Vibration Attenuation of a Thin Cantilevered Beam Using LQG-Based Controller and Inertial Actuator

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Abstract

This study considers the problem of attenuating the vibration at a certain location on a flexible cantilevered beam mounted on a vibrating base which is the tip in this case. Attenuation is achieved without the need for sensor placement at that location. A modal state-space model of the flexible beam is constructed from the beam's first ten modes of vibration. A reduced-order optimal observer is utilized to estimate the deflection of the beam's tip from measurements of vertical deflections at mid-span and actuator locations. An inertial actuator is mounted on the beam itself, provides the control effort necessary for attenuating the tip vibration, resulting from shaker excitation. Experimental and simulation results have demonstrated the effectiveness of the proposed control technique.

Keywords: Inertial actuator, optimal control, LQG, estimation, cantilevered beam, modal analysis.

NOTATION

$\mathbf{A}$	dynamics matrix of full order modal model
$\mathbf{A}_c$	dynamics matrix of the controller
$\mathbf{A}_\mathbf{r}$	dynamics matrix of reduced-order modal model
$\mathbf{A}_a$	dynamics matrix of augmented system
b	actuator damping coefficient
$\boldsymbol{B}_f$	input vector of transmitted force
$\mathbf{B}_\mathbf{m}$	input matrix of full order modal model
$\mathbf{B}_\mathbf{r}$	input matrix of reduced-order modal model
$\mathbf{B}_\mathbf{rc}$	control distribution modal matrix
$\mathbf{B}_\mathbf{c}$	controller input vector
$\mathbf{C}_c$	controller output matrix
$\mathbf{C}_\mathbf{md}$	modal displacement output matrix
$\mathbf{C}_{\mathbf{m}v}$	modal rate output matrix
$\mathbf{C}_\mathbf{r}$	output matrix of reduced-order system

$\mathbf{D}$	nodal damping matrix
$\mathbf{D}_{ur}$	reduced-order feed through matrix
$\boldsymbol{e}_2$	model reduction error
F_i	Excitation force from the shaker (support)
F_c	actuator control force
G_c	Actuator transfer function
G_r	transfer function of reduced model
$\mathbf{I}$	identity matrix
J	quadratic cost performance index
k	actuator stiffness
$\mathbf{K}$	nodal stiffness matrix
$\mathbf{K}_c$	controller modal Matrix of gains
$\mathbf{L}$	Estimator gain matrix
$\mathbf{M}$	nodal mass matrix
m	number of actuators
m_o	mass of inertial actuator
n	dimension of nodal model
N	dimension of modal model
N_r	dimension of reduced order modal model
p	number of sensors
$\mathbf{P_c}$	solution of the controller algebraic Riccati equation
$\mathbf{P_e}$	solution of the estimator algebraic Riccati equation
$\mathbf{Q}$	Controller weighting matrix
$\boldsymbol{q}$	vector of nodal displacements
$\mathbf{R}$	controller weighting matrix
s	Laplace variable
t	time
u	input to transmitted force model
$\mathbf{U_c}$	control law
$\mathbf{V}$	estimator weighting matrix
v	measurements noise
$\boldsymbol{w}$	vector of external inputs
$\mathbf{W}$	estimator weighting matrix
$\mathbf{W_c}$	controllability grammian
$\mathbf{W_o}$	observablility grammian
$y(t)$	displacement at actuator location
$\boldsymbol{Y}_r$	nodal output vector of reduced model
$\boldsymbol{Y}_s$	nodal output vector of full model
$\boldsymbol{z}$	modal states
$\mathbf{\Delta}$	modal damping matrix
$\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}$	modal displacement and velocity vectors
$\mathbf{\Phi}$	matrix of eigenvectors

Ω diagonal matrix of eigenvalues

1. INTRODUCTION

Accuracy of the model of a flexible structure plays a major role in the design of an appropriate control system that will force the structure to perform certain tasks or attenuate structural vibration. Flexible structures are inherently distributed parameter structures with infinite degrees of freedom. A model of such structure will have to be a reduced order model but adequate enough to yield a controller that will carry out the necessary manipulation and/or vibration attenuation.

Research efforts in this field have provided a wealth of techniques for modeling and control of flexible structures. Once the model is constructed, the accuracy and precision of control systems, needed to manipulate the structure, or to perform certain tasks, depends solely on the accuracy of the structure's model. Since, models of flexible structures are reduced order, the control scheme has to either, focus on certain dynamic characteristics and try to control the structure based on the available model, or it has the ability to somewhat estimate or predict the un accounted for dynamics. In either case, the model and the controller will have limited ability in predicting the actual structural behavior **[1].** As it is known, any active control scheme must keep track of the behavior of the physical system in order to adjust its effort to suite the current status of the structure and perform the predefined requirements on that structure. This also poses another problem in large flexible structures, since sensor placement on large flexible structures increases the complexity of both the model and the controller. Therefore, to actively control a flexible structure, one has to keep in mind all the aforementioned challenges.

From modeling standpoint, many research efforts have utilized FEM to obtain models of flexible structures that can be used for control purposes. However, modal models obtained by FEM are not sufficient for providing a reliable model for control purposes for many reasons, one of which is that, FEM provides no information about damping in the structure **[1].** This aspect is crucial to the model since damping dictates the transient behavior of the structure as well as the phase of the control effort in relation to structural vibration.

In modeling of flexible structures, FEM technique is widely used for constructing analytical models of flexible structures. This method is well established but may not be accurate for controller design due to drawbacks mentioned above **[2, 3].** Another approach for modeling flexible systems is to determine the model directly from experimental data, such as experimental modal analysis [**2**]. Analytical methods have also been used to obtain working models for flexible structures **[4-6].**

As for the control of flexible structures, considerable research has been done in this field with various approaches and techniques. A common form of vibration control of flexible structure is done using passive means such as using viscoelastic materials, and passive damping **[7, 8].** Other control approaches utilize various forms of feedback control techniques to achieve certain manipulation, attenuation, suppression or isolation requirements **[9-13].** Other techniques involve robust control, optimal and fuzzy control **[14-19]**. Control techniques of flexible structures focus mainly on improving the agility, efficiency and bandwidth of the controller in the presence of noise as well as modeling and excitation uncertainties. A comprehensive review of shape control of flexible structures with emphasis on smart structures using piezoelectric actuators is presented by **[20, 21].**

