1}\right]^{\frac{r}{2}}}\right| \qquad (18)$$

Therefore, the Absolute Relative Error(ARE) appears to be very dependent on the sample size in regardless with the sample z-score as given in Table_2. This table indicates that the error associated with the standardized moments(Skewness and Kurtosis) of the statistical packages technique is very large compared to the simplified one especially when the sample size is less than 300.Figure_2 shows the variation. Therefore, computing standardized moments using simplified technique is recommended especially when the sample size is less than 600.

Sample size(n)	Skewness			Absolute Relative Error (ARE)`		
	Exact	Simplified	Statistical package	Simplified		Statistical Package
				Pract.	Comp.	
20	-0.37911	-0.37736	-0.41057	0.4608	0.4608	8.2977
30	-0.24103	-0.24053	-0.25390	0.2060	0.2060	5.3420
50	-0.81154	-0.81094	-0.83686	0.0744	0.0744	3.1197
100	0.19241	0.19237	0.19535	0.0186	0.0186	1.5293
200	-0.11239	-0.11239	-0.11324	0.0046	0.0046	0.7572
400	0.21474	0.21474	0.21555	0.0011	0.0011	0.3768
600	0.01677	0.01677	0.01682	0.0005	0.0005	0.2508
1000	0.05781	0.05781	0.05790	0.0001	0.0001	0.1502
1400	-0.12846	-0.12846	-0.12860	9.5e-5	9.5e-5	0.10728
2000	-0.02750	-0.02750	-0.02753	4.6e-5	4.6e-5	0.07507
2600	0.03271	0.03271	0.03273	2.7e-5	2.7e-5	0.05773
3000	-0.01793	-0.01793	-0.01795	2.1e-5	2.1e-5	0.05003
3600	-0.02616	-0.02616	-0.02617	1.4e-5	1.4e-5	0.041688
4000	-0.01818	-0.01818	-0.01819	1.1e-5	1.1e-5	0.037517
4500	0.005310	0.005310	0.005312	9.2e-6	9.2e-6	0.033347
5000	-0.04197	-0.04197	-0.04198	7.5e-6	7.5e-6	0.030011
5500	-0.04199	-0.04199	-0.04200	6.2e-6	6.2e-6	0.027282
6000	0.033432	0.033432	0.033440	5.2e-6	5.2e-6	0.025007
8000	-0.00851	-0.00851	-0.00851	2.9e-6	2.9e-6	0.018754
10000	-0.00057	-0.00057	-0.00057	1.8e-6	1.8e-6	0.015002

TABLE 2: a)Skewness (ARE)

Sample size(n)	CPU time (Second)		
	Exact	Simplified	Statistical package
20	0.000185	0.000126	0.000146
30	0.000189	0.000133	0.000145
50	0.000200	0.000156	0.000154
100	0.000213	0.000153	0.000163
200	0.000234	0.000181	0.000185
400	0.000301	0.000257	0.000239
600	0.000394	0.000302	0.000357
1000	0.000487	0.000407	0.000457
1400	0.000584	0.000513	0.000518
2000	0.000746	0.000676	0.000690
2600	0.000989	0.000883	0.000909
3000	0.001096	0.001046	0.001002
3600	0.001233	0.001164	0.001162
4000	0.001396	0.001329	0.001375
4500	0.001489	0.001018	0.001355
5000	0.001594	0.001559	0.001574
5500	0.001785	0.001250	0.001683
6000	0.001891	0.001286	0.001916
8000	0.002164	0.001500	0.002286
10000	0.002318	0.001802	0.003122

TABLE 2: a)Skewness(CPU)

Sample size(n)	Kurtosis			Absolute Relative Error(ARE)		
	Exact	Simplified	Statistical Package	Simplified		Statistical Package
				Pract.	Comp.	
20	3.0034	2.9816	3.377	0.725	0.725	12.439
30	2.897	2.8876	3.1077	0.32593	0.32593	7.2722
50	2.628	2.6249	2.7182	0.1184	0.11840	3.4341
100	2.6438	2.643	2.6878	0.0298	0.0298	1.6648
200	3.0312	3.031	3.0626	0.00747	0.00747	1.0361
400	2.8435	2.8434	2.8566	0.00187	0.00187	0.4634
600	2.967	2.967	2.9768	0.00083	0.00083	0.32998
1000	2.932	2.932	2.938	0.00029	0.00029	0.19384
1400	3.066	3.066	3.071	0.00015	0.00015	0.14793
2000	3.023	3.023	3.027	7.5e-5	7.5e-5	0.1014
2600	2.947	2.947	2.949	4.4e-5	4.4e-5	0.0749
3000	2.963	2.963	2.965	3.3e-5	3.3e-5	0.06553
3600	2.882	2.882	2.884	2.3e-5	2.3e-5	0.05220
4000	2.976	2.976	2.978	1.8e-5	1.8e-5	0.04946
4500	2.952	2.952	2.954	1.4e-5	1.4e-5	0.04341
5000	3.010	3.010	3.011	1.1e-5	1.1e-5	0.04024
5500	2.998	2.998	2.999	9.9e-6	9.9e-6	0.03636
6000	3.073	3.073	3.074	8.3e-6	8.3e-6	0.03454
8000	3.041	3.041	3.042	4.6e-6	4.6e-6	0.02552
10000	2.911	2.911	2.912	2.9e-6	2.9e-6	0.01909

TABLE 2: b) Kurtosis(ARE)

Sample size(n)	CPU time (Second)		
	Exact	Simplified	Statistical Package
20	0.000164	0.000125	0.000187
30	0.000169	0.000127	0.000196
50	0.000170	0.000153	0.000214
100	0.000192	0.000151	0.000216
200	0.000216	0.000176	0.000248
400	0.000290	0.000256	0.000302
600	0.000327	0.000295	0.000363
1000	0.000547	0.000430	0.000457
1400	0.000563	0.000543	0.000554
2000	0.000737	0.000832	0.001109
2600	0.000967	0.000914	0.000962
3000	0.001020	0.000989	0.001173
3600	0.001187	0.001181	0.001231
4000	0.001338	0.000944	0.001073
4500	0.001426	0.001482	0.001643
5000	0.001128	0.001598	0.001619
5500	0.001138	0.001732	0.001868
6000	0.001826	0.001290	0.001893
8000	0.002628	0.001832	0.002436
10000	0.002571	0.001830	0.002922

TABLE 2: b) Kurtosis(CPU)

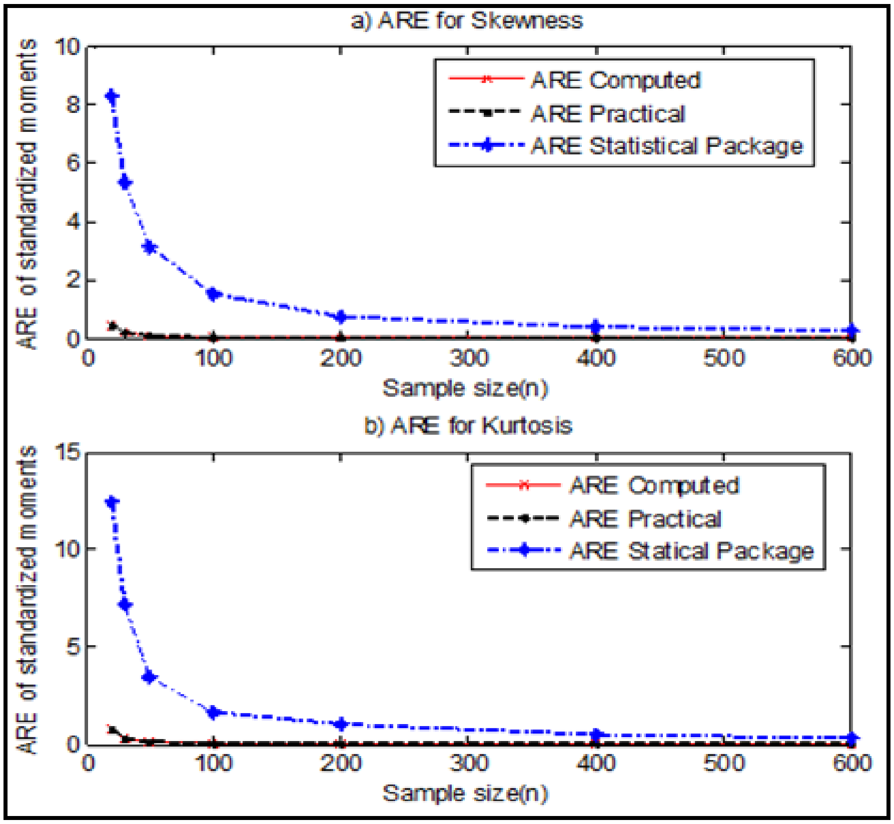

FIGURE 2: Absolute Relative Error of standardized moments

The percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one of the standardized moment is given by:

$$\delta E = \left| \frac{ARE\ of statistical\ package - ARE\ of simplified}{ARE\ of statistical\ package} \right| * 100 \qquad (19)$$

Where:

δE isthe percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one.

