

Using Imperialist Competitive Algorithm to Find the Optimum Shape Design of Internally Pressurized Torispherical Dome Ends Based on Buckling Pressure

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Abstract

In this paper, Imperialist Competitive Algorithm (ICA) and Genetic Algorithm (GA) are used to find the optimal form for torispherical dome ends under internal pressure load. According to fabrication and strength of material requirements, a group of compromised counters are studied. According to ASME Section VIII and BS5500 pressure vessel codes, a reasonable buckling pressure limit is proposed. Four-centered ellipse method is used to describe the geometry of the torispherical dome end that this method is commonly used in engineering drawing. A minimum weight optimization problem based on buckling pressure is studied. Two different size torispherical dome end examples are selected and studied. Imperialist Competitive Algorithm is found to be very efficient and easy to use for the applications, such as torispherical dome end and subjected to internally pressurized loading.

Keywords: Imperialist Competitive Algorithm, Genetic Algorithm, Four-Centered Ellipse Method, Torispherical, Buckling Pressure, Internal Pressure

1. INTRODUCTION

Pressure vessels are very important in shell structures and a majority of them are axisymmetric. Torispherical dome ends are frequently used as end closures on pressure vessels and different types of cylindrical containers which can be found in various fields, such as aerospace, food processing, chemical, nuclear, oil industries, and so on. In this paper, we are only concerned with the structural honesty of the dome ends of the pressure hull.

Designers are seeking for maximum strength of structure with minimum weight. Minimum weight design of cylindrical and conical shells have been studied in details [1-3]. It seems that, for external pressurized cased, the optimization of torispherical dome ends under buckling

constraints was attempted in [4-6]. But studies concerned with the optimal shape design of torispherical dome end which under internal pressure loading are still limit. In this paper, we looked for an optimal design of torispherical dome end according to elastic – plastic buckling pressure.

The British pressure vessel code BS5500 gives the design pressure P_w of internally pressurized dome ends which is not allowed to exceed 1/8.33 of the elastic plastic buckling pressure P_b , while for the ASME section VIII it is 1/19.4. Therefore, in this paper, reasonable buckling pressure range is proposed between $8.33P_w$ and $19.4P_w$.

By using four-centered ellipse, three commonly used dome geometries that namely hemispherical dome, ellipsoidal dome, and torispherical dome can be described [7]. This method is commonly used in engineering drawing and it is very useful in the design and construction of the dome structure.

Imperialist Competitive Algorithm (ICA) is a new socio – politically motivated global search strategy that has recently been introduced for dealing with different optimization task [8]. This evolutionary optimization strategy has shown great performance in both convergence rate and better global optima achievement [9-11]. In this paper, we used imperialist Competitive Algorithm in optimizing torispherical dome ends and compared it with the genetic algorithm results.

2. GEOMETRY OF TORISPHERICAL DOME ENDS

At first, consider an elastic torispherical dome end with constant thickness t (Figure 1) under static internal pressure. It can be assumed that there is not any flange and the torispherical dome end is fully clamped at the edge.

The geometry of torispherical dome end can be constructed by using the four-centered ellipse method (Figure 1).

If we know the ratio $K = \frac{b}{a}$, then the other parameter θ, ϕ, r , and R can be determined from the following equation:

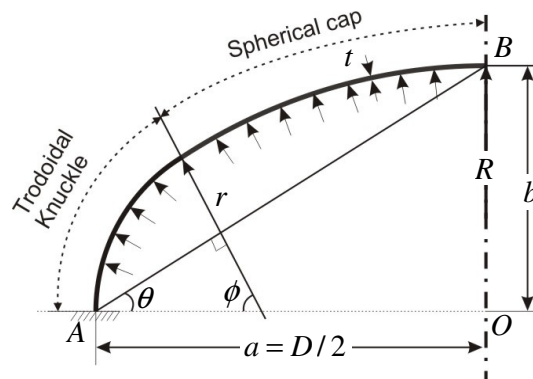


FIGURE 1: Geometry of torispherical dome end

$$\theta = \tan^{-1} \left(\frac{b}{a} \right) = \tan^{-1} K \quad (1)$$

$$\phi = \frac{\pi}{2} - \theta \quad (2)$$

$$r = \frac{a}{2} \left[\frac{1 + \sin \theta - \cos \theta}{\cos^2 \theta} \right] \quad (3)$$

$$R = \frac{a}{2 \sin \theta} \left[1 + \frac{1 - \sin \theta}{\cos \theta} \right] \quad (4)$$