In this work, vibration attenuation of the tip of a cantilevered beam subjected to excitation at the support is presented. A novel control technique uses a non-collocated sensors and actuators to attenuate the vibration at the beam's tip. The control effort is determine by an LQG-based controller that uses feedback measurements at locations other than the tip to determine the required control force needed to attenuate the vibration at the tip. A non-reactive actuator (i.e. inertial actuator) is mounted close to the support and used to apply an appropriate control force to attenuate the tip vibration. The sensor-actuator non-collocation is utilized in this work to accommodate situations where sensor placement at the control location is prohibitive. Inertial actuators react directly off a mass; therefore, they can be placed directly on the vibrating structure without the need for a reactive base **[11, 12, 22]**. LQG is utilized to estimate the control effort needed without the need for sensor placement at the control location. LQG-based controller reduces computational cost and provides robustness in the presence of model uncertainties as well as measurements and excitation noise. Dynamic modal model of the flexible structure is constructed from the first ten modes of vibration obtained via finite element analysis of the structure. To improve the agility of the controller and reduce its sensitivity, the proposed control technique uses a reduced order modal model of the structure. The latter eliminates the least effective modes of vibration at the control location, i.e., modes with the least H_2 norms are truncated from the modal model [**23].** Both simulation and experiments are carried out to verify the integrity of the proposed control technique.

2. CONTROL STRATEGY DESCRIPTION

In the following sections, the linear quadratic Gaussian (LQG) based controller is constructed. The LQG has two parts, namely, an optimal observer, and an optimal controller. The observer generates estimates of the vertical nodal displacement of three locations on the beam, which are the actuator location, mid-span, and the beam tip. The controller output is a control force lateral to the beam axis. As for the inputs, the observer is subject to all external inputs that the actual beam is subject to. Those inputs are the force applied to the beam at the support F_i, and the vertical control force F_c. Moreover, optimal observers in general require feedback of errors between state estimates and state measurements.

The demand on LQG is to reduce the vibration of the beam tip. To fulfill this demand, the LQG has to perform the two nearly-simultaneous tasks of, (a) producing optimal estimates of the vibration at three locations along the beam's span and (b) generating a control vector that will drive the inertial actuator to minimize the force transmitted to the tip and subsequently reduce its vibration.

3. INERTIAL ACTUATOR

To better explain the control scheme proposed by this study. The layout of the dynamic system is shown in Figure (1-a). In Figure (1-b) a free body diagram of the beam is shown where the inertial actuator is applying force F_c to cancel out the effect of force F_i on the section of the beam to the right of the inertial actuator. The actuator is particularly targeting the attenuation of the vibration at the tip through transferring a force to the beam with required magnitude and phase such that the tip's vibration is minimized. Modeling the actuator as a spring mass damper system, the transmitted force from actuator to the beam can be expressed in the form

$$-m_o \ddot{z} = F_c \tag{1}$$

where m_o is the mass of the actuator and z is the actuator's mass displacement. Therefore,

$$F_c = c(\dot{z} - \dot{y}) + k(z - y) \tag{2}$$

where y is the displacement of the beam at the actuator's location. Assuming that the ultimate objective of the controller is to nullify the displacement of the actuator mass relative to the beam (i.e., $z == 0$), will yield an a required actuator force of the form,

$$F_C = c(-\dot{y}) + k(-y) \tag{3}$$

With an actuator stiffness that is relatively large compared to the stiffness of the beam, and assuming only natural damping in the beam and actuator, the force acting on the actuator mass will vanish only if the vibration of the beam at the actuator location vanishes. However, since the purpose of the control is to eliminate the vibration at the beam tip not the actuator location, the controller will assume the actuator's mass vibration is indirectly caused by the vibration of the tip, and therefore, must keep track of the tip's vibration despite the fact that physical measurements at that tip are inaccessible. Therefore, if estimates of the tip vibration is fed to the actuator, the result will be a control force that will minimize the tip vibration with F_c in Equation (3) replaced by

$$F_C = c(-\dot{\hat{y}}_t) + k(-\hat{y}_t) \tag{4}$$

Where $\hat{y}_t$ is the estimate of the tip's vibration and the transfer function mapping the tip's displacement estimates to the control force is,

$$G(s) = \frac{F_s(s)}{\hat{Y}_t(s)} = -\left(bs + k\right). \tag{5}$$

this resembles a PD controller with proportional and derivative gains equal to the stiffness and damping of the actuator, respectively.

It is clear, that actual implementation of the inertial actuator will cause the dynamics of the latter to be part of the overall dynamics of the system. Therefore, actuator dynamics will be later on augmented with those of the structure (i.e., beam) during the design of the LQG-based controller.

4. STATE-SPACE REPRESENTATION OF THE BEAM

To implement the proposed concept of controlling the tip vibration using inertial actuator, a state-space model is formulated, in this case, for a distributed parameter system such as the beam shown in Figure (1). The beam is considered as an *n*-dimensional model of a modally damped flexible structure having (*m*) actuators and (*p*) sensors, not necessarily collocated. The beam system retrofitted with an actuator is shown in Figure

(1). The beam's structure is represented in nodal coordinates by the following second-order matrix differential equation:

$$\mathbf{M}\ddot{\boldsymbol{q}} + \mathbf{D}\dot{\boldsymbol{q}} + \mathbf{K}\boldsymbol{q} = \mathbf{B}\boldsymbol{w} \tag{6.a}$$

$$\boldsymbol{Y}_s = \mathbf{C}_\mathbf{d}\boldsymbol{q} + \mathbf{C}_\mathbf{v}\dot{\boldsymbol{q}} \tag{6.b}$$

In the above equation $\boldsymbol{q}$ is the $n \times 1$ displacement vector, $\boldsymbol{w}$ is the $m \times 1$ external input vector, $\boldsymbol{Y}_s$ is the $p \times 1$ nodal output vector, $\mathbf{M}$, $\boldsymbol{D}$, and $\mathbf{K}$ are the $n \times n$ mass, damping, and stiffness matrices, respectively. $\mathbf{C}_\mathbf{d}$, and $\mathbf{C}_\mathbf{v}$ are respectively, the $p \times n$ output displacement and output velocity matrices. The mass matrix is positive definite and the stiffness and damping matrices are positive semidefinite. The damping matrix $\mathbf{D}$ is assumed to be proportional to the stiffness matrix $\mathbf{K}$ without any significant effect on the integrity of the model [**23, 24**]. Dynamic model such as the one shown in Equation (6) is usually obtained from finite-element codes and has the dimension *n* which is unacceptably high to use in producing a state-space model suitable for structural control. Therefore an alternative approach is to use an *N*-dimensional second-order modal model of the system where $N << n$.