Table_3 shows the percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one for different sample size. This table indicates that the simplified technique of the standardized moments gives reduction in ARE by approximately 96.7% compared to the statistical package technique especially when the sample size is less than 100.Figure_3 shows the variation.

The squared error(E_r) between the standardized and simplified moments is given by:

$$E_r = (standardised - simplified)^2$$

$$= (ARE * standardised)^2 \qquad (20)$$

Sample size(n)	Skewness (r=3)		Kurtosis (r=4)	
	Error percentage(%)	Error reduction(%)	Error percentage(%)	Error reduction(%)
20	5.553	94.447	5.828	94.172
30	3.856	96.144	4.482	95.518
50	2.385	97.615	3.448	96.552
100	1.216	98.784	1.790	98.210
200	0.608	99.392	0.721	99.279
400	0.292	99.708	0.404	99.596
600	0.199	99.801	0.252	99.748
1000	0.067	99.933	0.150	99.850
1400	0.089	99.911	0.101	99.899
2000	0.061	99.939	0.074	99.926
2600	0.047	99.953	0.059	99.941
3000	0.042	99.958	0.050	99.950
3600	0.034	99.966	0.044	99.956
4000	0.029	99.971	0.036	99.964
4500	0.028	99.972	0.032	99.968
5000	0.025	99.975	0.027	99.973
5500	0.023	99.977	0.027	99.973
6000	0.021	99.979	0.024	99.976
8000	0.015	99.985	0.018	99.982
10000	0.012	99.988	0.015	99.985
Mean	0.73 %	99.27 %	0.879%	99.121%

TABLE 3: Error reduction of standardized moments

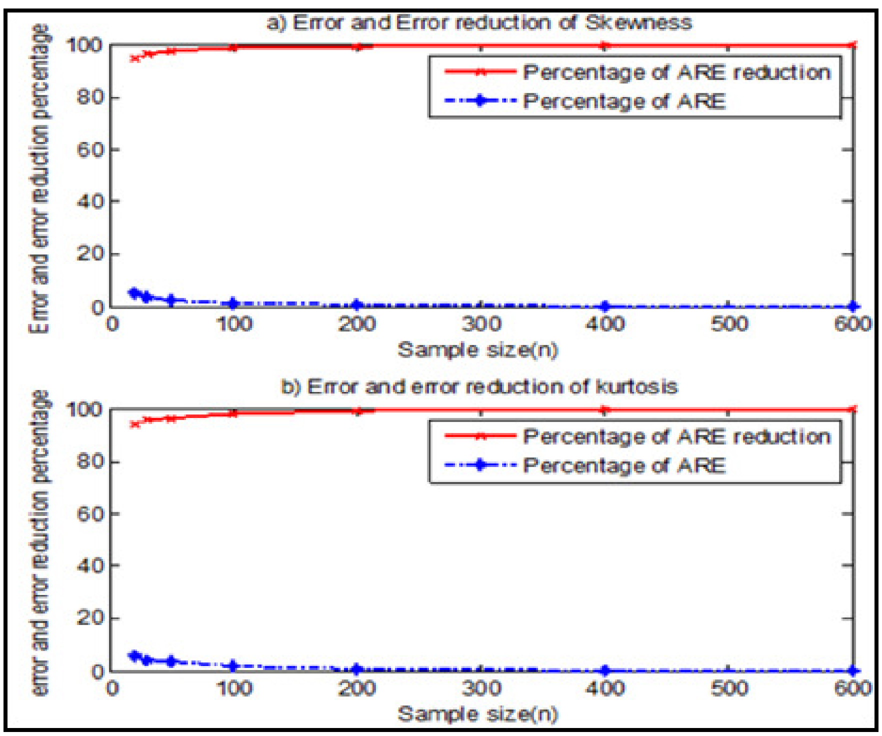

FIGURE 3: Error and error reduction of standardized moments

3. POPULATION EXAMPLE

A data set of 10000 points was randomly generated to have a mean of 100 and a standard deviation of 10. The histogram for this data is shown in figure_4 and looks fairly bell-shaped. A different sample size was randomly selected from the data set to calculate the two statistics(skewness and kurtosis).

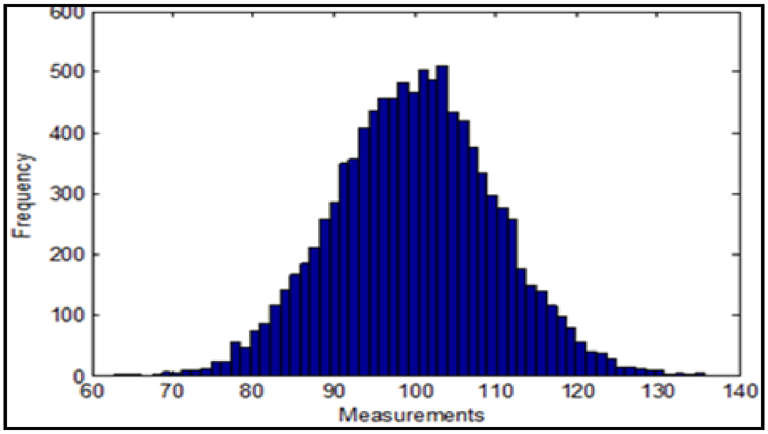

FIGURE 4: Histogram of 10000 points randomly generated(μ=100,σ =10)

4. IMPACT OF SAMPLE SIZE ON SKEWNESS AND KURTOSIS

The 10000 point data set above was used to explore what happens to skewness and kurtosis based on sample size. There appears to be a lot of variation in the results based on sample size. The results are shown in Table_2.Figure_5shows how the skewness and kurtosis changed with sample size.

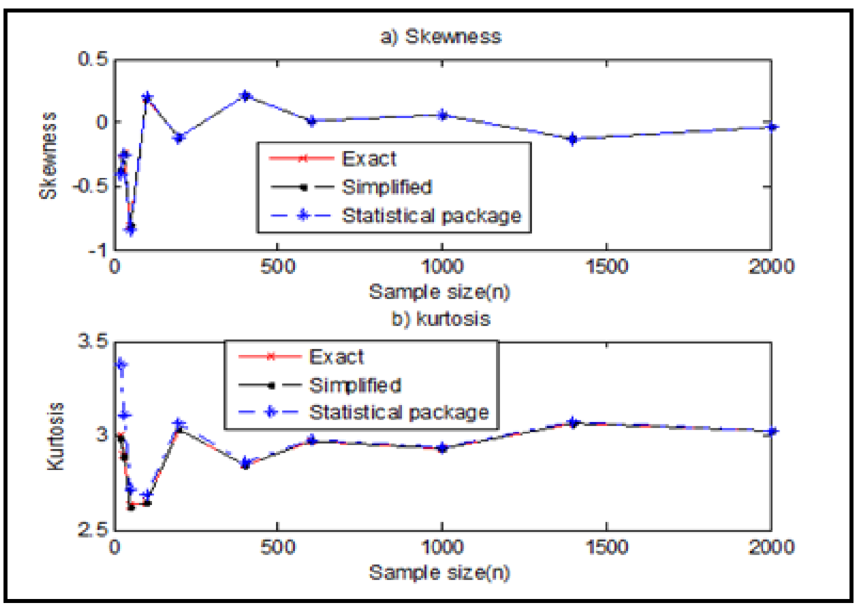

FIGURE 5: Impact of size sample on skewness and kurtosis

5. PROCESSING TIME OF STANDARDIZED MOMENTS

The processing time required for Computing the skewness and kurtosis is executed by LaptopDELL-inspiron-1520.Table_2 indicates that the processing time required for computing the skewnessusing the simplified technique is minimum than other especially when the sample size increases. Figure_6 shows the variation.

