Multi-Response Optimization For Industrial Processes

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Abstract

Process optimization is a very important point in modern industry. There are many classical optimization methods, which can be applied when some mathematical conditions are verified. Real situations are not very simple so that classical methods may not succeed in optimizing; as in cases when the optimization has several contradictory objectives (Collette, 2002).

The purpose of this work is to propose an optimization method for industrial processes with multiple inputs and multiple outputs (MIMO), for which the optimization objectives are generally contradictory and for which some objectives are not maximum or minimum but performance criteria.

The first step of this method is modeling each process response by a quadratic model. After establishing the model, we use a simplified numerical optimization algorithm in order to determine values of the parameters allowing optimizing the different responses, for MIMO processes.

This method will also allow finding optimum target values for multiple inputs single output processes.

Keywords: Multi-Response, Optimization, Discrete, Numerical, Modeling.

1. INTRODUCTION

A multi-objective optimization problem for an industrial process implies simultaneously minimizing some criteria defined in the same space, such as minimizing costs while maximizing performance. These optimization criteria are contradictory and the solution is a balance between the two objectives, as shown in figure 1 (Pareto,1896).

Pareto line (boundary) contains all balanced solutions. In figure 1, A and B are two points of Pareto line: A does not dominate B, B does not dominate A, but both of them dominate C. The purpose of multi-objective optimization is to find the Pareto line for a given problem (Gräbener, 2008). The dominant solutions of an optimization problem are those represented by the points on the Pareto boundary. Therefore for n objective functions there are C_n^2 boundaries to compute and the solutions are to be found in the domain limited by these C_n^2 boundaries.

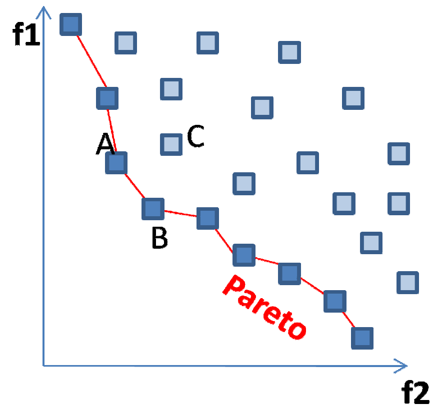

FIGURE 1: Pareto Line (boundary).

2. MULTI-OBJECTIVE OPTIMIZATION

Collette classified optimization methods in two categories, the scalar methods which transform the multi-objective problem in a mono-objective one and heuristic methods (Collette, 2002), which are generally stochastic iterative algorithms leading to a global optimum. A third method, using the desirability notion, was introduced by E.C Harrington (Harrington, 1965) and developed by G. Derringer (Derringer, 1980), in order to compensate for disadvantages of classic scalar methods.

2.1 Scalar Methods

These methods propose an a priori resolution by simplifying multi-objective problems in mono-objective ones. The scalar methods are weighting method (Coello, 2000), the ε-constraint method (compromise method)(Miettinen, 1999) and the goal method (Dean and Voss, 2000).

The weighting method computes a weighted sum of the objectives.

The problem becomes then:

$$\begin{cases} minF(X) = \sum_{i=1}^{m} W_i f_i(X) \\ \sum_{i=0}^{m} W_i = 1 \; and \; W_i \geq 0 \end{cases} \quad (1)$$

The weights Wi values are chosen by the designer. By giving a greater value to a weight W_i, the function fi will have a greater influence in the weighted sum. Generally it is interesting to solve some multi-objectives problems by considering some weights sets, but this type of solution become expensive in computing time.

In the case of a two objectives problem the equation (1) becomes:

$$f_2(X) = \frac{1}{w_2} F(X) - \frac{w_1}{w_2} f_1(X) \quad (2)$$

Since we want to obtain a minimum for F(X), we look for a line of directory coefficient $-\frac{w_1}{w_2}$ with the smallest ordinate and tangential to the set of Pareto optimal solutions.

The weighting method allows finding only the solutions existing on the convex Pareto boundary (Geoffrion, 1968). The ε-constraint method does not present this disadvantage. In this method, one of the functions is considered the optimization objective. The remaining functions are considered constraints and the problem becomes:

$$\begin{cases} \min_{X \in \mathbb{R}^n} f_{i0}(X) \\ f_i(X) \leq \varepsilon_i \; for \; i \neq 0 \end{cases} \quad (3)$$

As in the weighting method, it is possible to solve successively some mono-objective optimization problems with constraints, using each time different εi sets.

