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EDITORIAL PREFACE

The International Journal of Scientific and Statistical Computing (IJSSC) is an effective medium for interchange of high quality theoretical and applied research in Scientific and Statistical Computing from theoretical research to application development. This is the *First* Issue of *Third* Volume of IJSSC. International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas.

The initial efforts helped to shape the editorial policy and to sharpen the focus of the journal. Started with Volume 3, 2012, IJSSC appears with more focused issues. Besides normal publications, IJSSC intend to organized special issues on more focused topics. Each special issue will have a designated editor (editors) – either member of the editorial board or another recognized specialist in the respective field.

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IJSSC editors understand that how much it is important for authors and researchers to have their work published with a minimum delay after submission of their papers. They also strongly believe that the direct communication between the editors and authors are important for the welfare, quality and wellbeing of the Journal and its readers. Therefore, all activities from paper submission to paper publication are controlled through electronic systems that include electronic submission, editorial panel and review system that ensures rapid decision with least delays in the publication processes.

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Movement of Share Prices and Sectoral Analysis: A Reflection Through Interactive and Dynamic Graphs

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Abstract

Interaction in graphs gives the user with an advantage to analyze the data in greater depth. With the help of interactive graphics users can get better insight of the data in comparison to the static graphical tools. This paper introduces an interactive graphical tool consisting of two graphs, a line diagram complemented by a boxplot. The line diagram helps to understand how successive values of a variable are related to time and box plot can help the visual comparison of several such variables. Here the line diagram is used to visualize share prices of a company corresponding to a number of days and the boxplot displays the position of the Share price of all companies in a particular sector. An investor in share market needs to consider a number of factors before making any decision about investment. Some of the factors influencing the decision are the performance of the particular security in recent past, its position in terms of share price in its own sector. The graphical technique used in this software tool shall be helpful while making investment decision.

Keywords: Boxplot, Line Diagram, Share Market, Statistical Graphics.

1. INTRODUCTION

Interactive and dynamic graphs can be prepared easily with the help of advanced graphical software. Interactive graphics allows analyst to draw better inferences in comparisn to the non-interactive methods. Through live visual displays, interactive graphics offer insights which are impossible to achieve with traditional statistical tools [1]. The present state of computer software and hardware technology encourages to draw graphs in delightfull way to portray the insight of available data. Today's advanced state of information technology has opened up opportunities to communicate statistical information in a way that can be easily understood not only by experts but even by the layman [2]. Application of interactive graphics can be found in various fields like national statistics [3], national economy [4], time series data [5] etc. Use of dynamic and interactive graphs can be found in the field of financial analysis. A good number of web based applications like online technical charts [6] and standalone charting application software can be found that are used to analyze stock market information in dynamic and interactive way.

This paper presents a software developed using Microsoft Visual Basic 6.0 for drawing two graphical tools viz. line diagram and boxplot. The line diagram is drawn taking the closing share price of a particular company for last few days. It shall depict how the share price of the said company has been moving during those days. The boxplot, on the other hand, shall depict the position of all the companies in a particular sector.

2. FLUCTUATIONS IN SHARE MARKET

Investment is the sacrifice of current consumption for the future benefit. There are various avenues for investment. Some of the avenues provide certain return and there are certain avenues providing uncertain return. The investment instruments that provide certain return includes bank fixed deposits, corporate bonds, government securities etc. However, there are another set of instruments that does not guarantee the return and one such instrument is equity share. The return from the share is dependent upon the performance of the company. The return from the equity shares depends upon the price of the equity shares in the market. Price of a share at every given moment is an "efficient" reflection of the expected value. The fluctuation in the price of the shares is dependent on many factors. Some of the factors are performance of the company, market conditions, general economic conditions, cyclic fluctuations, condition of the industry/sector to which company belongs etc. Besides, interest rates, inflation, quarterly earnings reports, news on corporate events, crime and fraudulent activities, energy prices, war and terrorism, and other local and worldwide political factors also play significant role in determining the share prices [7].

Another theory of share price determination comes from the field of Behavioral Finance. According to Behavioral Finance, humans often make irrational decisions particularly, related to the buying and selling of securities. It is based upon fears and misperceptions of outcomes. The irrational trading of securities can often create securities prices which vary from rational, fundamental price valuations [8].

All the above factors affect the supply and demand for the shares. The price of a stock is determined by supply and demand. The price of the stock moves in order to achieve and maintain equilibrium. For example, if there are more people wanting to buy a stock than to sell it, the price will be driven up because those shares are rarer and people will pay a higher price for them and vice versa. Even if there is nothing wrong with a company, a large shareholder who is trying to sell millions of shares at a time can drive the price of the stock down, simply because there are not enough people interested in buying the stock he/she is trying to sell.

3. INDUSTRIAL OR SECTORAL ANALYSIS IN SHARE MARKET

The fields of fundamental analysis and technical analysis attempt to understand market conditions that lead to price changes, or even predict future price levels. Fundamental analysis is the examination of the underlying forces that affect the well being of the economy, industry groups, and companies. As with most analysis, the goal is to derive a forecast and profit from future price movements. At the company level, fundamental analysis may involve examination of financial data, management, business concept and competition. At the industry level or sector level, there might be an examination of supply and demand forces for the products offered. To forecast future stock prices, fundamental analysis combines economic, industry, and company analysis to derive a stock's current fair value and forecast future value [7].

By analyzing the sector or industry, an investor would want to consider the overall growth rate, market size, and importance to the economy. Although, the individual company is still important but its industry group is likely to exert just as much, or more, influence on the stock price [7]. When stocks move, they usually move as groups; there are very few lone guns. Sometimes it is said that it is more important to be in the right industry/sector than in the right stock. There are certain things that are more relevant to be seen from the sectors point of view rather than that of the individual company. These are product position, competitive advantage, leader in the sector, barriers to entry in the industry etc. Each industry/sector has differences in terms of its customer base, market share among firms, industry-wide growth, competition, regulation and business cycles. Learning about how the industry or sector works will give an investor a deeper understanding of a company's financial health.

In doing the sectoral analysis, one can compare a stock price to other companies in the same sector. It stands to reason that two or more publicly traded companies in the same sector should be roughly similar in stock price, but this is rarely the case. By analyzing an entire business sector (airlines, banking, construction, IT etc.), one gets a feel for which are the best performing stocks in that particular sector. Comparing the stock prices side by side often reveals which companies are best poised for growth in that sector. There are certain factors in sectoral analysis which have influence in the performance of individual stocks belonging to the same sector [9]. These are given below:

(i) Industry Growth
(ii) Competition within the Industry
(iii) Regulatory Framework within the Industry

Industry analysis is the examination made in a specific sector or industry. Industry analysis investigates the general fundamentals of the equities within the industry but more importantly investigates the state of external factors and how they should affect the particular industry or sub sector. Different macroeconomic data and other statistics have a particular bearing on certain industries and analysts gauge to see how these data will affect them. Furthermore industry analysts also investigate the level of demand such as consumer tastes and supply such as competition within the industry and how stock prices should get affected by them.

4. THE NEED FOR INDUSTRY ANALYSIS

To summarize, these are the reasons why industry analysis is important:

(i) Generally the performance of a company is a function of the performance of the industry. For example, if raw material in a particular industry has gone up then all the companies in the industry will get affected.
(ii) If an industry suddenly gets in vogue or if a sudden change in the news is perceived to be good or bad for an industry the price of the stock will be affected mainly from what the average investor believes and most investors will follow industry trends.
(iii) Choosing a stock after sectoral analysis has its advantages. It is a lot easier to pick a stock from a specific industry, especially if one has to develop a long/short position with the purpose of removing industry and market risk.

A popular tool for analysing the share price of a particular organization is the Japanese Candlesticks. There are number of websites (e.g., [10], [11]) and standalone software packages (e.g., [12]) where one can draw the Japanese candlesticks for analysing the share price of a particular organization. A candlestick displays the opening price, closing price, maximum price and minimum price for a single day for a particular organization. For a period of time; for every single day, a candlestick is drawn. Thus a candlestick pattern provides visualization of performance of a single company for selected number of days.

One drawback with the Japanese Candlestick patterns is that it provides visualization of a single company only. i.e., one can visualize the performance of a single company for a selected period of time. It is difficult to find the summary of data. Also studying a sector is difficult using the candlestick patterns. To overcome this drawbacks of candle sticks this paper proposes a new software tool developed using Microsoft Visual Basic 6.0; that will consist of two graphical techniques viz. a line diagram and a boxplot. The line diagram shall present the closing share price of a single company for period of time and the box plot shall present the data summaries of a particular sector for those days depicted by corresponding line diagram.

5. GENERAL OVERVIEW OF THE SOFTWARE

As Microsoft Excel is the most widely used package for entering, editing and storing data; the software also uses Microsoft Excel for the same purpose. The software has two windows viz. entry and main window as shown in the Figure 1 and Figure 2. The user needs to enter the location

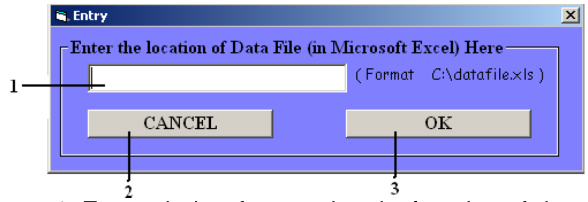

Figure 1: Entry window for entering the location of dataset
1. Text box for entering location of data file. 2. Cancel button. 3. Ok button

of the Microsoft Excel file; where dataset is stored, in the text box in the entry window. In the main window there is one file menu, two dropdown menus, one list menu, two command buttons two picture boxes and a legend as shown in Figure 2.

6. THE ENTRY WINDOW

The entry widow is designed as the starting point of the software tool. The user needs to enter the location of the Microsoft Excel file in which the dataset is stored. The format for entering the location is given in the right hand side of the textbox for reference. It is quite easy to enter the location; the user needs to open the folder where the file is stored and copy the address from the address bar and paste it to the textbox. After this the user needs to put a back slash and the file name. This completes the steps for entering location of the dataset. After this, user can click on the command button OK to proceed to drawing the graphs.

7.1 The File Menu

The main window, as shown in the Figure 3 has a file menu. Under file menu it has four different sub menus viz. New, Save Line Diagram, Save Box Plot and Exit. The save Line Diagram and Save Box Plot submenus can be used to save the graphs line diagram and box plot respectively. The exit submenu can be used to terminate the software.

7.2 Data Option

In the data option group there are two drop down menus and a list box. The first dropdown box under the title *Sector* lists all the sectors present in the data set. The user needs to select a sector first to start the drawing process. After selecting the sector it is time to select the appropriate company name from the data set. This is option is offered by the dropdown

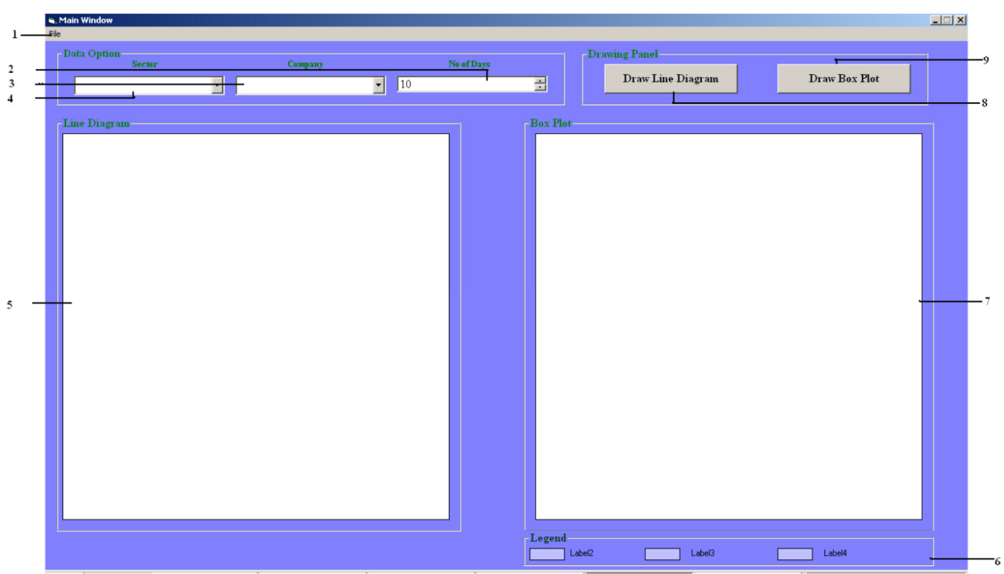

FIGURE. 2: The main window of the Software

1. File Menu, 2. Combo – Box for selecting no of days. 3. Dropdown menu for selecting company 4. Dropdown menu for selecting sector. 5. Picture box for drawing line diagram. 6. Legend. 7. Picture box for drawing boxplot. 8. Command button for drawing Line Diagram. 9. Command button for drawing boxplot.
2.

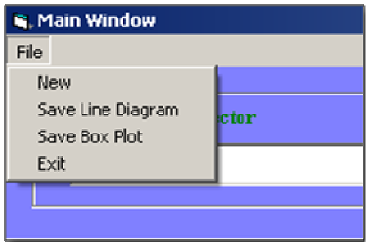

FIGURE 3: File Menu

menu company. The company dropdown menu presents the names of all the companies present under the selected sector in the dataset. The user needs to single out one company to draw the graphs. In the example shown in the figure 4.1 above, the data set contains two sectors stored viz. Banking and IT. So when user tries to select a sector by clicking on the dropdown menu *Sector,* he/she gets two options. Assuming that the user selects banking Sector and proceeds for choosing a bank, he/she gets all the names of the banking sector stored in the dataset as shown in the Figure 4.2. The list box under the title *No of Days* comes into play after drawing the graphs. This list box is used for selecting the number of days for which the graphs need to be drawn.

7.3 Drawing Panel

There are two command buttons in the group Drawing Panel with names Draw Line Diagram and Draw Box Plot. As the names suggest the buttons are used to draw the diagrams respectively. On clicking the Draw Line Diagram the user can draw a line diagram for the selected company and clicking on the Draw Box Plot user can draw box plots. The graphs are drawn in the two picture boxes named Line Diagram and Box Plot. The figure 5 below shows two buttons Draw Line Diagram and Draw Box Plots.

FIGURE. 4.1: Sector dropdown menu

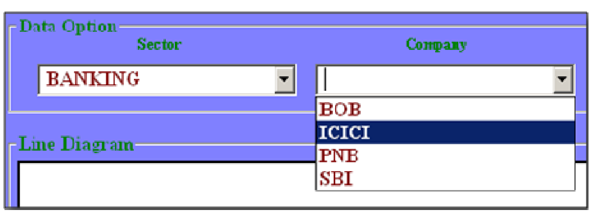

FIGURE 4.2: Sector and Company dropdown menu

The figure 6.1 depicts a line diagram drawn using the software. The line diagram helps us to understand how successive values of the variable are related to specified point of time [13]. In a line diagram values of one variable are plotted against x axis and the corresponding values of the other variable are plotted against y axis and the points are connected by a line.