Finally, the total weight of torispherical dome end with constant thickness is [5, 6]

$$W = 2\pi\gamma t \left[r\omega \left(\frac{\pi}{2} - \Phi \right) + r^2 \cos \Phi + (1 - \cos \Phi) R^2 \right] \quad (5)$$

Where

$$\omega = \frac{D}{2} - r = a - r \quad (6)$$

$$\Phi = \sin^{-1} \left(\frac{\omega}{R - r} \right) \quad (7)$$

And γ is the specific gravity.

In the [5, 6] Thickness t and the ratios $r/D, R/D$ were selected for the design variables of the optimum shape design of torispherical dome end. However, in this article, when the four-centered ellipse method is used, the radius of spherical cap R and the radius of toroidal knuckle r have certain relations which can be expressed as function of the angle of spherical cap θ and the ratio K . Therefore the thickness t and the ratio K are selected for design variables.

The choice of boundary values for the thickness t and the ratio K depends on practical application. From Harvey's study [14] in order to minimize the hoop stresses, the torispherical dome end must should a large knuckle radius. Most of pressure vessel contraction codes recognize this fact and therefore specify a minimum permissible knuckle radius. For instance, the British pressure vessel codes BS5500 and the ASME version VIII specify the minimum value of the knuckle radius as 6% of the crown radius. Therefore, the lower limit on the knuckle shallowness ($r/D = 0.06$) is chosen in this article. Blachut [5] showed that an unexpected drop in the buckling strength with the range $0.45 \leq r/D \leq 0.5$. Therefore, the range of r/D is selected to be $0.06 \leq r/D \leq 0.45$ in this paper.

3. MATHEMATICAL FORMULATION

An internally pressurized torispherical dome end is designed for the minimum weight objective satisfying specified design requirement. The manufacturing limitations on the geometric parameters and the mechanical properties of the shell material are the preassigned parameters for this design problem. The following optimization problem can then be formulated.

3.1 Cost function

Accompanying engineering demands for high pressure vessels with and without large sizes are often the economic ones of weight reduction to save materials, to enhance shipping and erection procedures and reduce fabrication costs. Therefore, cost function $F(\tilde{x})$ for weight is considered to be minimized in this paper

$$\text{Minimize } F(\tilde{x}) = W_d / W_s$$

Where W_s is the weight of hemispherical dome end ($W_s = \frac{1}{2}\pi\gamma D^2 t$) and W_d is the weight of torispherical dome end.

3.2 Design variables

The dimensions of the torispherical dome end are selected as the independent design variables. These variables are:

- (a) The thickness of torispherical dome end, $x_1 = t$
- (b) The ratio of minor axis to the major axis, $x_2 = K = b/a = \tan \theta$
- (c) The major axis of the torispherical dome end, $x_3 = a$

According to engineering design consideration, the length of major radius a is specified which must coincide with cylindrical shell at the junction. Therefore, there are two independent design variables taken for this problem. Therefore the vector \tilde{x} will be given as

$$\tilde{x}_{opt} = (t, K)$$

3.3 Constraints

The lower and upper bounds are imposed on all the design variables. Based on the standard of practical design, a reasonable buckling pressure range is proposed. The BS5500 design pressure is not allowed to exceed $P_b/8.33$, and the corresponding ratio for ASME is $P_b/19.4$.

The constraints g_i are represented by the following:

- (a) The lower bound on buckling pressure

$$g_1 = \frac{(P_L - P_b)}{P_b} \leq 0$$

Where P_b is the critical buckling pressure and P_L is the lower bound of critical buckling pressure, ($P_L = 8.33P_w$ where P_w is the maximum work pressure).

