INTERNATIONAL JOURNAL OF SCIENTIFIC AND STATISTICAL COMPUTING (IJSSC)

VOLUME 6, ISSUE 1, 2015

EDITED BY
DR. NABEEL TAHIR

ISSN (Online): 2180-1339
International Journal of Scientific and Statistical Computing (IJSSC) is published both in traditional paper form and in Internet. This journal is published at the website http://www.cscjournals.org, maintained by Computer Science Journals (CSC Journals), Malaysia.

IJSSC Journal is a part of CSC Publishers
Computer Science Journals
http://www.cscjournals.org
EDITORIAL PREFACE

The International Journal of Scientific and Statistical Computing (IJSSC) is an effective medium for interchange of high quality theoretical and applied research in Scientific and Statistical Computing from theoretical research to application development. This is the First Issue of Sixth Volume of IJSSC. International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas.

The initial efforts helped to shape the editorial policy and to sharpen the focus of the journal. Started with Volume 6, 2015, IJSSC appears with more focused issues. Besides normal publications, IJSSC intend to organized special issues on more focused topics. Each special issue will have a designated editor (editors) – either member of the editorial board or another recognized specialist in the respective field.

This journal publishes new dissertations and state of the art research to target its readership that not only includes researchers, industrialists and scientist but also advanced students and practitioners. The aim of IJSSC is to publish research which is not only technically proficient, but contains innovation or information for our international readers. In order to position IJSSC as one of the top International journal in computer science and security, a group of highly valuable and senior International scholars are serving its Editorial Board who ensures that each issue must publish qualitative research articles from International research communities relevant to Computer science and security fields.

IJSSC editors understand that how much it is important for authors and researchers to have their work published with a minimum delay after submission of their papers. They also strongly believe that the direct communication between the editors and authors are important for the welfare, quality and wellbeing of the Journal and its readers. Therefore, all activities from paper submission to paper publication are controlled through electronic systems that include electronic submission, editorial panel and review system that ensures rapid decision with least delays in the publication processes.

To build international reputation of IJSSC, we are disseminating the publication information through Google Books, Google Scholar, Directory of Open Access Journals (DOAJ), Open J Gate, ScientificCommons, Docstoc, Scribd, CiteSeerX and many more. Our International Editors are working on establishing ISI listing and a good impact factor for IJSSC. I would like to remind you that the success of the journal depends directly on the number of quality articles submitted for review. Accordingly, I would like to request your participation by submitting quality manuscripts for review and encouraging your colleagues to submit quality manuscripts for review. One of the great benefits that IJSSC editors provide to the prospective authors is the mentoring nature of the review process. IJSSC provides authors with high quality, helpful reviews that are shaped to assist authors in improving their manuscripts.
EDITORIAL BOARD

Associate Editor-in-Chief (AEiC)

Dr Hossein Hassani
Cardiff University
United Kingdom

EDITORIAL BOARD MEMBERS (EBMs)

Dr. De Ting Wu
Morehouse College
United States of America

Dr Mamode Khan
University of Mauritius
Mauritius

Dr Costas Leon
César Ritz College
Switzerland

Assistant Professor Christina Beneki
Technological Educational Institute of Ionian Islands
Greece

Professor Abdol Soofi
University of Wisconsin-Platteville
United States of America

Assistant Professor Yang Cao
Virginia Tech
United States of America
<table>
<thead>
<tr>
<th>Pages</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 14</td>
<td>Use Fuzzy Midrange Transformation Method to Construction Fuzzy Control Charts limits</td>
<td>Kawa M. Jamal Rashid, Suzan S. Haydar</td>
</tr>
</tbody>
</table>
Use Fuzzy Midrange Transformation Method to Construction Fuzzy Control Charts limits

Kawa M. Jamal Rashid & Suzan S. Haydar
School of Administrations and Economics/ Department of Statistics
Sulamani University
Sulamani, Iraq

Abstract
Statistical Process Control (SPC) is approach that uses statistical techniques to monitor the process. The techniques of quality control are widely used in controlling any kinds of processes. The widely used control charts are $\bar{X} - R$ and $\bar{X} - S$ charts. These are called traditional variable control chart, which consists of three horizontal lines called Centre Line (CL), Upper Control Limit (UCL) and Lower Control Limit (UCL) are represented by numeric values. The center line in a control chart denotes the average value of the quality characteristic under study. A process is either "in control" or "out of control" depending on numeric observation values. In the consideration of real production process, it is assumed that there are no doubts about observations and their values. But when these observations include human judgments, evaluations and decisions, a continuous random variable (xi) of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. So, linguistic terms can be used instead of an exact value of continuous random variable. In this context fuzzy set theory is useful tool to handle this uncertainty. Numeric control limits can be transformed to fuzzy control limits by using membership function, therefore; the concept of fuzzy control charts with $\alpha$ cuts by using $\alpha$-level fuzzy midrange with trapezoidal fuzzy number (TRN) is proposed. The fuzzy control charts for arithmetic mean ($\bar{X}$), and range ($\bar{R}$) are developed. Fuzzy control limits provide a more accurate and flexible evaluation. In this paper through a real illustrative data from Sulaimani Company for Cement in the city of Sulaimani, shows the designing of fuzzy control chart for process average of quality.