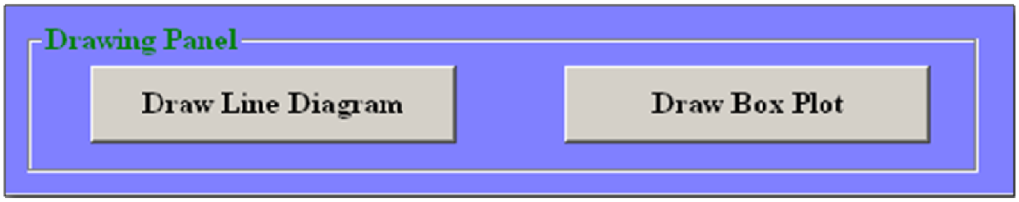

FIGURE 5 Drawing Panels for drawing the graphs

For the purpose of demonstrating the working of software the researcher has taken share prices of four banks viz. State Bank of India (SBI), Bank of Baroda (BOB), Punjab National Bank (PNB) and ICICI and four organizations from IT Sector viz. Tata Consultancy Services (TCS), Wipro, Infosys (INFY) and Patni Computer Services Limited (PCS) for the period of September 01, 2011 to October 17, 2011 for discussion. In the Figure 6.1 the share prices of ICICI bank for the period of September 01, 2011 to November 17, 2011 are plotted against the number of days. The number of days are plotted along the x axis and the share prices of the bank are plotted along y axis. As seen in the Figure the share prices vary between 779 and 919.3 in the mentioned period.

On clicking the command button *Draw Box Plot* the user gets a set of four mean box plots, one for each bank in the sector. The names appearing above the box plots means that the box plot is drawn for that particular bank. A mean box plot is a graphical tool that represents summaries of datasets. The bottom of the box extends to mean minus twice standard deviation and the top of the box extends to mean plus twice standard deviation. The line segment within the box is drawn at the mean value of the data [14]. As shown in the figure 6.2 the bold line in the middle represents mean of values, the rectangle above the line represents mean plus standard deviation and the line below the mean line represents mean minus standard deviation.

Thus a box plot of a particular bank represents mean value of the share price and it's fluctuation during the selected period of time. From the Figure. 6.2 one can visualize that the share price of SBI is having the maximum mean value and that of BOB is the minimum. Also the SBI is having the longest box and BOB has the shortest box. That means variation in the share price of SBI is maximum in comparison to the others in the group for the selected number of days and that for BOB is the minimum.

After drawing the line diagram and the boxplot, one can select the number of days for which he/she whishes to draw the graphs. This option is available in the panel *data option* under *no of days*. Once the user selects the number of days; both the graphs will be drawn taking the latest data of that many days from the dataset. That is if a user selects 15 from *no of days*; the line diagram and the boxplots will be drawn using the last 15 days data from the dataset. The minimum value available under this option is 10 and the maximum is 30. i.e., one can check how a particular company's share price changed during last 10 days to last 31 days. This facilitates the user to draw inference about a company's performance and its position in its group during the selected number of days.

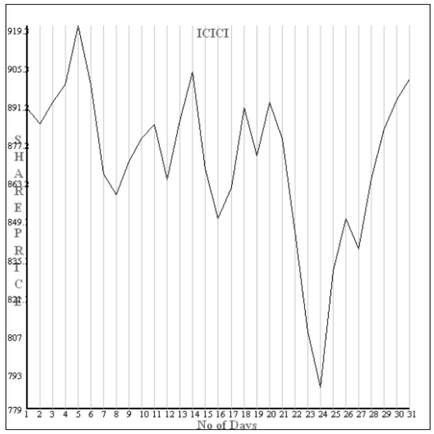

FIGURE 6.1 Line diagram

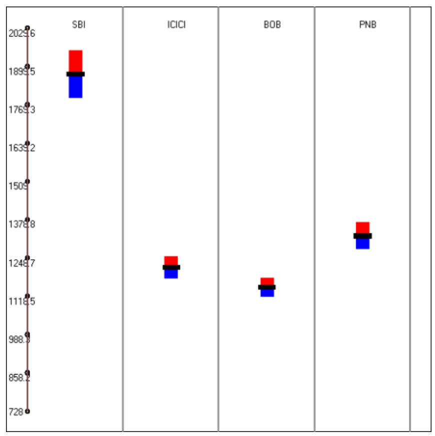

FIGURE 6.2 Boxplot

8. CONCLUSION

The graphical tools presented in this paper shall be handy for making better investment decision. The line diagram shall be useful for exploring the performance of a single company for a period of time while the box plots shall depict summaries of performance of different companies belonging to the same sector. The line diagram shall present how the share price of a company has been moving during the selected period of time. From the box plot one can draw the inference about the average share price and the fluctuations of every single company belonging to that group. The user also has the advantage of interacting with the graphs by selecting the number of days; that empowers the user to analyze the performance of the companies for the time various periods. Hence, the user would be able to make better decision using this software tool.

REFERENCES

[1] A. Unwin, How interactive graphics will revolutionize statistical practice . The Statistician , vo 41, n0 3 pp365-369 (1992).

[2] H. J. Mittag,. Interactive Visualization of Statistical information. Seminar on Dynamic Graphics, (p. 1). ISTAT headquarters, ISTAT headquarters, Rome, Italy (2007)

[3] A. Smith, Interactive Visualization of Statistical information. Seminar on Dynamic Graphics, (p. 1). ISTAT headquarters, Rome, ISTAT headquarters, Rome, Italy (2007)

[4] F. V. Ruth, The Statistics Netherlands Business Cycle Tracer; Visualising the State of the Economy, Seminar on Dynamic Graphics, ISTAT headquarters, Rome, Italy (2007)

[5] V. Breneman, Dynamic Data Displays: Experiences from USDA's Economic Research Service, Seminar on Dynamic Graphics, (p. 1). ISTAT headquarters, Rome, Italy (2007)

[6] Free Technical Charts, Internet: http://www.stockmarketindian.com/technical_charts/free_technical_charts_analysis.html, Retrieved 12 12, 2012

[7] D. E. Fischer, & R. J. Jordan, Security Analysis and Portfolio Management. New Delhi: Prentice Hall of India, 2006

[8] R. Singh, Behavioural Finance - The Basic Foundations. ASBM Journal of Management , vol 89 – 98, 2009.

[9] McClure, B. (n.d.). Fundamental Analysis: Qualitative Factors - The Industry, Internet: http://www.investopedia.com/university/fundamentalanalysis/fundanalysis3.asp#axzz1gJ0a24Xp, Retrieved 12 12, 2012

[10] Free Technical Charts, Internet: http://www.freestockcharts.com/?gclid=CLubqp_y2K0CFc566wodnHgEwg, January 18, 2012

[11] Learn Basics of Technical Analysis, Internet: http://www.stockmarketindian.com/technical_charts/free_technical_charts_analysis.html, January 18, 2012

[12] Technical Analysis and Stock Charting Software Stocktech™, Internet: http://www.technicalanalysisofstocks.in/technicalanalysissoftware.html, January 18, 201

[13] D. Bhattacharjee, & K. K. Das, 101 Graphical Technique, New Delhi: Asian Books Private Limited, pp 149-150, 2008.

[14] D. Bhattacharjee, Algorithm for Drawing a Mean Box Plot. Contributions to Applied and Mathematical Statistics , vol 4, pp 65-70, 2009.

A Framework for Statistical Simulation of Physiological Responses (SSPR)

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Abstract

The problem of variable selection from a large number of variables to predict certain important dependent variables has been of interest to both applied statisticians and other researchers in applied physiology. For this purpose, various statistical techniques have been developed. This framework embedded various statistical techniques of sampling and resampling and help in Statistical Simulation for Physiological Responses under different Environmental condition. This framework will facilitates the researchers to work on simulated population. It will also solve the problem of small sample size by providing resampling module. The generation of simulated population and other statistical calculations are based on the inputs provided by the user as mean vector and covariance matrix and the data. This framework is developed in a way that it can work for the original data as well as for simulated data generated by the software. **Approach:** The mean vector and covariance matrix are sufficient statistics when the underlying distribution is multivariate normal. This framework uses these two inputs and is able to generate simulated multivariate normal population for any number of variables. The software changes the manual operation into a computer-based system to automate the study, provide efficiency, accuracy, timelessness, and economy. **Result:** A complete framework that can statistically simulate any type and any number of responses or variables. Simulated data when analyzed using statistical techniques; results of such analysis will be the same as that using the original data. The system provides solution for missing data on some variables. **Conclusion:** The proposed system makes it possible to carry out the physiological studies and statistical calculations even if the actual data is not present and also in the case when sample size is small.

Keywords: SSPR(Statistical Simulation of Physiological Responses),Population Generation (PG), Sample Selection (SS), Simple Random Sampling With Replacement (SRSWR), Simple Random Sampling Without Replacement (SRSWOR) , Probability proportional to the Size With Replacement (PPSWR), Probability proportional to the Size Without Replacement(PPSWOR), Data Flow Diagram(DFD)

1. INTRODUCTION

The process of designing a model of a real system and conducting experiments with this model for the purpose either of understanding the behavior of the system or of evaluating various strategies (within the limits imposed by a criterion or set of criteria) for the operation of the system."

- R.E Sahnnon

The need for the statistical information is endless in the modern society for planning development and growth. One of the most important modes of data collection for satisfying such needs is sample survey, i.e. a partial investigation of finite population. Simulation is a process of designing a model of a real system & conducting experiments with this model on the computer for understating the behavior of the system or evaluating various strategies for operation of the system. The model can be defined as a presentation of a real system which can be controlled by various parameters.

The advanced computer programs can simulate weather conditions, chemical reactions, atomic reactions, even biological processes. In theory, any phenomena that can be reduced to mathematical data and equations can be simulated on a computer. In practice, however, simulation is extremely difficult because most natural phenomena are subject to an almost infinite number of influences. One of the tricks to developing useful simulations, therefore, is to determine which the most important factors are. There are various studies for building mathematical models for physiological processes. Some authors proposed the framework for modeling and simulation of physiological models to improve the modeling process. Phy-SIM is a modeling, integration and simulation environment for physiological processes from tissue level models to organ-organism levels [1].In this author proposed a layered approach modular design principles. A framework for cardiovascular system based on ontology was described by Daniel in 2006[10].But no attempts have been made to use the technique of simulation on physiological systems/ responses in the area of heat physiology and cold physiology etc. Therefore, in the present study, statistical techniques will be used to simulate some of these important physiological functions at sea level and high altitude. SSPR provide solution to overcome the problem of small sample size by embedding the resampling techniques. The Simulated data generated by SSPR can be used further for Meta analysis, statistical modeling and it can also be used for the development of physiological index. There are many organizations public and private that are working in the field of life sciences. They collect the data either by experiments or by other techniques as interview, questionnaire, and measurements by instruments or from previous studies etc. Many times in human studies problem of non-response encountered for example, when administering a survey people may answer some questions and not others due to this reason it is not possible to collect data that is complete. There are many reasons for missing values as missing by design; or not asked or not applicable. This missing data causes a problem for researchers specially when using structural equation modeling (SEM) techniques for data analysis. Because SEM and multivariate methods require complete data [2][3]. All over the world there are several organizations, working on sensitive information regarding the entities or subjects. These organizations use several different approaches to prevent disclosure of the sensitive information. These include restricting access to specific individuals, using database control techniques, masking the data prior to providing access, releasing interval data rather than individual data points, etc. [4]. In such situations use of simulated data rather than the original data is safe, easy and time saving. One can easily calculate mean and covariance matrix of all the parameters by avoiding missing values. When literature survey of previous studies is done only mean and covariance matrix of the parameters is available but not the actual data. Sometimes it is not possible for researchers to work on the entire population in that case they have to work on the subset of individuals from within a population to yield some knowledge about the whole population. The three main advantages of sampling are that the cost is lower, data collection is faster, and since the data set is smaller it is possible to ensure homogeneity and to improve the accuracy and quality of the data**.**[3][5]. Thus, there is a need to develop software that can help the individual to generate the simulated population of desired size by supplying mean and co-variance matrix as input and also be able to select random samples by different sampling and resampling techniques and further for using that sample for some basic statistical calculation as mean, median, mode standard deviation etc.

The whole system is divided into four major modules as shown below

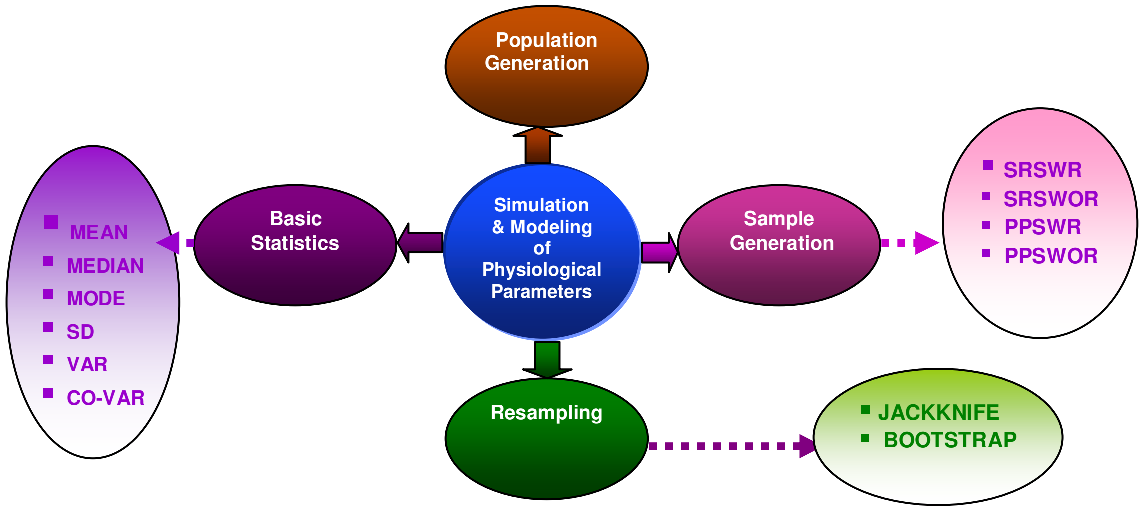

FIGURE 1: Statistical Simulation of Physiological Responses (SSPR) Modules.

The population generation module will help the researches who want to continue old studies but does not having the actual data by giving them the simulated population , This will also help the researchers who does not want to reveal sensitive information by generating the simulated population based on mean and variance covariance matrix. The Sample Selection module will help in sample selection by using various sampling techniques. The Resampling module will help the researches who are not able to continue their research due to small size of samples. The Basic statistics module will facilitates basic statistical calculations with graphical output as well.

2. MATERIALS AND METHODS

Before designing the SSPR, necessary objectives of the system were established. The objectives were created after the detailed analysis of organization work, limitations and concerns in the existing manual system. The various necessary details about the population generation, sample selection techniques and resampling techniques also gathered from the concerned authorities and users. This helped us to plan an effective SSMPP system.

System Analysis

System Analysis by definition is a process of systematic investigation for the purpose of gathering data, interpreting the facts, diagnosing the problem and using this information to either build a completely new system or to recommend the improvement to the existing system. As the new system is going to be developed, the requirement analysis a preliminary investigation, feasibility study for the required system was done. All the available resources with respect to software requirement were checked, these all step helped us in making the data flow diagram (DFD) for the SSPR as shown in the Figure: 2.

