Language as a renewable resource: Import, dissipation, and absorption of innovations*

Bengt-Arne Wickström *wickstr@hu-berlin.de*
Institut für Finanzwissenschaft
Humboldt-Universität zu Berlin
DE-10099 Berlin, Germany
and
School of Economics and Management
Free University of Bozen-Bolzano
IT-39100 Bozen-Bolzano, Italy

Abstract

The structural stability of different languages subject to the import of external elements is analyzed. We focus on the temporal side of the different processes interacting to produce a change in the structure of the language. That is, the rate of import and dissipation of new elements is seen in relation to the rate at which a language absorbs such new elements into its structure. The analysis leads to a model that in the steady state is formally similar to the standard model used to analyze the extraction of renewable natural resources.

This model is applied to different sociolinguistic situations and we speculate about how the structural type of a language might influence its rate of adaptation of the external innovations and how the cultural and social status of the idiom (partially) determines the rate of import of such innovations. Conditions that might lead to attrition and decay of the linguistic system, are characterized and some policy implications are drawn.

The model is presented as a theoretical model with only a few illustrative simulations. However, the structure is such that it can easily be adapted to computational methods and used in simulations. With obvious extensions, sociolinguistically more complex (and realistic) situations can be modeled.

Keywords: Language contact, borrowing, language structure, language status, language shift, language attrition, language planning, renewable resource.

1. INTRODUCTION

Languages have always changed and influenced one another. The vocabulary of high-status languages, especially, has entered and enriched languages of a lower status. The influence of Latin and Greek (directly or via other languages) on the Germanic languages, for instance, has been enormous and we would today be unable to manage in everyday life without using this "imported" vocabulary. It has become an integral part of the language. No normal user of English finds anything foreign or strange in words like "language", "change", "influence", "special", "vocabulary", "status", or "enter", just to mention a few, all taken from the first two sentences of this essay.

All these words have been nostrified into English and are today normal English words. To what degree the structure of English has changed in the process, and how sudden such changes were, is a question that we cannot discuss in detail here. However, there is some evidence that the transfer from Anglo-Saxon to English was not quite smooth. What we want to analyze in this essay is, how languages

*I am indebted to Helmar G. Frank, who gave me the idea leading up to the basic analytic approach of this essay. Some of the phenomena analyzed here were also discussed in a much simpler framework in German in [1]. I gratefully acknowledge the constructive suggestions of Jens Barthel and Sjur Flåm, which have considerably contributed to improving the quality of the analysis. I had the opportunity to work on this essay during a stay at Fudan University. I thank my host Li Weisen and his colleagues for an interesting and inspiring visit under excellent working conditions. Last, but not least, I am very grateful to Sonja Boden and Judith Wickström for halting and reversing the attrition of my English idiolect in this essay.

manage to import external elements and incorporate these into the structure without causing sudden structural breaks.

We will focus on two aspects of the imported elements, which we will call innovations. Initially, they are introduced by some individuals using them. Through imitation more and more individuals make use of these elements in their social intercourse and the innovations become more and more common. Then the process of absorption into the structure of the language sets in and at the end the innovations are integral parts of the idiom. The diffusion of the intruding elements is modeled as a random-walk stochastic process. The nostrification we see as a Poisson process.

We show that, depending on the parameter values of the model, a language can develop smoothly or be subject to sudden structural changes. We also speculate on the issue which structures are more vulnerable to external intruding elements than others.

The rest of the essay is organized as follows. In section 2, we give a brief overview of some relevant findings in the area of contact linguistics. This is primarily based on some well-known recent standard texts. A formal model is constructed and analyzed in section 3. In the main text, the model is presented in a verbal, non-technical manner, and all technical derivations are relegated to three appendices. Some predictions based on the model are presented in section 4 and the essay closes in section 6 with an outlook.

2. LANGUAGE CONTACT

In the literature on contact linguistics, several different phenomena and approaches related to the influence of one language on another are discussed and analyzed.[1] One can divide the analyses of contact linguistics into three broad categories: "borrowing" of both lexical and structural material from one language into another; language shift in bilingual situations; and the emergence of new languages through the fusion of two (or more) languages. A classic example of a language shift is when – in the community of Hungarian speakers in Burgenland – individuals first became bilinguals in Hungarian and German and then monolinguals in German over a few generations, abandoning the use of Hungarian in one domain after the other.[2] The emergence of new languages, we find mainly in the rise of pidgins and creoles.[3]

In this essay, we are concerned with the "borrowing" from other languages. Various terms are used in the literature to discuss changes in the structure of the language system due to the import of elements from other languages: code switching, code mixing, borrowing, transference etc.[4] We will talk about imported elements as externally induced innovations in the lexicon, phonology, morphology, and other aspects of the structure of the language.[5] Our focus is on the temporal process the innovation goes through: it enters the language, is dissipated among the speakers, being initially felt by them to be a "foreign" element which over time is slowly absorbed, "nostrified", into the linguistic system (or rejected and disappearing).

We assume that the flow of foreign elements entering a language and spreading among its users is primarily determined by the sociolinguistic situation, whereas the structure of the affected language to a large extent determines the rate at which the elements are absorbed into it. In this way, we attempt to provide a synthesis of the apparent opposite viewpoints of the determination of the acceptance of foreign material into a language. In order to model the importation in this manner, we need to focus on the time dimension of the import process. This process is made up of two distinct sub-processes. On the one hand, there is the diffusion of the usage of the imported elements in the language commu-

[1] We do not attempt to provide a systematic or representative review of this field here. Only the concepts relevant to our analysis are referred to. For a systematic overview, the reader is referred to a comprehensive treatise on the field of contact linguistics, for instance to [2–4] and the many references therein.

[2] See the study of [5]. A comprehensive treatise on – among other aspects of language change – language shift of immigrants in Australia is [3]. In this area there also exists a number of formal models, notably: [6–11].

[3] A standard text is [12, 13].

[4] See [3], chapter 3.

[5] [2] in chapter 4 discuss the various levels of borrowing as a function of the cultural pressure the dominant language exerts on the recipient one.

nity, and on the other hand, there is the absorption of the imported material into the structure of the language. As it turns out, the exogenous, sociologically determined, influx of new material can – in a dynamic steady-state – be analytically separated from the endogenous, structurally determined, rate of nostrification of the material and analyzed in a very simple model.

In the literature, language change and language shift often go hand-in-hand. The influence of a dominant language forces a minority tongue to leave one domain after the other, leading to attrition and decay as the speakers slowly stop using the language and switch to the dominant one. In this essay, we are primarily interested in how resistant the importing language and its speakers are to such external influences. This resistance we see as determined both by the rate of influx and diffusion (the sociolinguistic aspect) and the rate of absorption (determined both by the linguistic structure and the social rôle of the language). There can, however, be considerable, but stable changes in a language over time without attrition or decay as the well-known examples of the Balkan Sprachbund[6] or Cappadocian Greek[7] demonstrate. By focusing on language change, we are not directly concerned with the shift aspect of the problem. In many cases, where the innovations lead to attrition and decay, however, the shift is implied.[8] This might very well potentially be the most interesting application of this essay. The detailed modeling of the diffusion process is very flexible and can easily be modified to approximate many real-life situations.

2.1 Rate of borrowing

In the literature, two main explanations of borrowing are discussed. On the one hand, the social and cultural situation is seen as the most important factor behind the import of features from one language into another, as a rule from a dominant "high-status" language to an idiom of lower social or cultural status.[9] On the other hand, also the structure of the importing language is regarded as a determinant of the ease of import of different linguistic material.[10] This latter aspect, we call absorption. We provide a simple framework where both of these aspects are taken into account and interacting with one another.

We, hence, analytically separate the rate of influx of innovations from their nostrification. It is then natural to model this influx primarily as a function of the relative social and cultural status of the donor and recipient languages.

2.2 Diffusion

As already mentioned, the time aspect is very important for our arguments. Hence, the diffusion of an innovation in the population as a function of time is at the core of the model. The classical treatment of the diffusion of innovations in the social-science literature is [16]. In sociolinguistics one of the first models using diffusion methods studies the spread of different sound changes in Chinese syllables that are traced from one type of syllable to another.[11] This author finds the typical S-shaped curve with the change first slowly spreading to a few syllables then accelerating and then slowing down again as it affects the last non-affected syllables (or stops before the change is universal). This can be called W-diffusion: a certain property is spread from one part of the lexicon to another. The spread from speaker to speaker can be termed S-diffusion.[12] This is what we are concerned with.