Keywords: Statistical Process Control, Fuzzy Number, Fuzzy Control Charts, Membership Function, $\alpha$-cut and $\alpha$ - Level Fuzzy Midrange.

1. INTRODUCTION
Quality control is a process employed to ensure a certain level of quality in a product or service. It may include whatever actions a business deems necessary to provide for the control and verification of certain characteristics of a product or service. The basic goal of quality control is to ensure that the products, services, or processes provided meet specific requirements and are dependable and satisfactory [1], [2]. The fuzzy set theory is a more suitable tool for handling attribute data since these data may be expressed in linguistic terms such as "very good", "good", "medium", "bad", and "very bad" [3]. The fuzzy set theory was first introduced by Zadeh (1965). Many studies were done to combine statistical methods and fuzzy set theory. The fuzzy numbers are a reasonable way to analyze and evaluate the process. some measures of central tendency in descriptive statistics are used in variable control charts. These measures can be used to
convert fuzzy sets into scalars which are fuzzy mode, $\alpha$-level fuzzy midrange, fuzzy median and fuzzy average [1], [3].

2. THE STRUCTURE OF CONTROL LIMITS FOR FUZZY CONTROL CHART

The X-bar chart is most widely used chart for controlling the process mean quality level as well as the process variability can be controlled by either a control chart for the range, called R-chart or a control chart for the standard deviation, called S-chart.

Montgomery [2005] has proposed the control limits for $\bar{X}$ control chart based on sample range is given below:

$$UCL_\tau = \bar{X} + A_2 \bar{R}$$  \hspace{1cm} (1)  

$$CL_\tau = \bar{X}$$  \hspace{1cm} (2)  

$$LCL_\tau = \bar{X} - A_2 \bar{R}$$  \hspace{1cm} (3)  

where $A_2$ is a control chart coefficient, and $\bar{R}$ is the average of $R_i$'s that are the ranges of samples [2], [4], [5].

In the fuzzy case, which is used in this paper, each sample is represented by a trapezoidal (or triangular) fuzzy number $(a, b, c, d)$ where $a \leq b \leq c \leq d$, has the membership function by the following equation [5]:

$$\mu_\lambda(x) = \begin{cases} 
0, & \text{if } x \leq a \text{ or } x \geq d \\
\frac{x-a}{b-a}, & \text{if } a < x \leq b \\
\frac{b-a}{1}, & \text{if } b < x \leq c \\
\frac{d-x}{d-c}, & \text{if } c < x \leq d 
\end{cases}$$

as shown in Figure 1.

![Figure 1: Representation of a sample by trapezoidal fuzzy numbers][6]
The main purpose of this study is to define a general architecture of fuzzy control chart with fuzzy control limits, which is provide a more accurate and flexible evaluation by each elementary component. Numeric control limits can be transformed to fuzzy control limits by using membership function. In this study, trapezoidal fuzzy numbers are represented as $(X_a, X_b, X_c, X_d)$ for each observation. Note that a trapezoidal fuzzy number becomes triangular when $(b = c)$. The control limits of fuzzy $\bar{X}$ control charts with ranges based on fuzzy trapezoidal number are calculated as follows:

The upper control limit is:

$$U\bar{CL}_x = \bar{CL}_x + A_2 \bar{R} = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) + A_2 (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$= (\bar{X}_a + A_2 \bar{R}_a, \bar{X}_b + A_2 \bar{R}_b, \bar{X}_c + A_2 \bar{R}_c, \bar{X}_d + A_2 \bar{R}_d)$$

$$= (U\bar{CL}_1, U\bar{CL}_2, U\bar{CL}_3, U\bar{CL}_4)$$

The central limit is:

$$\bar{CL}_x = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) = (\bar{CL}_1, \bar{CL}_2, \bar{CL}_3, \bar{CL}_4)$$