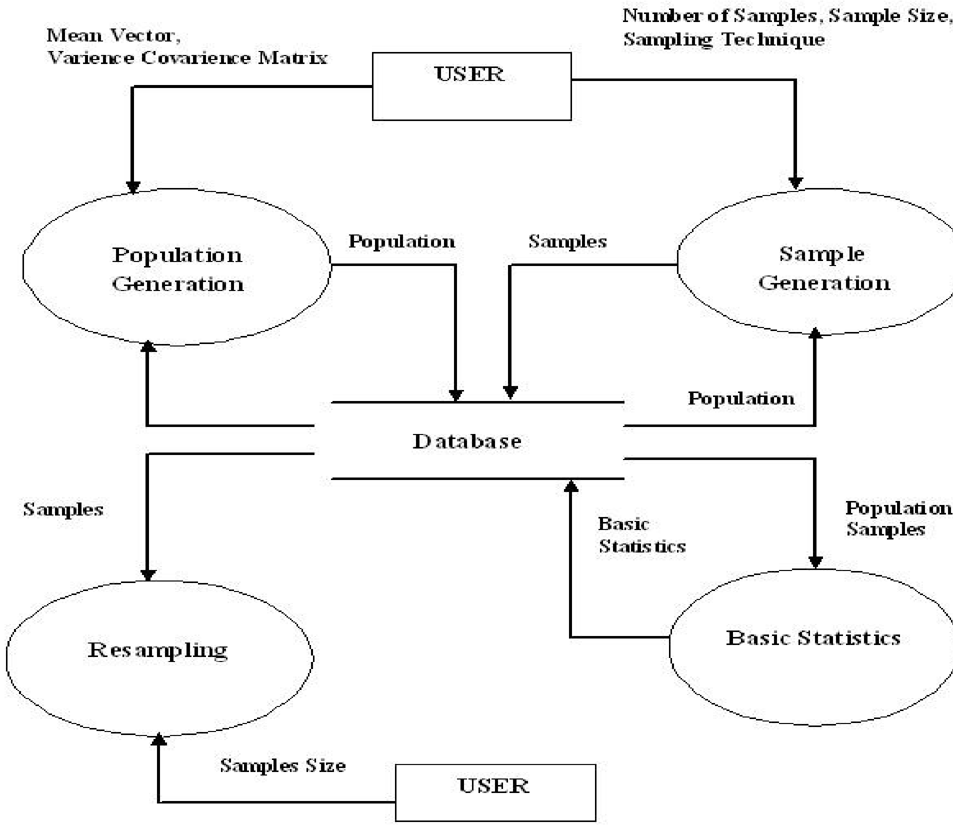

FIGURE 2: DFD for SSPR

The above diagram shows how the data flows from one process to others and from databases to the others process and vice –versa.

In the real world, most data collection schemes or designed experiments will result in multivariate data. When multivariate data are analyzed, the multivariate normal model is the most commonly used model. Many statistical techniques focus on just one or two variables but sometimes there is a need to analyze more than two variables. Multivariate analysis (MVA) techniques allow more than two variables to be analysed at once. The multivariate normal distribution is the generalized form of the one-dimensional i.e. univariate normal distribution to higher dimensions. The multivariate normal distribution of a k-dimensional random vector $X = [X_1, X_2, \ldots, X_k]$ can be written in the following notation[6]:

$$X \sim \mathcal{N}(\mu, \Sigma),$$

or to make it explicitly known that X is k-dimensional,

$$X \sim \mathcal{N}_k(\mu, \Sigma).$$

with k-dimensional mean vector

$$\boldsymbol{\mu} = [\mathrm{E}[X_1], \mathrm{E}[X_2], \ldots, \mathrm{E}[X_k]]$$

and *k x k* covariance matrix

$$\Sigma = [\mathrm{Cov}[X_i, X_j]], i = 1, 2, \ldots, k; j = 1, 2, \ldots, k$$

Sampling refers to the process of choosing a sample of elements from a total population of elements. Simple random sampling refers to a sampling method that has the following properties: The population consists of *N* objects. The sample consists of *n* objects. All possible samples of *n* objects are equally likely to occur. The main benefit of simple random sampling is that it guarantees that the sample chosen is representative of the population. This ensures that the statistical conclusions will be valid. Sampling can be done with replacement or without replacement [7][8]. **Sampling with replacement** is accomplished by "tossing" population members back into the mix after they have been selected. In this way, all *N* members of the population have an equal chance of being selected *at each draw*. In other words -Sampling is called with replacement when a unit selected at random from the population is returned to the population and then a second element is selected at random. Whenever a unit is selected, the population contains all the same units. A unit may be selected more than once. There is no change in the size of the population at any stage. Let us assume that a sample of any size can be selected from the given population of any size. This is only a theoretical concept and in practical situations the sample is not selected by using this scheme of selection. Suppose the population size N=5 and sample size n=2, and sampling is done with replacement. Out of 5 elements, the first element can be selected in 5 ways. The selected unit is returned to the main lot and now the second unit can also be selected in 5 ways. Thus in total there are 5X5=25 samples or pairs which are possible. In contrast, **sampling without replacement** is done so that once a population member has been drawn; this person is removed from further sampling. Thus, once a population member has been drawn, their subsequent probability of selection is zero and the probability that someone else is selected goes up a little. In other words -Sampling is called without replacement when a unit is selected at random from the population and it is not returned to the main lot. First unit is selected out of a population of size N and the second unit is selected out of the remaining population of N-1 units and so on. Thus the size of the population goes on decreasing as the sample size n increases. The sample size n cannot exceed the population size N. The unit once selected for a sample cannot be repeated in the same sample. Thus all the units of the sample are distinct from one another. In simple words , when a population element can be selected more than one time, the sampling is known as sampling with replacement and when a population element can be selected only one time then the sampling is known as sampling without replacement**.** In sampling with replacement the two sample values are independent. Practically, this means that what we get on the first one doesn't affect what we get on the second. Mathematically, this means that the covariance between the two is zero. In sampling without replacement the case is just reverse of this, the two sample values aren't independent. In this sampling technique what we got on the for the first one affects what we can get for the second one and the covariance between the two isn't zero.

Probability Proportional to Size

Probability proportional to size (PPS) is a sampling technique mostly used with surveys where the probability of selection of a sampling unit (city, any village, district etc) is proportional to the size of its population. It gives a probability (i.e., random, representative) sample[5]. It is most useful when the sampling units vary considerably in size because it assures that those in larger sites have the same probability of getting into the sample as those in smaller sites, and vice verse. This method also facilitates planning for field work because a pre-determined number of respondents is interviewed in each unit selected, and staff can be allocated accordingly [8].

Sometime the situation occur when the population distribution is unknown and the sample size is small, in that case resampling plays a very important role to continue the study. **Resampling:** Resampling means that inference is based upon repeated sampling within the same sample.For more than a century the inherent difficulty of formula-based inferential statistics has baffled scientists, induced errors in research, and caused million of students to hate the subject. In place of the formidable formulas and mysterious tables of parametric and non-parametric tests based on complicated mathematics and arcane approximations, the basic resampling tools are simulations, created especially for the task at hand by practitioners who completely understand what they are doing and why they are doing it. Resampling lets you analyze most sorts of data, even those that cannot be analyzed with formulas [9]. **Bootstrapping** is proposed by Bradly Efron in 1979. It is a statistical method for estimating the confidence interval of a parameter with or without assumption on data distribution. It also estimates the bias of an estimator. It may also be used for constructing hypothesis tests. It is often used as a robust alternative to inference based on parametric assumptions when those assumptions are in doubt, or where parametric inference is impossible or requires very complicated formulas for the calculation of standard errors[9]. **Jackknifing** is proposed by Quenouille in 1949 and in 1958 tukey named it as Jackknife .It is similar to bootstrapping, is used in statistical inference to estimate the bias and standard error (variance) of a statistic, when a random sample of observations is used to calculate it. The basic idea behind the jackknife variance estimator lies in systematically recomputing the statistic estimate leaving out one or more observations at a time from the sample set. From this new set of replicates of the statistic, an estimate for the bias and an estimate for the variance of the statistic can be calculated [10]. It is mainly proposed to reduce the bias of an estimator. It is only adaptive for estimators which are smooth function of the observation.

3. DISCUSSION

3.1 MODULE 1 : POPULATION GENERATION (PG)

PG is the module of the system SSPR, this takes mean, variance covariance matrix and population size as inputs and generates the simulated population of the size given by the user. The output of this module is the desired size simulated population matrix.

The function used to generate the Multivariate normal random numbers is mvnrnd function of MATLAB

Syntax

R = mvnrnd(MU,SIGMA)

Description

R = mvnrnd(MU,SIGMA) returns an n-by-d matrix R of random vectors chosen from the multivariate normal distribution with mean MU, and covariance SIGMA. MU is an n-by-d matrix, and mvnrnd generates each row of R using
the corresponding row of mu. SIGMA is a d-by-d symmetric positive semi-definite matrix, or a d-by-d-by-n array. If SIGMA is an array, mvnrnd generates
each row of R using the corresponding page of SIGMA, i.e., mvnrnd computes R(i,:) using MU(i,:) and SIGMA(:,:,i). If MU is a 1-by-d vector, mvnrnd replicates it to match the trailing dimension of SIGMA.[11]

Step1: Click on the button read mean Vector
Step2: Click on the button read Cov- Var Matrix
Step3: Enter the size of population you want to generate.
Step 4: Click on OK button.

Population of entered size get generated and saved in excel file name pop.xls in the MyOutput Folder (automatically get created) of C drive.

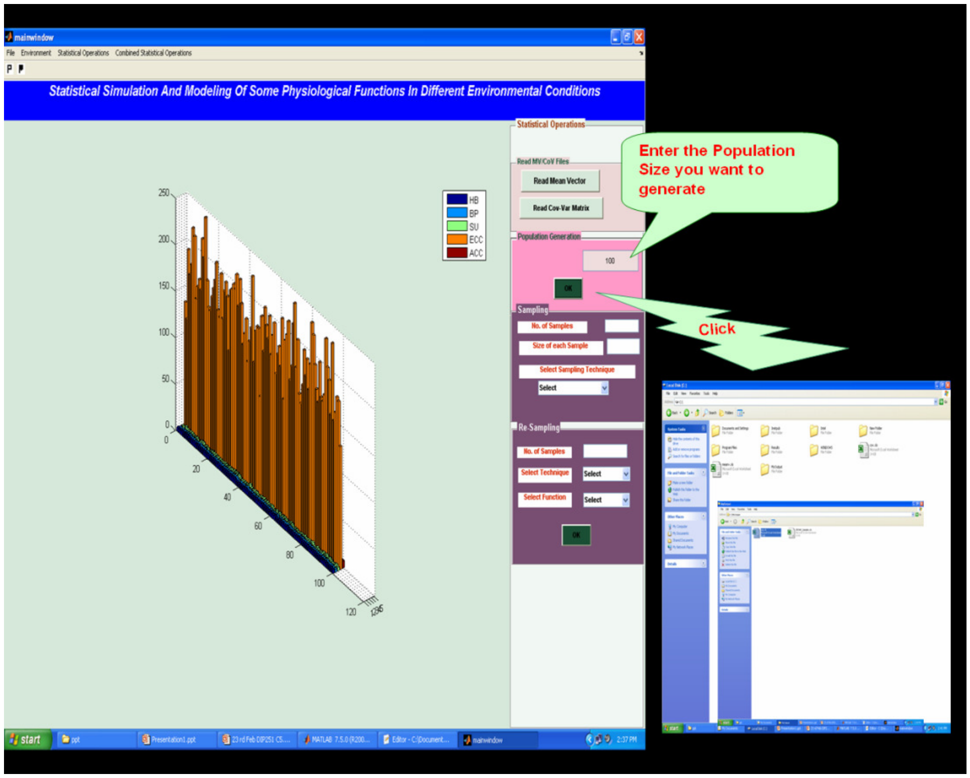

FIGURE 3: Population Generation Window

3.2 MODULE 2: SAMPLE SELECTION (SS)

Sample Selection : In this module the population generated by the module PG is sampled by the different sampling techniques selected by the user. Here the samples can be generated by Simple Random Sampling with and without replacement (SRSWR & SRSWOR) technique and also by Probability proportional to the size with and without replacement (PPSWR & PPSWOR) technique. In this module sample size, number of samples and sampling technique are the input and samples which get generated by this are the output.

Syntax

y = randsample(population,size,replace)

y = randsample(population,size,replace) returns a sample of the given size from the population . If replace is true then the samples will be taken with replacement or without replacement if replace is false [11].

Pre Requirement: Simulated population should exist in the file for example pop.xls, from which samples has to be selected.

Step 1: Enter the number of samples you want to generate (Ex 5,10,100 etc)
Step 2: Enter the size of each samples.
Step 3: Select one of the Sampling Technique given below

- SRSWR
- SRSWOR
- PPSWR
- PPSWOR

The samples get selected from the given population (SRSWOR.xls in case of SRSWOR sampling technique) and saved in the same directory as for population.

3.3 MODULE 3: BASIC SATISTICS

When performing statistical analysis on a set of data, the mean, median, mode, and standard deviation are all helpful values to calculate. The mean, median and mode are all estimates of where the "middle" of a set of data is. These values are useful when creating groups or bins to organize larger sets of data.In SSMPP the Basic Statistics module provide the basic statistics calculations like mean, SD, median, mode of the given population or samples. The methods which have used in software are as mean(A), median(A),mode(X),std(tsobj),var(X) etc.

- **Mode** - The mode of a distribution is simply defined as the most frequent or common score in the distribution. The mode is the point or value of *X* that corresponds to the highest point on the distribution. For this mode function of matlab is used.
- **Median**- The median is the score that divides the distribution into halves; half of the scores are above the median and half are below it when the data are arranged in numerical order. M = median(A) to calculate the median of the given array or matrix.
- **Mean** - The mean is the most common measure of central tendency and the one that can be mathematically manipulated. It is defined as the average of a distribution is equal to the $\sum X / N$. tsmean = mean(tsobj) computes the arithmetic mean of all data in all series in tsobj and returns it in tsmean.
- **Variance/Co-varience** - The variance is a measure based on the deviations of individual scores from the mean. The two functions used to calculate the varience and co-varience of the variables are var and cov.
- **Standard deviation** (SD)- The standard deviation (s or σ) is defined as the positive square root of the variance. The variance is a measure in squared units and has little meaning with respect to the data. Thus, the standard deviation is a measure of variability expressed in the same units as the data. The method used for it is std method.

Step 1: Go to the menu bar click on Statistical operations.
Step2: Select the menu item basic statistical operations. A new window will come out as shown below.
Step 3: Click on the operation you want to perform as Mean, Median Mode, SD, Covariance. A pop up window for file selection will come; select the file on which you want to perform basic statistical calculation.

This will provide the calculated values on the left side and the respective graphical output in the space provided for the graph. User can save the generated graph and the result in the excel sheet.