Most models are driven by an assumption that diffusion occurs by contact and imitation. These models are as a rule deterministic, modeling changes in fractions of users of innovations deterministically.[13] A consequence of this is that an innovation continuously spreads until it is adopted by all potential users.[14]

[6] Cf. [4], chapter 3.
[7] Cf. [4], chapter 7.
[8] Decreolisation, that [14] in chapter 11 refers to as "language suicide ", is an extreme example.
[9] See [15]; [2], chapters 2, 3, and 4; as well as [4], chapter 2.
[10] [4], chapter 2. In addition, there is an "implication table" of the order at which various types of elements are imported as the influence of the exporting language on the importing one grows: first lexicon, then phonology, and, finally, syntax and morphology. See the "borrowing scale" in [2], chapter 4.
[11] See [17].
[12] See [18].
[13] This is also the case in [1].
[14] Some fraction of speakers might be totally resistant and under no circumstances accept the innovation.

The process only goes one way. By modeling the diffusion as a stochastic process, we avoid this problem and the fraction of speakers using the innovation at any time is a stochastic variable taking a random walk, whose expected value is the average number of users.[15] In addition, using a stochastic process, would allow us to model different individual behavior in a more realistic fashion. It is a well-established fact that innovations spread at a different rate in different social groups and cross the borders of different social groups with different propensities.[16] In the stochastic modeling, this is easily accommodated by choosing different adoption and rejection probabilities for different sociologically determined groups of persons and by making the probabilities of encounters between individuals belonging to different groups group dependent.

2.3 Absorption

With the absorption of an innovation, we imagine the step from adoption to adaptation.[17] The phenomenon is treated in the literature,[18] but we know of no study investigating the temporal side of this process. Our assumption is that with repetitive usage individuals adapt the innovation phonologically, morphologically etc. until it has become an integral part of the receiving language in the view of its speakers. In the absence of any specific information about the absorption process, we make the simplest possible assumption about the expected time it takes for an innovation to be absorbed: it is directly determined by the frequency of its usage. That is, at each encounter involving the innovation there is a certain given probability that it will be adapted to the structure of the receiving language. The speed of this process is supposed to capture the various structurally determined constraints on the import of external elements.

We speculate that the resistance to adaptation is both a sociolinguistic issue and a matter of the structure of the receiving language. The greater the number of steps (phonological, morphological etc.) an innovation has to go through to be integrated into the language, the slower the adaptation process is assumed to be.

3. THE FORMAL MODEL

We first model how an intrusion or an innovation dissipates through society. At any time, for a given innovation, there are three types of individuals in the language community. F persons use the innovation, but consider it a foreign element in their language; A individuals use the innovation and consider it an integral part of the language; and R persons do not use it. With P we denote the sum of F and A and N is the total size of the language community:

$$N = P + R = F + A + R \tag{3.1}$$

For the sake of analysis, we assume that the contacts between the individuals occur pairwise and are consecutively numbered by θ. By assuming that there is a fixed number of transactions per unit of time, we will transfer the model into continuous time. At each encounter, the individuals mutate between the three groups with certain probabilities. Per unit of time, an exogenously determined number of innovations enter the language. We define the heterogeneity of the language as the sum of the number of individuals using an innovation without considering it an integral part of the idiom, *i. e.*, the sum of the F over all innovations.

We look for steady states of this system.

[15] See figures 1 and 2.
[16] See [15].
[17] See [14], chapter 8.
[18] See, for instance, [4], chapter 2.

3.1 Spread of innovations

An innovation – a new word, say – is assumed to enter the language from outside creating an F individual.[19] This individual interacts with other speakers and the innovation is then adopted with a certain probability by such a person after an encounter. Specifically, an RR encounter does not influence the spread of the innovation; a PP encounter does not influence the spread of the innovation, but it can influence the nostrification, *i. e.* the relative sizes of A and F. A PR encounter, on the other hand, influences the spread of the innovation: with probability α, the R individual mutates into a P (F or A) individual and with probability β, the P individual mutates into an R individual; with probability $(1-\alpha-\beta)$, no mutation takes place. We further define γ as β/α.

That is, the spread of innovations is due to imitation. In appendix A the dynamics of the probability density of the distribution of the P's, $\delta^P(\theta)$, and the size of the expected value of P, $\bar{P}$, are found, as well as the fraction $\bar{p} := \bar{P}/N$:

$$\bar{P}(\theta+1) - \bar{P}(\theta) = 2\alpha(1-\gamma)\left[\bar{P}(\theta)\,\frac{N-\bar{P}(\theta)}{(N-1)\,N} - \frac{\sigma_P^2(\theta)}{(N-1)\,N}\right] \tag{3.2}$$

and, as a function of the "age" τ of the innovation in the language:

$$\dot{\bar{p}}(\tau) = 2\omega\alpha(1-\gamma)\,\frac{N}{N-1}\left\{[1-\bar{p}(\tau)]\,\bar{p}(\tau) - \sigma_p^2(\tau)\right\} \tag{3.3}$$

The parameter ω is the number of encounters per unit of time and number of individuals in the population.

It is clear that $\bar{p}(\tau)$ has the expected S-form. Figure 1 shows the probability density of $\bar{P}$ for different values of θ. Here, we have set $N = 10, \alpha = 0.075$, and $\beta = 0.0075$. It is interesting to note that the mass of the probability density is concentrated at the lower end as well as at the higher end of the distribution. With time the concentration at the higher end increases and at the lower end decreases. That is, a typical innovation is either used by a few people or disappears or is, after a short time, used by virtually everyone. This is in agreement with the general findings in diffusion analysis; an innovation spreads slowly in the initial phase, then very rapidly in a middle phase, and at the end of the diffusion process it reaches the last potential users very slowly.[20] In figure 2 the expected number of users of the innovation, *i. e.*, the resulting average dissipation of an innovation as a function of the number of encounters is depicted for different values of α and β.

3.2 Absorption

The absorption is modeled as a spontaneous mutation of an individual who has adopted the innovation into an individual who has adopted it and for whom it is no more a foreign element of the language.[21] If the probability of such a mutation of an F individual in any period is given by λ and the number of individuals having adopted the innovation at the beginning of period θ with probability $\delta^P(\theta)$ is $P(\theta) = F^*(\theta) + A^*(\theta)$, then at the end of the period the expected number of $F(\theta)$ is $(1-\lambda)F^*(\theta)$ and the expected number of A is $A^*(\theta) + \lambda F^*(\theta)$. In other words, since the absorption process is stochastically independent of the diffusion process, the expected value of F can be written as:

$$\bar{F}(\theta) = \sum_{P=0}^{N} \delta^P(\theta)(1-\lambda)^\theta P = (1-\lambda)^\theta \sum_{P=0}^{N} \delta^P(\theta) P = (1-\lambda)^\theta \bar{P}(\theta) \tag{3.4}$$

[19] Of course, there are also innovations from the "inside". These are part of the system of the language, however, and do not need to be absorbed into the language system. They start as an A individual and can be assumed to spread in the same way as external innovations.

[20] See [16].

[21] This leads to a Poisson process. Such processes are used, for instance, to model nuclear decay.

