

# Vibration Analysis of Micro Scale Pipes Containing Internal Fluid Flow with Standing Beam Excited by Piezoelectric Material

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## Abstract

In this paper, interaction between fluid and structure as a numerical simulation of flow through a two-dimensional micro channel with an embedded elastic structure is studied. The micro system vibration exigencies by applied magnetic field to the piezoelectric material of micro standing beam and motion of the wind system has been studied. The system includes a micro-channel and standing beam that is tied from bottom. First the eigenvalues of the system gain and therefore the resonance frequencies will be obtained. Finally with changing in initial and boundary conditions of system, the frequency response of system analyzed and with getting maximum frequency, it compared with the system resonance frequencies.

**Keywords:** Fluid Structure Interaction, Micro Channel, Micro-beam, Piezoelectric Material, Frequency Response.

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## 1. INTRODUCTION

The purpose of this paper is getting to response of nonlinear equation for a micro beam with arbitrary condition in interaction with fluid flow aerodynamic force and the other forces induced by the magnetic field on the piezoelectric material.

Micro channels are usually integrated in micro systems, it is important to know the characteristics of the fluid flow in these micro channels for better design of various micro-flow devices. The characteristics of fluid flow and heat transfer in micro channels are remarkably different from those for conventional sized channels.

Interaction between an elastic structure and a fluid has been the subject of intensive investigations in recent years [1], [2], [3], [4]. Since analytical solutions are available only for very simple problems, numerical approaches, which can be formulated in time or frequency domain, have to be employed. Vonestorff et al. in [5] investigated the coupled fluid-structure systems subjected to dynamic loads using the finite element and boundary element methods. Similar method is used by Olson in [6] to analyze fluid-structure interaction. Many researchers have attempted to derive variational principles for different classes of the fluid-structure interaction problems.

Pinsky and Abboud in [7] proposed two mixed variational principles for transient and harmonic analyses of non-conservative coupled exterior fluid-structure interaction systems.

Kock and Olson [8] presented a finite element formulation directly derived from a variational indicator based on Hamilton's principle. Zeng et al. [9] developed an energy-based symmetric coupled finite element and boundary integral method which is valid for all frequencies. Seybert [10] employed Ritz vectors and eigenvectors along with a combination of finite element and boundary element methods to reduce the problem size.

Equations include non-permanent continuity equation, navier-stokes and differential equations describing the vibrations micro beam that must be solved couple. For solving system of equations COMSOL software is used with giving initial conditions and applied voltage to the system. In conventional work the fluid convey in micro beam [11] and study vibration effect for micro beam [12]. But In this paper, standing micro beam actuated by piezoelectric will be analysis. In this paper interaction between air flow and a beam in the micro channel is analyzed. The length of micro channel is  $300\ \mu\text{m}$  and the height of that is  $100\ \mu\text{m}$  and a vertical structure is located  $100\ \mu\text{m}$  away from the channel inlet, the beam width is  $12.5\ \mu\text{m}$  and height is  $60\ \mu\text{m}$ . The micro beam is made of 2 aluminum 300-H18 layer and between them is PZH-5H piezoelectric material that is indicated with quiet colour in Figure 1 And for more weigh and corresponding momentum a alloy of aluminum and copper indicate in the head of micro beam.

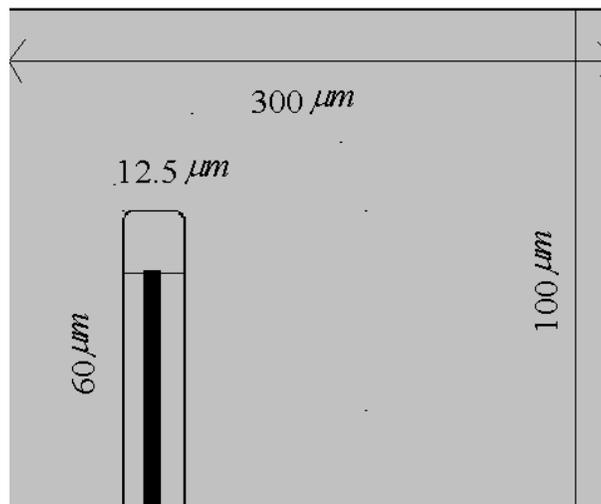


FIGURE 1: Figure of System.

In the Figure 1. Fluid flow come in from left side and goes out to right side. In equation 3 the fluid flow equation in indicated. The fluid in the channel has the property of air with a density of  $1\ \text{kg/m}^3$ .