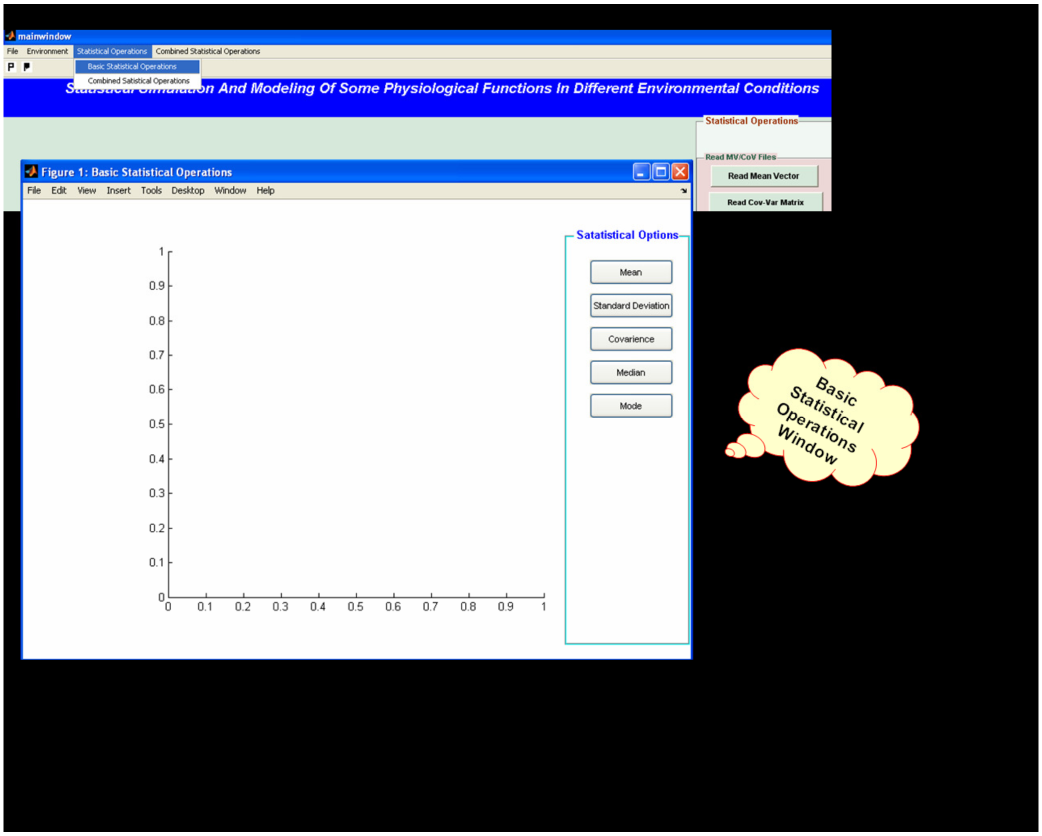

FIGURE 4 : Window for Basic Statistics

3.4 MODULE 4: RESAMPLING

Many researchers face the situation when they are not having the sample of sufficient size to do statistical analysis, at that point of time they has to close their research just because of lack of records or data. Resampling, the advance statistical technique is very useful in this situation. Resampling is also the type of sampling but in this the repeated sampling is done within the same sample, this is the reason it is called resampling. There are many techniques for resampling, common resampling techniques include bootstrapping, jackknifing and permutation tests. The Resampling module perform resampling using Jackknife Technique and Bootstrap Technique on samples generated by the module SS. Under this module user has to select the resampling technique from the drop down list and also the resample function , then he has to click on OK button. The output will save in the same folder MyOutput in C drive.

3.5 RESULT

The simulated population generation takes mean, variance- covariance matrix and the population size as input and produces the simulated population as output. The generated simulated population get stored in the excel worksheet a saved in the MyOutput folder under C drive. The second module Sample selection (SS) takes the population generated by the PG module or some other existing population, Number of Samples and Sample size as input and gives Samples as output based on the sampling techniques selected by the user. The third module is the basic statistics (BS) that can do all the basic statistical calculation as mean, median, mode, standard deviation on the selected file. This input file can be the population or samples of any size. The last module of SSMPP is the resampling. This module is based on two resampling techniques as jackknife and bootstrap.

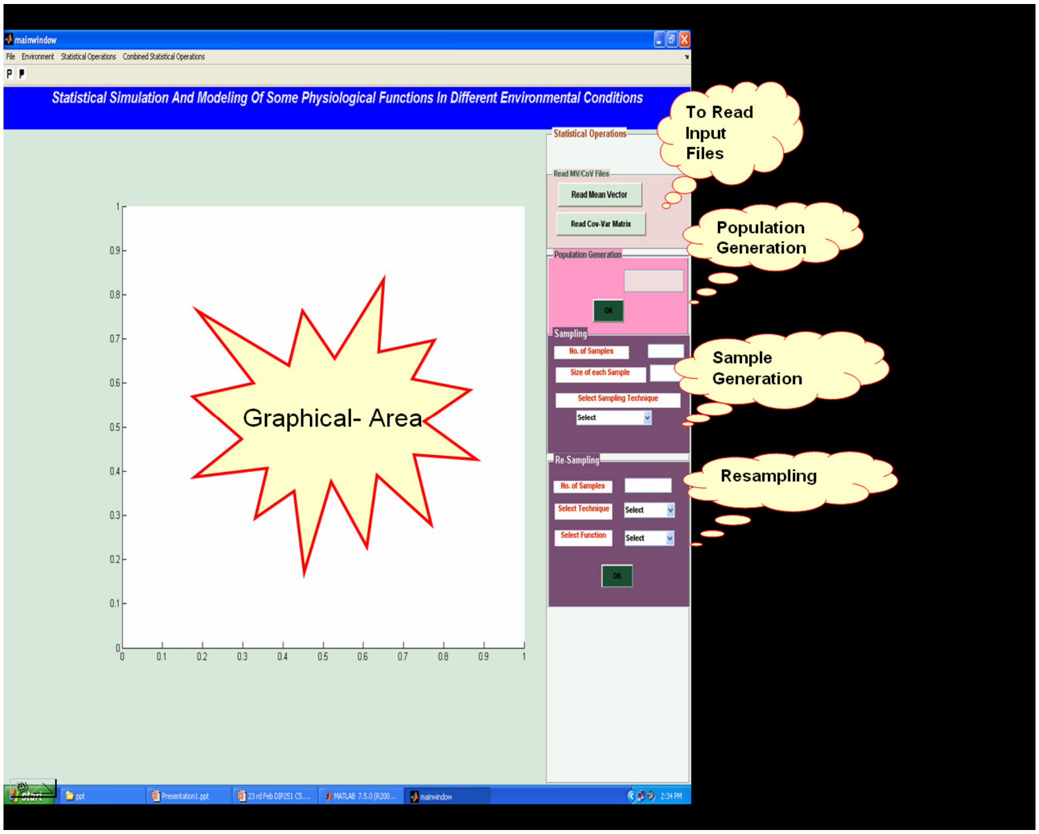

FIGURE 5: Main Window of SSPR

The results generated by this can further be used for statistical modeling, Multivariate analysis and Meta analysis. This framework is designed and developed using MATLAB an interactive numerical computing environment which is very reliable in terms of mathematical computing.

4. CONCLUSION

Kelton and Maria (1997) describe method to design the run for simulation models and interpreting their output. Statistical methods are described for several different purposes and related problem like comparisons , variance reduction etc. as has also been done in the present paper[3][12].Our work is similar to the work done by Thomson(1996) in estimating the uncertainty in physiological based pharmacokinetics model output by using Monte Carlo simulation to furnish random sample values for model parameters for further statistical analysis[13].Rubin(2006) created an ontologically guided methodology for representing a physiological model of circulation. Ontology provided a framework to construct a graphical representation of the model. Providing a simpler visualization than the large set of mathematical equations [10].In this study software SSPR that can do statistical simulation of some physiological functions in different environmental conditions for DIPAS is introduced. The study elaborates the system analysis, software design and system development. With the given inputs this system is able to do basic statistical calculations, generate simulated population, sampling and resampling by using different techniques as SRSWR, SRSWOR, PPSWR, PPSWOR, jackknife etc. In addition, it is a GUI based system so it is very user friendly and controls the data effectively. It uses the MS Excel as a database and also produces the results in the excel format, which can be easily readable in other statistical software, where further advance statistical analysis can be done. There is a less scope of manual error when this software is used for statistical operations. It is very flexible in terms of development. This framework will help the researchers and scientists who are working on sensitive information and wants statistical analysis without revealing the information. Researcher can generate the simulated population with respect to their data using PG module and can perform statistical analysis. SSPR also help the researchers who want to continue the research based on old studies and are having the some descriptive statistics only. SSPR also help in the situation when sample of sufficient size is not available by providing the Resampling module. The result generated by SS and Resampling Module can further be used for statistical modeling and meta analysis by exporting the output file into other statistical software. The output provided by the software is the graphical output as well as in excel sheet which is compatible with all statistical software. In future SSPR can be upgraded by including other statistical estimators, sampling and resampling techniques as like two stage sampling techniques stratified and unstratified etc.

REFERENCES

[1] E. Z. Erson, M. C. Cavusoglu. "Design of a Framework for Modeling, Integration and Simulation of Physiological Models" .In Proceedings of the International Conference of the IEEE Engineering in Medicine and Biology Society(EMBC '10),August 31 - September 4, 2010, Buenos Aires, Argentina.

[2] R.L. Carter. “Solutions for Missing Data in Structural Equation Modeling”. Research & Practice in Assessment Volume 1, Issue 1 March 2006.

[3] A. Maria “Introduction to Modeling and Simulation” . *Proceedingg of the 1997 Winter simulation conference ed.S.Andradottir, K.J.Healy, D.H. Withers, and B.L.nelson,* Atlanta, GA, USA ,1997.

[4] Krishnamurty Muralidhar and R. Sarathy. “Generating Sufficiency based Non Synthetic Perturbed Data”. *Transations on Data Privacy* 117-23, 2008.

[5] Therese McGinn ,2004. “Instructions for Probability Proportional to Size Sampling Technique”. RHRC Consortium monitoring and evaluation toolkit , October 2004. www.rhrc.org/.../55b%20PPS%20sampling%20technique.doc

[6] P. Bratley ,Bennett L.Fox, Linus E. Schrage.”A Guide to Simulation” ,Second Edition .*Springer, Eds. Publisher, New York*, 164-165, 1987.

[7] P.V. Sukhatme, B.V.Sukhatme ,S.Sukhatme ,C. Asok . “Sampling Theory of surveys with application.Indian society of agricultural statistics”, new delhi India , and IOWA State University Press Ames , USA, 21-25, 1984.

[8] William G.Cochran. “*Sampling Techniques”,* Third Edition. *Wiley Eastern Limited Publication New Delhi*, 1985, Pp-18-30, 250-259

[9] Wu, C.F.J. “Jackknife, Bootstrap and other resampling methods in regression analysis”. *The Annals of Statistics.* Vol. 14, 4, pp. 1261–1295, 1986.

[10] D.L. Rubin, D.Grossman, M. Neal, D.L. Cook. “Ontology-Based Representation of Simulation Models of Physiology”. *AMIA 2006 Symposium Proceedings* Page – 664-668.

[11] Matlab help Demo

[12] W. D.Kelton ,”Statistical Analysis of Simulation Output”. *Proceedings of the 1997 Winter Simulation Conference, Atlanta, GA, USA ,* 1997,Page-23-30.

[13] R. S. Thomas, W. E. Lytle, T. J. Keefe, Alexander A. Constan, And R. S. H. Yang “Incorporating Monte Carlo Simulation into Physiologically Based Pharmacokinetic Models Using Advanced Continuous Simulation Language (ACSL): A Computational Method”. *Fundamental and applied Toxicology 31*, 1996.Page-19-28.

Modeling and Simulation of Spread and Effect of Malaria Epidemic

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Abstract

The purpose of this paper is to consider malaria infection *(A)* and the control of malaria *(B)* as the two sets of soldiers engage in a war. The principal objectives are to see if it is possible with time to reduce and eradicate malaria in our environment taking reasonable precaution. The methodology approach is to model a mathematical equation using battling method approach to find the time*(t)* that control malaria in our environment will conquer the malaria infection i.e. when $A(t)=0$. The number of provided facilities *(n)* for the protection of malaria is also considered and varied. The result shows that as the number of malaria control increases the control time is decreasing

Keywords: Mathematical Modeling, Plasmodium Falciparum, Battling Methods & Merozoites.

1. INTRODUCTION

Despite considerable effort over the years to control malaria in our environment, many aspects of malaria and particularly the interactions between the parasite and its host which result in the disease are still poorly understood in our country.

A malaria infection starts with the bite of an infected female anopheles mosquito which introduces a few sporozoite stages that migrate to and infect liver cells. Asexual replication in the liver cell results in the release of thousands of merozoites that initiate the blood stage of the infection. This stage is responsible for the pathology associated with malaria (anaemia, fever, malase, anorexia est.). Merozoites infect red blood cells (RBCs) where they multiply to produce 8 – 32 new merozoites (depending on the malaria species), which are released by lysis of the RBCs. The released merozoites can infect new RBCs, causing a rapid increase in parasites and infected cells. During this initial or acute phase of infection, different parasite strains reach different densities and cause varying degrees of anaemia and other pathology [5], [4], [11], [13], [12]. While most studies agree that specific immunity does not play a major role in this initial dynamics, there is considerable controversy over which factors drive the dynamics shortly after infection [2], [8], [9], [14], [15]. Gives two reasons why it is difficult to determine the contribution of these different factors to the dynamics of acute infection. The first problem is that there is relatively limited data on the dynamics of the parasite and the loss of RBCs following infection of humans with human malaria parasites such as parasite falciparum and parasite vivax. One approach to overcome this limitation is to use data from well characterized model system such as infections of mice with species such that parasite chabondi or parasite yoeli. The second problem is that the dynamic of infection may involve many interacting populations which fall into three groups. (i) The parasite and its resource i.e merozoites, uninfected and infected RBCs (ii) The inmate immune response i.e macrophages, dendritic cells, crytokines e.t.c and (iii) The adaptive immune response i.e B & T cells and antibodies.

[15] Bring mathematical models of the early stages of malaria infection into contact with experimental data on the time course of both the parasite density and the loss of RBCs following infection. [17] Mathematical modeling has long been applied to malaria control and is particularly relevant to day in light of rapid country progress in reaching high intervention coverage targets and given the intensifying global efforts to achieve the malaria control among the children. Mathematical modeling has been a valuable decision making tools for vaccination strategies against infections disease in particular for those covered by the Expand Program on Immunization (EPI). [1]Compared with other organisms that cause infectious disease parasite falciparum has a complex life cycle, expressing many different potential targets for vaccines and various candidate vaccine targeting different stages of the parasites are in clinical development. [3] The history of infective or partially effective control of malaria and failed vaccination attempts has led to the assumption that the efficacy of a malaria vaccine is unlikely to approach 100%, but since parasite falciparum is one of the most frequent causes of morbidity and mortality in areas where it is endemic, [16], [4], [7] even a partially protective vaccines may be highly cost-effective and a critically important public health tool. Mathematical models of both the natural history and epidemiology of malaria are needed to guide the process malaria control. Malaria models have several roles that transcend their obvious limitation in making precise predictions. A mathematical model was developed elsewhere [10] taking into account some biological features related to malaria disease such as partial acquired immunity, immunology memory and duration of sporogony. From the model equilibrium points were determined and the stability of these points was analyzed.

In a subsequent study [10] the previously developed model was used to assess the effects of global warning and local socioeconomic condition on malaria transmission. These effects were assessing analyzing the equilibrium points calculated at different but fixed values of the parameters of the models. Regarding malaria transmission, it was observed that the effects of global warning posed a major challenge and the effects variation in local socioeconomic conditions are much stronger than the effect of the increasing global temperatures.

In this study, we bring mathematical modeling of malaria infection in our environment and its control addressing the challenges of the model to provide useful prediction on malaria control using detail data from World Heath Organization of malaria cases, malaria recorded due to infection and cured due to malaria control shown in Table1. Find out that in the nearest future the number of malaria cases in our environment will be dropped drastically.

2. MODEL

In this model, we shall use the following ,based on our knowledge of malaria Infection:

1. Malaria is caused through parasite i.e. Plasmodium.

2. The vector of malaria is female Anopheles Mosquito

The plasmodium has four species as regards to the region

1. Plasmodium Malaria A
2. Plasmodium Ovale
3. Plasmodium Vivax
4. Plasmodium Falciparum

If we consider malaria infections *(A)* and control of malaria *(B)* as two different set of soldiers engaging in a war. The reason is that no human being would be ready to die irrespective of their ages. But malaria infections as well known, it is a common disease that anybody can contact. So, for the fact that we are more prone to malaria infection in our region, we say that the control of malaria and malaria infections can engage in a battle war in which one would fight another one to finish.