- (b) The upper bound on buckling pressure

$$g_2 = \frac{(P_b - P_U)}{P_U} \leq 0$$

Where P_U is the lower bound of critical buckling pressure. ($P_U = 19.4P_w$)

- (c) The lower bound on design variables

$$g_3 = \frac{(x^{Li} - x_i)}{(x^{Ui} - x^{Li})} \leq 0$$

Where x_i are design variables ($i = 1, 2$), x^{Ui} is the upper bound value of design variable x_i and x^{Li} is the lower bound value of design variables x_i

- (d) The upper bound on design variables

$$g_4 = \frac{(x_i - x^{Ui})}{(x^{Ui} - x^{Li})} \leq 0$$

4. IMPERIALIST COMPETITIVE ALGORITHM

Imperialist competitive algorithm (ICA) is a new evolutionary algorithm for optimization. This algorithm starts with an initial population. Each population in ICA is called country. Some of the best countries in the population selected to be the imperialists and the rest form the colonies of these imperialists. In this algorithm the more powerful imperialist, have the more colonies. When the competition starts, imperialists attempt to achieve more colonies and the colonies start to move toward their imperialists. So during the competition the powerful imperialists will be improved and the weak ones will be collapsed. At the end just one imperialist will remain. In this stage the position of imperialist and its colonies will be the same. The flowchart of this algorithm is shown in Figure 2 [11]. More details about this algorithm are presented in [8-13].

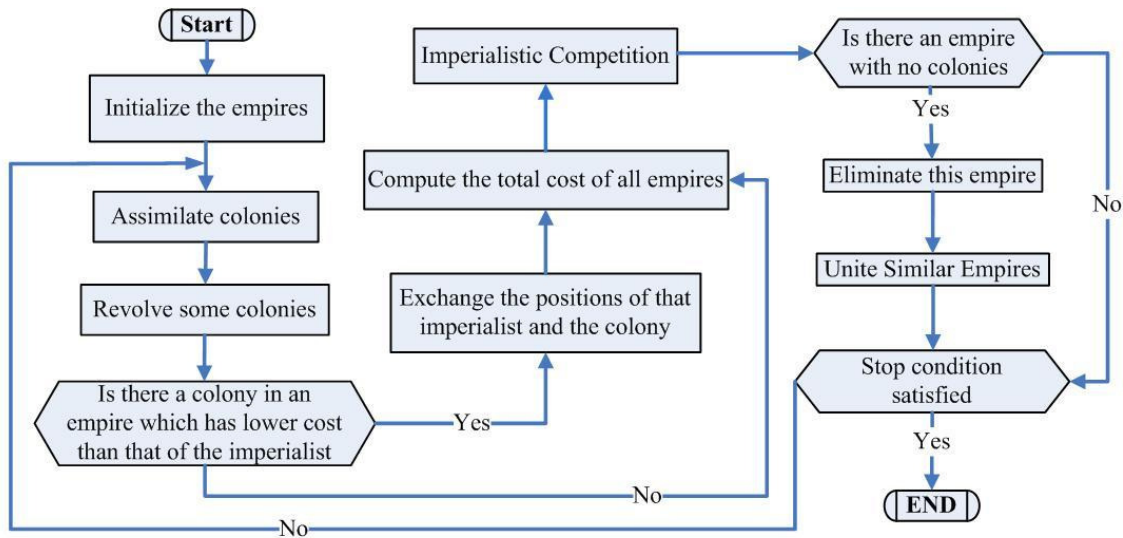


FIGURE 2: Flowchart of the imperialist competitive algorithm (ICA) [11].

5. MODEL DESCRIPTION

In this paper, two reference models of the torispherical dome end were considered. Both Models have the same poisson's ratio of 0.3 and the same elastic modulus of $3.0 \times 10^7 \text{ lbf} / \text{in}^2$. The fixed major axis a for model 1 is 63.98 in and 91.73 in for model 2. The principal dimensions for two references models are showed in table 1. The range of the ratio K is selected as $0.21 \leq K \leq 0.95$, and the range of thickness t as $50 \leq D/t \leq 150$. The preassigned lower and upper bound of design variables which were used in this paper are listed in table 2.

