Synchronization in the Genesio-Tesi and Coullet Systems Using the Sliding Mode Control

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Abstract

Chaotic behavior and control of Genesio-Tesi and Coullet is studied, in this paper. The Sliding Mode Control is proposed for synchronization in a pair of topologically inequivalent systems, the Genesio-Tesi and Coullet systems. The simulation result verifies the effectiveness of the proposed method.

Keywords: Chaos, Sliding Mode, Synchronization, Genesio-Tesi System, Coullet System

1. INTRODUCTION

Chaotic systems have recently been much considered due to their potential usage in different fields of science and technology particularly in electronic systems [17], secure communication [22] and computer [1]. Sensitive dependence on initial conditions is an important characteristic of chaotic systems. For this reason, chaotic systems are difficult to be controlled or synchronized. Control of these systems has been considered as an important and challenging problem [12]. Control of chaotic systems would have been supposed impossible with uncontrollable and unpredictable dynamic. The imagination was changed when three researchers (Ott, Grebogi, Yorke) have shown in [13] other vice. The effort has been progressed to control a chaos in great different areas, e.g. feedback linearization [2, 7, 20], Delay feedback control [3], OPF [9] and TDFC [16]. Over last two decades, due to the pioneering work of Ott et al. 1990, synchronization of chaotic systems has become more and more interesting in different areas. A very important case in chaotic systems is synchronizing the two identical systems with unequal initial conditions, but in these years, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two different chaotic systems [4, 10, 14]. The problem of designing a system, whose behavior mimics that of another chaotic system, is called synchronization. Two chaotic systems are usually called drive (master) and response (slave) systems respectively. Different control technique e.g. a chattering-free fuzzy sliding-mode control (FSMC) strategy for synchronization of chaotic systems even in presence of uncertainty has been proposed in [5]. In [6] authors have proposed an active sliding mode control to synchronize two chaotic systems with parametric uncertainty. An algorithm to determine parameters of active sliding mode controller in synchronizing different chaotic systems has been studied in [11]. In [19] an adaptive sliding mode controller has also been presented for a class of master–slave chaotic synchronization systems with uncertainties. In [15], a backstepping control was proposed to synchronize these systems. Genesio-Tesi and Coullet systems are topologically inequivalent [8]. It is difficult to synchronize two topologically inequivalent systems. In [8], a backstepping approach was proposed to synchronize these systems. In this paper a controller designed via Sliding Mode control to synchronize such systems. The organization of this paper is as follows: In Section 2, chaotic behaviors of two systems is studied. In Section 3, a controller is designed via Sliding Mode Control to synchronize Coullet and Genesio-Tesi systems. In Section 4, numerical simulations verify the effectiveness of the proposed method. Finally the paper will be concluded in section 5.

2. CHAOS IN GENESIO-TESI AND COULLET SYSTEMS

It is assumed that Genesio-Tesi system drives Coullet system. Thus, the master and slave systems are considered as follows:

$$\text{Master}:\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = x_3\\ \dot{x}_3 = a_3x_3 + a_2x_2 + a_1x_1 + x_1^2\end{cases} \tag{1}$$

And

$$\text{Slave}:\begin{cases}\dot{y}_1 = y_2\\ \dot{y}_2 = y_3\\ \dot{y}_3 = b_3y_3 + b_2y_2 + b_1y_1 - y_1^3\end{cases} \tag{2}$$

If we select the parameters of the systems as $(a_1,a_2,a_3)=(-1,-1.1,-0.45)$ and $(b_1,b_2,b_3)=(-0.8,-1.1,-0.45)$ the two systems chaotic behaviors will be occurred. Their chaotic attractors are shown in Fig.1 and Fig.2 respectively.