In a battle war, there would be some ammunition which each army engage in the war would use to fight or defend themselves. Malaria in its own side use any of the above four stated plasmodium to fight the control of malaria in our environment. In this sense, we represent malaria Infection by *A* while its ammunition is represented by 'ψ'. At time *(t),* we have A *(t)*

Also, control of malaria in our environment represented by *B* which has some ammunitions (management of malaria) to fight malaria infection. The ammunitions according to our research are:

1. The use of treated mosquito net
2. The use of ointment to rub body
3. Proper environmental sanitation
4. Proper drainage
5. Bush clearing
6. Use of mosquito insecticide
7. Use of chemicals on any stagnant water
8. Use of defensive drug / anti malaria medicine.

We regards all the above listed ammunitions as 'n' while this n can be varied according to the knowledge of individual on malaria. So, the killing power of malaria control on malaria infection is represented by 'θ'. At time 't' we have B (t).

Generally we all know that malaria infection is not an infectious disease likewise it can not be transmitted from one generation to another generation. Then, the control of malaria has more (n) weapons to fight malaria with one weapon.

3.0 Governing Equation

List of symbols

A – Malaria infection

B – Human malaria control

ψ - Malaria weapon

θ - Human weapon

n – No of malaria control

x – Malaria cases confirmed

y – Malaria death

z – Malaria cured

Combining all the factors approximately, we obtained model defining malaria infections over time as:

$$\frac{dA}{dt} = -n\theta B, \qquad n\geq 1 \qquad \text{.. (i)}$$

While a model defining control of malaria over time is

$$\frac{dB}{dt} = -\psi A \qquad \text{.................. (ii)}$$

Equation (i) and (ii) are first order linear O.D.E. and can be solved.

$$\psi A^2 = n\theta B^2 + (\psi A_0{}^2 - n\theta B_0{}^2) \qquad \text{................ (iii)}$$

Recall that: $A(0) = A_0$, $B(0) = B_0$

When will malaria control in human being conquer malaria infection totally?

$$\frac{d^2A}{dt^2} = -n\theta \frac{dB}{dt} \qquad \text{.................. (iv)}$$

$$\frac{d^2B}{dt^2} = -\psi \frac{dA}{dt} \qquad \text{.................. (v)}$$

Substitute for $\frac{dA}{dt}$ and $\frac{dB}{dt}$ into (iv) and (v) we have

$$\frac{d^2A}{dt^2} = n\psi\theta A \qquad A(0) = A_0 \qquad \text{................ (vi)}$$

$$\frac{d^2B}{dt^2} = n\psi\theta B \qquad B(0) = B_0 \qquad \text{................ (vii)}$$

Solving the second order O.D.E (vi) and (vii) we obtained

$$A(t) = \frac{1}{2}\left(\frac{n\theta B_0}{\sqrt{n\theta\psi}} + A_0\right)e^{\sqrt{n\theta\psi}\ t} + \frac{1}{2}\left(A_0 - \frac{n\theta B_0}{\sqrt{n\theta\psi}}\right)e^{-\sqrt{n\theta\psi}\ t} \qquad \text{................}$$

(viii)

Similarly

$$B(t) = \frac{1}{2}\left(\frac{\psi A_0}{\sqrt{n\psi\theta}} - B_0\right)e^{-\sqrt{n\psi\theta}\ t} + \frac{1}{2}\left(B_0 - \frac{\psi A_0}{\sqrt{n\psi\theta}}\right)e^{-\sqrt{n\psi\theta}\ t} \qquad \text{................ (ix)}$$

Note: Human malaria control will conquer malaria infection totally when A (t) =0

$$t = -\frac{1}{2\sqrt{n\psi\theta}}\log_e\left(\frac{\frac{n\theta B_0}{\sqrt{n\psi\theta}} + A_0}{A_0 - \frac{n\theta B_0}{\sqrt{n\psi\theta}}}\right) \qquad \text{............... (x)}$$

5. Validity of the Model / Example

Table 1:- Malaria cases recorded in Nigeria.

Source: - World Heath Organization (WHO)

Year	Malaria Cases Confirmed (x)	Malaria Death (y)	Malaria Cured (z)
2001	2,253,519	4,317	2,249,202
2002	2,605,381	4,092	2,601,289
2003	2,608,479	5,343	2,603,136
2004	3,310,229	6,032	3,304,197
2005	3,532,108	6,494	3,525,614
2006	3,982,372	6,586	3,975,786
2007	2,969,950	10,289	2,975,786
2008	2,834,174	8,677	2,825,497
	$\sum x$ **= 24,096,212**	$\sum y =$ **51,830**	$\sum z =$ **24,044,382**

$$A_0 = B_0 = \frac{\sum_{i=1}^{8} x}{t} = 3{,}012{,}027$$

$$\psi = \frac{\sum_{i=1}^{8} y}{\sum_{i=1}^{8} x} = 0.00215$$

$$\theta = \frac{\sum_{i=1}^{8} z}{\sum_{i=1}^{8} x} = 0.9978$$

Case 1

If n = 5, $A_0 = B_0$=3,012,027, ψ = 0.00215, θ = 0.9978

$$t = -4.8263 \log_e \left(\frac{148{,}060{,}315}{-142{,}036{,}254} \right)$$

$= 4.8263 \log_e (1.0424)$

= 0.20042year

$\approx$ 2months, 12days and 4hours

Case 2.

If n = 8, $A_0 = B_0$ = 3,012,027, ψ = 0.00215, θ = 0.9978

$$t = -3.8124 \log_e \left(\frac{186,338,022.6}{-180,313,968.6} \right)$$

$$= 3.8124 \log_e (1.0334)$$

$$= 0.12525\text{year}$$

$$\approx 1\text{month, 15days and 2hours}$$

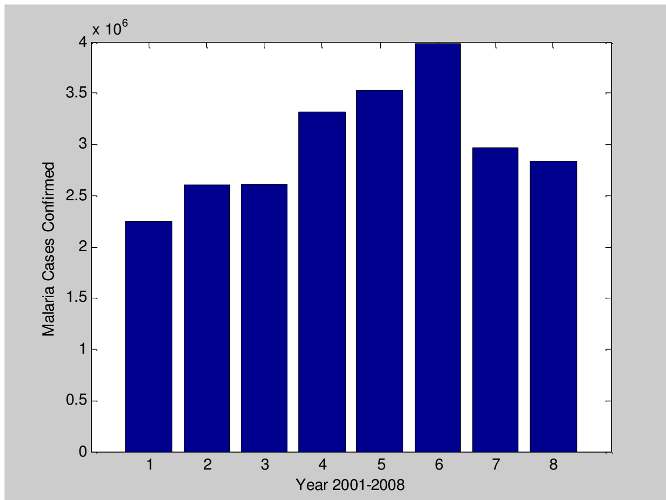

FIGURE1: Malaria Cases Confirmed

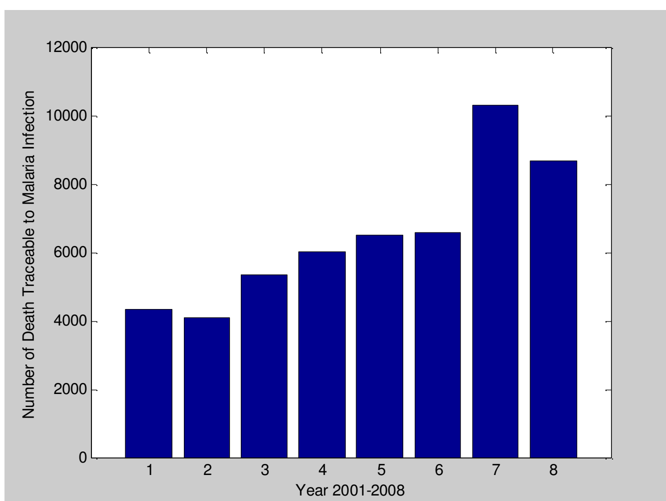

FIGURE 2: No of Death by Malaria

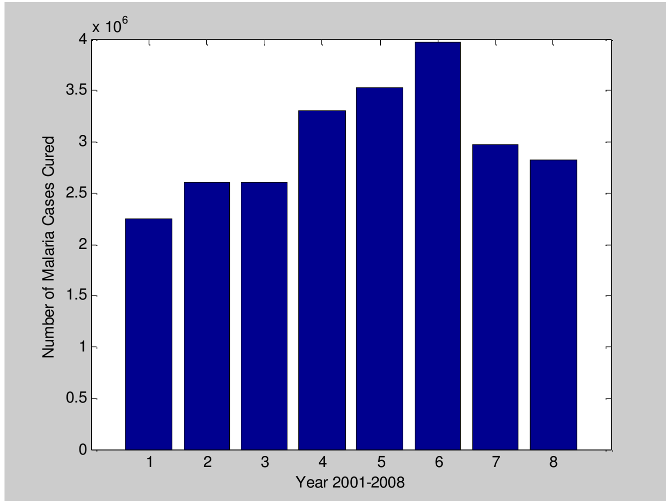

FIGURE 3: No of Malaria Infection Cured

REFERENCE

[1] Anderson, R.M. et al (1989). Non-linear phenomena in host-parasite interactions. Parasitology 99 (Suppl.), S59-S79

[2] Anderson, R.M., May, R.M. (1991). Infections Disease of Humans: Dynamics and control. Oxford, United Kingdom, Oxford University Press.

[3] Ballou, W.R. et al (2004). Update on the Clinical Development of Candidate Malaria Vaccines. Am J Trop Med Hyg 71 (2 suppl): pp239 – 247.

[4] Breman, J.G.; Egan, A.; Keusch, G.T. (2001). The intolerable burden of malaria: a new look at the numbers – Am J. trop med Hyg 64 (suppl): iv-vii.

[5] Field, J.W.; Niven, J.C. (1937). A note on prognosis in relation to parasites counts in acute subtertian malaria. Trans. R. Soc. Trop. Med. Hyg. 6, 569-574.

[6] Field, J.W. (1949). Blood examination and prognosis in acute falciparum malaria. Trans. R. Soc. Trop. Med. Hyg. 43, 33-48

[7] Greenwood, B. et al (2005). Malaria. Lancet: 1487- 1498

[8] Hellriegel, B. (1992). Modeling the immune response to malaria with ecological concepts: short- term behavior against long - term equilibrium. Proc. R. Soc. B 250, 249-256.

[9] Hetzel, C.; Anderson, R.M. (1996). The within – host cellular dynamics of blood stage malaria – theoretical and experimental studies. Parasitology 113, 25-38.

[10] Hyun, M Yang, (2001). A mathematical model for malaria transmission relating global warming and local socioeconomic conditions. Rev. saude public vol. 35 no 3 sao panlo June 2001.

[11] Kitchen, S.F. (1949a). Falciparum malaria in malariology (ed. M.F. Boyd), pp. 995-1016. London, UK: Saunders.

[12] Mackinnon, M.J.; Read, A.F. (2004). Virulence in Malaria: an evolutionary viewpoint. Phil Trans. R. Soc. B 359, 965 – 986.

[13] Molineaux, L. (2001). Plasmodium falciparum parasitaemia described by a new mathematical model. Parsitology 122, 379-391.

[14 Mc Queen, P.G.; Mckenzie, F.E. (2004). Age structured red blood cell susceptibility and the dynamics of malaria infections. Proc. Natl Acad. Sci USA 101, 9161 - 9166.

[15] Rustom Antia, et al. (2008). The dynamics of acute malaria infections .1. Effect of the parasites red blood cell preference. Proc R. Soc B 2008 275, 1449-1458.

[16] Snow, R.W. et al. (2005). The global distribution of clinical episodes of malaria. Nature 434:214-217

[17] Thomas Smith. et al (2006) Mathematical modeling of the impact of malaria vaccines on the clinical epidemiology and natural entory of plasmodrum falciparum malaria: Overview am. J Trop. Med. Hyg, 75 (suppl 2), 2006 pp 1-10

On Continuous Approximate Solution Of Ordinary Differential Equations

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Abstract

In this work the problem of continuous approximate solution of the ordinary differential equations will be investigated. An approach to construct the continuous approximate solution, which is based on the discrete approximate solution and the spline interpolation, will be provided. The existence and uniqueness of such continuous approximate solution will be pointed out. Its error will be estimated and its convergence will be considered. Finally, with the aid of modern PC and mathematical software, three practical computer approaches to perform above construction will be offered.
AMS Classification Subject: 34A45, 65L05, 65Y99

Key Words: Continuous Approximate Solution, Discrete Approximate Solution, Cubic Spline Interpolant.

1. INTRODUCTION

1.1 Presentation of Problem

Differential equations are often used to model, understand and predict the dynamic systems in the real world. The use of differential equations makes available to us the full power of Calculus. Modeling by differential equations greatly expands the list of possible applications of Mathematics.

Unfortunately, a wide majority of interesting differential equations have no closed form. i.e. the solution can't be expressed explicitly in terms of elementary functions such as polynomial,exponential, logarithmic or trigonometric functions even if it can be shown that a solution of the differential equation exists. Furthermore, in many case the explicit solution does exist, but the evaluation of the function may be difficult. Thus, we have to be content with an approximation of the solution of differential equations and the approximate methods for differential equations developed.

Generally, the approximate methods fall into two categories:

(1)Discrete approximate methods which produce a table of approximation of solution values corresponding to points of independent variable. This kind of methods provides quatitative information about solution even if we can not find the formula of the solution. There's also advantage that most of work can be done by machines. However, its disadvantage is that we obtain only approximation, not precise solution, and not a function.

(2)Continuous approximate methods on which much less work has been done. Although theoretically any discrete approximate methods can be converted to continuous approximate method by interpolating, but there remains a lot of problems not answered thoroughly, such as error, convergence,stability and computer approach . In this work, we attemp to solve some of these problems.

1.2 Description of Problem
In this work we'll study the problem of continuous approximate solution of initial value problem of ordinary differential equation:

$$y' = f(x,y) \quad \text{and} \quad y(a) = y_0 \quad x \text{ in } [a,b] \qquad (1.2.1)$$

Let f(x,y) be continuous and satisfy Lipschitz condition on D where
D={ (x,y): $a \le x \le b$ $-\infty < y < \infty$ } then (1.2.1) has unique solution:

$$y = y(x) \quad x \text{ in } [a,b]. \qquad (1.2.2)$$

Suppose we have a mesh of [a,b]: $a=x_0 < x_1 < ...... < x_n=b$

then its exact solution values at points in mesh are $y(x_i) = y_i$ i = 0, 1, ..., n .

If we use certain discrete approximate method of (1.2.1) to produce its discrete approximate solution:

$$\{ (x_i, w_i) : i = 0, 1, ...,n \} \qquad (1.2.3)$$

where w_i is the approximation of y_i with error $|y_i - w_i| = O(h_i)^p$.

In this work, we assume (1.2.1) is a scalar equation, but most theoretical and numerical consideration can be carried over to vector form -- the system of 1st order equations. And, we assume (1.2.1) satisfies stronger differentiation conditions as needed in theoretical analysis later.

1.3 Natural Cubic Spline Interpolant
In this work the natural cubic spline interpolant will be used to construct the continuous approximate solution of (1.2.1) which is defined as follows.