Torispherical dome	Model 1	Model 2
Length of major axis, a	63.98 in	91.73 in
Ratio of minor to major axis, $K = b/a$	0.61	0.53
Thickness, t	1.26 in	1.77 in
Radius of toroidal knuckle, r	29.28 in	34.35 in
Angle of toroidal knuckle, ϕ	58.62°	61.99°
Radius of spherical cap, R	95.91 in	156.83 in
Angle of spherical cap, θ	32.38°	28.01°

TABLE 1: The Principle Design Data of Reference Dome

Torispherical dome	Side constraint	Model 1	Model 2
Major axis, a	Fixed	63.98 in	91.73 in
Ratio of minor to major axis, $K = b/a$	Lower bound	0.21	0.21
	Upper bound	0.95	0.95
Thickness, t	Lower bound	0.85 in	1.22 in
	Upper bound	2.56 in	3.66 in

TABLE 2: The Upper and Lower Bounds of Design Variables

6. NUMERICAL RESULTS AND DISCUSSION

The numerical comparison between the results of the two optimal torispherical dome ends and two reference dome ends models are listed in table 3 and 4. Table 3. The results in table 3 are optimized by using Genetic Algorithm (GA) and in table 4 we use Imperialist competitive algorithm (ICA) for optimization. Also, the comparisons between optimal dome ends and referenced dome ends are displayed in Figure 3 and Figure 4. For a hemispherical dome end, the thickness effects of the dome end to weight and the effect of ratio K to weight are showed in Figure 5 and Figure 6. In additional, for a hemispherical dome end, the thickness effects of the dome end to the buckling pressure is studied and showed in Figure 7 and table 5. Also, if thickness is kept constant, the effect of ratio K to the buckling pressure of torispherical dome end is also studied and the results are showed in Figure 8 and table 6. If the ratio K is kept constant, the effect of thickness to the buckling pressure is also studied and the results are showed in Figure 9 and table 7.

Torispherical dome	Model 1		Model 2	
	Ref. dome	Opt. dome (GA)	Ref. dome	Opt. dome (GA)
Length of major axis, a	63.98 in	63.98 in	91.73 in	91.73 in
Ratio of minor to major axis, $K = b/a$	0.61	0.64	0.53	0.64
Thickness, t	1.26 in	0.87 in	1.77 in	1.24 in
Radius of toroidal knuckle, r	29.28 in	31.42 in	34.35 in	45.04 in
Angle of toroidal knuckle, ϕ	58.62°	57.39°	61.99°	57.39°
Radius of spherical cap, R	95.91 in	91.82 in	156.83 in	131.66 in
Angle of spherical cap, θ	32.38°	32.61°	28.01°	32.61°
Total weight of dome end	913.06 lbf	644.4 lbf	2486.9 lbf	1887.8 lbf
Elastic – Plastic buckling pressure, P_b	8216.6 psi	2741.7 psi	6273.85 psi	2715.9 psi

TABLE 3: The numerical comparison table of torispherical dome ends

Torispherical dome	Model 1		Model 2	
	Ref. dome	Opt. dome (ICA)	Ref. dome	Opt. dome (ICA)
Length of major axis, a	63.98 in	63.98 in	91.73 in	91.73 in
Ratio of minor to major axis, $K = b/a$	0.61	0.65	0.53	0.65
Thickness, t	1.26 in	0.85	1.77 in	1.22
Radius of toroidal knuckle, r	29.28 in	32.14 in	34.35 in	46.09 in
Angle of toroidal knuckle, ϕ	58.62°	56.98°	61.99°	56.98°
Radius of spherical cap, R	95.91 in	90.57 in	156.83 in	129.84 in
Angle of spherical cap, θ	32.38°	33.02°	28.01°	33.02°
Total weight of dome end	913.06 lbf	634.1 lbf	2486.9 lbf	1870.9 lbf
Elastic – Plastic buckling pressure, P_b	8216.6 psi	2737.7 psi	6273.85 psi	2742.8 psi