The lower control limit is:

$$L\bar{CL}_x = \bar{CL}_x - A_2 \bar{R} = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) - A_2 (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$= (\bar{X}_a - A_2 \bar{R}_a, \bar{X}_b - A_2 \bar{R}_b, \bar{X}_c - A_2 \bar{R}_c, \bar{X}_d - A_2 \bar{R}_d)$$

$$= (L\bar{CL}_1, L\bar{CL}_2, L\bar{CL}_3, L\bar{CL}_4)$$

Where $R_j = \sum_{i=1}^{m} R_{ij}$, $i = a, b, c, d$; $j = 1, 2, 3, ..., m$, the procedure for calculating $R_{ij}$ is as follows:

$$R_{aj} = X_{\text{max } aj} - X_{\text{Min } dj}$$

$$R_{bj} = X_{\text{max } bj} - X_{\text{Min } cj}$$

$$R_{cj} = X_{\text{max } cj} - X_{\text{Min } bj}$$

$$R_{dj} = X_{\text{max } dj} - X_{\text{Min } aj}$$

Where $(X_{\text{max } aj}, X_{\text{max } bj}, X_{\text{max } cj}, X_{\text{max } dj})$ is the maximum fuzzy number in the $j^{th}$ sample and $(X_{\text{min } aj}, X_{\text{min } bj}, X_{\text{min } cj}, X_{\text{min } dj})$ is the minimum fuzzy number in the $j^{th}$ sample, $j = 1, 2, 3, ..., m$ [7].
Then the trapezoidal fuzzy number is represented as follows figure:

3. THE STRUCTURE OF CONTROL LIMITS FOR $\alpha$-CUT FUZZY CONTROL CHART

An $\alpha$-cut comprises all elements whose membership degrees are greater than equal to $\alpha$. The set $A_\alpha = \{ x \in X; \mu_x (x) \geq \alpha, \ 0 \leq \alpha \leq 1 \}$. The $\alpha$-level sets $A_\alpha$ are also called the $\alpha$-cut sets [8]. Figure 3 shows a trapezoidal fuzzy number and it's $\alpha$-cut.

Applying an $\alpha$-cut to fuzzy $\tilde{X}$ control chart limits, then control limits based on ranges $(\tilde{UCL}, \tilde{CL}, \tilde{LCL})$ are determined as follows:

The upper control limit is:

$$U\tilde{CL}_X^\alpha = \tilde{C}\tilde{L}_X^\alpha + A_2 \bar{R}_X^\alpha = (\overline{X}_a, \overline{X}_b, \overline{X}_c, \overline{X}_d) + A_2 (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$= (\overline{X}_a + A_2 \bar{R}_a, \overline{X}_b + A_2 \bar{R}_b, \overline{X}_c + A_2 \bar{R}_c, \overline{X}_d + A_2 \bar{R}_d)$$

$$= (U\tilde{CL}_a^\alpha, U\tilde{CL}_b^\alpha, U\tilde{CL}_c^\alpha, U\tilde{CL}_d^\alpha)$$
The central limit is:

$$\tilde{C}L_{\alpha}^a = (\tilde{X}_a, \tilde{X}_b, \tilde{X}_c, \tilde{X}_d) = (\tilde{C}L_1, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4)$$

The lower control limit is:

$$L\tilde{C}L_{\alpha}^a = C\tilde{L}_{\alpha}^a - A_2 \tilde{R}_{\alpha}^a = (\tilde{X}_a, \tilde{X}_b, \tilde{X}_c, \tilde{X}_d) - A_2 (\tilde{R}_a, \tilde{R}_b, \tilde{R}_c, \tilde{R}_d)$$

$$= (\tilde{X}_a - A_2 \tilde{R}_a, \tilde{X}_b - A_2 \tilde{R}_b, \tilde{X}_c - A_2 \tilde{R}_c, \tilde{X}_d - A_2 \tilde{R}_d)$$

$$= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4)$$

Where:

$$\tilde{X}_a = \bar{X}_a + \alpha(\bar{X}_b - \bar{X}_a)$$

$$\tilde{X}_d = \bar{X}_d + \alpha(\bar{X}_d - \bar{X}_c)$$

$$\tilde{R}_a = \bar{R}_a + \alpha(\bar{R}_b - \bar{R}_a)$$

$$\tilde{R}_d = \bar{R}_d + \alpha(\bar{R}_d - \bar{R}_c)$$

The $\alpha$-cut fuzzy $\tilde{X}$ control limits based on ranges are shown in figure 4.

**FIGURE 4:** $\alpha$-Cut Fuzzy $\tilde{X}$ control limits based on ranges using fuzzy trapezoidal number.

4. **$\alpha$-CUT FUZZY $\tilde{X}$ CONTROL CHART BASED ON RANGES AT $\alpha$-CUT FUZZY MIDRANGE**

The $\alpha$-cut fuzzy midrange is one of the transformation techniques (among the four) used to transform the fuzzy set into scalar. It is used to check the production process, whether the
process is “in-control” or “out-of-control”. The control limits for $\alpha$-cut fuzzy midrange for $\alpha$-cut fuzzy $\tilde{X}$ control chart based on ranges can be obtained as follows:

$$U\tilde{C}L^\alpha_{\text{mr-}X} = C\tilde{L}^\alpha_{\text{mr-}X} + A_2 \left( \frac{\overline{R}_a + \overline{R}_d}{2} \right)$$

$$C\tilde{L}^\alpha_{\text{mr-}X} = f^\alpha_{\text{mr-}X}(C\tilde{L}) = \frac{\overline{X}_a + \overline{X}_d}{2}$$

$$L\tilde{C}L^\alpha_{\text{mr-}X} = C\tilde{L}^\alpha_{\text{mr-}X} - A_2 \left( \frac{\overline{R}_a + \overline{R}_d}{2} \right)$$