A second order modal-model of the system can be expressed as

$$\ddot{\boldsymbol{\eta}} + 2\mathbf{\Delta\Omega}\dot{\boldsymbol{\eta}} + \mathbf{\Omega}^2\boldsymbol{\eta} = \mathbf{B}_\mathbf{m}\boldsymbol{w}(t) \tag{7.a}$$

$$\boldsymbol{Y}_s = \mathbf{C}_\mathbf{md}\boldsymbol{\eta} + \mathbf{C}_\mathbf{mv}\dot{\boldsymbol{\eta}} \tag{7.b}$$

where $\boldsymbol{\eta} = \mathbf{\Phi}\boldsymbol{q}$, and $\mathbf{\Phi}$ is the $n \times N$ modal matrix, $\mathbf{\Omega}$ is the $N \times N$ diagonal matrix of modal natural frequencies, $\mathbf{\Delta}$ is the $N \times N$ modal damping matrix, $\mathbf{B}_\mathbf{m}$ is the $N \times m$ modal input matrix, $\mathbf{C}_\mathbf{md}$ and $\mathbf{C}_\mathbf{mv}$ are the $p \times N$ modal displacement and rate matrices, respectively. Defining the state vector $\boldsymbol{z} = [\boldsymbol{z_1} \quad \boldsymbol{z_2}]^T = [\boldsymbol{\eta} \quad \dot{\boldsymbol{\eta}}]^T$, a flexible structures having point force(s) as the input(s) and point displacement(s) as outputs will have the state space representation,

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -2\mathbf{\Delta\Omega} \end{bmatrix}\boldsymbol{z} + \begin{bmatrix} 0 \\ \mathbf{B}_\mathbf{m} \end{bmatrix}\boldsymbol{w} \tag{8.a}$$

$$\boldsymbol{Y}_s = [\mathbf{C}_\mathbf{md} \quad \mathbf{0}]\boldsymbol{z} + \mathbf{D}_\mathbf{u}\,\boldsymbol{u} \tag{8.b}$$

where, $Y_s(t)$ and $w(t)$ are the nodal output and input vectors, respectively. Matrix $\mathbf{D}_\mathbf{u}$ is the $p \times m$ feed through matrix. Equation (8) can also be represented by the following compact form,

$$\dot{\boldsymbol{z}} = \mathbf{A}(\theta)\boldsymbol{z} + \mathbf{B}(\theta)\boldsymbol{w} \tag{9.a}$$

$$\boldsymbol{Y}_s = \mathbf{C}(\theta)\boldsymbol{z} + \mathbf{D}_\mathbf{u}(\theta)\boldsymbol{w} \tag{9.b}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}_\mathbf{u}$ matrices are functions of the system (natural frequency, damping ratio, and mode shapes (i.e., $\theta = f(\omega_i, \zeta_i, \text{ and } \phi_i)_{i=1\Lambda N}$). The dimension of this state-space representation is $2N$ and it is much more manageable than the $2n$ state-space model obtained from the corresponding nodal model. Detailed information on constructing state-space model from natural frequencies and mode shapes is given in [**15, 23**].

5. MODEL REDUCTION

To show the effectiveness of the proposed control strategy, the $2N$-dimension state space model of the beam-hub system constructed in Equation (9) will be reduced even further to a $2N_r$-dimension state-space representation. This will reduce the computational cost and further validate the effectiveness of the proposed technique. In this study the reduction is based on retaining only half the number of modes (i.e., $N_r = (N/2)$) and the modes retained will be those with the largest H_2 norm [**25**]. In such case, the H_2 reduction error is expressed as:

$$e_2 = \|G - G_r\|_2 \tag{10}$$

where G is assumed to be the transfer function of the full model corresponding to equation (9), and G_r is the transfer function of the reduced model. The square of the mode norms are additive [**25**], therefore the norm of the reduced system with N_r modes is the root-mean-square sum of the mode norms

$$\|G_r\|_2^2 = \sum_{j}^{N_r} \|G_j\|_2^2 \tag{11}$$

and the reduction error is,

$$e_2^2 = \|G\|_2^2 - \|G_r\|_2^2 = \sum_{j=N_r+1}^{N} \|G_i\|_2^2 \tag{12}$$

The term $\|G_i\|_2$ is the modal cost of Skelton [**26**], which for the *j*th mode has the form

$$\|G_j\|_2 \cong \frac{\|B_j\|_2 \|C_j\|_2}{2\sqrt{\xi_j \omega_j}} \tag{13}$$

It can be seen from Equation (12) that near optimal reduction is attained if the truncated mode norms for $j = N_r + 1, \ldots N$ are the smallest. It should be noted that after truncation, modes should be rearranged such that the reduced model has a modal natural frequency $\Omega_f = diag[\omega_1, \ldots \omega_k]$ where, ω_1 has the highest H_2 norm. *This also implies rearranging the corresponding modal states as well*. The new state space representation of the truncated system is:

$$\dot{\mathbf{z}} = \mathbf{A_r}(\theta)\mathbf{z} + \mathbf{B_r}(\theta)\boldsymbol{w} \tag{14.a}$$

$$\boldsymbol{Y_r} = \mathbf{C_r}(\theta)\mathbf{z} + \mathbf{D_{ur}}(\theta)\boldsymbol{w} \tag{14.b}$$

where the subscript r denotes a reduced –order model. This state-space model has the dimension $2N_r$.

6. AUGMENTATION OF THE CONTROLLR WITH THE STRUCTURE

As it was mentioned in the Section (3) above, the dynamics of the actuator must be augmented with the dynamics of the beam because the actuator will e mounted on the beam itself. The augmented system will have the following state-space matrices,

$$A_a = \begin{bmatrix} A_c & 0 \\ B_r C_c & A_r \end{bmatrix}, B_a = \begin{bmatrix} B_c \\ B_r \times D_c \end{bmatrix}, C_a = \begin{bmatrix} D_{ur} \times C_c & C_r \end{bmatrix}, \textit{ and } D_a = [D_{ur} \times D_c] \tag{15}$$

Where A_c, B_c, C_c, D_c, are the state space matrices of the controller given in Equation (5). Notice that the transfer function in Equation (5) is unrealizable at its current state, but it can be with a second order filter in the denominator assuming that measurements of acceleration rather than displacement are to be fed to the actuator; see **[15]** for details.