TABLE 4: The numerical comparison table of torispherical dome ends

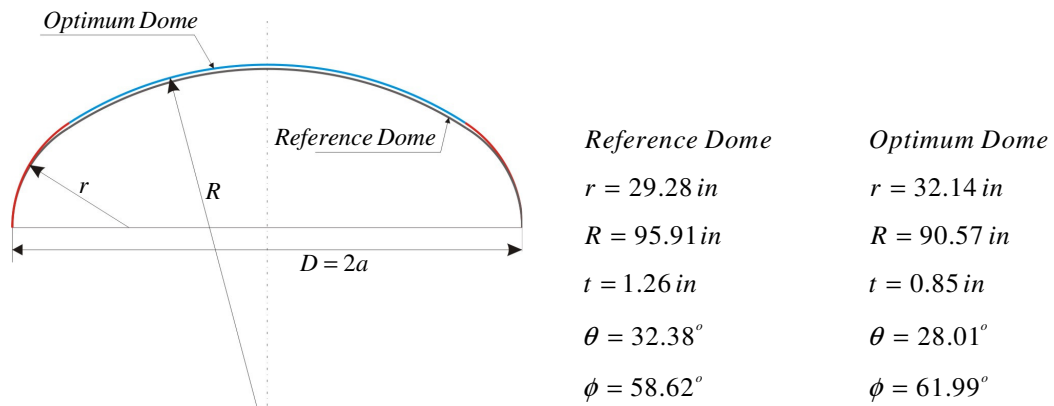


FIGURE 3: The dome configuration comparison of model 1 for minimum weight design

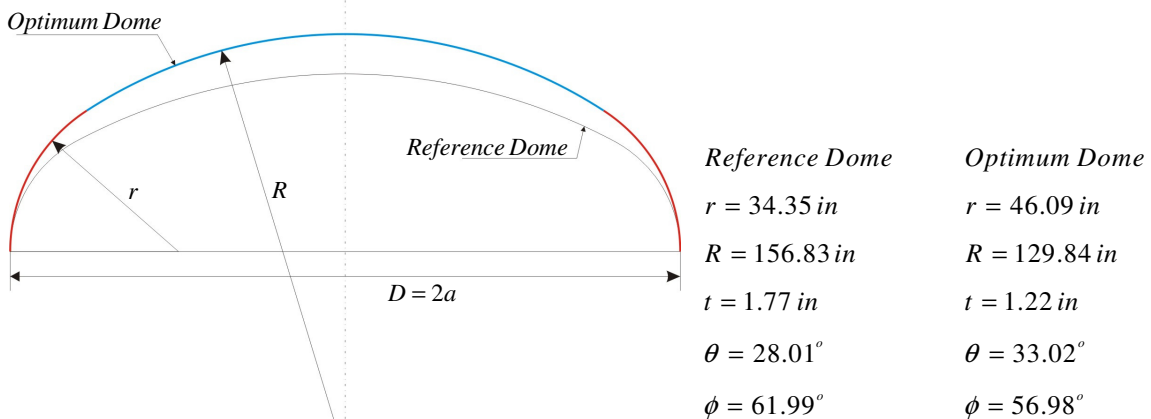


FIGURE 4: The dome configuration comparison of model 2 for minimum weight design

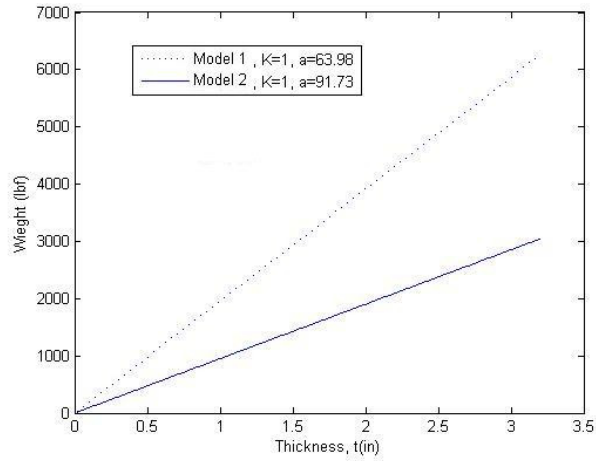


FIGURE 5: The weight via thickness of hemispherical dome end

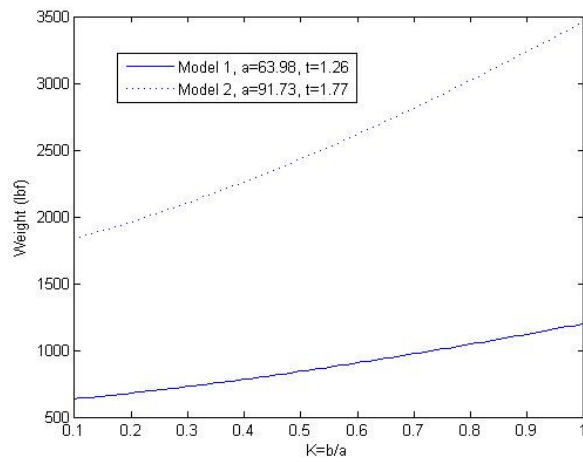


FIGURE 6: The weight via the slope K of torispherical dome end

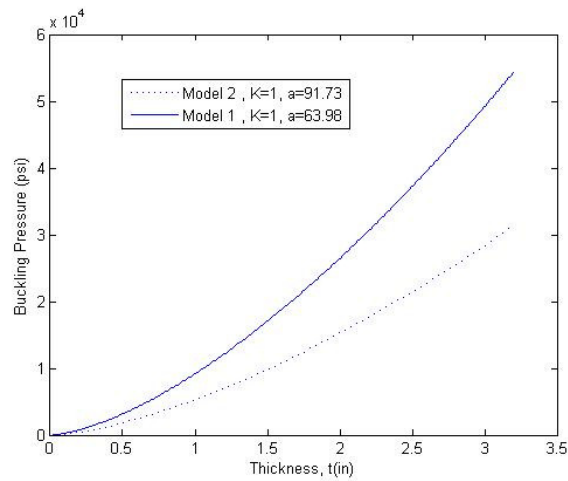


FIGURE 7: The buckling pressure via thickness of hemispherical dome end

Model 1 $K = 1, a = 63.98 \text{ in}$		Model 2 $K = 1, a = 91.73 \text{ in}$	
$t \text{ (in)}$	$P_b \text{ (psi)}$	$t \text{ (in)}$	$P_b \text{ (psi)}$
0.2	718.7	0.2	450.4