The definition of $\alpha$-cut fuzzy midrange of sample $j$ for fuzzy $\tilde{X}$ control chart is:

$$S^\alpha_{\text{mr-}X,j} = \frac{(X_{aj} + X_{dj}) + \alpha[(X_{bj} - X_{aj}) - (X_{dj} - X_{aj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \{ \text{incontrol}; \text{for } L\tilde{C}L^\alpha_{\text{mr-}X} \leq S^\alpha_{\text{mr-}X,j} \leq U\tilde{C}L^\alpha_{\text{mr-}X} \}$$

Based upon the value of $S^\alpha_{\text{mr-}X,j}$ for each sample, decision about the process can be made [4], [9].

5. FUZZY $\tilde{R}$ CONTROL CHART

The control limits for Shewhart $R$ control chart is given by:

$$UCL_R = D_4 \overline{R}$$

$$CL_R = \overline{R}$$

$$LCL_R = D_3 \overline{R}$$

Where $D_4$ and $D_3$ are control chart co-efficient, these co-efficient values are obtained by using the co-efficient table given by Montgomery [2005] [4], [6].

By using the traditional $R$ control chart procedure, the control limits for fuzzy $\tilde{R}$ control chart with trapezoidal fuzzy number is obtained as:

$$U\tilde{C}L_R = D_4(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$\tilde{C}L_R = (\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$L\tilde{C}L_R = D_3(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$
6. CONTROL LIMITS FOR $\alpha$-CUT FUZZY $\tilde{R}$ CONTROL CHART

The control limits of $\alpha$-cut fuzzy $\tilde{R}$ control chart based on trapezoidal fuzzy numbers are obtained as follows:

$$U\tilde{C}L_R^\alpha = D_4 (\tilde{R}_a^\alpha, \tilde{R}_b^\alpha, \tilde{R}_c^\alpha, \tilde{R}_d^\alpha)$$

$$\tilde{C}L_R^\alpha = (\tilde{R}_a^\alpha, \tilde{R}_b^\alpha, \tilde{R}_c^\alpha, \tilde{R}_d^\alpha)$$

$$L\tilde{C}L_R^\alpha = D_3 (\tilde{R}_a^\alpha, \tilde{R}_b^\alpha, \tilde{R}_c^\alpha, \tilde{R}_d^\alpha)$$

7. $\alpha$-CUT FUZZY $\tilde{R}$ CONTROL CHART AT $\alpha$–LEVEL FUZZY MIDRANGE

The control limits of $\alpha$-Level fuzzy midrange for $\alpha$-cut fuzzy $\tilde{R}$ control chart based on fuzzy trapezoidal numbers are defined by:

$$U\tilde{C}L_{mr-R}^\alpha = D_4 f_{mr-R}^\alpha (\tilde{C}L)$$

$$\tilde{C}L_{mr-R}^\alpha = f_{mr-R}^\alpha (\tilde{C}L) = \frac{\tilde{R}_a^\alpha + \tilde{R}_d^\alpha}{2}$$

$$L\tilde{C}L_{mr-R}^\alpha = D_3 f_{mr-R}^\alpha (\tilde{C}L)$$

Fuzzy transformation techniques are used for deciding if the process is “under-control” or “out-of-control” after calculating the control limits. The $\alpha$ - level fuzzy midrange of sample $j$ for fuzzy $\tilde{R}$ control chart can be transformed to crisp numbers with the fuzzy transformation techniques. In this paper, the fuzzy midrange transformation technique is used. The $\alpha$-level fuzzy midrange is defined as:

$$S_{mr-R,j}^\alpha = \frac{(R_{sj} + R_{dj}) + \alpha[(R_{bj} - R_{aj}) - (R_{dj} - R_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\begin{cases}
\text{Process in control; for } L\tilde{C}L_{mr-R}^\alpha \leq S_{mr-R,j}^\alpha \leq U\tilde{C}L_{mr-R}^\alpha \\
\text{Out of control; otherwise}
\end{cases}$$

The values of $S_{mr-R,j}^\alpha$ for all the samples are compared and decision about the process variability is made [4], [6], [9].

8. NUMERICAL ILLUSTRATION

In this section an application is considered to highlight the features of the above proposed fuzzy control charts. In this paper through a real illustrative data from Sulaimani Company for cement made in the city of Sulaimani, shows the designing of fuzzy control chart for process average of variable quality. The application was made on controlling the proportion of $CO_3$ component in the cement. Thirty samples with a sample size of 4 (the total measurement number is $4 \times 30 = 120$) were taken from the production process in Sulaimani Company. These measurements are converted into trapezoidal fuzzy numbers and given in Table 1.
TABLE 1: proportion of \( CO_3 \) in cement for 30 day.