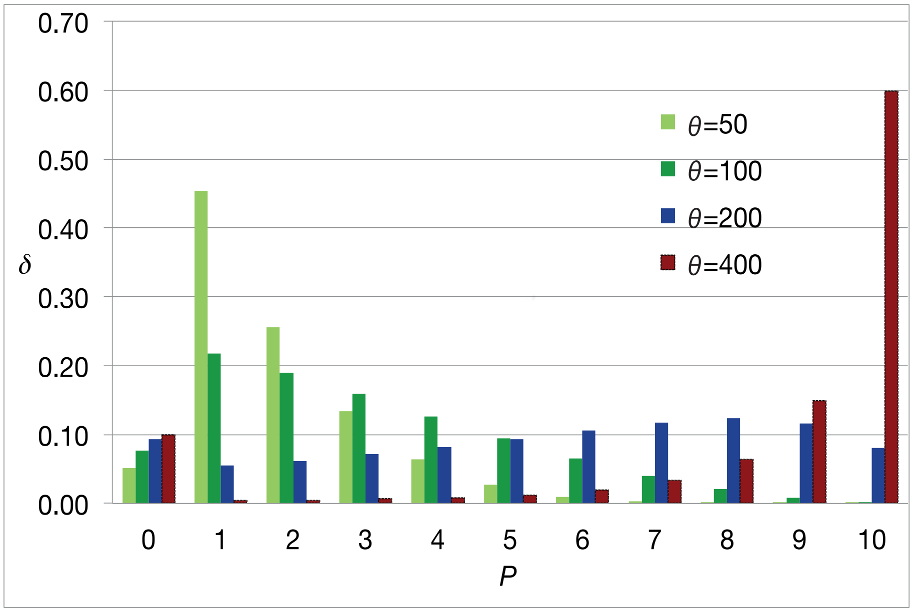

FIGURE 1: Probability densities of the diffusion of an innovation

This is the expected number of users of the innovation who consider it a foreign element. In figure 3 the expected value of $\bar{F}$ as a function of θ is shown for $\lambda = 0.002$ and the same values of α and β as in figure 2.

If λ is not stationary, but changes due to external influences from one encounter to the next, we have to number the encounters independently of the "encounter age" θ of each individual innovation. This absolute numbering of encounters we denote by η and equation 3.4 will then be written as:

$$\bar{F}(\theta, \eta) = \prod_{i=1}^{\theta} [1 - \lambda(\eta - i + 1)] \sum_{P=0}^{N} \delta^P(\theta) P \tag{3.5}$$

With a suitable choice of units, we write the expected fraction of the population using the innovation and considering it a foreign element as a function of time t and the "age" τ of the innovation in the language, $\bar{f}(\tau, t)$:

$$\bar{f}(\tau, t) = e^{Q(\tau,t)} \bar{p}(\tau) \tag{3.6}$$

The function Q is given by:

$$Q(\tau, t) := \rho \int_0^{\tau} \ln [1 - \lambda(t - \kappa)] \, d\kappa \tag{3.7}$$

The positive constant ρ in this expression is related to the number of encounters per unit of time.

3.3 Heterogeneity

We will call $\bar{f}$ the contribution of this innovation to the heterogeneity of the language. Since there is a steady stream of innovations at the rate $n(t)$ entering the language over time, the total heterogeneity at any time t, $H(t)$, can be defined as:

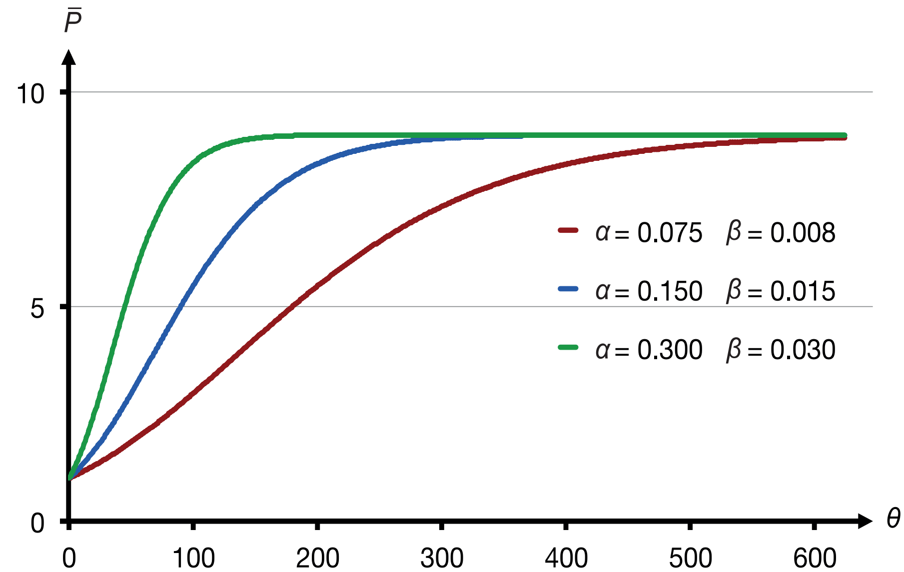

FIGURE 2: Average dissipation of an innovation

$$H(t) := \int_0^\infty n(t-\tau)\bar{f}(\tau,t)d\tau = \int_0^\infty n(t-\tau)\, e^{Q(\tau,t)}\bar{p}(\tau)d\tau \tag{3.8}$$

We assume that the rate of absorption depends on the heterogeneity. That is, a very heterogeneous language has a lower rate of absorption than a homogeneous language:

$$\lambda(t) = \tilde{\lambda}\left[H(t)\right], \qquad \frac{\partial\tilde{\lambda}}{\partial H} \leq 0 \tag{3.9}$$

Substituting ω for $t-\tau$, we find:

$$H(t) = \int_{-\infty}^{t} n(\omega)\, e^{Q(t-\omega,t)}\bar{p}(t-\omega)d\omega \tag{3.10}$$

We have in 3.10 an integral equation for the heterogeneity of the language. In the following, we will characterize its solution for an exogenous stream of innovations $n(t)$.

3.4 The dynamics of the heterogeneity

In appendix B it is shown that the dynamics of H can be expressed by:

$$\begin{aligned} \dot{H} \;=\; & \rho \ln\left\{1-\tilde{\lambda}\left[H(t)\right]\right\} H(t) - n(t)\rho \int_0^\infty \ln\left\{1-\tilde{\lambda}\left[H(t-\tau)\right]\right\} e^{Q(\tau,t)}\bar{p}(\tau)d\tau \\ + & \int_0^\infty \Bigg[n(t-\tau)\left(Q_2 + \rho\ln\frac{1-\tilde{\lambda}\left[H(t-\tau)\right]}{1-\tilde{\lambda}\left[H(t)\right]}\right) + n'(t-\tau) \\ & + \rho\Big[n(t)-n(t-\tau)\Big]\ln\left\{1-\tilde{\lambda}\left[H(t-\tau)\right]\right\}\Bigg] e^{Q(\tau,t)}\bar{p}(\tau)d\tau \end{aligned} \tag{3.11}$$

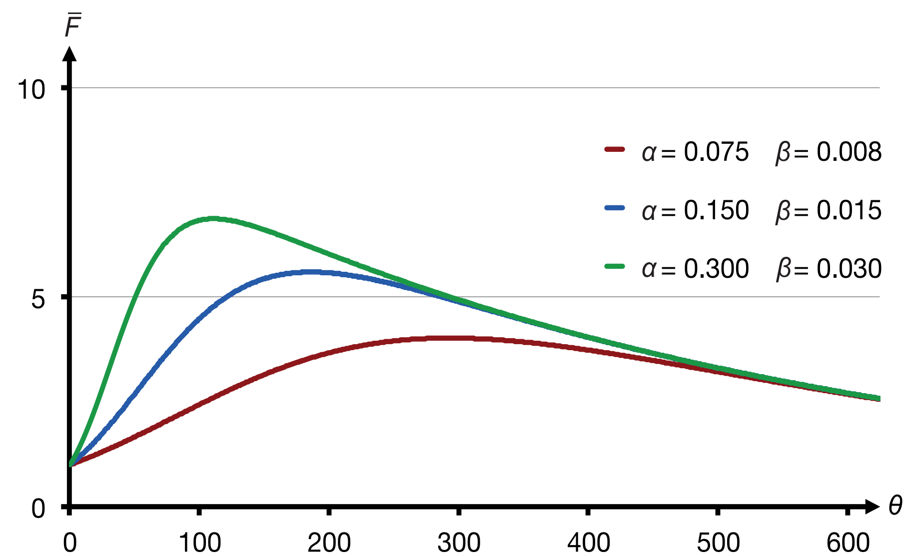

FIGURE 3: Expected number of users of a non-absorbed innovation with $\lambda = 0.002$

This expression is quite complicated and ultimately determined by the history of n. In other words, we would have to use the exogenous path of n to find the path of H from some initial value, H_0, with the help of this equation. To make the analysis tractable we will, however, limit ourselves to a comparison of the long-term steady states.