Definition of Natural Cubic Spline Interpolant
For a set of data { (x_i, w_i) i = 1, 2, ...,n } its natural cubic spline interpolant is a piece-wise cubic polynomial s(x), for x in $[x_i, x_{i+1}]$ $s(x) = s_i(x)$

where $s_i(x) = a_i \cdot (x - x_i)^3 + b_i \cdot (x - x_i)^2 + c_i \cdot (x - x_i) + d_i$ i = 0, 1, 2, ...,(n-1) (1.3.1)
which meet following conditions:

agreeing with data: $s_i(x_i) = w_i$ i = 0, 1, 2,......,(n-1) and $s_{n-1}(x_n) = w_n$ (1.3.2.a)
function values of adjacent 2 pieces are equal at joint points:

$$s_i(x_{i+1}) = s_{i+1}(x_{i+1}) \quad i = 0, 1, 2, ...,(n-2) \qquad (1.3.2.b)$$

1st derivative values of 2 adjacent pieces are equal at joint points:

$$s_i'(x_{i+1}) = s_{i+1}'(x_{i+1}) \quad i = 0, 1, 2, ...,(n-2) \qquad (1.3.2.c)$$

2nd derivative values of 2 adjacent pieces are equal at joint points:

$$s_i''(x_{i+1}) = s_{i+1}''(x_{i+1}) \quad i = 0, 1, 2, ...,(n-2) \qquad (1.3.2.d)$$

Boundary condition of natural spline: $s''(x_0) = s''(x_n) = 0$ (1.3.2.e)

Remark:

1. The natural cubic spline interpolant s(x) is in $C^2([a,b])$.
2. Generally, the cubic spline interpolant is not unique for a set of data. In order to get a unique cubic spline interpolant we need some boundary conditions. In this work we adopt the natural conditions, i.e. the 2nd derivative at two endpoints of interval are equal zero. There're several boundary conditions for cubic spline interpolant, but just cause a slight difference in the following numerical and theoretical consideration and results.

1.4 Computer Approach and Mathematical Software

One of advantage of discrete approximate methods is that it can be performed by computation machines and the machines can do most work of this kind of methods for us. In a long time it is difficult to perform a continuous approximate method by machines.

Now, modern computer technology makes it possible to perform the continuous approximate methods by PC and mathematical software. In this work we'll provide three computer approaches to construct the continuous approximate solution of (1.2.1).

These approach is based on PC and Mathcad 13. Mathcad 13 is a CAS and one of popular mathematical softwares in the world. We'll use its program function and routines to accomplish the construction of continuous approximate solution of (1.2.1)

2. Continuous Approximate Solution

In this part we'll provide an approach to construt the continuous approximate solution of (1.2.1) and discuss its existence and uniqueness. Also, the error will be estimated and the convergence will be considered.

2.1 Approach to Construct the Continuous Approximate Solution

For (1.2.1) and a simple mesh: $a = x_0 < x_1 < x_2 < \ldots\ldots < x_n = b$,

we can obtain a set of data { (x_i, w_i) i = 0,1,2, ...,n} where w_i is approximate solution values produced by certain discrete approximate method.

Then, we form a natural cubic spline interpolant (1.3.1) for the set of data { (x_i, w_i) i = 0,1,2, ...,n}

"To form a natural cubic spline interpolant" means "To determine $\{a_i, b_i, c_i, d_i$, i = 0,1, ...,n-1} in (1.3.1)".

Let $h_i = x_{i+1} - x_i$ i = 0, 1,......, n-1 , then

From (1.3.2.a) $d_i = w_i$ i = 0,1,......,n-1 and

$$a_{n-1}\left(h_{n-1}\right)^3 + b_{n-1}\left(h_{n-1}\right)^2 + c_{n-1}h_{n-1} + d_{n-1} = w_n$$

From (1.3.2.b) $a_i \cdot \left(h_i\right)^3 + b_i \cdot \left(h_i\right)^2 + c_i h_i + d_i = d_{i+1}$ i = 0, 1, ..., n-2 (2.1.0)

From (1.3.2.c) $3a_i\left(h_i\right)^2 + 2b_i h_i + c_i = c_{i+1}$ i = 0, 1, ..., n-2

From (1.3.2.d) $6a_i h_i + 2b_i = 2b_{i+1}$ i = 0, 1, ..., n-2

From (1.3.2.e) $2b_0 = 0$ and $6a_{n-1}h_{n-1} + 2b_{n-1} = 0$

This is a system of linear equations in 4n unknowns $\{a_i, b_i, c_i, d_i, i = 0, 1, ..., n-1\}$. In order to make it easy to solve, program and analyze theoritically we simlify above system as follows.

Let $s''(x_i) = u_i$ i = 0, 1, ...,n with $s''(x_0) = u_0 = 0$ and $s''(x_n) = u_n = 0$

then we have:

$$h_{i-1}\cdot u_{i-1} + 2\cdot(h_{i-1} + h_i)\cdot u_i + h_i\cdot u_{i+1} = 6\cdot\left(\frac{w_{i+1} - w_i}{h_i} - \frac{w_i - w_{i-1}}{h_{i-1}}\right) \qquad i = 1,2,......,n-1 \qquad (2.1.1)$$

also
$$a_i = \frac{u_{i+1} - u_i}{6h_i} \qquad i = 0,1,......,n-1 \qquad (2.1.2)$$

$$b_i = \frac{u_i}{2} \qquad i = 0,1,......,n-1 \qquad (2.1.3)$$

$$c_i = \frac{w_{i+1} - w_i}{h_i} - \frac{u_{i+1} + 2\cdot u_i}{6} h_i \qquad i = 0,1,......,n-1 \qquad (2.1.4)$$

$$d_i = w_i \qquad i = 0,1,......,n-1 \qquad (2.1.5)$$

Later we need the matrix form of (2.1.1) which is TU=W
Where

$$T=\begin{bmatrix} 2(h_1+h_2) & h_2 & 0 & . & . & . & . & . \\ 0 & h_2 & 2(h_2+h_3) & h_3 & 0 & . & . & . \\ ..... & .... & . & .... & . & . & .... & . \\ .... & 0 & h_i & 2(h_i+h_{i+1}) & h_{i+1} & . & 0 & .... \\ . & . & . & .... & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & .... & . & . & . \\ ..... & . & .... & . & 0 & . & h_{n-1} & 2(h_{n-1}+h_n) \end{bmatrix} \tag{2.1.6}$$

$$U=\begin{pmatrix} u_1 \\ u_2 \\ . \\ u_i \\ . \\ . \\ u_{n-1} \end{pmatrix} \quad (2.1.7) \qquad W=6\begin{pmatrix} \frac{w_2-w_1}{h_1}-\frac{w_1-w_0}{h_0} \\ \frac{w_3-w_2}{h_2}-\frac{w_2-w_1}{h_1} \\ . \\ \frac{w_{i+1}-w_i}{h_i}-\frac{w_i-w_{i-1}}{h_{i-1}} \\ . \\ . \\ \frac{w_n-w_{n-1}}{h_{n-1}}-\frac{w_{n-1}-w_{n-2}}{h_{n-2}} \end{pmatrix} \quad (2.1.8)$$

Summary:

The approach to construct a continuous approximate solution of (1.2.1) as follows:

1.Use a discrete approximate method to find an approximate solution { (x_i, w_i): i=0,1,...,n}

2.Solve system (2.1.1) for { u_i : i = 1, 2,....,n-1) with $u_0 = 0$ and $u_n = 0$

3.Find { a_i, b_i, c_i, d_i, i = 0,1,......,n-1 } by (2.1.2)--(2.1.5)

4 Form the natural cubic spline interpolant by (1.3.1) that is our desired continuous approximate solution of (1.2.1)

2.2 Existence and Uniqueness of Continuous Approximate Solution

The existence and uniqueness of such continuous approximate solution is stated in the following theorem.

Theorem 2.2.1 *For initial value problem (1.2.1) with discrete approximate solution (1.2.3) a unique continuous approximate solution, determined by the natural cubic spline interpolant (1.3.1), exists.*

Proof: The continuous approximate solution is the natural cubis spline interpolant (1.3.1) which is determined by its coeffients { (a_i, b_i, c_i, d_i) i = 0,1,...,n-1 }. These coefficients are solution of (2.1.1)--(2.1.5). (2.1.1) is a system of linear equations of (n-1)X(n-1) and the matrix T (2.1.6) is strictly diagonally dominant and nonsigular, so (2.1.1) has unique solution. Thus, The natural cubic spline interpolant exists and is unique.

2.3 Error of Continuous Approximate Solution

In this part, 1st we prove a lemma and then estimate the the error of such continuous approximate solution.

Lemma 2.3.1 *For a system of linear equations (2.1.1) TU = W then* $||U|| \le \dfrac{||W||}{2h}$ *(2.3.1)*

where || U || and || W || are max norm of U and W and h = min { h_i*, i = 0,1,......,n-1 },*

Proof: Let $||U|| = |u_i|$, then from i th equation of (1.2.1) we have:

$$|\, u_{i-1} h_{i-1} + 2u_i\left(h_{i-1} + h_i\right) + u_{i+1} h_i \,| = |\, w_i \,|$$

$$|\, u_i \,|\,|\, \frac{u_{i-1}}{u_i} h_{i-1} + 2\left(h_{i-1} + h_i\right) + \frac{u_{i+1}}{u_i} h_i \,| = |\, w_i \,|$$

Then, $\left|u_i\right|\left(h_{i-1} + h_i\right) \le \left|w_i\right|$ i.e $||U|| \le \dfrac{\left|w_i\right|}{h_{i-1} + h_i} \le \dfrac{||W||}{2h}$

Theorem 2.3.2 *For initial value problem (1.2.1) wlth discrete approximate solution (1.2.3) of order p if its continuous approximate solution s(x), determined by the natural cubic spline interpolant (1.3.1), then error with exact solution y(x):*

$$||y(x) - s(x)|| \le K \cdot \left|\left|y^{(4)}\right|\right| \cdot H^4 + H^p\left[c_3 \cdot \left(\frac{H}{h}\right)^3 + c_2 \cdot \left(\frac{H}{h}\right)^2 + c_1 \cdot \frac{H}{h} + c_0\right] \qquad (2.3.2)$$

Proof: | y(x) - s(x) | is continuous on [a,b], it has max and min on [a,b]. Assume at x in [x_i, x_{i+1}] , | y(x) - s_i(x) | is max, then || y(x) - s(x) || = | y(x) - s_i(x) | .

and we have: $$\left|y(x) - s_i(x)\right| \le \left|y(x) - g_i(x)\right| + \left|g_i(x) - s_i(x)\right| \qquad (2.3.3)$$

where $g_i(x) = a1_i\left(x - x_i\right)^3 + b1_i\left(x - x_i\right)^2 + c1_i\left(x - x_i\right) + d1_i$ The natural cubic spline interpolant for data set { (x_i, y_i) : i = 0,1,....,n } where y_i is the exact solution value of y(x) at x_i .

Let H = max{ h_i : i = 0,1,......,n-1} and h = min{ h_i : i = 0,1, ...,n-1 } then, Assume y(x) of (1.2.1) in $C^{(4)}[a,b]$ with $\left\|y^{(4)}\right\| \le M$ then the first term in (2.3.3) (see [1])

$$\left\|y(x) - g_i(x)\right\| \le K \cdot \left\|y^{(4)}\right\| \cdot H^4 \qquad (2.3.4)$$

Now, let's estimate the second term in (2.3.3)

$$\left|g_i(x) - s_i(x)\right| = \left|\left[a1_i \cdot (x - x_i)^3 + b1_i \cdot (x - x_i)^2 + c1_i \cdot (x - x_i) + d1_i\right] \ldots + \left[-a_i \cdot (x - x_i)^3 - b_i \cdot (x - x_i)^2 - c_i \cdot (x - x_i) - d_i\right]\right|$$

$$\left|g_i(x) - s_i(x)\right| \le \left|a1_i - a_i\right| \cdot H^3 + \left|b1_i - b_i\right| \cdot H^2 + \left|c1_i - c_i\right| \cdot H + \left|d1_i - d_i\right| \qquad (2.3.5)$$

where from (2.1.2) $\left|a1_i - a_i\right| = \left|\dfrac{u1_{i+1} - u1_i}{6h_i} - \dfrac{u_{i+1} - u_i}{6h_i}\right|$ $\quad \left|a1_i - a_i\right| \le \dfrac{\left|u1_{i+1} - u_{i+1}\right| + \left|u1_i - u_i\right|}{6h_i}$

from (2.1.3) $\left|b1_i - b_i\right| = \left|\dfrac{u1_i}{2} - \dfrac{u_i}{2}\right|$ $\quad \left|b1_i - b_i\right| \le \dfrac{\left|u1_i - u_i\right|}{2}$

from (2.1.4) $\left|c1_i - c_i\right| = \left|\left(\dfrac{y_{i+1} - y_i}{h_i} - \dfrac{u1_{i+1} + 2u1_i}{6} \cdot h_i\right) - \left(\dfrac{w_{i+1} - w_i}{h_i} - \dfrac{u_{i+1} + 2u_i}{6} \cdot h_i\right)\right|$

$$\left|c1_i - c_i\right| \le \frac{\left|y_{i+1} - w_{i+1}\right| + \left|y_i - w_i\right|}{h} + \frac{\left|u1_{i+1} - u_{i+1}\right| + 2\left|u1_i - u_i\right|}{6} \cdot H$$

from (2.1.5) $\left|d1_i - d_i\right| = \left|y_i - w_i\right|$ $\quad \left|d1_i - d_i\right| \le C \cdot H^p$

Since $u1_i$ is the solution of TU1=Y and u_i is the solution of TU=W, therefore $u1_i - u_i$ is the solution of T(U1-U)=Y-W and $\left\|U1 - U\right\| \le \dfrac{\left\|Y - W\right\|}{2h}$ by Lemma 2.3.1.

Thus, we have $\left\|U1 - U\right\| \le 6 \cdot \dfrac{\left\|\dfrac{y_{i+1} - w_{i+1}}{h_i} - \dfrac{y_i - w_i}{h_i} - \dfrac{y_i - w_i}{h_{i-1}} + \dfrac{y_{i-1} - w_{i-1}}{h_{i-1}}\right\|}{h_i + h_{i+1}} \le C \cdot \dfrac{H^p}{h^2}$ and

if we abuse the "C", then $\left|a1_i - a_i\right| \le C \cdot \dfrac{H^P}{h^3}$ $\quad \left|b1_i - b_i\right| \le C \cdot \dfrac{H^p}{h^2}$ $\quad \left|c1_i - c_i\right| \le C \cdot \dfrac{H^p}{h^3} + C \cdot \dfrac{H^P}{h^2} H$

Substituting into (2.3.5) we have:

$$\left|g_i(x) - s_i(x)\right| \le H^p\left[c_3 \cdot \left(\frac{H}{h}\right)^3 + c_2 \cdot \left(\frac{H}{h}\right)^2 + c_1 \cdot \frac{H}{h} + c_0\right]$$

Substituting (2.3.4) and (2.3.5) into (2.3.3) we get (2.3.2). This prove the theorem.

Remark of Theorem

!. The theorem shows: the error of continuous approxomate solution consists of 2 parts. One is caused by interpolation and another one is caused by discrete approximate method.

2. The accuracy of continuous approximate solution can not be higher than the accuracy of the spline interpolation. In this case, If p<4, then its accurary is p; if $p \ge 4$, then its accurary is 4.

2.4 Convergence of Continuous Approximate Solution

In this part we'll discuss the convergence of the continuous approximate solution. And, the result is stated in the following theorem.