For \( n = 4 \), the coefficient for different control charts are obtained from the Statistical Tables as \( A_2 = 0.729 \), \( D_4 = 2.282 \), \( D_3 = 0 \). These coefficients are used in constructing various control charts. By using fuzzy \( \bar{X} \) control chart based on ranges, we obtain the following results that the process is out of control for only 28\(^{th} \) and 29\(^{th} \) samples, otherwise, the process was under control with respect to \( \bar{X} \), figure (5) shows \( \bar{X} \)-chart of the average of \((X_a, X_b, X_c)\) with UCL, LCL. only point 27\(^{th} \) is out of control, the range between UCL and LCL is (1.07).
Figure 5: Shewhart $\bar{X}$ control chart based on rang.

Using equations of $\tilde{CL}$, $U\tilde{CL}$, $L\tilde{CL}$ for $\bar{X}$ control chart are determined as follows:
The fuzzy center line of the control data is:

$$\tilde{CL}\bar{X} = (\bar{X}_a, \bar{X}_b, \bar{X}_c) = (2.496, 2.706, 2.916)$$

$$\bar{R} = (\bar{R}_a, \bar{R}_b, \bar{R}_c) = (0.008, 0.428, 0.848)$$

The fuzzy control limits are:

$$U\tilde{CL}\bar{X} = \tilde{CL}\bar{X} + A_2 \bar{R} = (\bar{X}_a, \bar{X}_b, \bar{X}_c) + A_2 (\bar{R}_a, \bar{R}_b, \bar{R}_c) = (2.496, 2.706, 2.916) + 0.729(0.008, 0.428, 0.848) = (2.501, 3.018, 3.534)$$

$$L\tilde{CL}\bar{X} = \tilde{CL}\bar{X} - A_2 \bar{R} = (\bar{X}_a, \bar{X}_b, \bar{X}_c) - A_2 (\bar{R}_a, \bar{R}_b, \bar{R}_c) = (2.496, 2.706, 2.916) - 0.729(0.008, 0.428, 0.848) = (2.490, 2.393, 2.297)$$

Figure 6: The fuzzy control limits of $\bar{X}$ chart based on rang.
From Figure (6) it shows that the point (27th and 28th) out of control it clear that the rang between UCL, and LCL of X-chart less than fuzzy control chart.

By using $\alpha$ -Cut we get:

\[
\overline{X}_{a}^{0.65} = \overline{X}_{a} + 0.65(\overline{X}_{b} - \overline{X}_{a}) = 2.632
\]

\[
\overline{X}_{c}^{0.65} = \overline{X}_{c} + 0.65(\overline{X}_{c} - \overline{X}_{b}) = 2.779
\]

\[
\overline{R}_{a}^{0.65} = \overline{R}_{a} + 0.65(\overline{R}_{b} - \overline{R}_{a}) = 0.281
\]

\[
\overline{R}_{c}^{0.65} = \overline{R}_{c} + 0.65(\overline{R}_{c} - \overline{R}_{b}) = 0.575
\]

Then, $\tilde{CL}, U\tilde{CL}, L\tilde{CL} \ of \ \alpha - cut \ fuzzy \ \overline{X} \ control \ chart \ becomes:

The fuzzy center line of the control data is:

\[
\tilde{CL}_{\overline{X}}^{0.65} = (\overline{X}_{a}^{0.65}, \overline{X}_{b}^{0.65}, \overline{X}_{c}^{0.65}) = (2.632, 2.706, 2.779)
\]

\[
\overline{R} = (\overline{R}_{a}^{0.65}, \overline{R}_{b}^{0.65}, \overline{R}_{c}^{0.65}) = (0.281, 0.428, 0.575)
\]

The fuzzy control limits are:

\[
U\tilde{CL}_{\overline{X}}^{0.65} = \tilde{CL} + A_{2}\overline{R} = (\overline{X}_{a}, \overline{X}_{b}, \overline{X}_{c}) + A_{2} (\overline{R}_{a}, \overline{R}_{b}, \overline{R}_{c})
\]

\[
= (2.837, 3.018, 3.198)
\]

\[
L\tilde{CL}_{\overline{X}}^{0.65} = \tilde{CL} - A_{2}\overline{R} = (\overline{X}_{a}, \overline{X}_{b}, \overline{X}_{c}) - A_{2} (\overline{R}_{a}, \overline{R}_{b}, \overline{R}_{c})
\]

\[
= (2.427, 2.393, 2.3598)
\]