0.4	2257	0.4	1301
0.6	4193	0.6	2419
0.8	6519	0.8	3756
1.0	9173	1.0	5285
1.2	12124	1.2	6986
1.4	15349	1.4	8845
1.6	18828	1.6	10850
1.8	22546	1.8	12992
2.0	26490	2.0	15265
2.2	30648	2.2	17661
2.4	35013	2.4	20176
2.6	39574	2.6	22804
2.8	44325	2.8	25542
3.0	49260	3.0	28386

TABLE 5: The buckling pressure table at some corresponding thickness of hemispherical dome end

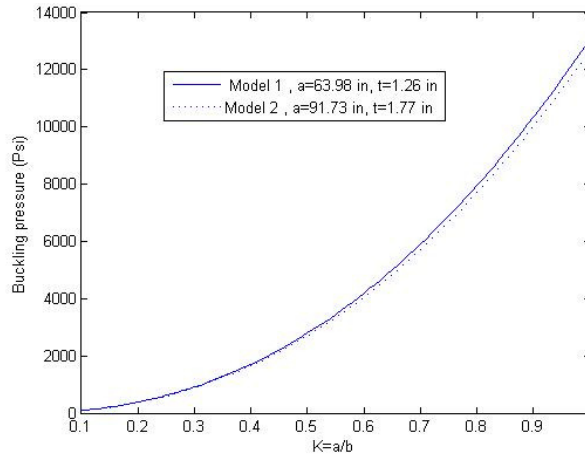


FIGURE 8: The Buckling Pressure via the Slope K of Torispherical Dome End

Model 1 $t = 1.26, a = 63.98 \text{ in}$		Model 2 $t = 1.77, a = 91.73 \text{ in}$	
$K = b/a$	$P_b \text{ (psi)}$	$K = b/a$	$P_b \text{ (psi)}$
0.1	91.97	0.1	89.14
0.2	384.2	0.2	372.3
0.3	910.6	0.3	882.6
0.4	1703	0.4	1651
0.5	2787	0.5	2702
0.6	4182	0.6	4054
0.7	5902	0.7	5721
0.8	7954	0.8	7709
0.9	10341	0.9	10023
1.0	13064	1.0	12662