## 2. MATERIALS AND METHODS

COMSOL is used to analyze the fluid-structure interaction as it provides a strong coupling between the dynamics of fluids and the dynamics of structures. At each computational step, the fluid flow field and the structure evolve as a coupled system. The interaction forces are immediately accounted for and their resultant motions enforced in each step.

In this paper Flow field is solved in a continuously deforming geometry using the arbitrary Lagrangian-Eulerian (ALE) technique. The fluid flows in the channel from left to right. An obstacle, however, forces it into the narrower path in the upper portion of the channel, and a force resulting from the viscous drag and fluid pressure is imposed at the walls.

The ALE method handles the dynamics of the deforming geometry and the moving boundaries with a virtual moving grid. It computes new mesh coordinates on the channel area based on the

movement of the boundaries of the structure. It reformulates the Navier-Stokes equations that solve the flow. The structural-mechanics portion of the model does not require the use of ALE, and it solves in a fixed coordinate system as usual. However, the strains computed are to be used for the computation of the deformed coordinates with ALE.

Following are the governing equations, which are solved using ALE technique:

$$\rho \frac{\partial u}{\partial t} - \nabla \cdot [-pI + \eta(\nabla u + (\nabla u)^T)] + \quad (1)$$

$$\rho((u - u_m) \cdot \nabla)u = F$$

$$-\nabla \cdot u = 0 \quad (2)$$

Where  $\rho$  is the fluid's density,  $u = (u, v)$  is the velocity field of the flow,  $p$  is the fluid pressure,  $I$  is the unit diagonal matrix, and  $F = (f_x, f_y)$  is the volume force affecting the fluid.

At the entrance the flow is fully developed with a parabolic velocity profile, but the flow amplitude changes with time. At first it increases rapidly, reaching its peak value at 0.215cm/s; thereafter the flow gradually decreases to its steady-state value of 5cm/s.

The centerline velocity  $u_{in}$  in the x-direction with the steady-state amplitude  $U$  is given as:

$$u_{in} = U.t^2 / \sqrt{t^8 - 0.07t^2 + 0.00016} \quad (3)$$

Pressure is specified at the channel outlet. For all other boundaries, no-slip condition is imposed. However, on boundaries where the fluid forces the structure to deform, the no-slip condition means that the fluid moves with the velocity of the adjacent fluid-structure boundary [13].

Equations for this purpose include Mass conservation equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0 \quad (4)$$

Navier - Stokes::Conservation of linear momentum equation is known as Newton's second law, which in fact represents the balance between the forces acting on a fluid element and its acceleration:

$$\rho \frac{D\vec{V}}{Dt} = \nabla \cdot \tau_{ij} \quad (5)$$

In equation 5, stress tensor for a Newtonian fluid obtain by equation 6

$$\tau_{ij} = -\tilde{P}\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda\nabla \cdot \vec{V} \quad (6)$$

And now the momentum equations in two-dimensional Cartesian coordinate can be obtained

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial \tilde{x}} + \tilde{\mu} \left( \frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \right) \quad (7)$$

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial \tilde{y}} + \tilde{\mu} \left( \frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2} \right) \quad (8)$$

The vibrating beam equation is as follows [14]:

Kinetic energy equation:

$$T = \int_0^L \frac{1}{2} \left[ \rho A \left( \frac{\partial y}{\partial t} \right)^2 + \rho I \left( \frac{\partial \psi}{\partial t} \right)^2 \right] dx \quad (9)$$

Potential energy equation

$$V = \int_0^L \frac{1}{2} \left[ EI \left( \frac{\partial \psi}{\partial x} \right)^2 + \frac{1}{2} kGA \left( \frac{\partial y}{\partial x} - \psi \right)^2 \right] dx \quad (10)$$

And finally the piezoelectric equation is as follows [15]:

$$\Delta L = d_{31} \cdot V \cdot \frac{L}{t} \quad (11)$$

$$\Delta W = d_{31} \cdot V \cdot \frac{W}{t} \quad (12)$$

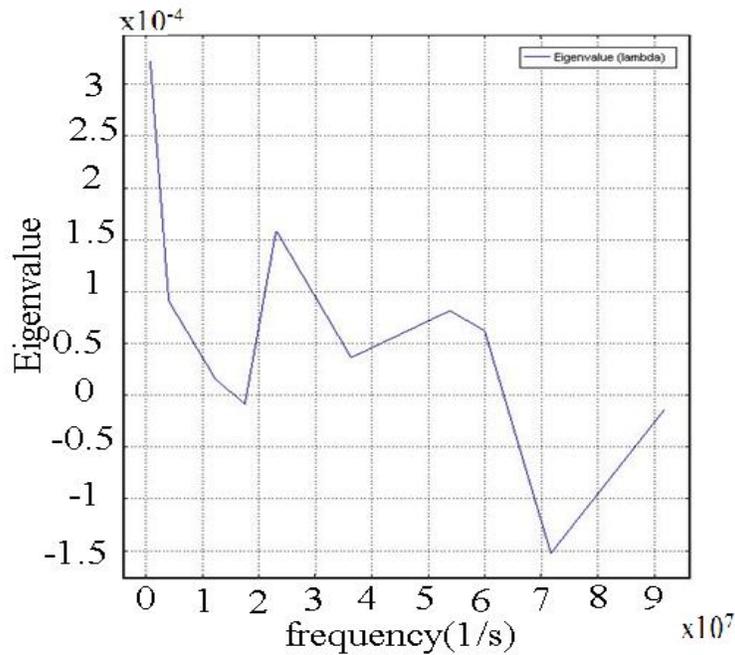
$$\Delta t = d_{33} \cdot V \quad (13)$$

### 3. RESULTS AND DISCUSSIONS

At first the resonance frequency and Eigen value of the micro-beam is obtained. This information presented in Figure 2 and table 1.

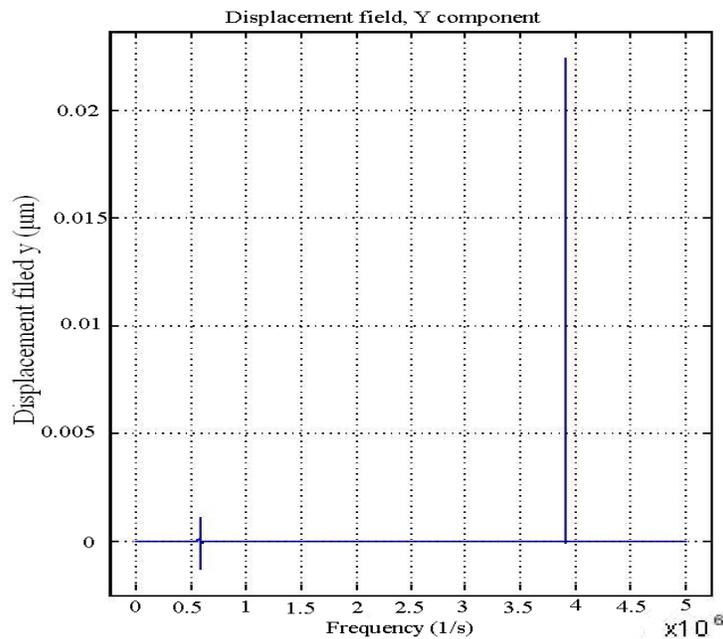
	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8	No.9	No.10
Eigen value	3.2e-4	9.2e-5	1.5e-5	-0.8e-6	1.5e-4	3.6e-5	8.2e-5	6.2e-5	-1.5e-4	-1.4e-5
Resonance frequency	5.79e5	3.91e6	1.21e7	1.74e7	2.29e7	3.62e7	5.62e7	5.99e7	7.15e7	9.17e7

TABLE 1: Frequency and Eigen value of the micro-beam.



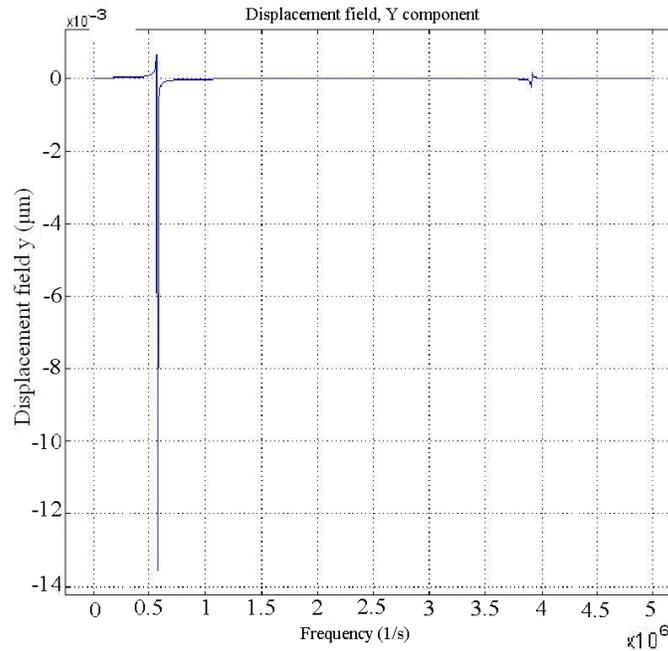
**FIGURE 2:** Frequency and Eigen Value of the micro-beam.