3.5 Steady state

In a steady state, n and, consequently, H are stationary. The condition for a sustainable steady state is then:

$$\ln\left[1 - \tilde{\lambda}(H)\right] H = n \int_0^\infty \ln\left[1 - \tilde{\lambda}(H)\right] e^{\tilde{Q}(\tau,H)} \bar{p}(\tau) d\tau \qquad (3.12)$$

Here, $\tilde{Q}(\tau, H)$ is defined by:

$$\begin{aligned} \tilde{Q}(\tau, H) &:= \rho \int_0^\tau \ln\left[1 - \tilde{\lambda}(H)\right] d\kappa \\ &= \rho\tau \ln\left[1 - \tilde{\lambda}(H)\right] \end{aligned} \qquad (3.13)$$

In appendix C it is shown that the steady-state condition reduces to:

$$n = g(H) \qquad (3.14)$$

The function $g(H)$ is defined in appendix C and describes the long-run capacity of the language to absorb innovations without increasing the heterogeneity. It will – under our assumptions – have the general form of figure 4.

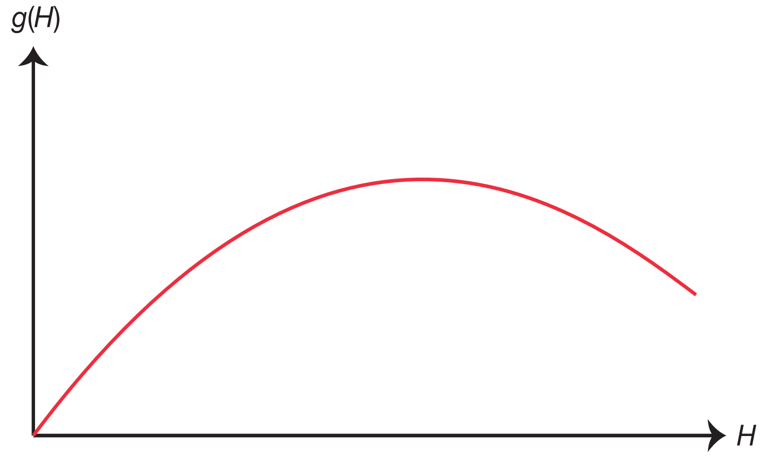

FIGURE 4: The absorption function

3.6 Dynamic equilibria

Not every steady state is a stable dynamic equilibrium. Intuitively, it is clear that if n exceeds $g(H)$, H will increase and inversely if n is smaller than $g(H)$. To show this in a stringent manner, we observe that for a constant n equation 3.12 takes the form:

$$\dot{H} = \rho \ln \left\{ 1 - \tilde{\lambda}[H(t)] \right\} H(t) + n \int_0^{\infty} \left[Q_2 - \rho \ln \left\{ 1 - \tilde{\lambda}[H(t)] \right\} \right] e^{Q(\tau,t)} \bar{p}(\tau) d\tau \qquad (3.15)$$

This equation relates the rate of change in H to the current value of H as well as to its history captured in Q. We note that Q_2 takes the sign of the rate of change in H and in a steady state is equal to zero. It reacts with a delay to changes in H, and a perturbation in H from a steady state will initially have a negligible influence on Q_2, but its value will change as time goes on if the value of H changes over time; becoming positive and growing if H grows and the opposite if H decreases.

The absorption function allows for two types of steady states, see figure 5. Now assume that there is a small perturbation in H moving it away from point A. An increase in H will make n smaller than $g(H)$, and the right-hand side of equation 3.15 becomes negative; H will decrease and move back towards point A.[22] A positive perturbation away from point B will make n greater than $g(H)$, and the right-hand side of equation 3.15 becomes positive; H will continue to grow and with time also Q_2 will become positive and grow, enforcing the growth rate of H. The system is unstable.

The corresponding results are obtained for a negative perturbation in H. A move away from point A will make the right-hand side of equation 3.15 positive, and H will return towards point A. A negative perturbation away from point B will make the rate of change negative, and H will continue moving away from point B. With time, Q_2 becomes negative and will be growing in absolute value. This will reinforce the motion away from B, and the decreasing H will overshoot point A before changing direction. The system will eventually came to a rest in point A after oscillating around this point due to the delayed reactions captured by Q.[23]

4. COMPARATIVE ANALYSIS

The model above provides us with a tool to discuss which languages might be threatened by structural decay and which will be dynamically stable. We hint at a classification of different languages (largely

[22] We ignore the effect of Q_2, which will introduce some oscillating behavior around the point A. See the discussion below.

[23] Formally, one cannot exclude an ever stronger amplitude of these oscillations without specifying the limits on the dependency of λ on H. Such an exploding behavior, however, can in any sensible specification of this functional relationship be excluded.

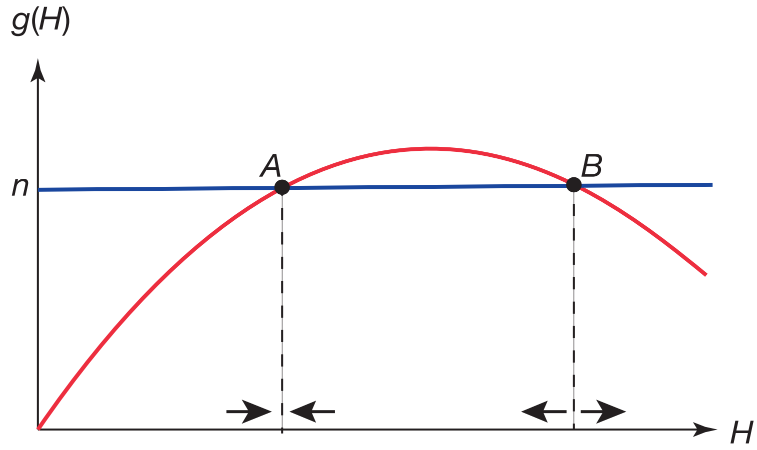

FIGURE 5: Possible steady states

based on anecdotal evidence) according to their structural types and degree of normalization, as well as cultural and social status.[24]

There are, in essence, two parameters of the model that are crucial for our analysis: the rate of import of innovations, n, and the rate of nostrification, captured by λ. As mentioned above, the rate of import is assumed to be mainly a result of the (relative) status of the language, whereas the nostrification rate is taken to be determined by the structure and degree of normalization. The interaction of these aspects is analyzed in a simple diagram comparing possible long-run steady-state equilibria. Finally, some (very speculative) policy implications will be drawn.

4.1 Rate of innovation

The rate of innovation, $n(t)$, is assumed to be exogenously given and to depend primarily on the relative cultural and social status of the donor and recipient languages, as we noted above in section 2.1. The question here is how this relationship is determined by other factors and how it can be altered through a conscious language policy.

An external factor that has become increasingly important in recent years is globalization, be it due to expanding trade, the increased spread of culture from one land to another through new media and reduced transaction costs, or easy direct access to individuals all over the world with the help of the internet. Especially the accelerating dominance of (the American variant of) English in many international domains has lead to an increased borrowing from American of both vocabulary and structural elements in virtually any language of the world.

Some countries, like France or Iceland, try to counteract this borrowing with corpus planning. The degree of success seems to be variable. If the rate of borrowing in a minority language from the majority tongue depends on the relative status of the two idioms, the obvious way to influence the borrowing rate is through status planning. Giving the minority language some official status would presumably also increase its cultural and social status. Also corpus planning, however, could have an influence here.

4.2 Language structure

As noted in section 2, the structure of the language might not directly influence the borrowing, but could affect the rate of nostrification. Languages that are similar might more easily incorporate elements from one another, than languages that are far apart. Also, adapting an imported verb, say, into an isolating language like Chinese, might be easier, basically only requiring a phonological adaptation, than adapting an imported verb into a highly inflected language like Russian, where in addition to phonetics also a

[24] A further discussion of the various classification possibilities can be found in [1].