7. OPTIMAL OBSERVER-CONTROLLER SYNTHESIS

In this section, the optimal observer-controller is formulated. The formulation is based on the augmented model of Equation (15). Measurements of the beam's acceleration in the vertical direction are assumed to be available at actuator location and at beam's mid-span. The beam support is assumed to be mounted on a shaker the cause the excitation at the support. The optimal observer-controller will be an LQG type for which the plant is described as follows:

$$\dot{z} = \mathbf{A}_a z + \mathbf{B}_{rc} \mathbf{U_c} + \mathbf{B}_a w \tag{16.a}$$

$$Y_a = \mathbf{C}_a z + \mathrm{v} \tag{16.b}$$

The plant model of equation (19) is perturbed by the external input w, and the output Y_a is corrupted with noise v. The term D_{ur} is dropped because the denominator of Equation (5) has been replaced by the second order filter. The optimal control problem is formulated by finding the linear quadratic regulator (LQR) gains and the Kalman estimator gains that will minimize a quadratic performance index,

$$J = \int_0^\infty (\mathbf{z}^T \mathbf{Q} \mathbf{z} + \mathbf{U_c}^T \mathbf{R} \mathbf{U_c}) dt \tag{17}$$

The estimator dynamics are given by the following state-space matrix differential equation

$$\dot{\hat{z}} = \mathbf{A}_a \hat{z} + \mathbf{B}_{rc} U_c + \mathbf{L}(Y_r - \mathbf{C}_a \hat{z}) + B_a w \tag{18.a}$$

$$\hat{Y}_a = \mathbf{C}_a \hat{z} \tag{18.b}$$

Here $\hat{z}$ is the vector of estimated modal states, B_{rc} is the modal feedback control distribution matrix, and $\boldsymbol{U}_\mathbf{c} = -\mathbf{K_c} \hat{\boldsymbol{z}}$, where K_c, and L are respectively, the LQR and Kalman gains. The feedback gain vector $\mathbf{K_c}$ is give by

$$\mathbf{K_c} = \mathbf{R}^{-1} \mathbf{B}_{rc}^{\mathbf{T}} \mathbf{P_c} \tag{19}$$

where $\mathbf{P_c}$ is the solution of the controller algebraic Riccati equation

$$\mathbf{A}_a{}^{\mathbf{T}}\mathbf{P_c} + \mathbf{P_c}\mathbf{A}_a - \mathbf{P_c}\mathbf{B}_{rc}\mathbf{R}^{-1}\mathbf{B}_{rc}^{\mathbf{T}}\mathbf{P_c} + \mathbf{Q} = \mathbf{0}, \quad (20)$$

and the estimator gain matrix $\mathbf{L}$ is given by

$$\mathbf{L} = \mathbf{P_e}\mathbf{C}_a^{\mathbf{T}}\mathbf{V}^{-1} \quad (21)$$

where $\mathbf{P_e}$ is the solution of the estimator algebraic Riccati equation

$$\mathbf{A}_a\mathbf{P_e} + \mathbf{P_e}\mathbf{A}_a{}^{\mathbf{T}} - \mathbf{P_e}\mathbf{C}_a{}^{\mathbf{T}}\mathbf{V}^{-1}\mathbf{C}_a\mathbf{P_e} + \mathbf{B}_a\mathbf{W}\mathbf{B}_a{}^{\mathbf{T}} = \mathbf{0} \quad (22)$$

In this study, Equation (18) produces the vector $\hat{Y}_a = \begin{bmatrix}\hat{q}_c & \hat{q}_{tip} & \hat{q}_{mid-span} & \hat{q}_{act\_location}\end{bmatrix}^T$ where, q_c are the controller states. It should be noted that the matrix $\mathbf{K_c}$ is the LQR modal gains and the portion of $\hat{Y}_a$ namely; $\begin{bmatrix} & \hat{q}_{tip} & \hat{q}_{mid-span} & \hat{q}_{act\_location} & \end{bmatrix}^T$ is the vector of beam's nodal displacements, and thus, before utilizing the optimal feedback, this portion has to be transformed back to the modal domain using the modal matrix $\mathbf{\Phi}$. Matrices $\mathbf{R}$ and $\mathbf{V}$ are positive definite while matrices $\mathbf{Q}$ and $\mathbf{W}$ are positive semidefinite. The latter four matrices can be treated both as weighting matrices and as tuning parameters for the LQG [**26, 27**].

In this study it is assumed beforehand that q_{tip} is not available for measurements, therefore, no estimate error for this state is available for feedback. However, estimate errors for the actuator location and the mid-span states are available and must be fedback to the estimator through the Kalman matrix of gains $\mathbf{L}$. This implies that poles of the closed loop system depicted by Equation (18) are placed in such manners that only optimal LQG estimates of actuator location and mid-span displacements are obtained. This is a major deficiency and it will cause an arbitrary change in the tip state due to the lack of information available to the estimator about measured tip displacements. Such estimator will not be of much use if the tip deflection is to be controlled. Therefore, the last term of equation (18) (i.e., $\mathbf{B}_a w$) is utilized to both eliminate this deficiency and achieve the demand on the LQG. In other words, the term $B_a w$ will have to enable the LQG to (a) force subsequent estimates of tip modal states (i.e., $\hat{z}$) to be dependent on previous estimates of tip deflection and (b) generate a control force that will minimize the difference between the actuator mass and tip deflections through manipulation of the actuator location only. If the inertial actuator utilized to generates a force that is a function of the difference between the actuator mass displacement (i.e., zero in this case based on the reasoning in Equations (3-5)) and tip deflections, then equation (18) will have $w = \begin{bmatrix}F_i & F_c\end{bmatrix}^T$ where, F_i *and* F_c are the excitation and control forces, respectively. we obtain,

$$\mathrm{F}_i = C_c(\mathbf{sI} - \mathbf{A}_c)^{-1}B_c(0 - \hat{\mathrm{q}}_{act\_location}), \quad (26)$$

where s is the Laplace variable, $\mathbf{I}$ is a 2×2 identity matrix.