Theorem 2.4.1 *For initial value problem (1.2.1) if*

(1) f(x,y) is in C^4 *([a,b]X*$(-\infty,\infty)$*),*

(2) M_n *is a sequence of quasi-uniform simple mesh of [a,b],* M_n *: a =* $x_{0,n} < x_{1,n} < \dots\dots\dots < x_{n,n}$ *= b*

with $\frac{H_n}{h_n} \le c < \infty$ *where* $h_{i,n} = x_{i+1,n} - x_{i,n}$ *i = 0,1,....,n-1* H_n*=max{* $h_{i,n}$ *}* h_n*=min {* $h_{i,n}$ *}*

(3) { $(x_{i,n}, w_{i,n})$*, i = 0,1,.....,n } is an approximate solution on* M_n *produced by a discrete approximate method with error order p*

(4) s_n*(x) is the natural cubic spline interpolant for data set in (3)*

then $s_n(x)$ *----> y(x) on [a,b] as* H_n *----> 0*

Proof: From (2.3.2) we have:

$$\left|\left|y(x) - s_n(x)\right|\right| \le K\cdot\left|\left|y^{(4)}\right|\right|\cdot\left(H_n\right)^4 + \left(H_n\right)^p\left[c_3\cdot\left(\frac{H_n}{h_n}\right)^3 + c_2\cdot\left(\frac{H_n}{h_n}\right)^2 + c_1\cdot\frac{H_n}{h_n} + c_0\right]$$

It is obvious: || y(x) - $s_n(x)$ || ----> 0 as H_n ----> 0

So, $s_n(x)$ convergences to y(x).

3. PC Approach for Constructing a Continuous Approximate Solution

In this part we'll offer three approaches to perfom the construction of the continuous approxi- mate solution in Section 2.1 which are based on PC and mathematical software "Mathcad 12 ".

3.1 The Approach Based on Program Function and Cubic Spline's Routines

This approach consists of 2 steps: first write a program RK4 which performs the Runge-Kutta method of order 4 to get discrete approximate solution and second use a built in cubic spline routine "lspline and interp" to get continuous approximate solution.

The program is as follows:

$$
\mathrm{RK4}(f,a,\alpha,b,h) := \left|
\begin{array}{l}
n \leftarrow \dfrac{b-a}{h} \\
x_0 \leftarrow a \\
w_0 \leftarrow \alpha \\
\text{for } i \in 0..n-1 \\
\quad \left|
\begin{array}{l}
x_{i+1} \leftarrow x_i + h \\
k1 \leftarrow h \cdot f(x_i, w_i) \\
k2 \leftarrow h \cdot f\left(x_i + \dfrac{h}{2}, w_i + \dfrac{k1}{2}\right) \\
k3 \leftarrow h \cdot f\left(x_i + \dfrac{h}{2}, w_i + \dfrac{k2}{2}\right) \\
k4 \leftarrow h \cdot f(x_i + h, w_i + k3) \\
w_{i+1} \leftarrow w_i + \dfrac{1}{6} \cdot (k1 + 2k2 + 2k3 + k4)
\end{array}
\right. \\
s \leftarrow \mathrm{augment}(x, w) \\
s
\end{array}
\right.
$$

In this program,
Input are function f(x,y), endpoints of interval [a,b], initial function value and step size h;
Output is a matrix of (n+1)X2 which 1st column is x-values and 2nd column is y-values.

Now, let's work out an example to illustrate this approach.

Example 3.1.1 Given: initial value problem y' $= 3 \cdot \cos(y - 3x)$ and y(0) = /2 x in I = [0,2]

Find:its continuous approximate solution on I with step size h = 0.2.

Solution: 1st Use RK4 to find its discdete approximate solution

Input: $f(x,y) := 3 \cdot \cos(y - 3 \cdot x)$ $\quad a := 0 \quad b := 2 \quad \alpha := \dfrac{\pi}{2} \quad h := 0.2$

Call RK4 and define its discrete approximate solution by a matrix B: $B := \mathrm{RK4}(f,a,\alpha,b,h)$

Then, we get the discrete approximate solution :

$$B^T = \begin{pmatrix} 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 \\ 1.571 & 1.718 & 2.054 & 2.486 & 2.972 & 3.49 & 4.028 & 4.58 & 5.142 & 5.71 & 6.284 \end{pmatrix}$$

2nd, Use built-in routine "lspline" to get natural cubic spline interpolant.

$$vx := B^{\langle 0 \rangle} \qquad vy := B^{\langle 1 \rangle} \qquad vs := \text{lspline}(vx, vy)$$

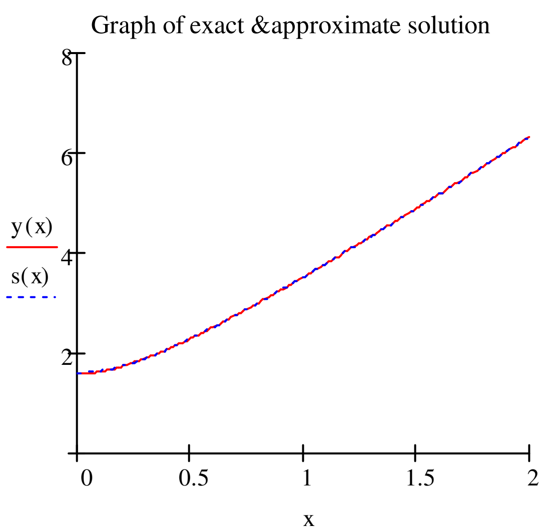
3rd, use "interp"to define the continuous approximate solution by above natural cubic spline interpolant.

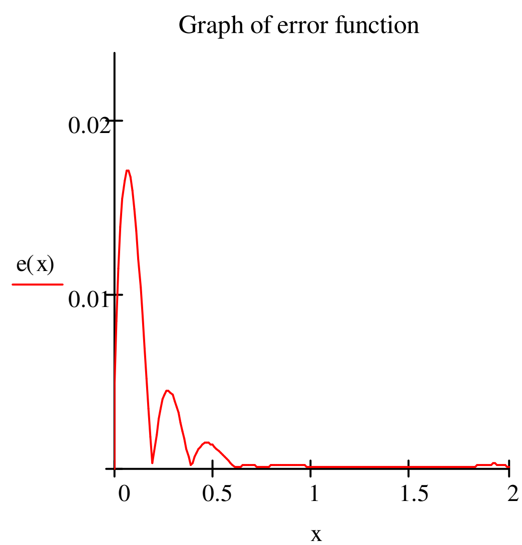
This is our desired continuous approximate solution: $s(x) := \text{interp}(vs, vx, vy, x)$

In this example the IVP has a exact solution: $y(x) := 3 \cdot x + 2 \cdot \text{acot}(3 \cdot x + 1)$

Now, let's compare them by graphing both functions and their error function: $e(x) := |y(x) - s(x)|$

$x := 0, 0.01 .. 2$





We find some error: $e(0.12) = 0.014$ $\qquad e(0.5) = 1.31 \times 10^{-3}$ $\qquad e(1.85) = 1.366 \times 10^{-4}$

Remark:

1.In this approach the program RK4 can be replaced by built-in routine of differential equation solver "rkfixed" or "rkadpt". The approach even is simple.
2 In Mathcad there're 3 built-in routines for cubic spline interpolation: "lspline" for natural spline; "pspline" for parabolic endpoints; "cspline" for cubic endpoints(or "not a knot conditions). We can choose appropriate one according to the boundary conditions.
3.Although we have a function s(x) as the continuous approximate solution, we have no expression of the function s(x). This is a defect.

Now, we use an example to illustrate above remark 1. This example is same as Example 3.1.1 but we use the routine "rkfixed" to find discrete approximate solution rather than program Rk4.

Example 3.1.2 Given: initial value problem y' $= 3\cdot\cos(y - 3x)$ and y(0) = □/2 x in I = [0,2]
Find:its continuous approximate solution on I with step size h = 0.2.
Solution: 1st use "rkfixed" to find discrete approximate solution of given IVP.

$$y_0 := \frac{\pi}{2} \qquad\qquad D(x,y) := 3\cdot\cos\left(y_0 - 3\cdot x\right)$$

Call the routine and the discrete approximate solution is indicated in matrix W: $W := \text{rkfixed}(y, 0, 2, 10, D)$

then, we get same result in B:

$$W^T = \begin{pmatrix} 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 \\ 1.571 & 1.718 & 2.054 & 2.486 & 2.972 & 3.49 & 4.028 & 4.58 & 5.142 & 5.71 & 6.284 \end{pmatrix}$$

2nd, We use this discrete approximate solution and "lspline" to find the natural cubic spline interpolant.

$$vx := W^{\langle 0 \rangle} \qquad vy := W^{\langle 1 \rangle} \qquad vs := \text{lspline}(vx, vy)$$

3rd, use this result and "interp" to get our desired continuous approximate solution.

$$s1(x) := \text{interp}(vs, vx, vy, x)$$

Remark:
Writing the program for discrete approximate solution still is necessary although there're some routines available since not every discrete approximate method's routine is available. Moreover, sometime we need the discrete approximate method with order p greater than 4 to guarantee our desired accuracy.

3.2 The Approach Based on Program Finction and Routine of "Solve Block"

This approach consists of following steps: 1. Write a program to get discrete approximate solution; 2. Define the natural cubic spline interpolant by (1.3.1) and get the system of linear equations in 4n unknown coefficients by (2.1.0); 3. Use built-tn routine "solve block" to solve above system; 4. Use the result to construct the natural cubic spline interpolant -- the desired continuous approximate solution.

Now, let's work out an example to illustrate this approach.

Example 3.2 Given: $y' = 1 + (x - y)^2$ and $y(2) = 1$ x in I = [2,3]

Find: its continuous approximate solution on I with step size h = 0.25

Solution: 1st, get its discrete approximate solution by RK4 in section 3.1

$$
\begin{aligned}
\mathrm{RK4}(f,a,\alpha,b,h) := \;& n \leftarrow \frac{b-a}{h} \\
& x_0 \leftarrow a \\
& w_0 \leftarrow \alpha \\
& \text{for } i \in 0..n-1 \\
& \quad x_{i+1} \leftarrow x_i + h \\
& \quad k1 \leftarrow h \cdot f(x_i, w_i) \\
& \quad k2 \leftarrow h \cdot f\left(x_i + \frac{h}{2}, w_i + \frac{k1}{2}\right) \\
& \quad k3 \leftarrow h \cdot f\left(x_i + \frac{h}{2}, w_i + \frac{k2}{2}\right) \\
& \quad k4 \leftarrow h \cdot f(x_i + h, w_i + k3) \\
& \quad w_{i+1} \leftarrow w_i + \frac{1}{6} \cdot (k1 + 2k2 + 2k3 + k4) \\
& s \leftarrow \mathrm{augment}(x, w) \\
& s
\end{aligned}
$$

In this program,
Input are function f(x,y), endpoints of interval [a,b], initial function value and step size h;
Output is a matrix of (n+1)X2 which 1st column is x-values and 2nd column is y-values.

Input: $f(x,y) := 1 + (x-y)^2$ $\quad a := 2$ $\quad \alpha := 1$ $\quad b := 3$ $\quad h := 0.2$

Call RK4 and define its discrete approximate solution by B: $B := \mathrm{RK4}(f,a,\alpha,b,h)$

Then, the discrete approximate solution is:

$$B^T = \begin{pmatrix} 2 & 2.25 & 2.5 & 2.75 & 3 \\ 1 & 1.45 & 1.833 & 2.179 & 2.5 \end{pmatrix}$$

$x_0 := \left(B^{\langle 0 \rangle}\right)_0$ $\quad x_1 := \left(B^{\langle 0 \rangle}\right)_1$ $\quad x_2 := \left(B^{\langle 0 \rangle}\right)_2$ $\quad x_3 := \left(B^{\langle 0 \rangle}\right)_3$ $\quad x_4 := \left(B^{\langle 0 \rangle}\right)_4$

$w_0 := \left(B^{\langle 1 \rangle}\right)_0$ $\quad w_1 := \left(B^{\langle 1 \rangle}\right)_1$ $\quad w_2 := \left(B^{\langle 1 \rangle}\right)_2$ $\quad w_3 := \left(B^{\langle 1 \rangle}\right)_3$ $\quad w_4 := \left(B^{\langle 1 \rangle}\right)_4$

$h_0 := x_1 - x_0$ $\quad h_1 := x_2 - x_1$ $\quad h_2 := x_3 - x_2$ $\quad h_3 := x_4 - x_3$

2nd, Using above data we define the natural cubic spline interpolant s(x) which is equal to

s0(x) = $a_0(x-x_0)^3 + b_0(x-x_0)^2 + c_0(x-x_0) + d_0$ x in $[x_0, x_1]$

s1(x) = $a_1(x-x_1)^3 + b_1(x-x_1)^2 + c_1(x-x_1) + d_1$ x in $[x_1, x_2]$

s2(x) = $a_2(x-x_2)^3 + b_2(x-x_2)^2 + c_2(x-x_2) + d_2$ x in $[x_2, x_3]$

s3(x) = $a_3(x-x_3)^3 + b_3(x-x_3)^2 + c_3\cdot(x-x_3) + d_3$ x in $[x_3, x_4]$

Now, the problem is reduced to finding these coefficients from the system which come from (1.3.2.a)----(1.3.2.e).

3rd, Use "solve block" to find these coefficients which structure is "Guess -- Given -- Find".