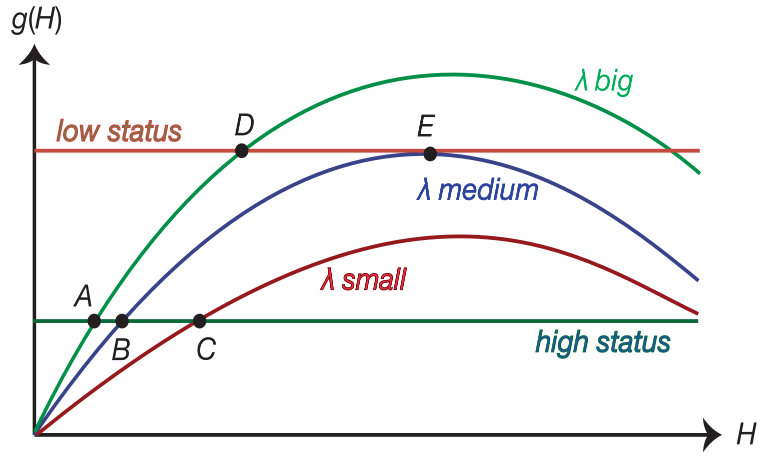

FIGURE 6: Possible stability scenarios

considerable adaptation to a rich set of conjugation forms is necessary.

Adaptation might also be facilitated by clear rules. Simplifying, we could say that codified written languages more rapidly absorb imported elements than languages with mainly an oral tradition. That is, nostrification might come faster in languages with a long written tradition than in languages that are used primarily orally.

4.3 Structural instability

The previous discussion can be summarized in figure 6. We see that languages possessing a stabilized written form, which are conjectured to have a high λ, can be assumed to have a stable structure, a long-run equilibrium in points A or D, depending on their status. On the other hand, a language without a written codification could be border-line unstable or unstable if it has a low status, point E. Many creole languages seem to fit this image. Jamaican Creole or Hawai'ian Creole seem to be unstable going through a process of decreolization. Other creole languages, like Tok Pisin, Bislama or Haitian, on the other hand, seem to stabilize due to a higher status as official languages, point B.

4.4 Policy implications

The policy conclusions that can be drawn from this seem to be that increasing the status of a language, for instance giving it an official status, might stabilize it, moving it from E to B or from free fall to C. Also corpus planning, providing a written norm might help, inducing a movement from free fall to E, or from E to D.

5. OUTLOOK AND EXTENSIONS

As mentioned before, the detailed modeling of the diffusion process can be extended to include more complex social structures. One can define different social groups with different contact probabilities between individuals in the group and with individuals outside the group. The adoption probabilities of innovations can differ for different individuals and groups. This type of analysis can easily be accomplished in a computational version of the model, wherein real-life situations can be approximated and simulated.[25]

A next step would, hence, be to implement a computational version of the model and make simulations. This type of simulations could prove to be a valuable method in analyzing language death through attrition, a phenomenon that threatens a considerable portion of the world's 6000 or so languages. To understand the individual processes leading to this attrition, might not be a sufficient condition for reversing the process of language death in most cases, but it might well be a necessary condition.

[25] [15] describes many such stratified situations.

6. CONCLUDING REMARK

There is an extensive amount of literature discussing language death in terms of language shift. With this essay we try to focus on language death through unstable structures. In many cases the two effects go hand-in-hand. Language shift lowers the status of a language and as a consequence might make the structure unstable. The details of these processes are largely unknown, though. It is especially the different rates of change that have not been extensively studied.

Many arguments in this essay are to a large extent rather speculative and intuitive. We know a bit about how languages adapt innovations and make them part of the system. However, we know very little about how fast the adaptation processes are and what determines their speed. We can only make some general assumptions based on anecdotal evidence or introspection.

What we have attempted to demonstrate, though, is that due to the interaction of the process of adopting external innovations with the process of their internal adaptation, the issue of time and the relative velocity of these processes are of considerable importance in the analysis of language attrition and decay, and consequently of language shift and death. If the structural properties of languages are important for these rates of adjustment, then we cannot ignore the structure in analyzing language shift.

7. REFERENCES

[1] B.-A. Wickström. "Die Sprache als erneuerbare Ressource: Die Kapazität verschiedener Sprachen, fremde Elemente zu nostrifizieren." In: *Florilegium Interlinguisticum: Festschrift für Detlev Blanke zum 70. Geburtstag*. Ed. by C. Brosch and S. Fiedler. Frankfurt am Main: Peter Lang, 2011, pp. 193–208.

[2] S. G. Thomason and T. Kaufman. *Language Contact, Creolization and Genetic Linguistics*. Berkeley: University of California Press, 1988.

[3] M. Clyne. *Dynamics of Language Contact: English and Immigrant Languages*. Cambridge Approaches to Language Contact. Cambridge: Cambridge University Press, 2003.

[4] D. Winford. *An Introduction to Contact Linguistics*. Vol. 33. Language in Society. Oxford: Blackwell, 2003.

[5] S. Gal. *Language Shift: Social Determinants of Linguistic Change in Bilingual Austria*. New York: Academic Press, 1979.

[6] D. M. Abrams and S. H. Strogatz. "Modelling the dynamics of language death." *Nature*, vol. 424, p. 900, 2003.

[7] J. W. Minett and W. S.-Y. Wang. "Modelling endangered languages: The effects of bilingualism and social structure." *Lingua*, vol. 118, pp. 19–45, 2008.

[8] B.-A. Wickström. "Can bilingualism be dynamically stable? A simple model of language choice." *Rationality and Society*, vol. 17, no. 1, pp. 81–115, 2005.

[9] C. Fernando, R.-L. Valijärvi, and R. A. Goldstein. "A model of the mechanisms of language extinction and revitalization strategies to save endangered languages." *Human Biology*, vol. 82, no. 1, pp. 47–75, Feb. 2010.

[10] M. Patriarca and T. Leppänen. "Modeling language competition." *Physica A*, vol. 338, pp. 296–299, 2004.

[11] X. Castelló, L. Loureiro-Porto, V. M. Eguíluz, and M. San Miguel. "The fate of bilingualism in a model of language competition." In: *Advancing Social Simulation: The First World Conference*. Ed. by S. Takahashi, J. Sallach, and Rouchier. Tokyo: Springer-Verlag, 2007, pp. 83–94.

[12] J. A. Holm. *Pidgins and Creoles: Volume 1, Theory and Structure*. Cambridge Language Surveys. Cambridge: Cambridge University Press, 1988.

[13] J. A. Holm. *Pidgins and Creoles: Volume 2, Reference Survey*. Cambridge Language Surveys. Cambridge: Cambridge University Press, 1989.

[14] A. M. S. McMahon. *Understanding Language Change*. Cambridge: Cambridge University Press, 1994.

[15] W. Labov. *Principles of Lingustic Change, Volume 2: Social Factors*. Vol. 29. Language in Society. Oxford: Blackwell, 2001.

[16] E. M. Rogers. *Diffusion of Innovations*. 5th ed. New York: The Free Press, 2003. (orig. pub. as: *Diffusion of Innovations*. New York: The Free Press, 1962.)

[17] M. Chen. “The time dimension: Contribution toward a theory of sound change.” *Foundations of Language*, vol. 8, no. 4, pp. 457–498, Jul. 1972.

[18] W. S.-Y. Wang and J. W. Minett. “The invasion of language: Emergence, change and death.” *Trends in Ecology and Evolution*, vol. 20, no. 5, pp. 263–269, 2005.

APPENDICES

A. THE DISPERSION FUNCTION

The dispersion of an innovation is a stochastic process. We assume that there is one random encounter in each period. The periods are denoted by θ. The number of users of the innovation at the beginning of encounter θ, $\widetilde{P}(\theta)$, is a stochastic variable, which is realized as $P(\theta) \in \{0, 1, 2, ..., N\}$. Let the probability density be $\delta^P(\theta)$. The probability that the number of users of the innovation changes from encounter number θ to encounter number $\theta + 1$ depends on whether in the encounters of two people, one is a user of the innovation and the other isn't. Let the probability that after such an encounter both use the innovation be α and the probability that neither use it β. If P persons use the innovation and $N - P$ do not, the probability that the next encounter is of this type, is given by:

$$\xi(P) = 2\frac{(N-P)\,P}{(N-1)\,N} \tag{A.1}$$

The number of users will then increase by one person with probability $\xi\alpha$, decrease by one person with probability $\xi\beta$, and remain constant with probability $1 - (\alpha + \beta)\,\xi =: 1 - \alpha\,(1 + \gamma)\,\xi$, where γ is defined as β/α.