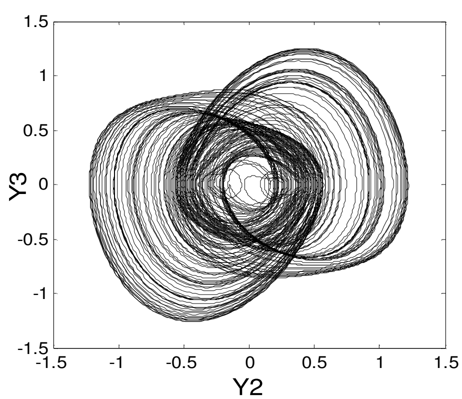

Fig.1: Chaotic attractors Y2-Y3 of Genesio-Tesi System

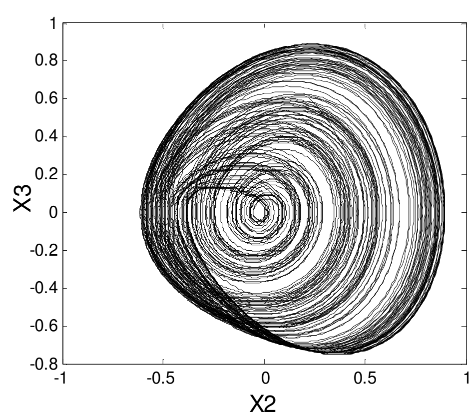

Fig.2: Chaotic attractors X2-X3 of Coullet system

3. SLIDING MODE CONTROLLER DESIGNATION

The aim in this section is to design a Sliding Mode Controller to achieve synchronization between two systems. The procedure of designing will be briefly shown here. The controller has a duty to synchronization between two inequivalent systems. In general in the sliding mode control terminology, a controller is designed such that states approach to zero in finite time [21], Thereafter states have to stay in the location for the rest of time. To gain the benefit of sliding mode control, a sliding surface has to be defined first. This surface introduces a desired dynamic and the route of approaching the states towards a stable point. This also defines the switching of the sliding control (as a compliment of control law) in the surface. Any outside state in a finite time approaches to the surface.

In this work this surface defines based on the error of stats two inequivalent systems. Choosing $a_3x_3 + a_2x_2 + a_1x_1 + x_1^2 = F(x)$, the equation of Genesio-Tesi system, Eq. (1) will be rewritten as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = F(X) \end{cases} \tag{3}$$

Choosing $b_3y_3 + b_2y_2 + b_1y_1 - y_1^3 = G(Y)$, the Eq. (2) can be rewritten as:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = G(Y) + U_c \end{cases} \tag{4}$$

Where U_c is controller which will be designed via Sliding Mode Control to synchronize the Coullet system and Genesio-Tesi system. The design of the Controller for this system will be completed in two stages:

1- Design of surface that explanatory dynamics of system
2- Compietion of control law until states of system slide upon to the surface

The essential work in this procedure is to determine control law, so the error of states between two systems and sliding surface are defined as follow:

$$e_i = Y_i - X_i \tag{5}$$

And

$$S(t) = \sum_{i=1}^{n} c_i e_i(t) \tag{6}$$

Where n is the number of stats and c_i will be chosen according to the sliding dynamics. When any state reaches the surface, it is told the sliding mode is taken place. At this time, the state dynamics will be controlled via sliding mode dynamics, so selection of c_i is important [18]. After the reaching, the state must stay in the surface. The sliding mode control needs two stages of:

Approaching phase to the surface $S(t) \neq 0$

A sliding phase to $S(t) = 0$

To verify the stability requirements function $V = \frac{1}{2}s^2$ is candidate as a Lyapunov function, where S is the sliding surface. To guarantee the stability the differentiation of Lyapunov function has to be negative definite. A sufficient condition of transition from the first phase to the second will be defined by the sliding condition as Differentiation of V when approaches to:

$$\dot{V} = s\dot{s} < 0 \tag{7}$$

Differentiation of (6) yields:

$$\dot{S}(t) = c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 = c_1(y_2 - x_2) + c_2(y_3 - x_3) + c_3[G(Y) - F(X) + U] \quad (8)$$

Replacing Eq. (8) into (7) achieves:

$$\dot{V}(t) = s(t)\{c_1(y_2 - x_2) + c_2(y_3 - x_3) + c_3[G(Y) - F(X) + U]\} \quad (9)$$

The selection $\dot{s} = -K\,\mathrm{sgn}(s)$ meets the sliding condition and yields the control law in the following form:

$$U_1 = -\frac{K}{c_3}.\mathrm{sgn}(S(t)) - \frac{c_1}{c_3}(y_2 - x_2) - \frac{c_2}{c_3}(y_3 - x_3) - [G(Y) - F(X)] \quad (10)$$

Where

$$u_c = \frac{K}{c_3}.\mathrm{sgn}(S(t)) \quad (11)$$

And

$$u_{eq} = \frac{c_1}{c_3}(y_2 - x_2) - \frac{c_2}{c_3}(y_3 - x_3) - [G(Y) - F(X)] \quad (12)$$

u_c and u_{eq} are the corrective control law and the equivalent control law, respectively, whereas $K > 0$ is the switching coefficient. An equivalent control $u_{eq}(t)$ causes the system dynamics to approach to the sliding surface and u_c is corrective control law which completes the control law in $u_{eq}(t)$.