Guess

$a_0 := 0$ $\quad b_0 := 0$ $\quad c_0 := 0$ $\quad d_0 := 0$ $\quad a_1 := 0$ $\quad b_1 := 0$ $\quad c_1 := 0$ $\quad d_1 := 0$

$a_2 := 0$ $\quad b_2 := 0$ $\quad c_2 := 0$ $\quad d_2 := 0$ $\quad a_3 := 0$ $\quad b_3 := 0$ $\quad c_3 := 0$ $\quad d_3 := 0$

Given

$d_0 = w_0$ $\quad d_1 = w_1$ $\quad d_2 = w_2$ $\quad d_3 = w_3$ $\quad a_3(h_3)^3 + b_3(h_3)^2 + c_3 h_3 + d_3 = w_4$

$a_0(h_0)^3 + b_0(h_0)^2 + c_0 h_0 + d_0 = d_1$ $\quad a_1(h_1)^3 + b_1(h_1)^2 + c_1 h_1 + d_1 = d_2$

$a_2(h_2)^3 + b_2(h_2)^2 + c_2 h_2 + d_2 = d_3$

$3a_0(h_0)^2 + 2\cdot b_0 h_0 + c_0 = c_1$ $\quad 3a_1(h_1)^2 + 2\cdot b_1 h_1 + c_1 = c_2$ $\quad 3a_2(h_2)^2 + 2\cdot b_2 h_2 + c_2 = c_3$

$6a_0 h_0 + 2\cdot b_0 = 2\cdot b_1$ $\quad 6a_1 h_1 + 2\cdot b_1 = 2\cdot b_2$ $\quad 6a_2 h_2 + 2\cdot b_2 = 2\cdot b_3$

$2\cdot b_0 = 0$ $\quad 6a_3 h_3 + 2\cdot b_3 = 0$

$C := \mathrm{Find}(a_0, b_0, c_0, d_0, a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, a_3, b_3, c_3, d_3)$

$C^T = ▮$

$a_0 := C_0$ $\quad b_0 := C_1$ $\quad c_0 := C_2$ $\quad d_0 := C_3$ $\quad a_1 := C_4$ $\quad b_1 := C_5$ $\quad c_1 := C_6$ $\quad d_1 := C_7$

$a_2 := C_8$ $\quad b_2 := C_9$ $\quad c_2 := C_{10}$ $\quad d_2 := C_{11}$ $\quad a_3 := C_{12}$ $\quad b_3 := C_{13}$ $\quad c_3 := C_{14}$ $\quad d_3 := C_{15}$

4th, Use these coefficient to construct the natural cubic spline interpolant.

$$s0(x) := -0.996(x-2)^3 + 0(x-2)^2 + 1.862(x-2) + 1 \qquad x \text{ in } [2,2.25]$$

$$s1(x) := 0.731(x-2.25)^3 - 0.747(x-2.25)^2 + 1.675(x-2.25) + 1.45 \qquad x \text{ in } [2.25,2.5]$$

$$s2(x) := -0.027(x-2.5)^3 - 0.212(x-2.5)^2 + 1.436(x-2.5) + 1.833 \qquad x \text{ in } [2.5,2.75]$$

$$s3(x) := 0.31(x-2.75)^3 - 0.233(x-2.75)^2 + 1.324(x-2.75) + 2.179 \qquad x \text{ in } [2.75,3]$$

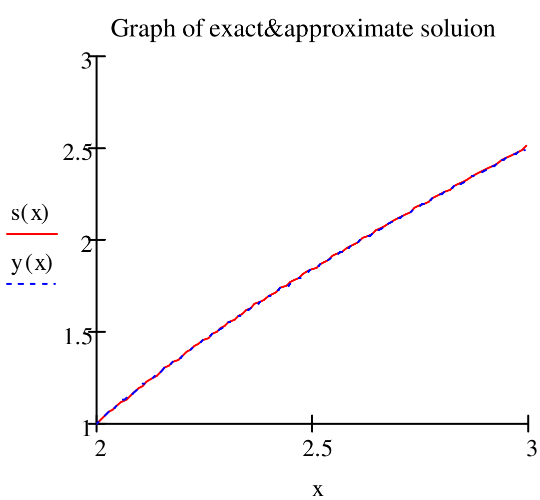
Or we can combine them into one function by built-in routine "if". We define:

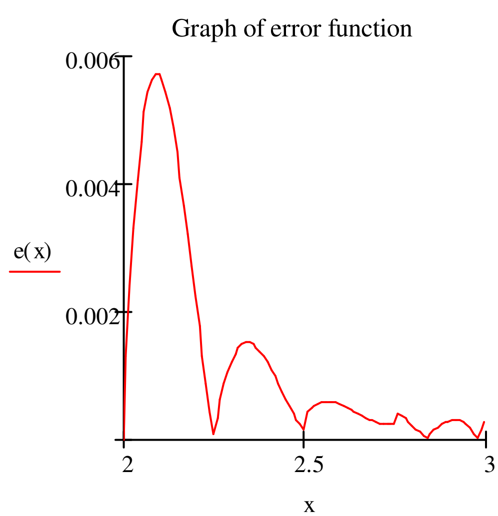
$$s(x) := \text{if}(2 \le x \le 2.25, s0(x), \text{if}(2.25 \le x \le 2.5, s1(x), \text{if}(2.5 \le x \le 2.75, s2(x), s3(x))))$$

Also, this initial value problem has unique solution: $y(x) := x - \dfrac{1}{x-1}$

Now, we compare them by graphing them and their error function: $e(x) := |y(x) - s(x)|$

$x := 2, 2.01 .. 3$





Remark:

1. This approach can get an expression of the continuous approximate solution but need to do more work ourself .
2. This approach is limited by the capacity of software. For example Mathcad can solve the system of equations up to 60 unknowns so it can find a cubic spline of at most 15 pieces.
3. We can use another routine "solve" of symbolic operation instead of "solve block" in this approach.

3.3 The Approach Based on Two Programs

This approach consists of 3 steps: 1. Use a program to get discrete approximate solution of initial value problem; 2. Use second program and above data to get coefficients of the natural cubic spline interpolant; 3. Use the result to construct the continuous approximate solution.

Now, let's work out an example to illustrate this approach.

Example 3.3 Given: $y' = \frac{y}{x} - \left(\frac{y}{x}\right)^2$ and $y(1) = 1$ x in $I = [1,3]$

Find: its continuous approximate solution on I with step size h = 0.2

Solution: 1st, we use RK4 in previous section to get its discrete approximate solution.

Input: $f(x,y) := \frac{y}{x} - \left(\frac{y}{x}\right)^2$ $\quad a := 1 \quad \alpha := 1 \quad b := 3 \quad h := 0.2$

$$
RK4(f,a,\alpha,b,h) := \left|
\begin{array}{l}
n \leftarrow \frac{b-a}{h} \\
x_0 \leftarrow a \\
w_0 \leftarrow \alpha \\
\text{for } i \in 0..n-1 \\
\quad \left|
\begin{array}{l}
x_{i+1} \leftarrow x_i + h \\
k1 \leftarrow h \cdot f(x_i, w_i) \\
k2 \leftarrow h \cdot f\left(x_i + \frac{h}{2}, w_i + \frac{k1}{2}\right) \\
k3 \leftarrow h \cdot f\left(x_i + \frac{h}{2}, w_i + \frac{k2}{2}\right) \\
k4 \leftarrow h \cdot f(x_i + h, w_i + k3) \\
w_{i+1} \leftarrow w_i + \frac{1}{6} \cdot (k1 + 2k2 + 2k3 + k4)
\end{array}
\right. \\
w
\end{array}
\right.
$$

In this program,
Input are function f(x,y), endpoints of interval [a,b], initial function value and step size h;
Output is a matrix of (n+1)X1 which is approximate solution values of y(x).

Then, we have the discrete approximate solution in B: $B := RK4(f,a,\alpha,b,h)$

$$B^T = (1 \;\; 1.015 \;\; 1.048 \;\; 1.088 \;\; 1.134 \;\; 1.181 \;\; 1.23 \;\; 1.28 \;\; 1.33 \;\; 1.38 \;\; 1.43)$$

2nd, in this example the step sizes are same h = 0.2 , so the interval is divided into 10 subintervals i.e. the natural cubic spline interpolant consists of 10 piece of cubic polynomial. And, we need to find 40 coefficients for constructing it. We use the program Cf which solve the system of (2.1.1) -- (2.1.5), to get the coefficients of the natural cubic spline interpolant. This program applies LU factorization to solve (2.1.1) to get u_i and then find { a_i, b_i, c_i, d_i } by u_i from (2.1.2) -- (2.1.5).

The Cf program is as follows:

$$
\mathrm{Cf}(f,a,\alpha,b,h) := \left|
\begin{array}{l}
n \leftarrow \dfrac{b-a}{h} \\
\text{for } i \in 0..n \\
\quad w_i \leftarrow B_i \\
\text{for } i \in 1..n-1 \\
\quad v_i \leftarrow \dfrac{6}{h}\cdot\left(w_{i+1} - 2\cdot w_i + w_{i-1}\right) \\
l_0 \leftarrow 1 \\
\mu_0 \leftarrow 0 \\
z_0 \leftarrow 0 \\
\text{for } i \in 1..n-1 \\
\quad \left|
\begin{array}{l}
l_i \leftarrow 2\cdot h - h\cdot\mu_{i-1} \\
\mu_i \leftarrow \dfrac{h}{l_i} \\
z_i \leftarrow \dfrac{v_i - h\cdot z_{i-1}}{l_i}
\end{array}
\right. \\
l_n \leftarrow 1 \\
z_n \leftarrow 0 \\
u_n \leftarrow 0 \\
\text{for } j \in (n-1)..0 \\
\quad u_j \leftarrow z_j - \mu_j\cdot u_{j+1} \\
\text{for } k \in 0..(n-1) \\
\quad \left|
\begin{array}{l}
a_k \leftarrow \dfrac{u_{k+1} - u_k}{6\cdot h} \\
b_k \leftarrow \dfrac{u_k}{2} \\
c_k \leftarrow \dfrac{w_{k+1} - w_k}{h} - \dfrac{u_{k+1} + 2\cdot u_k}{6}\cdot h \\
d_k \leftarrow w_k
\end{array}
\right. \\
s \leftarrow \mathrm{augment}(a,b,c,d) \\
s
\end{array}
\right.
$$

We get these coefficients in the matrix which is indicated by E: $E := \mathrm{Cf}(f, a, \alpha, b, h)$

$$E^T = \begin{pmatrix} 1.409 & -2.023 & 1.473 & -1.422 & 1.126 & -0.962 & 0.721 & -0.527 & 0.304 & -0.099 \\ 0 & 0.846 & -0.369 & 0.515 & -0.338 & 0.337 & -0.24 & 0.193 & -0.123 & 0.06 \\ 0.018 & 0.075 & 0.219 & 0.18 & 0.26 & 0.215 & 0.267 & 0.232 & 0.262 & 0.242 \\ 1 & 1.015 & 1.048 & 1.088 & 1.134 & 1.181 & 1.23 & 1.28 & 1.33 & 1.38 \end{pmatrix}$$

3rd, We use above result to construct the continuous approximate solution s(x) which is equal to:

$s0(x) := 1.409(x-1)^3 + 0\cdot(x-1)^2 + 0.018(x-1) + 1$ x in [1,1.2]

$s1(x) := -2.203(x-1.2)^3 + 0.846(x-1.2)^2 + 0.075(x-1.2) + 1.015$ x in [1.2,1.4]

$s2(x) := 1.473(x-1.4)^3 - 0.369(x-1.4)^3 + 0.219(x-1.4) + 1.048$ x in [1.4,1.6]

$s3(x) := -1.422(x-1.6)^3 + 0.515(x-1.6)^2 + 0.18(x-1.6) + 1.088$ x In [1.6,1.8]

$s4(x) := 1.126(x-1.8)^3 - 0.338(x-1.8)^2 + 0.26(x-1.8) + 1.134$ x in [1.8,2]

$s5(x) := -0.962(x-2)^3 + 0.337(x-2)^2 + 0.215(x-2) + 1.181$ x in [2,2.2]

$s6(x) := 0.721(x-2.2)^3 - 0.24(x-2.2)^2 + 0.267(x-2.2) + 1.23$ x in [2.2,2.4]

$s7(x) := -0.527(x-2.4)^3 + 0.193(x-2.4)^2 + 0.232(x-2.4) + 1.28$ x in [2.4,2.6]

$s8(x) := 0.304(x-2.6)^3 - 0.123(x-2.6)^2 + 0.262(x-2.6) + 1.33$ x in [2.6,2.8]

$s9(x) := -0.099(x-2.8)^3 + 0.06(x-2.8)^2 + 0.242(x-2.8) + 1.38$ x in [2.8,3]

Or, we can combine by built-in routine "if".

$u1(x) := \mathrm{if}(1 \le x \le 1.2, s0(x), \mathrm{if}(1.2 \le x \le 1.4, s1(x), \mathrm{if}(1.4 \le x \le 1.6, s2(x), s3(x))))$

$u2(x) := \mathrm{if}(1.8 \le x \le 2, s4(x), \mathrm{if}(2 \le x \le 2.2, s5(x), \mathrm{if}(2.2 \le x \le 2.4, s6(x), s7(x))))$

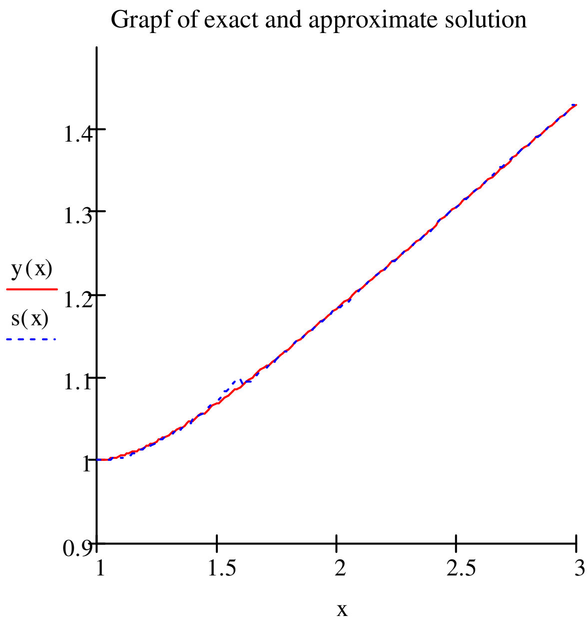
$u3(x) := \mathrm{if}(2.6 \le x \le 2.8, s8(x), s9(x))$

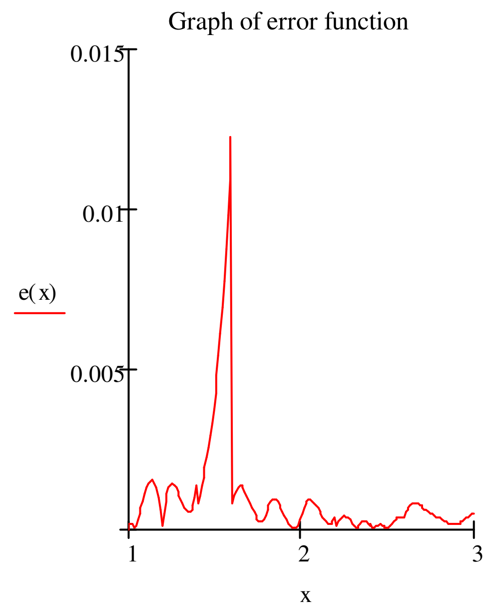
$s(x) := \mathrm{if}(1 \le x \le 1.8, u1(x), \mathrm{if}(1.8 \le x \le 2.6, u2(x), u3(x)))$

This is our desired continuous approximate solution.
And, this initial value problem has an exact solution y(x).

Now, let compare the continuous approximate solution and exact solution by graphing them and their error function e(x).

$x := 1, 1.01..3$ $y(x) := \dfrac{x}{1+\ln(x)}$ $e(x) := |y(x) - s(x)|$





Remark:

1. This approach can find the expression of the continuous approximate solution.
2. This approach is based on two programs. In 2nd program we use LU factorization to solve (2.1.1), of cause we can use other ways and other program.

4. REFERENCE

[1] Birhoff, Garrett & De Boor, Carl: "Error Bounds for Spline Interpolation", Journal of Mathematics and Mechanics, Vol 13, No 5, 1964.

[2] Burden, Richard L & Faires, J. Douglas: "Numerical Analysis" 8th edition, Brooks/Cole, Tomson Learning Inc. 2004.

[3] Davis,P.J.: "Interpolation and approximation", Blaisdell, New York, 1963.

[4] De Boor, Carl:"A practical guide to spline", Spring Verlag Inc., New York, 2001.

[5] E.Hairer, S.P.Norsett, G.W.Wanner: "Solving Ordinary Differential Equstion I" (nonstiff problem) Springer, 1993 Vol. 1

[6] E.Hairer, S.P.Norsett, G.W.Wanner: "Solving Ordinary Differential Equstion II" (stiff and differential algebraic problem) Springer, 1996.

[7] Loscalzo, F.R. 7 Tallot,T.D.:"Spline function approximation for solutions of ordinary differential equations", SIAM J., Numerical Analysis 4, 1967, p433 - 445.

[8] Schoenberg, I. J: " On spline functions", In Tech Summary Rep. 625. Mathematics Research Center, University of Wisconsin, Madison, 1966.

[9] Wu, D.T.:"Teaching Numerical Analysis with Mathcad", The International Journal of Computer Algebra in Mathematical Edication, 10 No 1, 2003.

[10] WU, D.T.: "Mathcad's Program Function and Application in teaching of Mathmatics" in Electronic Proceeding of ICMCM 16, Aug. 2004

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