We can now find the probability density $\delta^P(\theta + 1)$ after encounter $\theta + 1$. $P = 0$ can occur in two ways: If P in period θ were zero, P stays equal to zero; if P were one and this person gives up the usage, P becomes zero which happens with probability $\xi(1)\,\beta$. Hence:

$$\delta^0(\theta + 1) = \delta^0(\theta) + \alpha\gamma\xi(1)\,\delta^1(\theta) \tag{A.2}$$

or

$$\delta^0(\theta + 1) - \delta^0(\theta) = \alpha\gamma\xi(1)\,\delta^1(\theta) \tag{A.3}$$

Similarly, we find:

$$\delta^1(\theta+1) = [1-\alpha(1+\gamma)\,\xi(1)]\,\delta^1(\theta) + \alpha\gamma\xi(2)\,\delta^2(\theta) \tag{A.4}$$

or

$$\delta^1(\theta+1) - \delta^1(\theta) = \alpha\left[-(1+\gamma)\,\xi(1)\,\delta^1(\theta) + \gamma\xi(2)\,\delta^2(\theta)\right] \tag{A.5}$$

For $2 \leq P \leq N-2$, the expression becomes:

$$\delta^P(\theta+1) = [1-\alpha(1+\gamma)\,\xi(P)]\,\delta^P(\theta) + \alpha\gamma\xi(P+1)\,\delta^{P+1}(\theta) + \alpha\xi(P-1)\,\delta^{P-1}(\theta) \tag{A.6}$$

or

$$\delta^P(\theta+1) - \delta^P(\theta) = \alpha\left[\xi(P-1)\,\delta^{P-1}(\theta) - (1+\gamma)\,\xi(P)\,\delta^P(\theta) + \gamma\xi(P+1)\,\delta^{P+1}(\theta)\right] \tag{A.7}$$

For $P = N-1$, we have:

$$\delta^{N-1}(\theta+1) = [1-\alpha(1+\gamma)\,\xi(N-1)]\,\delta^{N-1}(\theta) + \alpha\xi(N-2)\,\delta^{N-2}(\theta) \tag{A.8}$$

or

$$\delta^{N-1}(\theta+1) - \delta^{N-1}(\theta) = \alpha\left[\xi(N-2)\,\delta^{N-2}(\theta) - (1+\gamma)\,\xi(N-1)\,\delta^{N-1}(\theta)\right] \tag{A.9}$$

Finally, for $P = N$, the expression is:

$$\delta^N(\theta+1) = \delta^N(\theta) + \alpha\xi(N-1)\,\delta^{N-1}(\theta) \tag{A.10}$$

or

$$\delta^N(\theta+1) - \delta^N(\theta) = \alpha\xi(N-1)\,\delta^{N-1}(\theta) \tag{A.11}$$

We note that the system of difference equations is scaled by α. Hence, an increase in α by constant γ will make the process go faster, but will in no other way influence it. The parameter γ will determine how many innovations survive in the end. In order to find the success rate of innovations, we combine the equations above substituting each one into the next for increasing values of θ, to find comparable expressions for all the δ's.

Since $\delta^0(0) = 0$, we find from A.2:

$$\delta^0(\theta) = \alpha\gamma\xi(1)\left[\sum_{\tau=0}^{\theta-1}\delta^1(\tau)\right] \tag{A.12}$$

Similarly for δ^1, noting that $\delta^1(0) = 1$:

$$\delta^1(\theta) = 1 + \alpha\left[-(1+\gamma)\,\xi(1)\sum_{\tau=0}^{\theta-1}\delta^1(\tau) + \gamma\xi(2)\sum_{\tau=0}^{\theta-1}\delta^2(\tau)\right] \tag{A.13}$$

In general, for $2 \leq P \leq N-2$, since $\delta^P(0) = 0$, we find:

$$\delta^P(\theta) = \alpha\left[\xi(P-1)\sum_{\tau=0}^{\theta-1}\delta^{P-1}(\tau) - (1+\gamma)\,\xi(P)\sum_{\tau=0}^{\theta-1}\delta^P(\tau) + \gamma\xi(P+1)\sum_{\tau=0}^{\theta-1}\delta^{P+1}(\tau)\right] \tag{A.14}$$

for $P = N-1$, the result is:

$$\delta^{N-1}(\theta) = \alpha \left[\xi(N-2) \sum_{\tau=0}^{\theta-1} \delta^{N-2}(\tau) - (1+\gamma)\, \xi(N-1) \sum_{\tau=0}^{\theta-1} \delta^{N-1}(\tau) \right] \tag{A.15}$$

and finally for $P = N$, the expression becomes:

$$\delta^{N}(\theta) = \alpha \xi(N-1) \sum_{\tau=0}^{\theta-1} \delta^{N-1}(\tau) \tag{A.16}$$

As $\theta \to \infty$, the limiting values are:

$$\begin{array}{lll} \delta^0(\theta) & \to & \delta^0 \geq 0 \\ \delta^P(\theta) & \to & 0, \qquad\qquad\qquad 1 \leq P \leq N-1 \\ \delta^N(\theta) & \to & \delta^N = 1 - \delta^0 \geq 0 \end{array} \tag{A.17}$$

Using this fact and defining $\Delta(P) := \xi(P) \sum_{\tau=0}^{\infty} \delta^P(\tau)$, we rewrite equations A.12 through A.16 as:

$$\delta^0 = \alpha\gamma\Delta(1) \tag{A.18}$$

$$\alpha\,(1+\gamma)\,\Delta(1) = 1 + \alpha\gamma\Delta(2) \tag{A.19}$$

$$\Delta(P-1) = (1+\gamma)\,\Delta(P) - \gamma\Delta(P+1) \tag{A.20}$$

$$\Delta(N-2) = (1+\gamma)\,\Delta(N-1) \tag{A.21}$$

$$\alpha\Delta(N-1) = 1 - \delta^0 \tag{A.22}$$

We first substitute equation A.19 into equation A.18:

$$\delta^0 = \frac{\gamma}{1+\gamma} + \frac{\alpha\gamma^2}{1+\gamma}\Delta(2) \tag{A.23}$$

and then find $\Delta(2)$ from A.20:

$$
\begin{aligned}
\Delta(2) &= (1+\gamma)\,\Delta(3) - \gamma\Delta(4) \\
&= (1+\gamma)\,[(1+\gamma)\,\Delta(4) - \gamma\Delta(5)] - \gamma\Delta(4) \\
&= \left(1+\gamma+\gamma^2\right)\Delta(4) - (1+\gamma)\,\gamma\Delta(5) \\
&= \left(1+\gamma+\gamma^2+\gamma^3\right)\Delta(5) - \left(1+\gamma+\gamma^2\right)\gamma\Delta(6) \\
&= \sum_{i=0}^{P-2}\gamma^i\Delta(P) - \sum_{i=1}^{P-2}\gamma^i\Delta(P+1) \qquad \text{(A.24)} \\
&= \sum_{i=0}^{N-4}\gamma^i\Delta(N-2) - \sum_{i=1}^{N-4}\gamma^i\Delta(N-1) \\
&= \left[(1+\gamma)\sum_{i=0}^{N-4}\gamma^i - \sum_{i=1}^{N-4}\gamma^i\right]\Delta(N-1) \\
&= \frac{1-\delta^0}{\alpha}\sum_{i=0}^{N-3}\gamma^i \\
&= \frac{1-\gamma^{N-2}}{1-\gamma}\frac{1-\delta^0}{\alpha}
\end{aligned}
$$

Substituting this into A.23 and solving, we finally arrive at the value of δ^0:

$$
\begin{aligned}
\delta^0 &= \frac{\gamma}{1+\gamma} + \frac{\gamma^2}{1+\gamma}\frac{1-\gamma^{N-2}}{1-\gamma}\left(1-\delta^0\right) \qquad \text{(A.25)} \\
\delta^0 &= \gamma\frac{1-\gamma^{N-1}}{1-\gamma^N} = \gamma - \frac{1-\gamma}{1-\gamma^N}\gamma^N
\end{aligned}
$$

For a large N and $\gamma < 1$ this, of course, reduces to:

$$
\delta^0 = \gamma \qquad \text{(A.26)}
$$

That is, a fraction γ of the innovations does not survive in the long run.