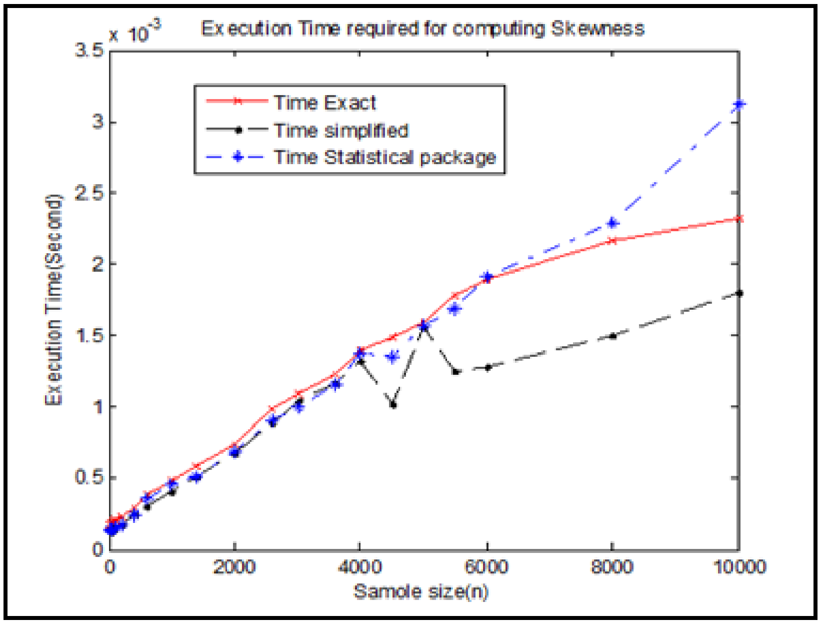

FIGURE 6:a) Execution time required for computing skewness

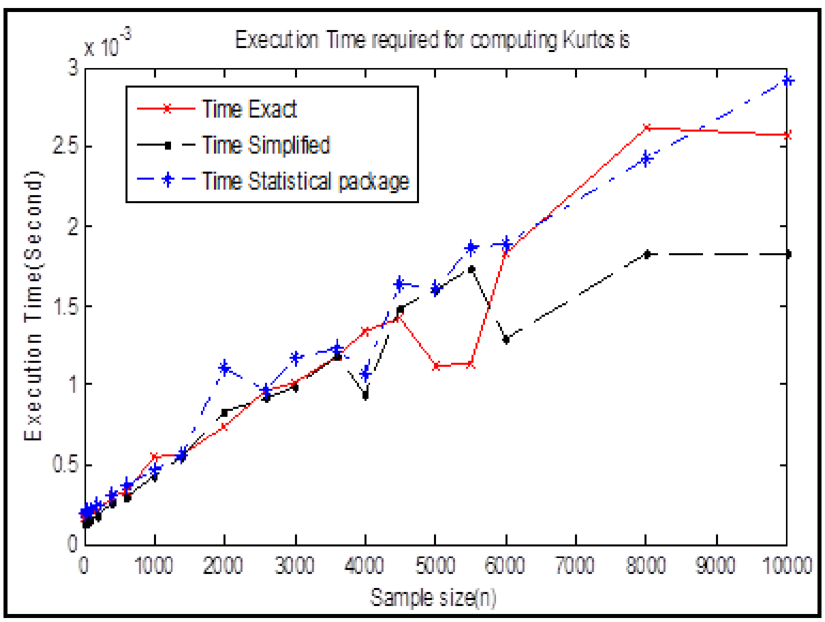

FIGURE 6: b) Execution time required for computing kurtosis

6. COMPUTATIONAL ENERGY OF STANDARDIZED MOMENTS

Computing the computational energy for standardized moments (skewness and kurtosis) requires the determination of the sample size(n), the square error(E_r), and the central processing time(CPU time). Therefore, consider the sample size(n) represents the resistance, the square error is measured in $[\text{volts}]^2$, and the CPU time in second. Then, the computational energy per sample size is given by:

$$CE = \frac{E_r * t_r}{n} \qquad (21)$$

Where:

CE is the computational energy per sample size.
E_r is the rth square error.
t_r is r^{th} CPU time.
n is the sample size.

The computational energy saved by the simplified technique compared to the exact one is given by:

$$\delta CE = \frac{CE_e - CE_s}{CE_e} * 100 \qquad (22)$$

Where:

δCE is the relative computational energy saved by the simplified technique.
CE_e is the computational energy for the exact technique.
CE_s is the computational energy for the simplified technique.

Table_4 shows the computational energy(CE) for each technique. This table indicates that the simplified technique saved computational energy by approximately 96.7% compared to the statistical package technique. Figure_7 shows the variation.

Sample size(n)	**CE-Skewness (r=3)**			
	CE Exact	**CE Simplified**	**CE Statistical Package**	**CE saved by simplified (%)**
20	2.82e-07	1.92e-07	7.22e-05	99.73
30	1.55e-08	1.09e-08	8.01e-06	99.86
50	1.45e-08	1.13e-08	1.97e-05	99.94
100	2.72e-11	1.95e-11	1.41e-07	99.98
200	3.12e-13	2.41e-13	6.69e-09	99.99
400	4.19e-14	3.58e-14	3.91e-09	99.99
600	4.61e-17	3.53e-17	1.05e-11	99.99
1000	1.62e-17	1.36e-17	3.44e-11	99.99
1400	6.21e-17	5.45e-17	7.02e-11	99.99
2000	5.96e-19	5.40e-19	1.47e-12	99.99
Mean	----	----	----	99.95%

TABLE 4: Computational Energy of standardized moments(a:Skewness)

Sample size(n)	**CE-Kurtosis (r=4)**			
	CE Exact	**CE Simplified**	**CE Statistical Package**	**CE saved by simplified (%)**
20	3.88e-05	2.96e-05	0.01304	99.77
30	5.02e-06	3.77e-06	2.89e-3	99.86
50	3.29e-07	2.96e-07	3.48e-4	99.91
100	1.19e-08	9.37e-09	4.18e-05	99.97
200	5.53e-10	4.51e-10	1.22e-05	99.99
400	2.04e-11	1.80e-11	1.31e-06	99.99
600	3.30e-12	2.98e-12	5.79e-07	99.99
1000	3.95e-13	3.10e-13	1.47e-07	99.99
1400	8.50e-14	8.20e-14	8.14e-08	99.99
2000	1.89e-14	2.13e-14	5.2e-08	99.99
Mean	----	-----	------	99.95%

TABLE 4: Computational Energy of standardized moments(b:Kurtosis)