For effective implementation of the proposed control strategy it is important to determine early on whether the system is controllable (i.e., actuators excite all modes) and observable (i.e., the sensors detect the motion of all modes). It is clear, however, that the proposed control strategy may very well have a value of N large enough to prohibit accurate calculation of the controllability and observability matrices of the system. To avoid such problem, the controllability and observability grammians are used instead of the original method proposed by Kalman [**22, 23**]. The grammians are determined from the following Lyapunov equations:

$$\mathbf{A}\mathbf{W_c} + \mathbf{W_c}\mathbf{A^T} + \mathbf{B}\mathbf{B^T} = \mathbf{0}, \\ \mathbf{A^T}\mathbf{W_o} + \mathbf{W_o}\mathbf{A} + \mathbf{C}\mathbf{C^T} = \mathbf{0} \tag{30}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, are the dynamic , input, output matrices of the overall model of the system, respectively. Matrices $\mathbf{W}_c$ and $\mathbf{W_o}$ are, respectively, the controllability and observability grammians. The system is controllable and observable if $\mathbf{W_c} = \mathbf{W_c}^{\mathbf{T}}$, and $\mathbf{W_o} = \mathbf{W_o^T}$. The synthesized controller is implemented and the results are presented in the next section.

8. EXPERIMENTAL AND NUMERICAL SIMULATION RESULTS

Vibration attenuation of the beam tip of Figure (1) is investigated. The beam-actuator system is subject to excitation shown in Figure (2) exerted on the beam's support. Dimensions and properties of the system under study are listed in Table 1. Actual behavior of the beam-actuator is constructed based on the first ten modes of vibration of the beam-actuator system for which the modal frequencies and modal norms are listed in Table (2). The reduced-order dynamics model used in the design of LQG is constructed from four modes only. The four modes are those with the largest H_2 norms as shown in Table 2, namely, modes 1, 2, 3, and 5. The reduced order model of the system is constructed from those four modes and the states are arranged according to the same sequence.

Using four modes of vibration to synthesize the LQG-based controller indicates that exact knowledge of the dynamics of the system is not necessary for successful implementation of the controller; rather a carefully reduced-order model is sufficient. The controller and estimator gain matrices, K_c, and *L* are generated in Matlab using weighting matrices $\mathbf{Q} = \alpha\mathbf{I}_{\mathbf{10\times10}}$, $\mathbf{R} = \mathbf{I}_{2\times2}$, $\mathbf{V} = 10\times\mathbf{I}_{2\times2}$, and $\mathbf{W} = \mathbf{I}_{\mathbf{3\times3}}$, $\alpha = 100$. Modal damping of 0.01 is used for all modes of vibration. Notice that the two extra states in K_c, and *L* belong to the controller as shown by Equation (15).

The control effort is tuned to reduce the relative vibration between the beam's tip and the actuator mass via (1) feedback of two error signals; which are the difference between measured and estimated displacements at actuator location and mid-span, which are used to produce better estimates of deflections including that of the tip. And (2) estimates of the tip deflection, are fed through the inertial actuator with a demand to produce the control effort necessary to reduce the tip vibration. Block diagram of the control scheme used in this simulation is shown in Figure (3).

The ideal outcome of the control strategy is to reduce the estimates of the beam tip vibration to zero. Figures 4 and 5 show significant reduction in the vibration both at the tip, and at the actuator location. Figure 6 shows the control force needed to achieve the attenuation and it is within the capabilities of the actuator.

To verify simulation outcomes, experiments are carried out under conditions identical to those used in the simulation. The experimental setup shown in Figure 7 is utilized to verify the integrity of the proposed technique. Significant reduction in the magnitude of the transfer function (approximately 30% at resonant frequencies) is attained as shown in Figure 8. No abnormal vibration is created at the actuator location when the controller is targeting the tip vibration as shown in Figure 9. Experimental actuator force is shown in Figure 10 which is reasonable and within the capabilities of the actuator used.

It should be pointed out that this control technique has draw backs mostly in its sensitivity to excitation changes, mass of the inertial actuator, and overestimates of the force which appear to be related to the choice of values for the weighting matrices of the controller and observer as well as the derivative term in the transfer function of Equation (5).

9. CONCLUSIONS

A novel approach for attenuating the vibration of a cantilevered beam mounted on a vibrating base has been presented. The proposed technique is particularly useful considering the fact that, real-life systems exhibits considerable variations in their properties. Thus, the characteristics of a structure corresponding to these properties show some stochastic variations. This makes it necessary to take into account the uncertainties of the system if a reliable control system is to be implemented. To this purpose, a robust and effective control system using optimal estimation and control techniques would be a suitable choice. The control strategy uses available measurements at various locations on a flexible structure to produce estimates of the vibration at inaccessible locations. Latter estimates are then used to control the vibration at those inaccessible locations using inertial actuator. The inertial actuator is non-collocated with the sensors, and can be mounted directly on the structure, providing flexibility in locating sensors and actuators according to needs and constraints.

Experimental and simulation results of the proposed method show significant reduction in the tip vibration without the need for any sensor(s) placement at the control location (the tip in this case). This is an important feature which could prove useful in applications where use of sensory devices at any location on the flexible structure is difficult to attain. The proposed strategy managed to reduce the vibration of the beam’s tip by approximately 30% at resonant frequencies using only estimates of the tip vibration rather than actual measurements.

Fine tuning of the weighting matrices should be carried to obtain the optimal results for the control force and the reduction in the tip vibration. Further work is needed in the implementation of this technique and two and three dimensions where the processes can be duplicated for the other two dimensions that were not discussed by this work.

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Table 1: Simulation and Experimental Data

Component	Properties	Dimensions
Thin beam	Aluminum 6061 $E = 6.89 \times 10^{10}\ Pa$ $\rho = 2710\ kg/m^3$ $Poisson's\ Ratio = 0.35$	Length =0.53 m Thickness =2.1 mm Depth=0.035 m
Actuator	ETREMA Terfenol-D Actuator	Stiffness 300 kN/mm Mass=0.203 kg Damping =11560 kg/s Excursion=10 micro-meter pk-pk
Excitation	B&K shaker	1-100 Hz chirp signal + Random input

Table 2: Beam-Actuator system modal frequencies and modal H_2 Norms

Mode Number	Frequency Hz	H_2 Norm
1	10.106	0.05043736496038
2	54.732	0.00179075345056
3	139.72	0.00021895088343
4	195.05	0.00021895088343
5	271.71	0.0000000000000012
6	457.76	0.00004501906852
7	461.14	0.00000000000000
8	590.08	0.00002132364349
9	720.38	0.00000521173674
10	1000.4	0