It is of some interest to know how the expected value $\bar{P}$ of $\tilde{P}$ changes with time:

$$
\begin{aligned}
\bar{P}(\theta+1) &:= \sum_{P=0}^{N}\delta^P(\theta+1)P \\
&= \delta^1(\theta) - \alpha\,(1+\gamma)\,\xi(1)\,\delta^1(\theta) + \alpha\gamma\xi(2)\delta^2(\theta) \\
&+ \sum_{P=2}^{N-2}\left[\delta^P(\theta)P - \alpha\,(1+\gamma)\,\xi(P)\,\delta^P(\theta)P\right. \\
&+ \left.\alpha\gamma\xi(P+1)\,\delta^{P+1}(\theta)P + \alpha\xi(P-1)\,\delta^{P-1}(\theta)P\right] \qquad \text{(A.27)} \\
&+ \delta^{N-1}(\theta)\,(N-1) - \alpha\,(1+\gamma)\,\xi(N-1)\,\delta^{N-1}(\theta)\,(N-1) \\
&+ \alpha\xi(N-2)\,\delta^{N-2}(\theta)\,(N-1) \\
&+ \delta^N(\theta)N + \alpha\xi(N-1)\,\delta^{N-1}(\theta)N
\end{aligned}
$$

or:

$$\begin{aligned}
\bar{P}(\theta+1) &= \bar{P}(\theta) \\
&- \alpha\,(1+\gamma)\left[\xi(1)\,\delta^1(\theta)+\sum_{P=2}^{N-2}\xi(P)\,\delta^P(\theta)P+\xi(N-1)\,\delta^{N-1}(\theta)\,(N-1)\right] \\
&+ \alpha\gamma\left[\xi(1)\,\delta^1(\theta)+\xi(2)\delta^2(\theta)2+\sum_{P=2}^{N-2}\xi(P+1)\delta^{P+1}(\theta)\,(P+1)\right] \\
&- \alpha\gamma\left[\xi(1)\,\delta^1(\theta)+\xi(2)\delta^2(\theta)+\sum_{P=2}^{N-2}\xi(P+1)\delta^{P+1}(\theta)\right] \qquad \text{(A.28)} \\
&+ \alpha\left[\sum_{P=2}^{N-2}\xi(P-1)\delta^{P-1}(\theta)\,(P-1)+\xi(N-2)\delta^{N-2}(\theta)\,(N-2)+\xi(N-1)\delta^{N-1}(\theta)\,(N-1)\right] \\
&+ \alpha\left[\sum_{P=2}^{N-2}\xi(P-1)\delta^{P-1}(\theta)+\xi(N-2)\delta^{N-2}(\theta)+\xi(N-1)\delta^{N-1}(\theta)\right]
\end{aligned}$$

Hence, using the fact that $\xi(N)=0$, one finds:

$$\begin{aligned}
\bar{P}(\theta+1) &= \bar{P}(\theta) \\
&+ [\alpha+\alpha\gamma-\alpha\,(1+\gamma)]\sum_{P=0}^{N}\xi(P)\,\delta^P(\theta)P \qquad \text{(A.29)} \\
&+ (\alpha-\alpha\gamma)\sum_{P=0}^{N}\xi(P)\,\delta^P(\theta)
\end{aligned}$$

This gives us:

$$\bar{P}(\theta+1)-\bar{P}(\theta)=\alpha\,(1-\gamma)\sum_{P=0}^{N}\xi(P)\,\delta^P(\theta) \qquad \text{(A.30)}$$

Substituting for ξ, we find:

$$\begin{aligned}
\sum_{P=0}^{N}\xi(P)\,\delta^P(\theta) &= \frac{2}{(N-1)\,N}\sum_{P=0}^{N}\delta^P(\theta)\,(N-P)\,P \\
&= \frac{2}{(N-1)\,N}\sum_{P=0}^{N}\delta^P(\theta)\left(NP-P^2\right) \\
&= \frac{2}{(N-1)\,N}\sum_{P=0}^{N}\delta^P(\theta)\left(NP+\bar{P}^2-2P\bar{P}-\left(P-\bar{P}\right)^2\right) \qquad \text{(A.31)} \\
&= 2\bar{P}(\theta)\,\frac{N-\bar{P}(\theta)}{(N-1)\,N}-2\frac{1}{(N-1)\,N}\sum_{P=0}^{N}\delta^P(\theta)\left(P-\bar{P}\right)^2 \\
&= 2\left[\bar{P}(\theta)\,\frac{N-\bar{P}(\theta)}{(N-1)\,N}-\frac{\sigma_P^2(\theta)}{(N-1)\,N}\right]
\end{aligned}$$

That is, the dynamics of the expected value of $\widetilde{P}$ is given by:

$$\bar{P}(\theta+1)-\bar{P}(\theta)=2\alpha(1-\gamma)\left[\bar{P}(\theta)\frac{N-\bar{P}(\theta)}{(N-1)N}-\frac{\sigma_P^2(\theta)}{(N-1)N}\right] \tag{A.32}$$

or:

$$\bar{p}(\theta+1)-\bar{p}(\theta)=2\alpha(1-\gamma)\frac{1}{N-1}\left\{[1-\bar{p}(\theta)]\bar{p}(\theta)-\sigma_p^2(\theta)\right\} \tag{A.33}$$

Here, $\widetilde{p}(\theta)$ is the fraction of the population that has adopted the innovation after θ encounters, and p is the realization of $\widetilde{p}$. If the number of encounters per unit of time is ωN, we can make the substitution $t\omega N=\theta$ and express the dynamics in time units, where it is understood that the variables are now functions of the age of the innovation in time units:

$$\dot{\bar{p}}(t)=2\omega\alpha(1-\gamma)\frac{N}{N-1}\left\{[1-\bar{p}(t)]\bar{p}(t)-\sigma_p^2(t)\right\} \tag{A.34}$$

B. THE DYNAMICS OF THE HETEROGENEITY

We want to separate terms that do not vanish in a steady state of the system from the rest. Differentiating 3.10 with respect to t, we find:

$$\begin{aligned}\dot{H} &= n(t)\,\bar{p}(0)\\ &+ \int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\frac{dQ}{dt}\,\bar{p}(t-\omega)d\omega\\ &+ \int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\,\bar{p}\,'(t-\omega)d\omega\end{aligned} \tag{B.1}$$

For the sake of simplicity, we denote the three terms of B.1 by A, B, and C.

A does not need any further discussion.

In B we add and subtract a term:

$$\rho\ln\left\{1-\widetilde{\lambda}[H(t)]\right\}\int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\,\bar{p}(t-\omega)d\omega \tag{B.2}$$

B can then be rewritten as:

$$\begin{aligned}B &= \int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\left(\frac{dQ}{dt}-\rho\ln\left\{1-\widetilde{\lambda}[H(t)]\right\}\right)\bar{p}(t-\omega)d\omega\\ &+ \rho\ln\left\{1-\widetilde{\lambda}[H(t)]\right\}\int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\,\bar{p}(t-\omega)d\omega\\ &= \int_{-\infty}^{t} n(\omega)\,e^{Q(t-\omega,t)}\left(\frac{dQ}{dt}-\rho\ln\left\{1-\widetilde{\lambda}[H(t)]\right\}\right)\bar{p}(t-\omega)d\omega\\ &+ \rho\ln\left\{1-\widetilde{\lambda}[H(t)]\right\}H(t)\end{aligned} \tag{B.3}$$

We evaluate the derivative dQ/dt:

$$\frac{dQ(t-\omega,t)}{dt} = Q_1 + Q_2 = \rho \ln\left\{1 - \tilde{\lambda}\left[H(\omega)\right]\right\} + Q_2 \tag{B.4}$$

Here, Q_1 and Q_2 denote the partial derivatives with respect to the first and second arguments, respectively, of the function Q.