In the goal method, the problem becomes a mono-objective one as follows:

$$\begin{cases} \min_{X \in \mathbb{R}^n} \alpha \\ f_i(X) - \alpha . d_i \leq z_i \; for \; i = 1, \dots, m \end{cases} \quad (4)$$

In this equation, z is a point of $\mathbb{R}_m$ and d a vector of $\mathbb{R}_m$

where m is the number of optimization criteria. In this method, a priori values are to be chosen for the point z and for the direction d. For a same point z, it is then necessary to solve more mono-objective optimization problems with different directions d. The computation is repeated subsequently for more values of z, increasing thus the computation time.

2.2 Evolutionary Methods

These methods are used for complex optimization and search problems. The metaheuristics are generally stochastic iterative algorithms leading to a global optimum (Holland, 1992), such as the simulated annealing, genetic algorithms, the tabu search or the ant colony optimization.

The main advantage of such methods is their capability to avoid local optimums (maximum or minimum), by allowing a momentary degradation of the situation, in contrast to classical methods (Collette, 2002).

2.3 The Desirability Function

The idea of desirability is based on a weighting of the objective functions as in scalar methods but by using a product and by transforming all responses in a unique dimensionless desirability scale (individual desirability). The desirability functions (d_i) values are between 0 and 1.

The desirability function method allows rewriting an optimization problem as a mono-objective problem by proposing a unique composed criterion from some simple criteria; using classical methods then solves the mono-objective optimization problem.

The individual desirability functions are defined as follows:

$$\begin{cases} \left[\frac{Y_i - Y_{i\,min}}{\mu_i - Y_{i\,min}}\right]^p \text{ if } Y_{i\,min} \leq Y_i < \mu_i \\ \left[\frac{Y_i - Y_{i\,max}}{\mu_i - Y_{i\,mnax}}\right]^q \text{ if } \mu_i \leq Y_i < Y_{i\,max} \\ 0 \text{ if } Y_i < Y_{i\,min} \text{ or } Y_i > Y_{i\,max} \end{cases} \quad (5)$$

Where Yi min is the lower limit for Y_i value ($d_i = 0$), Yi max is the upper limit for Y_i value ($d_i = 0$), μ_i is the target optimal value for Y_i ($d_i = 1$) and p and q are importance factors for the desirability function.

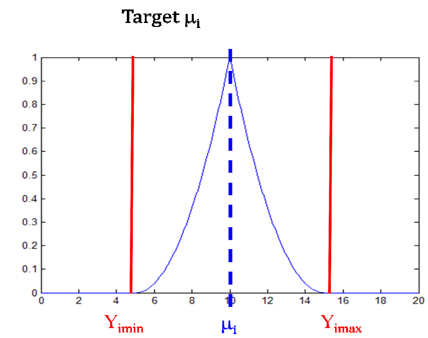

FIGURE 2: Desirability Function.

The set of individual desirability functions is used to compute a global desirability D (He and Zhu, 2008), by:

$$D_j = \left(\prod_{i=1}^{n} d_{i,j}\right)^{1/n} \qquad (6)$$

Since D is a geometric mean, it is equal to zero if one of the individual desirability functions is zero, rejecting thus a function in which one of the objectives is not at all attained, even if the other objectives are attained.

The maximal value for D is obtained when the combination of different responses is globally optimal.

3. LIMITATIONS OF USUAL METHODS

The scalar methods do not leave a choice to the user. They propose a unique solution, even if there are several possibilities. Moreover, sometimes-non-dominated solutions are impossible to obtain, whatever the coefficients (when the Pareto boundary is not convex). Finally, there are some non-additive quantities (Collette, 2002).

The metaheuristic approaches address some of these problems. However, constructing an efficient evolutionary algorithm is very difficult, since evolutionary processes are algorithm and parameter choice sensitive, and problem representation sensitive. Best such methods are based on sound knowledge and experience of the problem, on much creativity and on good comprehension of evolutionary mechanisms (Zhang, 2005).

Moreover, these methods may be less performing when applied to strongly constrained problems. Furthermore, they do not allow having some information on the Pareto boundary, and therefore it is impossible to evaluate the quality of the solutions (Terki, 2009).

In the case of the global desirability function, identical weights are generally used if all responses have the same importance. However, the optimum depends on the weights allocated to each response. The greatest difficulty is the choice of weights allocated to the individual desirability functions and of the model for the mono-objective optimization of the global desirability function.

For example, when considering a process with three criteria, corresponding to three different desirability functions, even if the individual desirability functions have values only of 10%, 20%, …, 90%, there are still 9!/(9-7)! = 504 different possible global desirability functions.

The choice of the mono-objective optimization model is complex due to the great number of possible choices and to the limitations of the different methods.