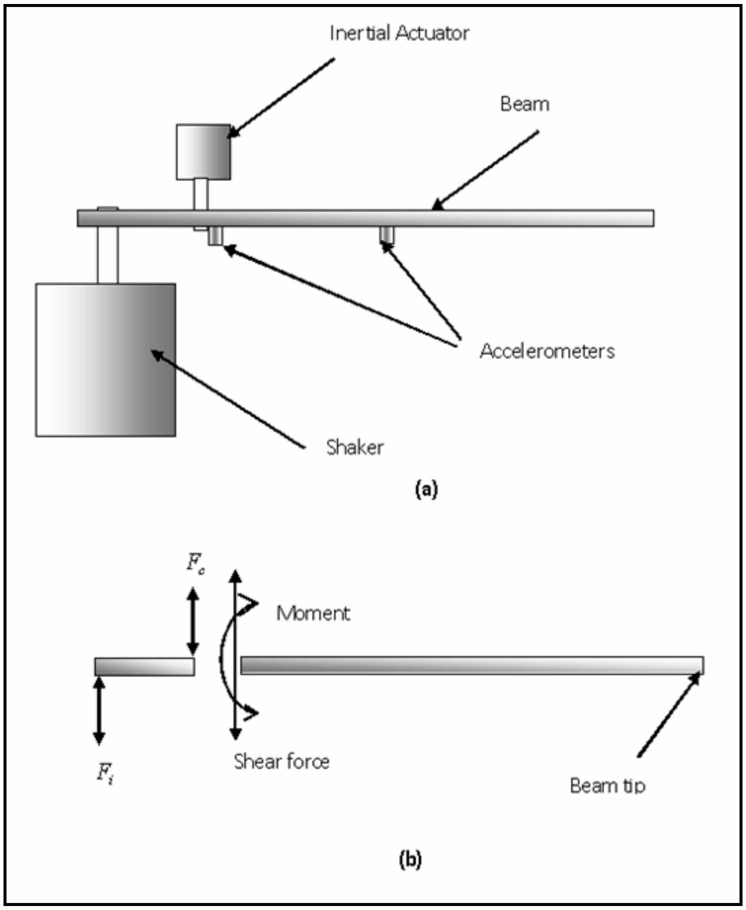

Figure 1: (a) Cantilevered beam mounted on a shaker (b) Free body diagram of the beam