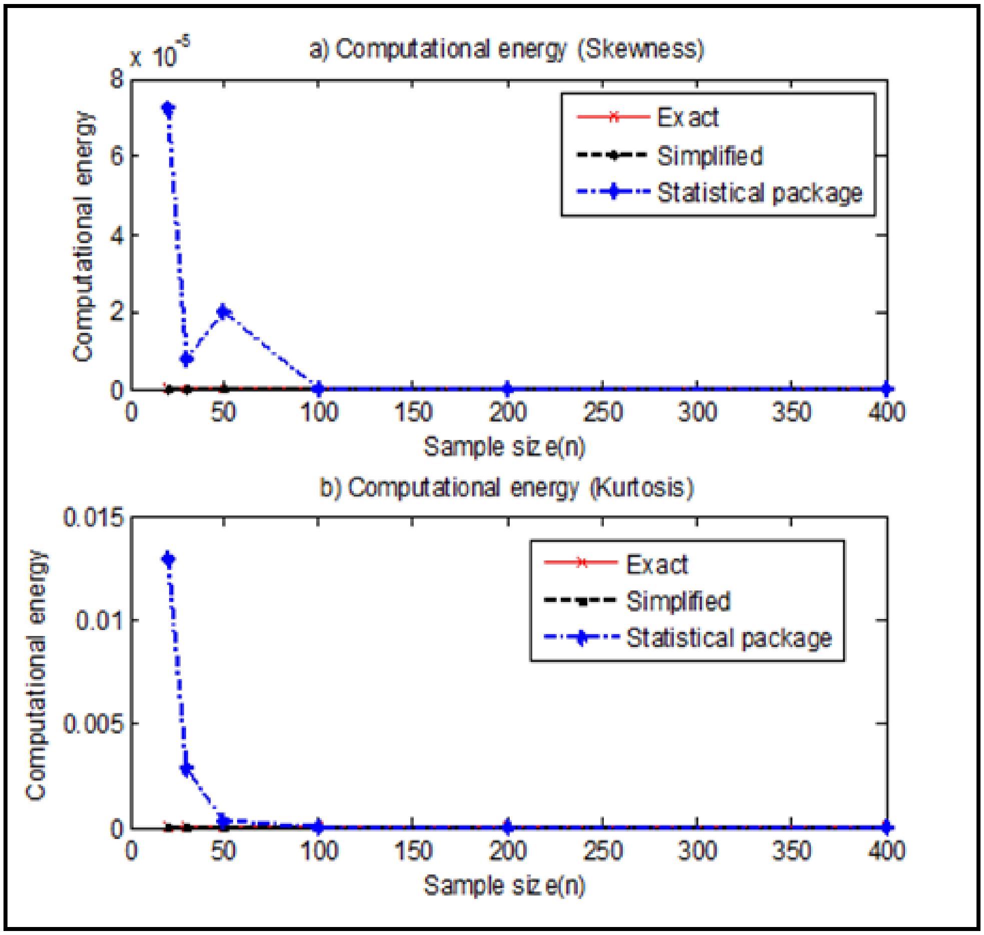

FIGURE 7: Computational Energy for standardized moments

7. MATLAB PROGRAMMING

A complete program can be obtained by writing to the author[4]. There is a part of MATLAB program shown here:

```
% Grenrate random data set of size (n) points with mean (mu) and a standard
% deviation (segma)and returns: (1) skewness and kurtosis,(2) cpu time,(3) ARE &
% SquareError,(4) Computational Energy(CE),(5) computational energy saved
% by the simplified technique compared to the exact one
options.Interpreter='tex';
prompt = {'Enter Sample size:','Enter mean(\mu) :','Enter std.dev.(\sigma) :'};
dlg_title = 'Generate random data set';
num_lines = 1;
def = {'','',''};
options.Resize='on';
options.WindowStyle='normal';
answer = inputdlg(prompt,dlg_title,num_lines,def,options);
ifisempty(answer)
error('No inputs were found!')
end
n=str2num(answer{1})
mu= str2num(answer{2})
sigma = str2num(answer{3})
if n< 3 || isempty(n)
error('n must be integer &>=2')
```

```
end
// Part of the program is omitted //
tic
            S_SP=(n/((n-1)*(n-2)))*sum(((s-mean(s))./std(s)).^r);
t_SP= toc;
tic
            S_E=(1/n)*(n/(n-1))^(r/2)*sum((zscore(s)).^r);
t_E = toc;
tic
            S_S=(1/n+r/(2*n^2))*sum((zscore(s)).^r);
t_S = toc;
   A_E=abs(((S_E-S_S)/S_E)*100);
  A_S=abs((((n/(n-1))^(r/2)-(1+r/(2*n)))/(n/(n-1))^(r/2))*100) ;
  A_SP=abs(((S_E-S_SP)/S_E)*100);
  SK=dataset({ S_E,'Exact'},{ S_S,'Simplified'},{ S_SP,'Stat_Package'} )
 ARE=dataset({ A_E,'Practical'},{ A_S,'Computed'}, {A_SP,'Stat_Package'} )
        // Part of the program is omitted //
```

8. CONCLUSIONS

Computer algorithms for fast implementation of standardized moments are an important continuing area of research.A new algorithm has been designed for the evaluation of the standardized moments. As a result the new technique offered four advantages over the current technique:

(1) It drastically reduces the CPU time for calculating the standardized moments especially when the sample size increases.
(2) It drastically reduces the absolute relative error(ARE) for calculating the standardized moments(Skewness and Kurtosis) by 99.27% compared to the current one.
(3) It gives minimum square error compared to the current algorithm.
(4) It has lowest computational energy.

The aforementioned features are combined in a mathematical formula to describe the system performance. This formula is called the computational energy. A quantitative study has been carried out to compute the computational energy for each technique. The results show that the simplified technique saved computational energy by 96.7% compared to the current one.

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A Computationally Efficient Algorithm to Solve Generalized Method of Moments Estimating Equations based on Secant-Vector Divisions Procedure

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Abstract

Generalized method of moment estimating function enables one to estimate regression parameters consistently and efficiently. However, it involves one major computational problem: in complex data settings, solving generalized method of moments estimating function via Newton-Raphson technique gives rise often to non-invertible Jacobian matrices. Thus, parameter estimation becomes unreliable and computationally difficult. To overcome this problem, we propose to use secant method based on vector divisions instead of the usual Newton-Raphson technique to estimate the regression parameters. This new method of estimation demonstrates a decrease in the number of non-convergence iterations as compared to the Newton-Raphson technique and provides reliable estimates. We compare these two estimation approaches through a simulation study.

Keywords: Quadratic Inference Function, Newton-Raphson, Jacobian, Secant Method, Vector Divisions.

1. INTRODUCTION

GENERALIZED method of moments (GMM) is a popular tool developed by Hansen (1982) to estimate regression parameters especially in settings where the number of equations exceeds the number of unknown parameters. Recently, Qu et al. (2000) and Qu and Lindsay (2003) have formulated a GMM function known as the quadratic inference function (QIF) to analyze the effects of explanatory variables on repeated responses. Since, in general, the correlation structure of repeated measures are unknown, Qu et al.(2000) assume a working structure which can be decomposed into several basis matrices. These basis matrices are then combined to form a score vector whose dimension is quite large. The objective is to use this score vector to estimate the vector of regression parameters. The authors proposed to construct a generalized moment estimating function based on this score vector and thereafter use calculus to optimize the function and obtain estimates of the regression parameters. The optimized function is non-linear and thus, the Newton-Raphson algorithm is implemented to solve iteratively the equation. However, we often remark in simulation studies that the Jacobian matrix of the Newton-Raphson iterative equation is close to singularity. This may lead to unreliable parameter estimates or a blockage of the iterative process. Our objective in this paper is to apply an alternative iterative method that omits the computation of the inverse Jacobian matrix. Yixun (2008) and Mamode Khan (2011) considered the secant method of estimation which is based on vector divisions. In this paper, we use this approach to estimate the regression parameters based on the GMM objective function and compare the Newton-Raphson estimation approach with the Secant method based vector

divisions. The comparison between these two techniques is made through simulating AR(1) correlated Poisson counts with different covariate designs.
The organization of the paper is as follows: In section 2, we provide the estimating equations of the Generalized method of moments and its estimation procedures. In section 3, we introduce the method of secant iterative scheme based on vector divisions following Mamode Khan (2011). In the next section, we present a simulation study whereby we generate AR(1) correlated Poisson counts and use GMM and Secant based on vector divisions to estimate the regression parameters. In the last section, we provide the conclusions and recommendations based on comparisons of these two techniques.

2. GENERALIZED METHOD OF MOMENTS

Qu et al. (2000) introduced a GMM of the form of a quadratic objective function that combines an extended score function with its covariance matrix.
The construction of the extended score function is based on the generalized estimating equation, that is

$$g(\beta)=\sum_{i=1}^{I}\left(\frac{\partial\mu_i}{\partial\beta^T}\right)^T V_i^{-1}(y_i-\mu_i)=0 \qquad (1)$$

where

$$V_i=A_i^{\frac{1}{2}}R(\alpha)A_i^{\frac{1}{2}} \qquad (2)$$

and $R(\alpha)$ is the working correlation structure. Their method of GMM models the inverse of the correlation structure $R(\alpha)^{-1}$ by a class of matrices

$$R(\alpha)^{-1}=\sum_{i=1}^{m}a_i M_i \qquad (3)$$

where $M_1, M_2,....,M_m$ are known basis matrices and $a_1, a_2,..., a_M$ are constants. Equation (3) can accommodate the popular correlation structures. Equation (1) can then be written as

$$g(\beta)=\sum_{i=1}^{I}\left(\frac{\partial\mu_i}{\partial\beta^T}\right)^T A_i^{-\frac{1}{2}}(a_1M_1+a_2M_2+.....+a_mM_m)A_i^{-\frac{1}{2}}(y_i-\mu_i)=0 \qquad (4)$$

Based on this representation, Qu et al. (2000) defined an extended score

$$g^*(\beta)=\frac{1}{I}\sum_{i=1}^{I}g_i(\beta)=\frac{1}{I}\begin{pmatrix}\sum_{i=1}^{I}\left(\frac{\partial\mu_i}{\partial\beta^T}\right)^T A_i^{-\frac{1}{2}}M_1A_i^{-\frac{1}{2}}(y_i-\mu_i)\\ \sum_{i=1}^{I}\left(\frac{\partial\mu_i}{\partial\beta^T}\right)^T A_i^{-\frac{1}{2}}M_2A_i^{-\frac{1}{2}}(y_i-\mu_i)\\ \vdots \\ \sum_{i=1}^{I}\left(\frac{\partial\mu_i}{\partial\beta^T}\right)^T A_i^{-\frac{1}{2}}M_mA_i^{-\frac{1}{2}}(y_i-\mu_i)\end{pmatrix} \qquad (5)$$