B now becomes:

$$\begin{aligned} B &= \int_{-\infty}^{t} n(\omega)\, e^{Q(t-\omega,t)} \left(Q_2 + \rho \ln \frac{1-\tilde{\lambda}\left[H(\omega)\right]}{1-\tilde{\lambda}\left[H(t)\right]}\right) \bar{p}(t-\omega) d\omega \\ &+ \rho \ln\left\{1 - \tilde{\lambda}\left[H(t)\right]\right\} H(t) \end{aligned} \tag{B.5}$$

C can be integrated by parts:

$$\begin{aligned} C &= -n(\omega)\, e^{Q(t-\omega,t)}\, \bar{p}(t-\omega)\Big|_{-\infty}^{t} \\ &+ \int_{-\infty}^{t} n'(\omega)\, e^{Q(t-\omega,t)}\, \bar{p}(t-\omega) d\omega \\ &+ \int_{-\infty}^{t} n(\omega)\, e^{Q(t-\omega,t)} \frac{dQ}{d\omega} \bar{p}(t-\omega) d\omega \end{aligned} \tag{B.6}$$

We evaluate the derivative $dQ/d\omega$:

$$\frac{dQ}{d\omega} = -Q_1 = -\rho \ln\left\{1 - \tilde{\lambda}\left[H(\omega)\right]\right\} \tag{B.7}$$

Evaluating the first term as well as adding and subtracting the term

$$n(t)\rho \int_{-\infty}^{t} e^{Q(t-\omega,t)} \ln\left\{1 - \tilde{\lambda}\left[H(\omega)\right]\right\} \bar{p}(t-\omega) d\omega \tag{B.8}$$

we find the expression:

$$\begin{aligned} C &= -n(t)\, \bar{p}(0) \\ &+ \int_{-\infty}^{t} n'(\omega)\, e^{Q(t-\omega,t)}\, \bar{p}(t-\omega) d\omega \\ &+ \rho \int_{-\infty}^{t} \left[n(t) - n(\omega)\right] e^{Q(t-\omega,t)} \ln\left\{1 - \tilde{\lambda}\left[H(\omega)\right]\right\} \bar{p}(t-\omega) d\omega \\ &- n(t)\rho \int_{-\infty}^{t} e^{Q(t-\omega,t)} \ln\left\{1 - \tilde{\lambda}\left[H(\omega)\right]\right\} \bar{p}(t-\omega) d\omega \end{aligned} \tag{B.9}$$

Expression B.1 can now be written as:

$$\begin{aligned}
\dot{H} &= \rho \ln\left\{1-\tilde{\lambda}[H(t)]\right\} H(t) - n(t)\rho\int_{-\infty}^{t} \ln\left\{1-\tilde{\lambda}[H(\omega)]\right\} e^{Q(t-\omega,t)}\,\bar{p}(t-\omega)d\omega \\
&+ \int_{-\infty}^{t}\left[n(\omega)\left(Q_2+\rho\ln\frac{1-\tilde{\lambda}[H(\omega)]}{1-\tilde{\lambda}[H(t)]}\right)+n'(\omega)\right. \\
&\qquad \left. +\rho\Big[n(t)-n(\omega)\Big]\ln\left\{1-\tilde{\lambda}[H(\omega)]\right\}\right] e^{Q(t-\omega,t)}\,\bar{p}(t-\omega)d\omega
\end{aligned} \tag{B.10}$$

Making the substitution $\tau = t-\omega$, we finally arrive at:

$$\begin{aligned}
\dot{H} &= \rho \ln\left\{1-\tilde{\lambda}[H(t)]\right\} H(t) - n(t)\rho\int_{0}^{\infty} \ln\left\{1-\tilde{\lambda}[H(t-\tau)]\right\} e^{Q(\tau,t)}\,\bar{p}(\tau)d\tau \\
&+ \int_{0}^{\infty}\left[n(t-\tau)\left(Q_2+\rho\ln\frac{1-\tilde{\lambda}[H(t-\tau)]}{1-\tilde{\lambda}[H(t)]}\right)+n'(t-\tau)\right. \\
&\qquad \left. +\rho\Big[n(t)-n(t-\tau)\Big]\ln\left\{1-\tilde{\lambda}[H(t-\tau)]\right\}\right] e^{Q(\tau,t)}\,\bar{p}(\tau)d\tau
\end{aligned} \tag{B.11}$$

The second integral vanishes in a steady state. The properties of the steady states are hence determined by the first two terms.

C. STEADY STATE

We take our point of departure in equation 3.12:

$$\ln\left[1-\tilde{\lambda}(H)\right] H = n\int_{0}^{\infty} \ln\left[1-\tilde{\lambda}(H)\right] e^{\tilde{Q}(\tau,H)}\,\bar{p}(\tau)d\tau \tag{C.1}$$

Using the fact that in the steady state

$$\begin{aligned}
e^{\tilde{Q}(\tau,t)} &= \exp\left[\rho\int_0^{\tau}\ln\left[1-\tilde{\lambda}(H)\right]d\kappa\right] \\
&= \exp\left[\rho\ln\left[1-\tilde{\lambda}(H)\right]\int_0^{\tau}d\kappa\right] \\
&= \exp\left[\ln\left[1-\tilde{\lambda}(H)\right]^{\rho\tau}\right] \\
&= \left[1-\tilde{\lambda}(H)\right]^{\rho\tau}
\end{aligned} \tag{C.2}$$

this equation, for $\tilde{\lambda}(H) > 0$, becomes:

$$H = n\int_0^{\infty}\left[1-\tilde{\lambda}(H)\right]^{\rho\tau}\bar{p}(\tau)d\tau \tag{C.3}$$

We define $\psi(\tau; H, \rho) > 0$ by:

$$\begin{aligned}
\psi(\tau; H, \rho) \quad &:= \quad \frac{\left[1-\tilde{\lambda}(H)\right]^{\rho\tau}}{\displaystyle\int_0^\infty \left[1-\tilde{\lambda}(H)\right]^{\rho\kappa} d\kappa} \\
&= \quad \frac{\left[1-\tilde{\lambda}(H)\right]^{\rho\tau}}{\left[1-\tilde{\lambda}(H)\right]^{\rho\kappa}\Big|_0^\infty} \rho \ln\left[1-\tilde{\lambda}(H)\right] \\
&= \quad -\rho \ln\left[1-\tilde{\lambda}(H)\right]\left[1-\tilde{\lambda}(H)\right]^{\rho\tau}
\end{aligned} \tag{C.4}$$

Clearly, $\psi(\tau; H, \rho)$ integrates to one:

$$\int_0^\infty \psi(\tau; H, \rho) d\tau = 1 \tag{C.5}$$

Multiplying both sides of equation C.3 by $-\rho \ln\left[1-\tilde{\lambda}(H)\right]$, we can rewrite it as:

$$-\rho \ln\left[1-\tilde{\lambda}(H)\right] H = n \int_0^\infty \psi(\tau; H, \rho)\, \bar{p}(\tau) d\tau \tag{C.6}$$

The integral multiplying n is, hence, a weighted average of $\bar{p}$ over the age of the innovation with the weights decreasing with increasing age. Since $0 < \bar{p} < 1$, the weighted average also lies between 0 and 1. If $\tilde{\lambda}$ increases, more weight is given to small values of τ. Since $\bar{p}(\tau)$ increases with τ, the value of the integral decreases with larger values of $\tilde{\lambda}$. The rate of absorption $\tilde{\lambda}$, however, decreases with an increase in H. Hence the value of the integral increases with an increase in H. We write the integral as $1 > \imath(H, \rho) > 0$, $\partial\imath/\partial H \geq 0$.

Equation C.3 now becomes:

$$-\rho \ln\left[1-\tilde{\lambda}(H)\right] H = n\, \imath(H, \rho) \tag{C.7}$$

or:

$$n = -\rho \ln\left[1-\tilde{\lambda}(H)\right] \frac{H}{\imath(H, \rho)} =: g(H, \rho) \tag{C.8}$$

The function that we have defined as $g(H, \rho)$ describes the long-run capacity of the language to absorb innovations without an increase in the heterogeneity. The form of g depends on how $\tilde{\lambda}$ behaves for large values of H. It is clear that $g(0, \rho) = 0$ if $\tilde{\lambda}(0) > 0$. Also, if for some value H^M of H the value of the function $\tilde{\lambda}(H^M) = 0$, then $g(H^M, \rho) = 0$. If $\tilde{\lambda}(H) \to 0$ as $H \to \infty$, the behavior of g depends on "how fast" $\tilde{\lambda}(H)$ approaches zero. It has to be faster than $1/H$ for g to approach zero for large values of H.

We will assume that either there exists an H^M such that $\tilde{\lambda}(H^M) = 0$ or that $\tilde{\lambda}(H)$ approaches zero "fast enough" for sufficiently high H. Then, the function g has the general form of figure 4.