Performance Analysis of MUSIC and ESPRIT DOA Estimation Algorithms for Adaptive Array Smart Antenna in Mobile Communication

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Abstract

Adaptive array smart antenna involves the array processing to manipulate the signals induced on various antenna elements in such way that the main beam directing towards the desired signal and forming the nulls towards the interferers. Such smart antennas are widely used in wireless mobile communications as they can increase the channel capacity and coverage range. In adaptive array smart antenna, to locate the desired signal, various direction of arrival (DOA) estimation algorithms are used. This paper investigates and compares MUSIC and ESPRIT, DOA estimation algorithms which are widely used in the design of smart antenna system. MUSIC and ESPRIT algorithms provide high angular resolution and hence they are explored much in detail by varying various parameters of smart antenna system .However simulation in this paper shows that MUSIC algorithm is highly accurate and stable and provides high angular resolution compared to ESPRIT and hence MUSIC algorithm can be widely used in mobile communication to estimate the DOA of the arriving signals.

Keywords: MUSIC, ESPRIT, Smart antenna, DOA**.**

1. INTRODUCTION

One of the most promising techniques for increasing the capacity in 3G cellular is the adaptive array smart antenna. The smart antenna technology is based on antenna arrays where the radiation pattern is altered by adjusting the amplitude and relative phase on the different elements. If several transmitters are operating simultaneously, each source creates many multipath components at the receiver and hence receive array must be able to estimate the angles of arrival in order to decipher which emitters are present and what are their angular locations. This information in turn can be used by the smart antenna to eliminate or combine signals for greater fidelity or suppress interferers to improve the capacity of cellular mobile communication . The various DOA estimation algorithms are Bartlett, Capon, Min-norm, MUSIC and ESPRIT. But MUSIC and ESPRIT algorithms are high resolution and accurate methods which are widely used in the design of smart antennas and hence their performance is evaluated in following sections. With use of MUSIC algorithms smart antennas add a new possibility of user separation by space through space division multiple access (SDMA).

2. DOA ESTIMATION ALGORITHMS

The purpose of DOA estimation is to use the data received by the array to estimate the direction of arrival of the signal. As shown in Fig. 1, the results of DOA estimation are then used by the array to design the adaptive beam former in such way as to maximize the power radiated towards the users and to suppress the interference. In short the successful design of adaptive array smart antenna depends highly on the performance of DOA estimation algorithm. In the design of adaptive array smart antenna for mobile communication the performance of DOA estimation algorithm depends on many parameters such as number of mobile users and their space distribution , the number of array elements and their spacing, the number of signal samples and SNR. Number of DOA estimation algorithms have been developed and categorized into two methods viz. conventional and subspace methods. Conventional methods also called classical methods which first compute a spatial spectrum and then estimate DOAs by local maxima of the spectrum. Methods that fall under this class are Bartlett, Capon methods. But these methods suffer from lack of angular resolution. For this reason high angular resolution subspace methods such as MUSIC and ESPRIT algorithms are most used.

2.1 Music Algorithm

MUSIC is an acronym which stands for Multiple Signal classification. It is high resolution technique based on exploiting the eigenstructure of input covariance matrix. MUSIC makes assumption that the noise in each channel is uncorrelated making correlation matrix diagonal. The incident signals are somewhat correlated creating non diagonal signal correlation matrix.

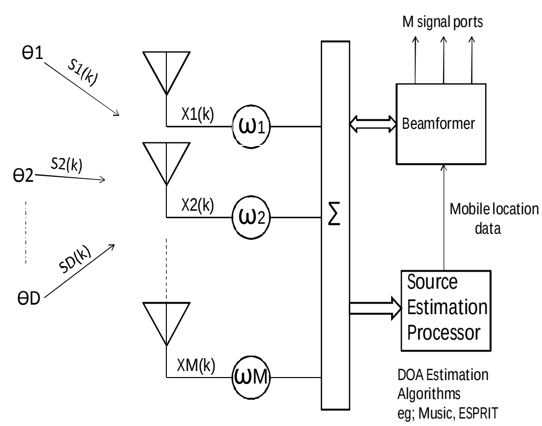

FIGURE 1: M element antenna array with D arriving signals.