In principle, the vector $g^*(\beta)$ contains more equations than parameters but they can be combined optimally following GMM to form a quadratic objective function of the form

$$S(\beta)=[g^*(\beta)]^T W^{-1}[g^*(\beta)] \qquad (6)$$

Where

$$W=\frac{1}{I^2}\sum_{i=1}^{I}g_i(\beta)g_i(\beta)^T \qquad (7)$$

The idea is to minimize $S(\beta)$. Qu et al. (2000) showed that asymptotically,

$$\dot{S}(\beta) = 2\left[\frac{\partial g^*}{\partial \beta^T}\right]^T W^{-1} g^* \tag{8}$$

$$\ddot{S}(\beta) = 2\left[\frac{\partial g^*}{\partial \beta^T}\right]^T W^{-1} \left[\frac{\partial g^*}{\partial \beta^T}\right] \tag{9}$$

Then the vector of regression parameters β is estimated iteratively using the Newton-Raphson technique

$$\hat{\beta}_{(j+1)} = \hat{\beta}_{(j)} - [\ddot{S}(\hat{\beta}_j)]^{-1}\, \dot{S}(\hat{\beta}_j) \tag{10}$$

Qu et al. (2000) showed that asymptotically $\hat{\beta}$ is consistent and its variance reaches the Cramer-Rao type lower bound. The algorithm works as follows: After assuming a working structure for the basis matrices, we construct $\dot{S}(\beta)$ and $\ddot{S}(\beta)$ for an initial of vector regression parameters $\hat{\beta}_0$. We replace in equation (10) to obtain an updated $\hat{\beta}_1$. We then use $\hat{\beta}_1$ to obtain $\ddot{S}(\hat{\beta}_1)$ and $\dot{S}(\hat{\beta}_1)$. However, this iterative equation may not be successful in estimating parameters for every type of setting. In fact, we carried out an experiment that involves the simulation of correlated Poisson counts and noted that while estimating the regression parameters, the Jacobian matrix $\ddot{S}(\hat{\beta})$ often turns out to be singular and ill-conditioned. This ultimately blocks the computation process. To remedy the situation, we propose an alternative approach known as the secant method based on vector divisions to estimate the parameters. In the next section, we introduce the secant method and show its iterative scheme.

3. SECANT METHOD

The traditional secant iterative formula to estimate a scalar parameter β is given by

$$\hat{\beta}_{r+1} = \hat{\beta}_r - \frac{F(\hat{\beta}_r)}{F[\hat{\beta}_r, \hat{\beta}_{r-1}]} \tag{11}$$

with

$$F[\hat{\beta}_r, \hat{\beta}_{r-1}] = \frac{F(\hat{\beta}_r) - F(\hat{\beta}_{r-1})}{\hat{\beta}_r - \hat{\beta}_{r-1}} \tag{12}$$

where $F(\beta) = 0$

However, this iterative formula cannot be applied directly to obtain the vector of regression parameters β in equation (8) since β is here multi-dimensional. To overcome this issue, Yixun Shi (2008) developed an iterative multi-dimensional secant formula using vector divisions. We adapt his procedures to solve equation (8). By letting $F(\beta) = \dot{S}(\beta)$, we estimate iteratively using

$$\hat{\beta}_{j+1} = \hat{\beta}_j - \frac{(\hat{\beta}_j - \hat{\beta}_{j-1})^T (\hat{\beta}_j - \hat{\beta}_{j-1})}{(\hat{\beta}_j - \hat{\beta}_{j-1})^T (F(\hat{\beta}_j) - F(\hat{\beta}_{j-1}))} F(\hat{\beta}_j) \tag{13}$$

or

$$\hat{\beta}_{j+1} = \hat{\beta}_j - \frac{(F(\hat{\beta}_j) - F(\hat{\beta}_{j-1}))^T F(\hat{\beta}_j)}{(F(\hat{\beta}_j) - F(\hat{\beta}_{j-1}))^T (F(\hat{\beta}_j) - F(\hat{\beta}_{j-1}))} (\hat{\beta}_j - \hat{\beta}_{j-1}) \tag{14}$$

The iterative process works as follows: For initial values of $\hat{\beta}_0, \hat{\beta}_1$, we calculate $\hat{\beta}_2$ using equation (13) or (14). Then using $\hat{\beta}_1$ and $\hat{\beta}_2$, we calculate $\hat{\beta}_3$. The iterative process continues until convergence, i.e, $\| \hat{\beta}_{t+1} - \hat{\beta}_t \| < 10^{-5}$. However, to ensure convergence, we can use a steepest direction coefficient following Mamode Khan (2011) and Yixun (2008).

4 SIMULATION STUDY

In this section, we generate AR(1) correlated Poisson counts following McKenzie (1986) with true mean parameters $\beta_0 = 1, \beta_1 = 1$. Note that in GMM, the bases matrices M_0, M_1 and M_2 are : M_0: The identity matrix, M_1 has one on the two main off-diagonals and M_2 has 1 on the corners $(1,1)$ and $(n.n)$. We consider different covariates designs:

Design 1: $$x_{it1} = 1, (i = 1, \dots, \frac{I}{4}), x_{it2} = 0, (i = \frac{I}{4} + 1, ..., \frac{3I}{4}), x_{it3} = 1, (i = \frac{3I}{4} + 1, ..., I) \quad (15)$$

Design 2: $$x_{it1} = rbin(3,0.5), (i = 1, \dots, \frac{I}{4}), x_{it2} = 0, (i = \frac{I}{4} + 1, ..., \frac{3I}{4}), x_{it3} = rbin(3,0.5), (i = \frac{3I}{4} + 1, ..., I) \quad (16)$$

Design 3: $$x_{it1} = rpois(3), (i = 1, \dots, \frac{I}{4}), x_{it2} = 0, (i = \frac{I}{4} + 1, ..., \frac{3I}{4}), x_{it3} = rpois(0.7), (i = \frac{3I}{4} + 1, ..., I) \quad (17)$$

and for the second covariate x_{it2}, we generate I standard normal values.

For each design, we run 5,000 simulations for $I = 20, 60, 100$ and 500. The following tables provide the simulated mean of the estimates and the number of non-convergent simulations under both techniques.

Size	$\hat{\beta}_{1,NR}$	$\hat{\beta}_{2\,NR}$	$\hat{\beta}_{1\sec}$	$\hat{\beta}_{2\sec}$	Number of non-convergent simulations in the Newton-Raphson	Number of non-convergent simulations in the Secant approach
20	0.9621	0.9872	0.9632	0.9881	2341	1550
60	0.9991	0.9999	0.9992	1.0054	1201	910
100	0.9990	1.0024	1.0014	0.9996	825	534
400	1.0010	0.9998	0.9999	0.9999	230	100
600	0.9999	0.9999	0.9999	0.9999	98	30
1000	1.0000	1.0000	1.0000	1.0000	54	10

TABLE 1: Estimates of GMM regression parameters under Newton-Raphson and Secant method: Design 1

Size	$\hat{\beta}_{1,NR}$	$\hat{\beta}_{2NR}$	$\hat{\beta}_{1\sec}$	$\hat{\beta}_{2\sec}$	Number of non-convergent simulations in the Newton-Raphson	Number of non-convergent simulations in the Secant approach
20	0.9943	0.9899	0.9942	0.9892	2562	1899
60	0.9993	0.9997	0.9992	0.9998	1666	1032
100	0.9990	1.0003	0.9999	0.9999	1321	998
400	1.0001	0.9998	0.9999	0.9999	344	223
600	0.9999	0.9999	0.9999	0.9999	142	97
1000	1.0000	1.0000	1.0000	1.0000	76	55

TABLE 2: Estimates of GMM regression parameters under Newton-Raphson and Secant method: Design 2

Size	$\hat{\beta}_{1,NR}$	$\hat{\beta}_{2NR}$	$\hat{\beta}_{1\sec}$	$\hat{\beta}_{2\sec}$	Number of non-convergent simulations in the Newton-Raphson	Number of non-convergent simulations in the Secant approach
20	0.9897	0.9899	0.9901	0.9900	1040	999
60	0.9997	0.9999	0.9998	1.0001	889	762
100	0.9999	1.0001	1.0000	0.9996	762	566
400	1.0001	0.9999	0.9999	0.9999	444	320
600	0.9999	0.9999	0.9999	0.9999	102	87
1000	1.0000	1.0000	1.0000	1.0000	65	34

TABLE 3: Estimates of GMM regression parameters under Newton-Raphson and Secant method: Design 3

For each design, we assume small initial values of the mean parameters to run the simulations. As noted, there is no huge discrepancy between the estimated parameters and the true value of the regression parameters. As the cluster size increases, the discrepancies become lesser under both estimation techniques in all of the designs. This is in accordance with the consistency properties of the estimators under the GMM approach. As regards to the number of non-convergent simulations, the Newton-Raphson technique reports a comparatively higher number of non-convergent simulations than the Secant method as the Jacobian matrix becomes close to singularity. This problem was noted in almost all cluster sizes. However, as the cluster size increases, the number of non-convergent simulations decreases significantly under both approaches. The non-convergence problem also occurs because of the choice of the steepest descent coefficient as reported by Mamode Khan (2011). Under some simulations, these coefficients were modified to yield convergence and to speed convergence. Based on the simulation results, we may conclude that GMM based on the secant method using vector divisions is a computationally fast and efficient approach. Also, its computational complexities compared with the Newton-Raphson method will be lesser. In the same context, Mamode Khan (2011) showed through simulation studies that the secant method is an efficient estimation approach from a computational perspective as it reduces the number of non-convergent simulations and provides equally consistent estimates.