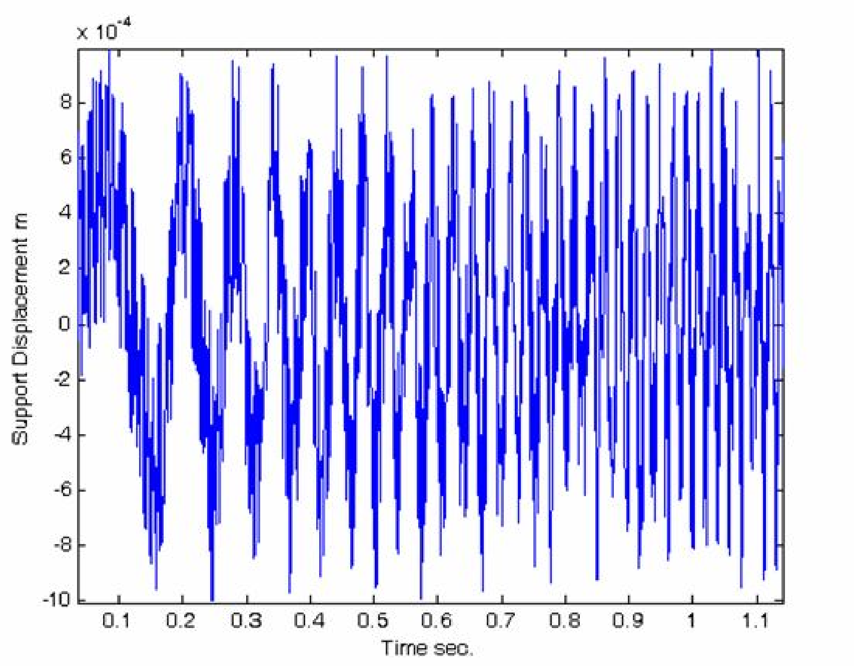

Figure 2: Excitation at the beam's support

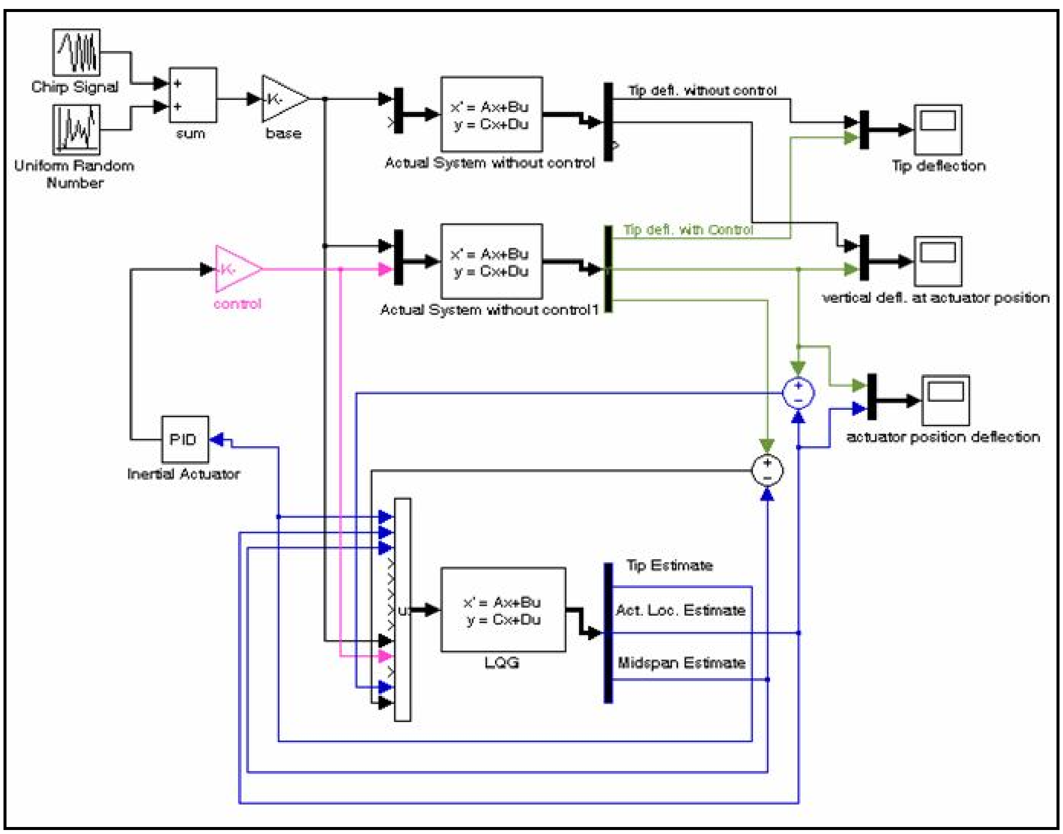

Figure 3: Block Diagram of the control scheme

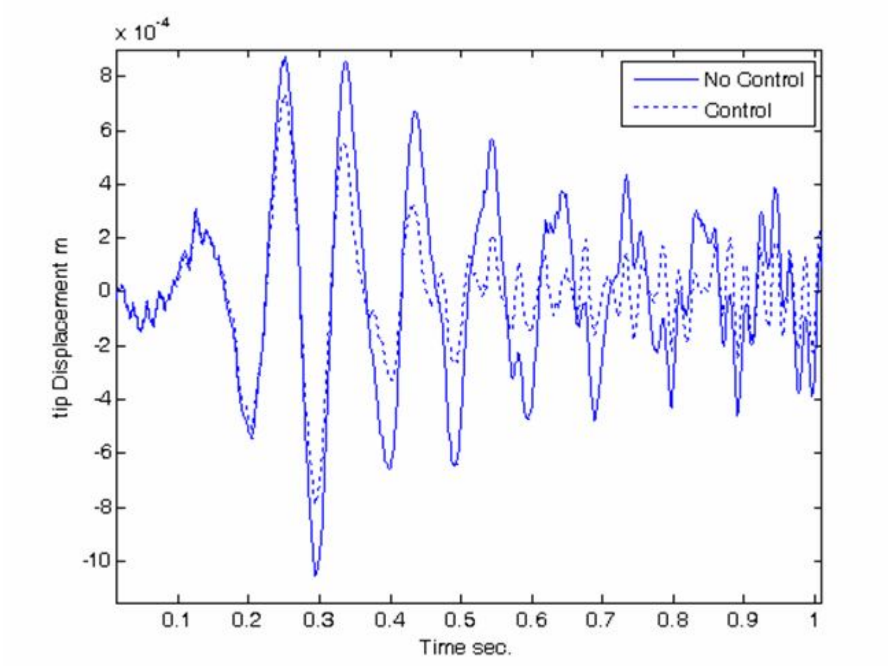

Figure 4: Tip vibration with and without control

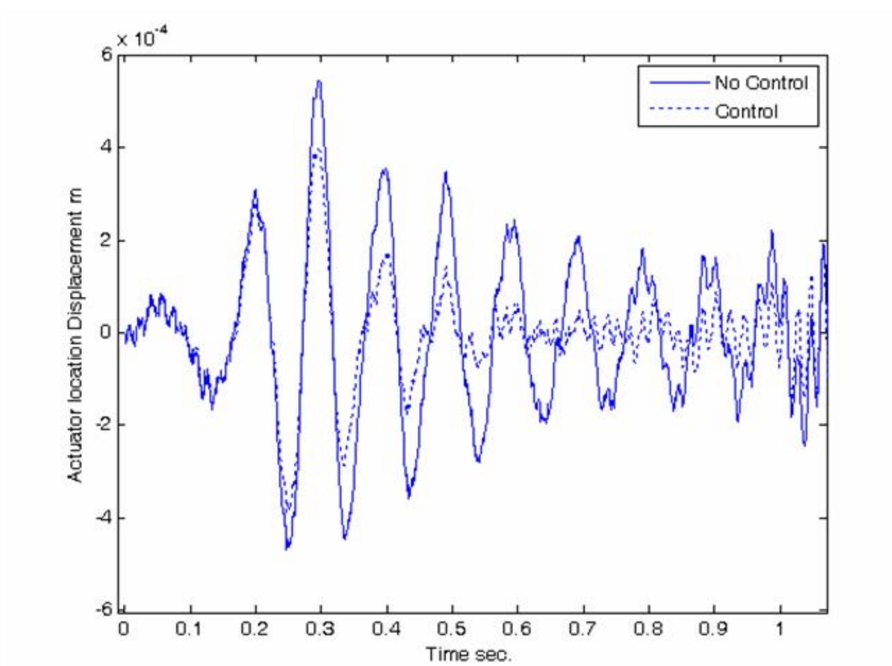

Figure 5: Vibration at the actuator location

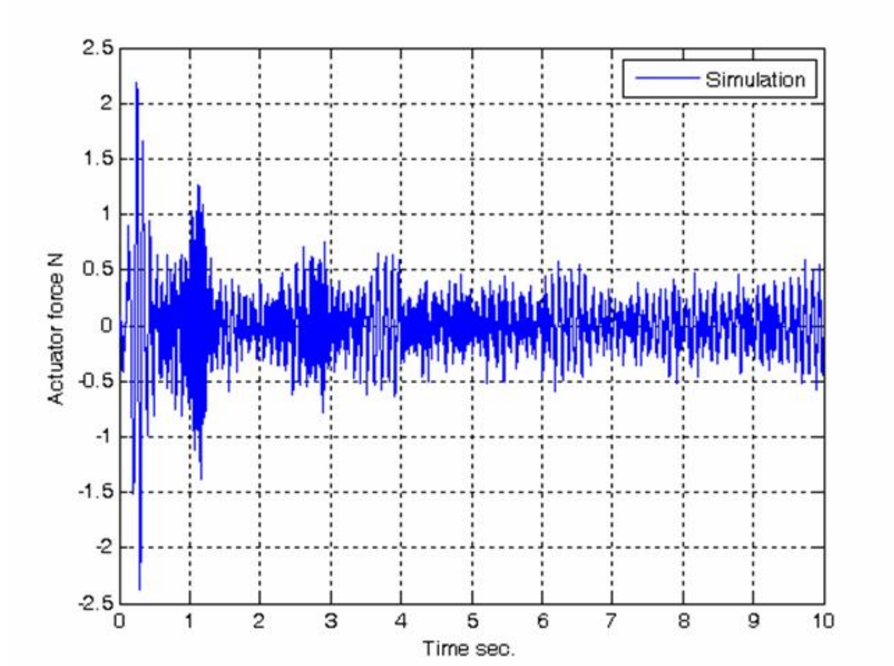