Now with changing input wind velocity, frequency response of system obtained. The average velocity is considered 1 m/s:



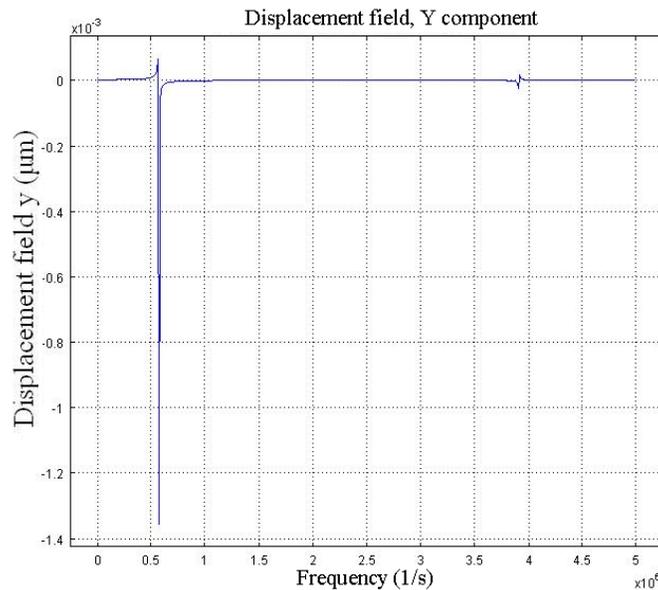
**FIGURE 3:** Frequency Response of System with 1 m/s Input Wind Velocity.

First resonance frequency is  $0.6 \times 10^6$  Hz and the second-resonance frequency is  $3.9 \times 10^6$  Hz. Therefore, the resonance frequency for system with 1 m/s average input wind velocity is different with Eigen value of system. And now the average wind velocity changes into 10 times of the first amount of wind velocity and then check the frequency response of system.



**FIGURE 4:** Frequency Response of System with 10 m/s Input Wind Velocity.

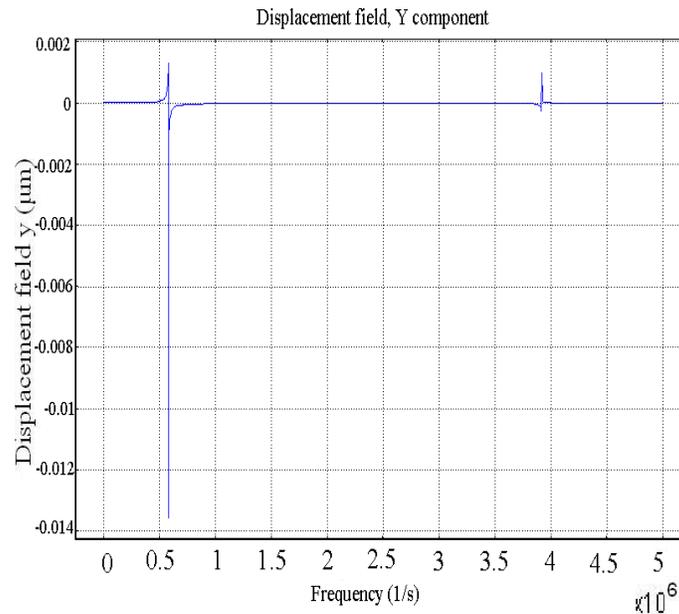
With this changing, the first resonance frequency is 0.6e6 Hz and the second-resonance frequency of is 3.9e6 Hz and the resonance frequency for system with 10 m/s average input wind velocity is different with Eigen value of system but these values are not different with the system while average of input wind velocity was 1 m/s. Now the average of input wind velocity changes into 100 m/s and the frequency response of system will be checked.



**FIGURE 5:** Frequency Response of System with 1 m/s Input Wind Velocity.

Consequently With this changing, the first resonance frequency still is 0.6e6 Hz and the second-resonance frequency is 3.9e6 Hz and the resonance frequency for system with 100 m/s average velocity input wind is different with Eigen value of system but these values are not different with

the system while average of input wind velocity was 1 m/s and 10 m/s. Now applied magnetic field to the system is changed into the ten times the first amount of magnetic field definition.



**FIGURE 6:** Frequency Response of System with Changing in Magnetic Field.

With changing the voltage to 10 V frequency response of system analyzed. Again, maximal frequency is like situations before with the same frequency.

#### 4. CONCLUSIONS

As was shown, the natural frequencies and frequency response of the system with different inputs analyzed and therefore amount of natural frequency is different with frequency response of system with various inputs. Consequently there is the lack of a significant difference in maximum frequency response for micro-fluid structure interaction with changing inputs.

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