Other limitations come from the inner nature of the problem: optimization means maximizing or minimizing an objective, while in industrial processes, it is necessary to obtain precise values for performance criteria, within established acceptance limits.

In order to treat the optimization problem globally, it is necessary to solve a multi equations multi variables system. Since analytical resolution is very difficult, we propose a numerical approach, described in next section.

4. NUMERICAL APPROACH

An industrial process optimization by the means of this approach has some advantages:

- The possible interval is given by acceptable limits;
- The continuous variables may be considered as being discontinuous, since measure instruments give discontinuous values, according to their limits;
- The number of variables affecting the response cannot exceed some limit in industrial processes;
- The targets of the processes are generally defined within limits.
- These properties allow us to propose a method with the following steps:
- In order to have discrete variables, the digitalization step is defined according to the acceptable limits of measure instruments.
- All responses satisfying the constraints (validity domain) are computed.
- The intersection of different validity domains of each objective function gives the global validity domain, for all objective functions, but the user makes the final choice of the optimal solution.

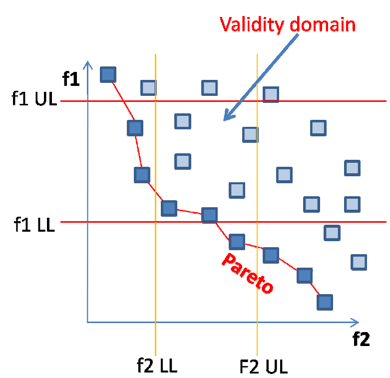

FIGURE 3: Validity Domain (LL: Lower Limit, UL: Upper Limit).

It is also possible to diminish the limits if the number of solutions is too big, or to increase them if there are not enough solutions.

This method has the advantage, when minimizing, to be closer to the optimum than the analytical methods, which generally use approximations.

Moreover, the use of complex mathematical formulations is not always well accepted in industry; it is thus interesting to have a simple optimization strategy, based on numerical calculation.

The proposed algorithm has also the advantage of proposing more possible solutions, depending on the conditions on the variables. Generally, when constraints are very restrictive, the number of proposed solutions diminishes. The solutions can be different, depending on the expressed need on input or response variables. The user can choose to have a solution as close as possible to the target, whatever the conditions on the input variables, or to favor the conditions on the input variables over the precision of the solution.

In order to diminish the number of proposed solutions, it is possible to set additional constraints, according to the objectives and initial constraints.

It is interesting to define the interval of acceptable values and the optimization algorithm to obtain several solutions, in order to have an appropriate choice.

For example, suppose the two solutions as follows:

a) $X_1 = 3$, $X_2 = 10$ and $X_3 = 100$, responses $Y_1 = 80\%$ and $Y_2 = 4$

b) $X_1 = 1$, $X_2 = 5$ and $X_3 = 60$, responses $Y_1 = 79\%$ and $Y_2 = 3.9$

If the objective is to have a unique optimum, the proposed solution will be a.

If the optimization objective is to propose different solutions, both solutions have approaching values for responses but different values of input variables. The choice depends on input variables values and constraints.

5. APPLICATION EXAMPLE

An optimization method is generally based on a mathematical model allowing expressing the objective function versus the influential parameters. The factorial and Taguchi experiments and the quadratic models allow modeling processes depending on several controllable factors and having objectives of product quality or costs (Montgomery, 2001), (Dean, 2000), (Fowlkes,1995). Our example process is modeled by quadratic functions.

The problem considered in the application example concerns a welding machine for chips bags and the objective is to optimize the welding process by finding the manufacturing conditions which give the best visual quality of the weld and weld strength close to 85 (Oueslati, 2001).

Depending on the behavior of the response versus input factors and depending on the optimization objectives, it is possible to obtain several vectors Xi satisfying these conditions or to find out that there is no such a vector.

The experiment is designed with three factors: the temperature (X_1), the pressure (X_2) and the tightening duration (X_3) and with two responses: the weld strength (Y_1) and the visual weld quality (Y_2).

The values for each input variable are given in the following table:

Level	Temperature (°C)	Pressure (Kg/dm^3)	Tightening duration (Seconds)
-1	120	50	0.2
1	180	150	2

The objective is to find out the values for X_1, X_2 and X_3 in the domain where Y_1 is close to 85 (with an acceptable limit of $\pm$ 5) and where Y_2 is bigger than 4.