TABLE 6: The buckling pressure table at some corresponding thickness of hemispherical dome end

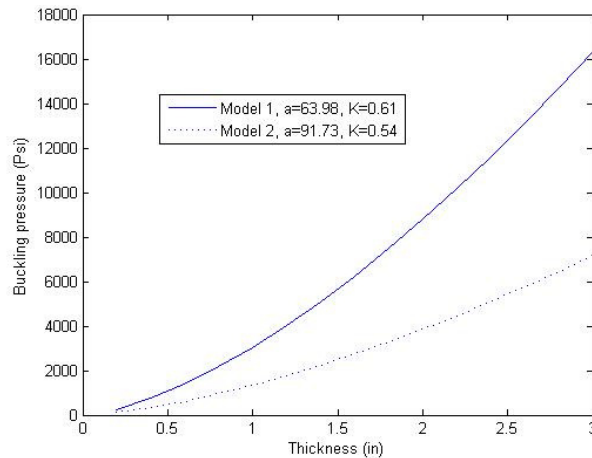


FIGURE 9: The buckling pressure via thickness of torispherical dome end

Model 1		Model 2	
$K = 1.61, a = 63.98 \text{ in}$		$K = 0.53, a = 91.73 \text{ in}$	
$t(\text{in})$	$P_b (\text{psi})$	$t(\text{in})$	$P_b (\text{psi})$
0.2	259.7	0.2	114.0
0.4	750.0	0.4	329.3
0.6	1394	0.6	612.4
0.8	2165	0.8	951.1
1.0	3047	1.0	1338
1.2	4027	1.2	1768
1.4	5099	1.4	2239
1.6	6254	1.6	2746
1.8	7490	1.8	3289
2.0	8800	2.0	3864
2.2	10182	2.2	4471
2.4	11632	2.4	5107
2.6	13147	2.6	5773
2.8	14725	2.8	6466
3.0	16365	3.0	7186

TABLE 7: The buckling pressure table at some corresponding thickness of hemispherical dome end

From table 3 and 4 and Figure 3 and 4, the ratio of K is found to be 0.64 for optimal model 1 and mode2 by using Genetic algorithm and it is found to be 0.65 for both of models when we use imperialist competitive algorithm (ICA), while the ratios of K are chosen as 0.61 and 0.53 for the reference models. The critical buckling pressure are 2741.7 psi for optimum model 1 and 2715.9 psi for optimum model 2 by using genetic algorithm and 2737.7 psi and 2742.8 psi for imperialist competitive algorithm (ICA) and for reference models being 8216.6 psi and 6273.8 psi .

According to this fact that the ratios of K for the optimum models are bigger than that of reference models, the shape of the optimum dome ends of two models tend to be bigger than the reference models, while the buckling pressures are lower than the reference models. Because of we set the preassigned buckling pressure bound ($8.33P_w \leq P_b \leq 10.4P_w$) and select the minimum weight type of cost function, the weight is reduced by 29.4% for model 1 and

24.1% for model 2 when we optimized by genetic algorithm and 30% for model 1 and 24.7% for model 2 while imperialist competitive algorithm is our optimization method.

From table 7 and Figure 9, the critical buckling pressure for a constant ratio K torispherical dome end is to be raised with the increase of the thickness. Also, the other important factor of the total weight of the torispherical dome end is thickness. However, the larger the thickness, the greater the weight and this situation must be taken into account.

7. CONCLUSION & FUTURE WORK

The four-centered ellipse method is a very easy and good method to describing the shape of the torispherical dome end and it makes the shape optimization problem very easy. This methodology uses only two design variables: thickness and the ratio of minor axis to major axis K . Imperialist competitive algorithm is a very efficient and easy to use algorithm for optimization problems such as the optimization of torispherical dome end subject to internally pressurized loading and the results of imperialist competitive algorithm are very close to the results of genetic algorithm.

For future study, the optimum design of composite torispherical dome end may consider such factor as buckling pressure, strength, space use, manufacturing cost, safety, weight, and so on.

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