Now using $\alpha$ - level fuzzy midrange techniques for fuzzy $\overline{X}$ control chart to transform to crisp numbers as:

\[
U\tilde{CL}_{mrx-\overline{X}}^{0.65} = \tilde{CL}_{mrx-\overline{X}}^{0.65} + A_{2} \left( \frac{\overline{R}_{a}^{0.65} + \overline{R}_{c}^{0.65}}{2} \right) = 3.018
\]

\[
\tilde{CL}_{mrx-\overline{X}}^{0.65} = f_{mrx-\overline{X}}^{0.65}(\tilde{CL}) = \left( \frac{\overline{X}_{a}^{0.65} + \overline{X}_{c}^{0.65}}{2} \right)
\]

\[
L\tilde{CL}_{mrx-\overline{X}}^{0.65} = \tilde{CL}_{mrx-\overline{X}}^{0.65} - A_{2} \left( \frac{\overline{R}_{a}^{0.65} + \overline{R}_{c}^{0.65}}{2} \right) = 2.39
\]
The definition of $\alpha$-cut fuzzy midrange of sample $j$ for fuzzy $\tilde{X}$ control chart is:

$$S_{\text{mr-}X,j}^\alpha = \frac{(X_{x_{ij}} + X_{y_{ij}}) + \alpha[(X_{x_{ij}} - X_{y_{ij}}) - (X_{y_{ij}} - X_{x_{ij}})]}{2}$$

We get our decision as follows in table (2):

<table>
<thead>
<tr>
<th>sample</th>
<th>$S_{\text{mr-}X,j}^\alpha$</th>
<th>2.393 $\leq S_{\text{mr-}X,j}^\alpha &lt; 3.018$</th>
<th>sample</th>
<th>$S_{\text{mr-}X,j}^\alpha$</th>
<th>2.393 $\leq S_{\text{mr-}X,j}^\alpha &lt; 3.018$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6875</td>
<td>In control</td>
<td>16</td>
<td>2.5625</td>
<td>In control</td>
</tr>
<tr>
<td>2</td>
<td>2.605</td>
<td>In control</td>
<td>17</td>
<td>2.7</td>
<td>In control</td>
</tr>
<tr>
<td>3</td>
<td>2.6625</td>
<td>In control</td>
<td>18</td>
<td>2.61</td>
<td>In control</td>
</tr>
<tr>
<td>4</td>
<td>2.4575</td>
<td>In control</td>
<td>19</td>
<td>3.05</td>
<td>Out control</td>
</tr>
<tr>
<td>5</td>
<td>2.7375</td>
<td>In control</td>
<td>20</td>
<td>2.8825</td>
<td>In control</td>
</tr>
<tr>
<td>6</td>
<td>2.8</td>
<td>In control</td>
<td>21</td>
<td>2.965</td>
<td>In control</td>
</tr>
<tr>
<td>7</td>
<td>2.5875</td>
<td>In control</td>
<td>22</td>
<td>2.585</td>
<td>In control</td>
</tr>
<tr>
<td>8</td>
<td>2.7775</td>
<td>In control</td>
<td>23</td>
<td>3.0175</td>
<td>In control</td>
</tr>
<tr>
<td>9</td>
<td>2.6575</td>
<td>In control</td>
<td>24</td>
<td>2.585</td>
<td>In control</td>
</tr>
<tr>
<td>10</td>
<td>2.4275</td>
<td>In control</td>
<td>25</td>
<td>2.475</td>
<td>In control</td>
</tr>
<tr>
<td>11</td>
<td>2.7475</td>
<td>In control</td>
<td>26</td>
<td>2.5675</td>
<td>In control</td>
</tr>
<tr>
<td>12</td>
<td>2.7425</td>
<td>In control</td>
<td>27</td>
<td>2.5725</td>
<td>In control</td>
</tr>
<tr>
<td>13</td>
<td>2.37</td>
<td>Out control</td>
<td>28</td>
<td>3.1075</td>
<td>Out control</td>
</tr>
<tr>
<td>14</td>
<td>2.82</td>
<td>In control</td>
<td>29</td>
<td>3.0775</td>
<td>Out control</td>
</tr>
<tr>
<td>15</td>
<td>2.965</td>
<td>In control</td>
<td>30</td>
<td>2.6625</td>
<td>In control</td>
</tr>
</tbody>
</table>

**TABLE (2):** The decision using $\alpha$-level fuzzy midrange of $\alpha$-cut fuzzy $\tilde{X}$.

As shown in the above table, the process is out of control for $13^{th}$, $19^{th}$, $28^{th}$, and $29^{th}$ samples, otherwise, the process was under control with respect to $S_{\text{mr-}X,j}^{0.65}$. This chart is shown in the following figure:

**FIGURE 7:** $\alpha$-cut fuzzy $\tilde{X}$ control chart based on ranges at $\alpha$-cut fuzzy midrange.
Another way to construct the fuzzy control limits is to use the sample range as an estimate of the variability of the process. Remember that the range is simply the difference between the largest and smallest values in the sample. The spread of the range can tell us about the variability of the data.