5 : CONCLUSION

Generalized method of moments is an efficient estimation approach that yields consistent and reliable estimates of regression parameters particularly in an over-determined system of non-linear equations but its estimation procedures often give rise to singular Jacobian matrices. This makes computation quite difficult. In this paper, we propose an alternative to Newton-Raphson known as the Secant method based on vector divisions. This approach omits the computation of the Jacobian matrix and provides equally consistent and reliable estimates than GMM under Newton-Raphson approach. Another advantage of this method is the computational complexities are lesser than Newton-Raphson as the inverse of a matrix requires quite a number of flop counts. Based on simulation results, we note that both Newton-Raphson and Secant method based on vector divisions yield consistent estimates but the secant method yields fewer non-convergent simulations than Newton-Raphson. However, care must be taken when choosing initial values of the parameters. Otherwise, the resulting estimates may be unreliable. Thus, we may conclude that GMM based Secant method is a more optimal estimation methodology.

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Application of Statistical Tool for Optimisation of Specific Cutting Energy and Surface Roughness on Surface Grinding of Al-SiC$_{35p}$ Composites.

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Abstract

In this paper, the effects and the optimization of machining parameters on surface roughness and specific cutting energy during surface grinding of 6061Al-SiC$_{35P}$ composites under different process parameters such as Vol% of SiC, feed and depth of cut were investigated using response surface methodology (RSM). The specific cutting energy and surface roughness are considered as performance characteristics. Experiments are conducted using standard RSM design called Central composite design (CCD). A second order response model was developed for specific cutting energy and surface roughness. The results identify the significant influence factors to minimise the specific cutting energy and surface roughness. Derringer's desirability function was then used for simultaneous optimization of specific cutting energy and surface roughness. The confirmation results demonstrate the practicability and effectiveness of the proposed approach.

Key words: Metal Matrix composites; Specific cutting energy; Surface Roughness; ANOVA; Response surface methodology; desirability function

1. INTRODUCTION

Discontinuously reinforced aluminium composites(DRAC's) is one of the important composites among the metal matrix composites, which have SiC particles with aluminium matrix is harder than tungsten carbide , which pose many problems in machining[1-2]. The aluminium alloy reinforced with discontinuous ceramic reinforcements is rapidly replacing conventional materials in various automotive, aerospace and automobile industries. But DRAC's grinding is one of the major problems, which resist its wide spread engineering application [3].

A fundamental parameter derived from the force measurements is the specific grinding energy, which is the energy per unit volume of material removal. Any proposed mechanisms of abrasive workpiece interactions must be consistent with the magnitude of the specific cutting energy and its dependence on the operating parameters [4]. While Al/SiC-MMC specimen slides over a hard cutting tool edge during grinding, due to friction, high temperature and pressure the particles of Al/SiC-MMC adhere to the grinding wheel which affects the surface quality of the specimen [5]. This also results in decreased uncut chip thickness and hence the increased specific cutting energy for grinding. Rowe et. al. Investigated the creep feed grinding of Nickel-based alloy and found that specific cutting energy is as high as 400 J/mm^3 for 150 mm^3 per mm width of metal removal [5]

A Di Ilio et.al [6] investigated the machining characteristics of Al2009-SiC-15P, Al2009-SiC-20P and Al2009-SiC-25P, concluded that composite shows better surface finish than the pure aluminium. They developed a model of the grinding process based on empirical relations and observed that workpiece surface roughness can be related with the equivalent chip thickness through a power relationship; it shows a decreasing linear trend as the hardness of workpiece material increases. Sanjay Agarwal et.al [7] conducted a study on surface and subsurface of the ground ceramic material and concluded that cutting force and specific cutting energy can considerably be reduced due to dislodgement of individual grains, resulting from microcracks along the grain boundaries. Brinksmeier et. al. [8] made an attempt to quantify the size effect and possibility of using this in grinding for controlled subsurface work hardening of metals. It is observed that, Main physical quantity characterizing the size effect is specific grinding energy which increases with decreasing chip thickness. Lowering cutting speed at a constant chip thickness shifts the chip formation mechanism towards micro-ploughing and thus additionally increases the specific grinding energy. Li et.al [9] investigated the effects of wheel wear on process responses and ground ceramic quality, particularly the flexural strength. Strong relationships between the wheel surface conditions and the process responses are found. During the initial stage of wheel wear, the surface density of diamond grits, surface roughness and flexural strength decreased, and the specific normal force, specific tangential force, force ratio, and specific cutting energy increased. Ren et.al [10] demonstrated the correlation of specific cutting energy with the grinding process parameters and the material property parameters for the tungsten carbides. The study also examines material-removal mechanisms and surface finish in grinding of such materials. Their study revealed that specific cutting energy is related not only to grinding process parameters, but also to the physical–mechanical properties of the workpiece material

Matheiu Barge et.al.[11] conducted scratching experiment on flat surface of AISI4140 steel and found that hardening and softening of the workpiece is key for the study of force and energy. Hwang et.al.[4] found that under a feed of 500 mm/min and for all the wheel speeds used, an increase in the wheel depth of cut from 0.1–2 mm slightly improved the ground surface finish, but greatly prolonged the wheel life. This increase did not deepen the subsurface damage layer for the alumina and alumina–titania, but resulted in a slightly deeper damage layer for the zirconia. Zhong et.al [12] conducted experiments on grinding of Al_2O_3 composites using SiC wheel and diamond wheel and found that SiC wheel is suitable for rough grinding and diamond wheel for finish grinding. Hood et.al.[13] used two separate L_9 taguchi fractional array for grinding of γ-TiAl alloy and BuRTi alloy and found that former require 10% less power and 25% less specific cutting energy compared to the later. They also observed that, high wheel speed, low depth of cut and low feed will result in improved surface roughness. Seeman et.al.[14] developed a second order response surface model for surface roughness and tool wear of Al/SiC composites. They concluded that formation of BUE will affect the tool wear and surface roughness. Krajnik [15] compared RSM and Genetic algorithm for centreless grinding of 9SMn28. Kwak and Kim [16] developed a second order response surface model for surface roughness and grinding force on grinding of Al/SiC/mg composites. They investigated that optimum content of SiC and Mg in AC8A aluminium alloy is 30vol% and 9vol% respectively. Kwak [17] presented the application of Taguchi and RSM for the geometric error. A second-order response model for the geometric error was developed and the utilization of the response surface model was evaluated with constraints of the surface roughness and the MRR. Box and Draper [18] proposed central composite rotatable design for fitting a second order response surface based on the criterion of rotatability.
From the above literature review it is evident that less amount of work is done to investigate the combined effect of specific cutting energy and surface roughness in grinding of Al-SiC composites. Hence in this study an attempt is made to optimise the specific cutting energy and surface roughness during grinding of Al-SiC_{35p} composites using desirability function in response surface methodology.

2. DESIGN OF EXPERIMENT BASED ON RESPONSE SURFACE METHODOLOGY

In order to investigate the influence of various factors on the Specific cutting energy (SE) and surface roughness (Ra), three principal factors such as the volume percentage of SiC (X_1), feed (X_2) and depth of cut (X_3) were taken. In this study, these factors were chosen as the independent input variables. The desired responses were the specific cutting energy (SE) and surface roughness (Ra) which are assumed to be affected by the above three principal factors. The response surface methodology was employed for modeling and analyzing the machining parameters in the grinding process so as to obtain the machinability performances of responses [2].