4. SIMULATION RESULTS

To verify the capability of the proposed method the Genesio-Tesi and Coullet systems with unequal initial condition is selected as:

$$X(0) = \begin{bmatrix} 0.21 \\ 0.22 \\ 0.61 \end{bmatrix} \quad , \quad \mathrm{Y(0)} = \begin{bmatrix} 0.1 \\ 0.41 \\ 0.31 \end{bmatrix} \quad (13)$$

The simulation is performed by MATLAB software. To avoid occurrence of chattering, saturation function is gained instead of the signum function in Eq. (10). This will be defined as:

$$sat(S) = \begin{cases} +1 & S(t) > \varepsilon \\ t & -\varepsilon \le S(t) \le \varepsilon \\ -1 & S(t) < -\varepsilon \end{cases} \quad (13)$$

The simulation results are shown in Fig. (3)-(5). Synchronization results and error of tracking are shown in Fig. (3) and Fig (4), respectively. The control input is shown in Fig. (5). It should be noted that the control is triggered at $t=100s$. These verify the performance of the sliding mode to stabilize the chaos and synchronize two inequivalent systems.

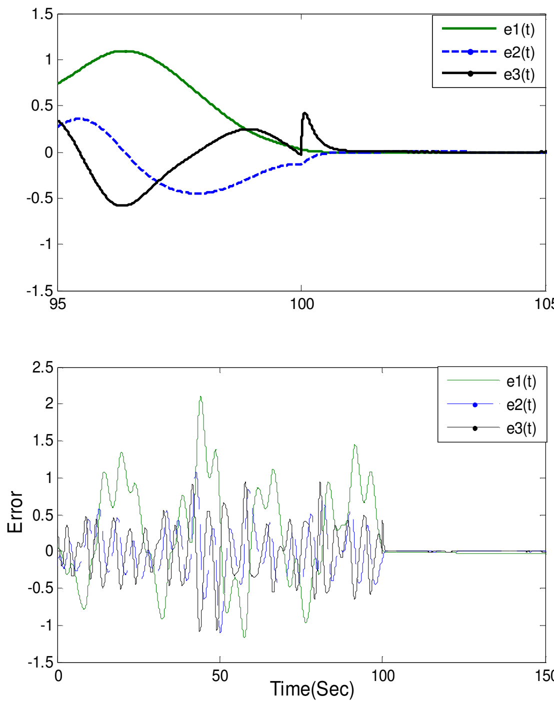

Fig. 4(a): The States Synchronization Error of Cuollet system and Genesti Tesi system

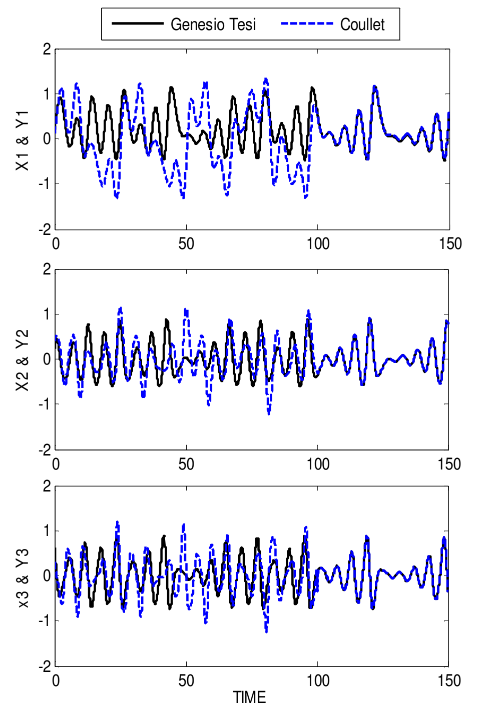

Fig. 3: Synchronization states of Collet system with Genesio-Tesi system

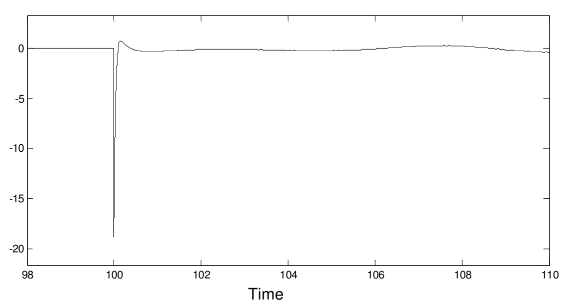

Fig. 5: The Signal Control Designed By Sliding Mode Controller

5. CONCLUSION

In this paper the synchronization between two different chaotic systems, i.e. Genesio-Tesi and Coullet systems is studied via Sliding Mode Controller. Although, different dynamics made more difficulty in synchronization control, a controller is designed based on Sliding Mode Control and Lyapunov stability theory which efficiently synchronized the Genesio-Tesi and Coullet systems.