The fuzzy control limits for Shewharts R control chart are given by:

\[
\tilde{C}_L_R = (\bar{R}_a, \bar{R}_b, \bar{R}_c) = (0.008, 0.428, 0.848)
\]

\[
U\tilde{C}_L_R = D_4 (\bar{R}_a, \bar{R}_b, \bar{R}_c)
\]

\[
= 2.282(0.008, 0.428, 0.848)
\]

\[
= (0.0183, 0.9767, 1.9351)
\]

\[
L\tilde{C}_L_R = D_3 (\bar{R}_a, \bar{R}_b, \bar{R}_c)
\]

\[
= 0(0.008, 0.428, 0.848)
\]

\[
= (0, 0, 0)
\]

By using \(\alpha\)-Cut we get:

\[
\bar{R}_{a}^{0.65} = \bar{R}_a + 0.65(\bar{R}_b - \bar{R}_a) = 0.281
\]

\[
\bar{R}_{c}^{0.65} = \bar{R}_c + 0.65(\bar{R}_c - \bar{R}_b) = 0.575
\]

Then, the control limits of \(\alpha\)-cut fuzzy \(\tilde{R}\) control chart given by:

\[
\tilde{C}_L_{R}^{0.65} = (\bar{R}_{a}^{0.65}, \bar{R}_{b}^{0.65}, \bar{R}_{c}^{0.65}) = (0.281, 0.428, 0.575)
\]

\[
U\tilde{C}_L_{R}^{0.65} = D_4 (\bar{R}_{a}^{0.65}, \bar{R}_{b}^{0.65}, \bar{R}_{c}^{0.65})
\]

\[
= 2.282(0.281, 0.428, 0.575)
\]

\[
= (0.6412, 0.9767, 1.312)
\]

\[
L\tilde{C}_L_{R}^{0.65} = D_3 (\bar{R}_{a}^{0.65}, \bar{R}_{b}^{0.65}, \bar{R}_{c}^{0.65})
\]

\[
= 0(0.281, 0.428, 0.575)
\]

\[
= (0, 0, 0)
\]
Now using $\alpha$-level fuzzy midrange techniques for fuzzy $\tilde{R}$ control chart to transform to crisp numbers as:

$$U\tilde{C}_L^{0.65} = D_4 f_{\mu_R}(\tilde{C}_L) = D_4 \left(\frac{\bar{R}_u^{0.65} + \bar{R}_c^{0.65}}{2}\right) = 0.9767$$

$$\tilde{C}_L^{0.65} = f_{\mu_R}(\tilde{C}_L) = \frac{\bar{R}_u^{0.65} + \bar{R}_c^{0.65}}{2} = 0.428$$

$$L\tilde{C}_L^{0.65} = D_3 f_{\mu_R}(\tilde{C}_L) = 0$$

The value of $\alpha$-level fuzzy midrange of $\alpha$-cut fuzzy $\tilde{R}$ control chart are given in table (3):

<table>
<thead>
<tr>
<th>sample</th>
<th>$S_{\mu_R}^{\alpha}$</th>
<th>$0 &lt; S_{\mu_R}^{\alpha} &lt; 0.98$</th>
<th>sample</th>
<th>$S_{\mu_R}^{\alpha}$</th>
<th>$0 &lt; S_{\mu_R}^{\alpha} &lt; 0.98$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17</td>
<td>In control</td>
<td>16</td>
<td>0.59</td>
<td>In control</td>
</tr>
<tr>
<td>2</td>
<td>0.22</td>
<td>In control</td>
<td>17</td>
<td>0.3</td>
<td>In control</td>
</tr>
<tr>
<td>3</td>
<td>0.07</td>
<td>In control</td>
<td>18</td>
<td>0.18</td>
<td>In control</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>In control</td>
<td>19</td>
<td>1.3</td>
<td>Out control</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>In control</td>
<td>20</td>
<td>0.41</td>
<td>In control</td>
</tr>
<tr>
<td>6</td>
<td>0.17</td>
<td>In control</td>
<td>21</td>
<td>0.4</td>
<td>In control</td>
</tr>
<tr>
<td>7</td>
<td>0.36</td>
<td>In control</td>
<td>22</td>
<td>0.79</td>
<td>In control</td>
</tr>
<tr>
<td>8</td>
<td>0.26</td>
<td>In control</td>
<td>23</td>
<td>1.4</td>
<td>Out control</td>
</tr>
<tr>
<td>9</td>
<td>0.16</td>
<td>In control</td>
<td>24</td>
<td>0.43</td>
<td>In control</td>
</tr>
<tr>
<td>10</td>
<td>0.52</td>
<td>In control</td>
<td>25</td>
<td>0.57</td>
<td>In control</td>
</tr>
<tr>
<td>11</td>
<td>0.3</td>
<td>In control</td>
<td>26</td>
<td>0.07</td>
<td>In control</td>
</tr>
<tr>
<td>12</td>
<td>0.97</td>
<td>In control</td>
<td>27</td>
<td>0.6</td>
<td>In control</td>
</tr>
<tr>
<td>13</td>
<td>0.37</td>
<td>In control</td>
<td>28</td>
<td>0.27</td>
<td>In control</td>
</tr>
<tr>
<td>14</td>
<td>0.43</td>
<td>In control</td>
<td>29</td>
<td>0.2</td>
<td>In control</td>
</tr>
<tr>
<td>15</td>
<td>0.4</td>
<td>In control</td>
<td>30</td>
<td>0.07</td>
<td>In control</td>
</tr>
</tbody>
</table>