In the RSM, the quantitative form of relationship between the desired response and independent input variables is represented as y= F(X_1, X_2, X_3)

Where y is the desired response and F is the response function (or response surface). In the procedure of analysis, the approximation of y was proposed using the fitted second-order polynomial regression model, which is called the quadratic model. The quadratic model of y can be written as given in equation (1) [19-22]:

$$\hat{y} = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} a_{ii} X_i^2 + \sum_{i-1}^{n} \sum_{j=1}^{n} a_{ij} X_{ij} ------(1)$$

Where a_0 is constant, a_i, a_i, and a_{ij} represent the coefficients of linear, quadratic, and interaction terms respectively. X_i reveals the coded variables that correspond to the studied factors.

The necessary data for building the response models are generally collected by the experimental design. In this study, the collections of experimental data were adopted using central composite design (CCD). The factorial portion of CCD is a full factorial design with all combinations of the factors at two levels (high, +1 and low, −1) and composed of the six axial points and six central points (coded level 0) which is the midpoint between the high and low levels[23]. The star points are at the face of the cubic portion on the design which corresponds to a value of α =1 and this type of design is commonly called the face-centered CCD.

3. DESIRABILITY FUNCTION

The desirability function approach to simultaneously optimizing multiple equations was originally proposed by Harrington (1965) and later improved by Derringer and Suich (1984) [24]. Essentially, the approach is to translate the functions to a common scale ([0, 1]), combine them using the geometric mean and optimize the overall metric. The method involves transformation of each predicted response, ŷ, to a dimensionless partial desirability function, di, which includes the researcher's priorities and desires when building the optimization procedure. One or two-sided functions are used, depending on whether each of the n responses has to be maximized or minimized, or has an allotted target value. If the response is to be minimised the response di can be defined as:

$$d(y) = \left[\frac{H - \hat{y}}{T - H}\right]^{wt} \rightarrow T < \hat{y} < H ------(2)$$

$$1 \rightarrow \hat{y} < T$$

$$0 \rightarrow \hat{y} > H$$

In Eq. (2), L, H and T are, respectively the lowest, highest and the target values and wt is the weight. The value of wt can be varied between 0.1 and 10. The value of one creates a linear ramp function between the low value, goal and the high value. Increased wt moves the result towards the goal or its decrease creates the opposite effect. The partial desirability function d_i ranges between 0, (for a completely undesired response), and 1, (for a fully desired response).

The partial desirability functions are then combined into a single composite response, the global desirability function D, defined as the geometric mean of the different d_i-values:

$$D= (d_1{}^{v}{}_{1} * d_2{}^{v}{}_{2} * d_n{}^{v}{}_{n})^{1/n} \quad (0 \leq D \leq 1) --------------(3)$$

In equation (3) v_i is the relative importance assigned to the response i. The relative importance v_i is a comparative scale for weighting each of the resulting d_i in the overall desirability product and it varies from the least important (v_i = 1) to the most important (vi = 5). It is noteworthy that the outcome of the overall desirability D depends on the v_i value that offers users flexibility in the definition of desirability functions.

4. EXPERIMENTAL PROCEDURE

Al-SiC specimens having aluminum alloy 6061 as the matrix and containing 8 vol.%,10 vol.% and 12 vol.% of silicon carbide particles of mean diameter 35μm in the form of cylindrical bars of length 120mm and diameter 20mm. The specimens were manufactured at Vikram Sarbhai Space Centre (VSSC) Trivandrum by Stir casting process with pouring temperature 700-710℃, stirring rate 195rpm. The specimen were extruded at 457℃, with extrusion ratio 30:1, and direct extrusion speed 6.1m/min to produce length 120mm and Ø22mm cylindrical bars. The extruded specimens were solution treated for 2 hr at a temperature of 540^{o}C in a muffle furnace; Temperatures were accurate to within ±2^{o}C and quench delays in all cases were within 20s. After solution treatment, the samples were water quenched to room temperature. Further the specimen is machined to 17mm square cross-section. Table-1 shows the chemical composition of Al 6061 alloy. Grinding method as machining process was selected. Experiments were conducted on 5 HP, 2880rpm, conventional surface grinding machine (Bhuraji make) with automatic (hydraulic) table-feed and Norton make diamond grinding wheel ASD76R100B2 with outer diameter 175mm, width of 12.5mm, thickness of 5mm and inner diameter of 31.75 which is generally used for finishing operation. The honing stick having specification GN0390220K7V7 is used for dressing the wheel. The experiments conducted under dry conditions.

Table-1: Chemical composition of Al 6061 alloy

Element	Cu	Mg	Si	Cr	Fe	Al
Volume %	0.25	1	0.6	0.25	0.2	Balance

The levels and factors selected for the experimentation are given in Table-2. Selection of factors for optimization was based on preliminary experiments, prior knowledge of the literature, and known instrumental limitations. The time required for machining the each specimen is measured. The volume of metal removed per unit time gives the metal removal rate. The surface roughness of the specimen is measured using Taylor/Hobson surtronoic 3+ surface roughness measuring instrument

TABLE 2: Levels of independent Factors

Factors	Levels		
	L	M	H
Vol % Percentage SiC (X_1)	8	10	12
Feed (mm/	60	70	80

s) (X_2)			
Depth of Cut (μm) (X_3)	8	12	16

5: RESULTS AND DISCUSSION

5.1 Development of Mathematical Model

The mathematical relationship between responses and grindingparameters were established using experimental test results from planned set of experiments; face-centered CCD. Table-3 and Table-4 Below shows coefficients of response surface regression and the corresponding p-value for specific cutting energy and surface roughness.

TABLE 3: Regression analysis for Specific cutting energy

Term	Coeffficients	P-value
Constant	543.669	0.020
X_1	18.469	0.424
X_2	-11.125	0.086
X_3	-37.725	0.001
X_1^2	- 1.587	0.148
X_2^2	0.089	0.053
X_3^2	0.948	0.004
X_1X_2	0.030	0.807
X_1X_3	0.628	0.060
X_2X_3	0.098	0.129

TABLE 4: Regression analysis for Surface roughness

Term	Coefficient	P-value
Constant	2.1966	0.003
X_1	- 0.2542	0.002
X_2	0.0078	0.643
X_3	- 0.0088	0.703
X_1^2	0.0095	0.008
X_2^2	0.00003	0.789
X_3^2	- 0.0001	0.874
X_1X_2	- 0.001	0.025
X_1X_3	0.0019	0.048
X_2X_3	0.00013	0.462

It is observed from Table-3 for the response surface regression analysis of specific cutting energy that, linear and square of depth of cut and square of feed are more significant as their P-value are less than 0.05. Similarly regression analysis of surface roughness from Table-4 shows that, linear and square of SiC volume percentage and interaction of SiC vol percentage with feed and depth of cut are more significant. Equation (4) and (5) represent the developed response surface regression equation for specific cutting energy and surface roughness respectively.

Regression equation for specific cutting energy

$$\hat{y}_1 = 543.669 + 18.469X_1 - 11.125X_2 - 37.725X_3 - 1.587X_1^2 + 0.089X_2^2 + 0.984X_3^2 + 0.03X_1X_2 + 0.628X_1X_3 + 0.098X_2X_3 \quad ----(4)$$

Regression equation for surface roughness

$$\hat{y}_2 = 2.1305 - 0.2204X_1 + 0.00576X_2 - 0.0147X_3 + 0.00874X_1^2 + 6.61E-05X_2^2 - 1.08E-04X_3^2 - 0.00117X_1X_2 + 0.00209X_1X_3 + 0.000168X_2X_3 \quad ------(12)$$

Where $\hat{y}_1$ and $\hat{y}_2$ are the responses for specific cutting energy and surface roughness respectively. X_1, X_2 and X_3 represents the decoded values of SiC volume percentage, Feed (mm/s) and depth of cut (microns) respectively.

5.2 Analysis of the Developed Mathematical Model

The ANOVA and F- ratio test have been performed to justify the goodness of fit of the developed mathematical models.

The calculated values of F- ratios for lack-of-fit have been compared to standard values of F-ratios corresponding to their degrees of freedom to find the adequacy of the developed mathematical models. Table-5 and Table-6 shows the ANOVA for specific cutting energy and surface roughness respectively. The standard percentage point of F distribution for 95% confidence level ($F_{0.05,5,5}$) is 5.05. Since the F-value for lack of fit is less than the standard value, both the models are adequate at 95% confidence level. R^2-value the measure of fitness of the model for specific cutting energy and surface roughness are 95.45% and 99.3% respectively. It indicates that model fits well with the experimental results

TABLE 5: Analysis of variance for specific cutting energy

S	D	S	A	F	P
R	9	1	1	2	0
Li	3	8	4	9	0
S	3	1	5	1	0
I	3	3	1	2	0
R	1	5	5		
L	5	4	8	4	0
P	5	9	1		
T	1	1			

.