Experiment:

	Temperature	Pressure	Duration	Resistance	Quality
1	-1	0	0	65.32	3.87
2	1	0	0	81.55	2.32
3	0	-1	0	91.45	3.14
4	0	1	0	93.29	4.36
5	0	0	-1	70.53	3.54
6	0	0	1	80.92	2.46
7	1	1	1	41.83	2.07
8	-1	1	1	89.97	3.01
9	1	-1	1	44.53	1.11
10	-1	-1	1	89.85	2.04
11	1	1	-1	91.53	2.77
12	-1	1	-1	11.25	4.48
13	1	-1	-1	92.94	2.04
14	-1	-1	-1	13.2	3.35
15	0	0	0	86.89	3.81
16	0	0	0	91.03	3.63
17	0	0	0	93.11	3.46
18	0	0	0	89.41	3.74
19	0	0	0	88.71	3.62

Response modeling:
Each response is defined by a quadratic model:

$$Y_i = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{23}X_2X_3 + a_{11}X_{12} + a_{22}X_{22} + a_{33}X_{32}$$

	a_0	a_1	a_2	a_3	a_{12}	a_{13}	a_{23}	a_{11}	a_{22}	a_{33}
Y_2	3.66	-0.64	0.50	-0.55	0.00	0.14	0.01	-0.49	0.00	-0.58
Y_1	90.47	8.28	0.00	6.77	0.00	-31.68	0.00	-16.72	0.00	0.00

Classical optimization:
By using a scalar method, the solution is:

X_1 = 143.94, X_2 = 115 1.38 and X_3 = 0.98; responses Y_1 = 85 and Y_2 = 4.0

One can see that the proposed solution is not optimal and that there is no other choice; for example, we could lower the constraints on input variables.

If the objective were to find a maximum for both functions, the classical optimization solution would be:

X_1 = 137.40, X_2 = 161 and X_3 = 0.43; responses Y_1 = 86.32 and Y_2 = 2.6

This solution is not optimal; the input variables values are not within the limits, there is no correspondence between response values and input values according to the model and, finally, the Pareto boundary is not computed.

Optimization using desirability function:
As is generally done, we used equal weights for individual desirability functions. The optimum of the global desirability function is graphically found. The solution obtained is:

X_1 = 143.1, X_2 = 150 and X_3 = 0.95; responses Y_1 = 84 and Y_2 = 4.36

The graphic research for the optimum is possible even for the three variable function, because after the statistical analysis of effects, one of the parameters is found as being non influent. However, this method cannot be used for more complex functions.

Numerical optimization:
Each response is submitted to some constraints. In order to diminish the number of possible solutions, there were diminished the acceptable limits on the responses:

	Objective	Lower Limit	Upper Limit
Y_1	85	84	86
Y_2	>4	4	

We begin by some fifteen iterative computations on the three variables. When the validity domain is found, it is possible to have another iterative computation on this domain, in order to be closer to the constraints on the input and output variables.

After the first iterative computation, four possible solutions were found:

	X_1	X_2	X_3	Y_1	X_4
Solution 1	138	150	1.15	85.5	4.3
Solution 2	142.5	150	1.01	85.9	4.3
Solution 3	147	150	0.61	84	4.3
Solution 4	147	150	0.74	85.5	4.3

In order to make a choice, the costs generated by the conditions may be used as additional criteria.

For instance, the difference between the tightening duration obtained from solutions 1 and 3 is of 0.54s, i.e. a time about 50% shorter than if one chooses the third solution, but one must mention that this last solution is 6% costlier in energy consumption.

One can also see that different solutions are obtained when changing the digitalization step.

In order to have a smaller number of possible solutions, the limits on the responses can still be diminished, for example to 0.5 for Y_1 and a minimum of 4.38 for Y_2. In this case, the proposed solution is: X_1 = 143.4, X_2 = 150, X_3 = 0.88; Y_1 = 84.5 and Y_2 = 4.38.

6. CONCLUDING REMARKS AND FUTURE WORK
The proposed method allows optimizing in several steps a multiple input/multiple output process, i.e.:

- modeling each response by a quadratic function or a Taguchi model;
- finding the variables values satisfying the objectives by an iterative numerical calculation of the responses;
- obtaining the global validity domain by the intersection of all domains previously found;

- making choices of acceptable solutions in this global domain (depending on costs or on another criterion) and finally;

- proposing an optimal solution, by imposing if necessary an additional constraint on the objectives.

The first interest of this optimization method is the possibility to make an optimization on more variables and more responses and getting closer to optimum values, instead of using an analytical model. Moreover, this method allows finding several different solutions. Finally, it is based on numerical calculation, being thus simpler than the analytical methods.

The numerical search of optimal solution assumes that the quadratic model of the responses is valid.

Future work will concern the verification of the model's validity and some more applications of the method on industrial cases.

Another interest will be the comparison of performance between quadratic or Taguchi models and algorithms such as artificial neural networks or genetic algorithms.

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