**TABLE (3):** The decision using $\alpha$-level fuzzy midrange of $\alpha$-cut fuzzy $\tilde{R}$.

Above table shows that the process was in control with respect to $S_{\mu_R}^{0.65}$ for each sample except samples $19^{th}$ and $23^{th}$ out of control, as shown in figure 8. So these fuzzy control limits can be used to control the production process and detect small deviations.

![FIGURE 8: $\alpha$-cut fuzzy $\tilde{R}$ control chart at $\alpha$-cut fuzzy midrange.](image-url)
We note that fuzzy observations and fuzzy control limits can provide more flexibility for controlling a process, since reveal small deviations in the production process in addition to large deviations which is important to reducing the deviations between observations.

9. CONCLUSION
This paper shows that fuzzy set theory is useful tool to handle uncertainty and it applicable on traditional variable control charts, such that fuzzy control charts developed for linguistic data that are mainly based on membership and probabilistic approaches and $\alpha$-cut control charts for limits chart are developed. Fuzzy control charts (Fuzzy control limits) is very effective to identify the signals in the variable control charts, it can provide more flexibility for controlling process and have more appropriate mathematical description frame than control chart approach and give more meaning results than traditional quality control charts. The aim of this study is to present the theoretical structure of the “$\alpha$-level fuzzy midrange for the $\alpha$-cut fuzzy control chart”, its reveal small deviations in the production process in addition to large deviations which is important to reducing the deviations between observations.

10. REFERENCES
[4] A. Pandurangan, and R. Varadharajan. “Construction of $\alpha$-cut Fuzzy $\bar{X} - \bar{R}$ and $\bar{X} - \bar{S}$ Control Charts Using Fuzzy Trapezoidal Number”. Vol. 9, Issue 1, pp. 100-111, 2011.
INSTRUCTIONS TO CONTRIBUTORS

*International Journal of Scientific and Statistical Computing (IJSSC)* aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas. Submissions must reflect important developments, extensions, and applications in statistical modeling. IJSSC also encourages submissions that describe scientifically interesting, complex or novel statistical modeling aspects from a wide diversity of disciplines, and submissions that embrace the diversity of scientific and statistical modeling.

IJSSC goal is to be multidisciplinary in nature, promoting the cross-fertilization of ideas between scientific computation and statistical computation. IJSSC is refereed journal and invites researchers, practitioners to submit their research work that reflect new methodology on new computational and statistical modeling ideas, practical applications on interesting problems which are addressed using an existing or a novel adaptation of an computational and statistical modeling techniques and tutorials & reviews with papers on recent and cutting edge topics in computational and statistical concepts.

To build its International reputation, we are disseminating the publication information through Google Books, Google Scholar, Directory of Open Access Journals (DOAJ), Open J Gate, ScientificCommons, Docstoc and many more. Our International Editors are working on establishing ISI listing and a good impact factor for IJSSC.

**IJSSC LIST OF TOPICS**
The realm of *International Journal of Scientific and Statistical Computing (IJSSC)* extends, but not limited, to the following:

- Annotated Bibliography of Articles for the Statistics
- Bibliography for Computational Probability and Statistics
- Current Index to Statistics
- Guide to Statistical Computing
- Solving Non-Linear Systems
- Statistics and Statistical Graphics
- Theory and Applications of Statistics and Probability
- Annotated Bibliography of Articles for the Statistics
- Bibliography for Computational Probability and Statistics
- Current Index to Statistics
- Annals of Statistics
- Computational Statistics
- Environment of Statistical Computing
- Mathematics of Scientific Computing
- Statistical Computation and Simulation
- Symbolic computation
- Annals of Statistics
- Computational Statistics
- Environment of Statistical Computing
CALL FOR PAPERS

Volume: 6 - Issue: 3

i. Paper Submission: April 30, 2015

ii. Author Notification: May 31, 2015

iii. Issue Publication: June 2015
CONTACT INFORMATION

Computer Science Journals Sdn Bhd
B-5-8 Plaza Mont Kiara, Mont Kiara
50480, Kuala Lumpur, MALAYSIA

Phone: 006 03 6204 5627
Fax: 006 03 6204 5628

Email: cscpress@cscjournals.org