If the number of signals impinging on M element array is D, the number of signal eigenvalues and eigenvectors is D and number of noise eigenvalues and eigenvectors is M-D. The array correlation matrix with uncorrelated noise and equal variances is then given by

$$R_{xx} = A^*R_{ss}^*A^H + \sigma_n^{\,2*}I \qquad (1)$$

where A= $[a(\theta_1)\ a(\theta_2)\ a(\theta_3)\ \text{---}\ a(\theta_D)]$ is M x D array steering matrix

$R_{ss}=[s_1(k)\ s_2(k)\ s_3(k)\ \text{----}\ s_D(k)]^T$ is D x D source correlation matrix

R_{xx} has D eigenvectors associated with signals and M – D eigenvectors associated with the noise. We can then construct the M x (M-D) subspace spanned by the noise eigenvectors such that

$$V_N = [v_1\ v_2\ v_3\ \text{-------}\ v_{M-D}] \quad (2)$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals $\theta_1, \theta_2, \theta_3, \theta_D$ and the MUSIC Pseudospectrum is given as

$$P_{MUSIC(\theta)} = 1/abs((a(\theta)^H * V_N * V_N^{H*} a(\theta)) \quad (3)$$

However when signal sources are coherent or noise variances vary the resolution of MUSIC diminishes. To overcome this we must collect several time samples of received signal plus noise, assume ergodicity and estimate the correlation matrices via time averaging as

$$R_{xx} = \frac{1}{K} \sum_{k=1}^{K} x(k)*x(k)^H \quad (4)$$

and

$$R_{xx} = A*Rss*A^H + A*R_{sn} + R_{ns}*A^H + R_{nn} \quad (5)$$

The MUSIC Pseudospectrum using equation (3) with time averages now provides high angular resolution for coherent signals.

2.2 ESPRIT Algorithm

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques which is another subspace based DOA estimation algorithm. It does not involve an exhaustive search through all possible steering vectors to estimate DOA and dramatically reduces the computational and storage requirements compared to MUSIC. The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure. ESPRIT assumes that there are D < M narrowband sources centered at the centre frequency f_0. ESPRIT further assumes multiple identical arrays called doublets and these arrays are displaced translationally but not rotationally. Fig. 2 shows four element linear array which is composed of two doublets.

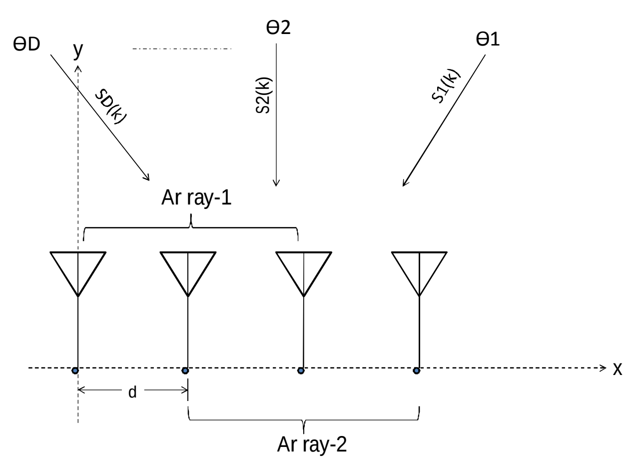

FIGURE 2: Four element linear array with two doublets.

The two subarrays , array-1 and array-2 are displaced by distance 'd'. The signals induced on each of the arrays are given by

$$x_1(k) = A_1 * s(k) + n_1(k)$$

and

$$x_2(k) = A_1 * \Lambda * s(k) + n_2(k)$$

where $\Lambda = \text{diag}\{e^{jkd\sin(\theta 1)}\ e^{jkd\sin(\theta 2)} \text{-----}\ e^{jkd\sin(\theta D)}\}$
= D x D diagonal unitary matrix with phase shifts between doublets for each DOA.

Creating the signal subspace for the two subarrays results in two matrices V_1 & V_2. Since the arrays are translationally related, the subspaces of eigenvectors are related by a unique nonsingular transformation matrix ф such that

$$V_1 \phi = V_2$$

There must also exist a unique nonsingular transformation matrix T such that

$V_1 = AT$ and $V_2 = A\Lambda T$ and finally we can derive

$$T \Phi T^{-1} = \Lambda \qquad (6)$$

Thus the eigenvalues of ф must be equal to the diagonal elements of Λ such that

$$\lambda_1 = e^{jkd\sin(\theta 1)},\ \lambda_2 = e^{jkd\sin(\theta 2)} \text{-----}\ \lambda_D = e^{jkd\sin(\theta D)}$$

Once the eigenvalues of ф, $\lambda_1, \lambda_2,$ ------ λ_D are calculated, we can estimate the angles of arrivals as

$$\theta_i = \sin^{-1}(\arg(\lambda_i)/kd) \qquad (7)$$

Clearly the ESPRIT eliminates the search procedure & produces the DOA estimation directly in terms of the eigenvalues without much computational and storage requirements. This eigenstructure method has shown excellent accuracy and resolution in many experimental and theoretical studies.

3. SIMULATION RESULTS

The MUSIC & ESPRIT methods of DOA estimation are simulated using MATLAB. A uniform linear array with M elements has been considered here.