6. REFERENCES

[1] Abdous F., A. Ranjbar, S.H. Hossein-Nia, Stabilizing the Hopf Bifurcation Using Time-delay Feedback Control in the Network Congestion, IEEE conference, WOSPA, 2008.
[2] Abdous F., A. Ranjbar N., S.H. Hosein Nia, A. Sheikhol Eslami, B. Abdous, The Hopf Bifurcation control in Internet Congestion Control System, Second International Conference on Electrical Engineering (ICEE), Lahore (Pakistan), 2008.
[3] Abdous, F. ,A. Ranjbar N., S.H. Hosein Nia, A. Sheikhol Eslami, Chaos Control of Voltage Fluctuations in DC Arc Furnaces Using Time Delay Feedback Control, Second International Conference on Electrical Engineering (ICEE) ,Lahore (Pakistan), 2008.
[4] Chua L. O., M. Itah, L. Kosarev and K. Eckert, Chaos synchronization in Chua's Circuits, J. Circuits, Syst. And Comp., 3 93-108, 1993.
[5] Deng W., C.P. Li, Chaos synchronization of the fractional Lu system, Phys., 353: 61–72, 2005.
[6] Deng W., C. Li, Synchronization of chaotic fractional Chen system, J. Phys.Soc. 74 (6):1645–1648, 2005.
[7] Gallegos JA., Nonlinear regulation of a Lorenz system by feedback linearization techniques. Dyn. Control 1994; 4: 277–98, 1994.
[8] Hu J. B., Y. Han, L. D Zhao, Synchronization in the Genesio-Tesi and Coullet systems using the Backstepping Approach", International Symposium on Nonlinear Dynamics (2007 ISND), Journal of Physics:Conference Series 96 012150 doi:10.1088/1742-6596/96/1/012150, 2007.
[9] Hunt E. R., Stabilization High-Period Orbits in a Chaos System, Phys. Rev. Lett. 1991; 67: 1953-55, 1991.
[10] Li D., J. A. Lu and X. Wu, Linearly coupled synchronization of the unified chaotic systems and the Lorenz systems, Chaos, Solitons & Fractals, Vol 23, Issue 1, Pages 79-85, January 2005
[11] Li C.P., W.H. Deng, D. Xu, Chaos synchronization of the Chua system with a fractional order, Phys. A 2006;360:171–185, 2006.
[12] Liu Shu-Yong, Xiang Yu and Shi-Jian Zhu, Study on the chaos anti-control technology in nonlinear vibration isolation system, Journal of Sound and Vibration, 310(4-5): 855-864, 2008.
[13] Ott. E., Grebogi, C., York, J. A, Controlling Chaos, Phys. Rev. Lett. ; 64: 1196-99, 1990.
[14] Park J. H., Stability criterion for synchronization of linearly coupled unified chaotic systems, Chaos, Solitons & Fractals, Vol. 23, Issue 4, Pages 1319-1325, February 2005.
[15] Phuah J., J. Lu, and T. Yahagi, Chattering free sliding mode control in magnetic levitation system, IEEJ Trans. EIS, 125(4):600–606, 2005.
[16] Pyragas K., Continuous control of chaos by self controlling feedback, Phys. Rev. Let. 67: 1953-55, 1991.
[17] Roy R., T. W. Murphy، T. D. Maier، Z. Gillis and E. R. Hunt, Dynamical Control of a chaotic Layser: Experimental stabilization of a Globally Coupled System, Physc. Rev. Lett, vol. 68، pp. 1259-1262, 2000.
[18] Slotine J. E., S. S. Sastry, Tracking control of nonlinear systems using sliding surface with application to robotic manipulators, Int. J.Contr. ,vol. 38, pp. 465 492, 1983.
[19] Utkin V. I., Variable structure systems with sliding mode, IEEE Trans.Automat. Contr., 22: 212–222, 1977.
[20] Yassen MT., Controlling chaos and synchronization for new chaotic system using linear feedback control, Chaos Solitons Fractal, 26: 913–20, 2005.
[21] L.-C. Yao, J.-S. Chen, C,-Y, Hsu, A Mode Switching Sliding-mode Controller with Observer based State-Dependent Boundary Layer and Its Application, International Journal of Engineering, vol. 1(1), pp. 39:53, 2007 .
[22] Yang T., "A survey of chaotic secure communication systems", Int. Journal of Compational Cognition, vol. 2. No. 2. pp. 81-130, 2004.