TABLE 6: Analysis of variance for Surface roughness

S	D	S	A	F	P
R	9	0	0	1	0
L	3	0	0	6	0
S	3	0	0	7	0
I	3	0	0	4	0
R	1	0	0		
L	5	0	0	1	0
P	5	0	0		
T	1	0			

Based on the response surface equation (4) and (5) contour plots for specific cutting energy and surface roughness are plotted. Fig-1 and Fig-2 shows the contour plot for specific cutting energy and surface roughness respectively. From Fig-1 it is observed that, specific energy increase with increase in feed. It is mainly due to the reason that increase in feed will decrease the contact time between the wheel and the workpiece which results in ploughing of wheel on the workpiece. Increased ploughing will increase the surface temperature and hence specific cutting energy [25]. Higher the specific cutting energy higher will be the heat dissipated and poor will be the surface finish [26].Moreover increase in feed will increase the cutting force which results in increased specific cutting energy. It is also observed that with depth of cut up to 13 to 14 microns specific cutting energy decreases. But increase in depth of cut beyond 14microns will results in increase of specific cutting energy. The initial decrease was, due to the increase in the maximum chip thickness with the increase in depth of cut, which resulted in decrease in specific cutting energy. The increase in specific cutting energy beyond certain value of depth of cut could be due to the reduction in friction between the wheel and the work and brittle fracture of the material [6]. Fig-2 shows the contour plot for surface roughness. It is observed from the figure that surface roughness improves with decrease in depth of cut and also with increase in volume percentage of SiC. It may be due to the reason that material becomes harder with increased volume percentage of SiC, which results in improved surface roughness.

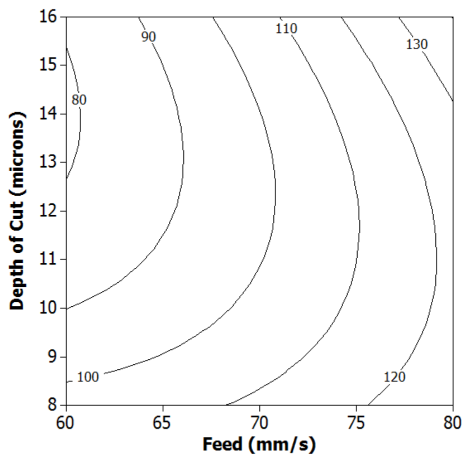

FIGURE 1: Contour plot for Specific energy

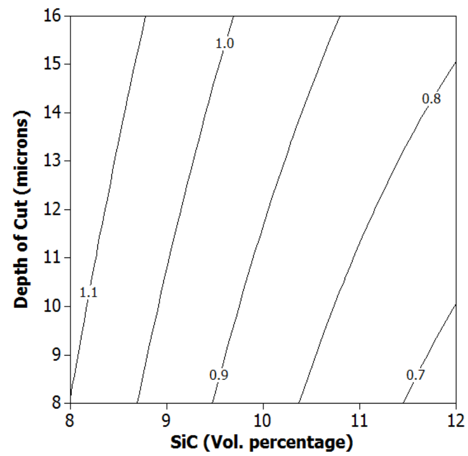

FIGURE 2: Contour plot for Surface Roughness

5.3 Analysis for the Optimisation of Response

Desirability function method popularised by Derringer and Suich [27] is used for the optimisation of specific cutting energy and surface roughness. The general approach is to first convert each response Y into an individual desirability function d_i that varies over the range. $0 \le d_i \le 1$
Where, if the response Y ^ is at its goal or target, then d_i=1, and if the response is outside an acceptable region, d_i=0.

The weight of the desirability function for each response defines its shape. The individual desirability functions are combined to provide a measure of the composite or overall desirability of the multi response system. This measure of composite desirability is the weighted geometric mean of the individual desirability for the responses [28]. The optimal operating conditions can then be determined by maximizing the composite desirability.

Fig-3 Shows the optimisation plot for of specific cutting energy and surface roughness. The goal is to minimise specific cutting energy and surface roughness. The upper value and target value for specific cutting energy have been fixed at 150 and 70 respectively. Similarly for the surface roughness the upper and target values are fixed at 1.3 and 0.65 respectively. Both the responses are assigned a weight of 3 and importance of 3. The optimisation plot shows that composite desirability is almost nearer to 1. The optimum value of specific cutting energy and surface roughness are 69.99J/mm^3 and 0.6505 microns respectively for machining Al-6061 SiC12 vl% specimen with feed 60mm/s and depth of cut 9.05 microns.

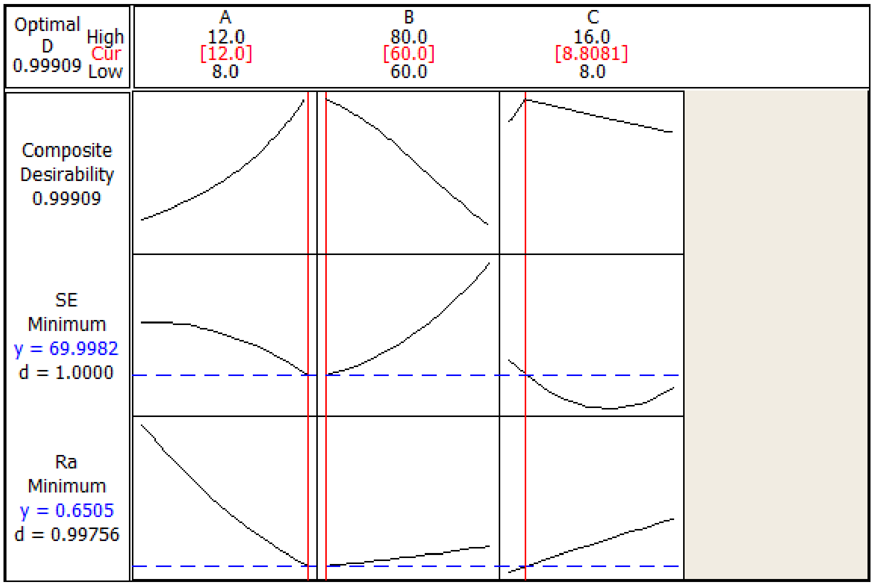

FIGURE 3: Optimum results for minimum specific cutting energy and minimum surface roughness

6. MODEL VALIDATION RUN

The response surface model developed in equation (4) and (5) were validated by the set of test runs. Table-7 gives the results obtained from experimental test, and the results obtained by the developed response surface model. The parentage error for specific cutting energy is within 9.5% and for surface roughness is within 2.5%. Hence it can be concluded that fitted model agrees very close to the experimental results.

TABLE 7: Validation of the results

		Experimental value		Response Surface model value		Percentage error	
	[illegible]	Specific energy (J/mm^3	Surface roughness (mic	Specific energy (J/mm^3	Surface roughness (mic	Specific energy	Surface roughness

		)	rons)	)	rons)		
		84.65	0.65	93.3597	0.653	9.33	0.46
		77.17	1.07	71.8131	1.083	6.94	1.21
		129.6	1.24	126.833	1.214	2.166	2.1
		76.436	77.47	0.62	0.6305	1.33	1.69

7. CONCLUSION

In this study, the Response surface methodology was applied for analyzing Specific cutting energy and surface roughness in the surface grinding of DRACs. Based on experimental results, following conclusions were drawn from the above experimental work.

i. It is observed that specific cutting energy increase with increase in feed. It may be due to the reason that all the cutting energy is dissipated in to heat at increased feed.

ii. Specific cutting energy is lower with increase in SiC weigt percentage of the specimen. This phenomenon is attributed to the fact that specific cutting energy associated with the ductile material removal process is much higher than that with a brittle removal mode.

iii. Surface roughness improves with increased SiC volume percentage of specimen and decrease in depth of cut. It is mainly due to the fact that, increase in vol% of SiC will increase the hardness of the specimen, which results in decrease ploughing of the wheel during grinding.

iv. Response surface regression is used to develop a second order equation for specific cutting energy and surface roughness. For 95% confidence level, it is observed that fitted value is very close to the experimental value.

v. Desirability function approach is applied to find the optimal cutting condition for minimum specific cutting energy and minimum surface roughness. Maintaining the feed at 60mm/s and depth of cut at 9 microns while machining Al6061-12%volSiC will produce a minimum specific cutting energy of 69.99J/mm^3 and a minimum surface roughness of 0.6505 microns.

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