3.1 Simulation results of MUSIC DOA estimation algorithm

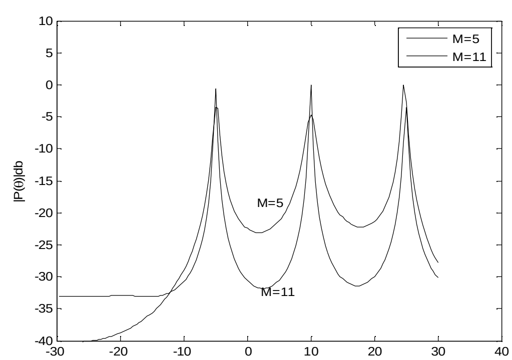

FIGURE 3: MUSIC spectrum for DOA= -5, 10, 25 degrees

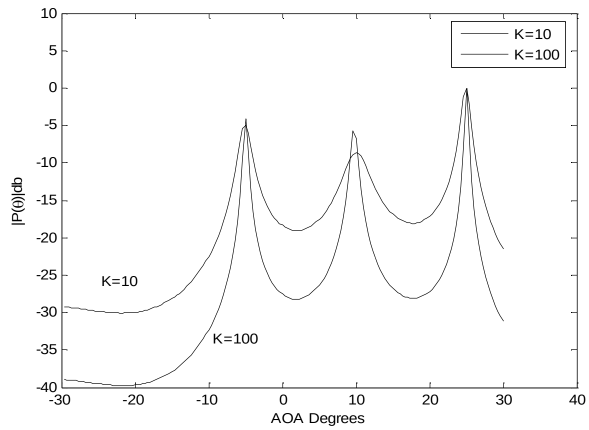

FIGURE 4: MUSIC spectrum for varying number of snapshots

SNR (dB)	MUSIC (θ)	ESPRIT(θ)
-20	21.66	-39.08
-19	24.00	23.47
0	24.66	26.02
9	25.00	25.67
20	25.00	25.21
21	25.00	24.99

TABLE 1: DOA Estimation by MUSIC & ESPRIT for varying SNR.
(Input θ =25 deg, M=4, k=100)

Fig. 3 shows the MUSIC spectrum for uniform linear array with varying elements and SNR=20 dB, K=100 for direction of arrivals -5, 10, 25 degrees. Spacing between elements is assumed to be 0.5λ.When number of elements in array is increased to 11 MUSIC spectrum takes the form of sharper peaks in which angular resolution is improved. The number of signal snapshots used to generate realistic signal model is a key factor in the realization of practical antennas . Fig.4 shows the music spectrum obtained for snapshots equal to 10 and 1000. Increased snapshots leads to sharper MUSIC spectrum peaks indicating more accurate detection and better resolution.

3.2 Simulation results of ESPRIT DOA estimation algorithm

Simulation of ESPRIT algorithm for linear uniform array with four elements is carried out by varying different parameters of linear array.

Sr,no,	θ Input (deg)	θ ESPRIT (deg)
1	10	9.43
	25	23.94
2	20	20.02
	80	80.27

TABLE 2: DOA Estimation Using ESPRIT For varying Angular Separation (M=4, SNR=20 dB, K=100)

K=10	
θ Input (deg)	**θ ESPRIT (deg)**
80	79.39
20	19.92
K=1000	
80	80.04
20	20.00

TABLE 3: DOA Estimation using ESPRIT for Varying Number of Snapshots K. (M=4, SNR=20 dB)

No. of signals	θ Input (deg)	θ ESPRIT (deg)
2	20	20.08
	40	39.96
4	10	24.46
	30	32.34
	50	-44.52
	70	-18.82

TABLE 4: DOA estimation using ESPRIT for varying number of signals.(M=4, SNR=20dB,K=100)

Table 2 illustrates that the percentage error in DOA detection using ESPRIT algorithm decreases as angular separation between arriving signals increases. Table 3 illustrates that when number of samples are increased the error becomes small. Table 4 illustrates how the ESPRIT algorithm can successfully detects 2 incident signals on array of 4 elements and how it completely fails if the number of incident signals increased to 4. Table 1 indicates DOA estimation by both MUSIC and ESPRIT with respect to SNR. This table clearly indicates that MUSIC provides high resolution and accurate detection of an angle of arrival than that of ESPRIT.

4. CONCLUSION

This paper presents the results of direction of arrival estimation using ESPRIT & MUSIC. These two methods have greater resolution and accuracy and hence these are investigated much in detail. The simulation results of both MUSIC and ESPRIT show that their performance improves with more elements in the array, with large snapshots of signals and greater angular separation between the signals. These improvements are seen in form of the sharper peaks in the MUSIC and smaller errors in angle detection in the ESPRIT. However Table 1 shows that there are more errors in DOA estimation by using ESPRIT compared to the MUSIC algorithm. Clearly MUSIC is more stable and accurate and provides high resolution and this adds new possibility of user separation through SDMA and can be widely used in the design of smart antenna system.

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