Figure 6: Control force, simulation

Figure 7: Experimental Setup

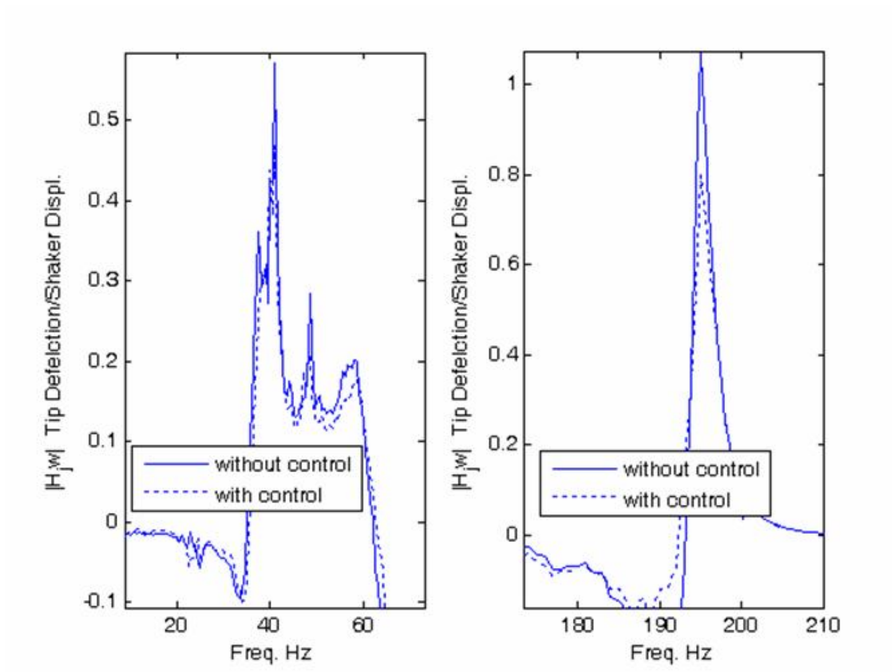

Figure 8: Frequency response function of the beam: tip displacement/shaker displacement

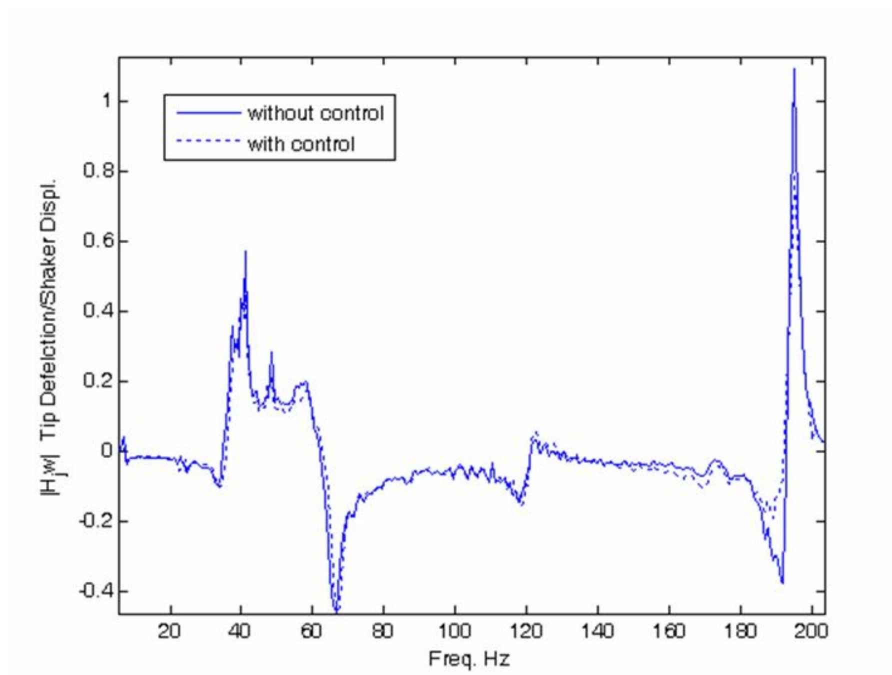

Figure 9: FRF of the beam with and without control

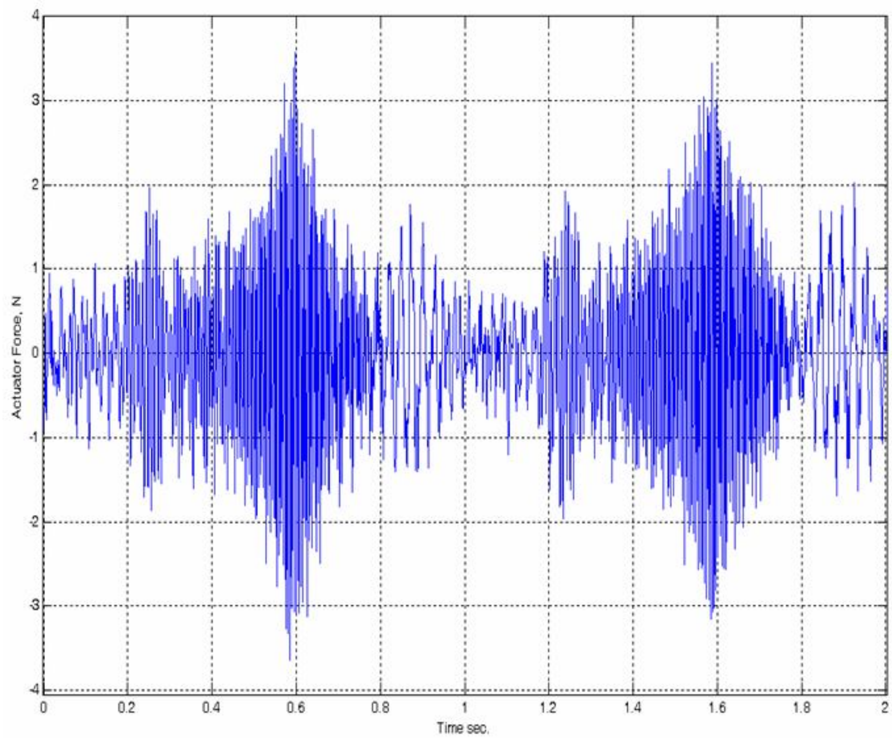

Figure 10: Control force, experimental