

Color Image Segmentation Based On Principal Component Analysis With Application of Firefly Algorithm And Gaussian Mixture Model

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Abstract

In this paper we propose a segmentation method for multi-spectral images in the HSV space, based on the Principal Component Analysis to generate grayscale images. Then the Firefly Algorithm has been applied on the gray-level images in a histogram-based research of cluster centroids. The FA is a metaheuristic optimization algorithm, centered on the flashing behaviour of fireflies. The Firefly Algorithm is performed to determine automatically the number of clusters and to select the gray levels for partitioning pixels into homogeneous regions. Successively, these gray values are employed during the initialization step of a Gaussian Mixture Model for estimation of parameters, evaluated through the Expectation-Maximization technique. The coefficients of the linear super-position of Gaussians can be regarded as the prior probabilities of each component. Applying the Bayes rule, the posterior probabilities have been estimated and their maxima are used to assign each pixel to the clusters, according to their gray values.

Keywords: Color Image Segmentation, Image Clustering, Firefly Algorithm, Gaussian Mixture Model.

1. INTRODUCTION

Image segmentation is the process of separating an image into disjoint and homogeneous components. Because of the enormous variety and complexity of color images, robust and efficient segmentation approaches are far from satisfying, especially when applied on images of the real world. Color image segmentation attracts more and more interest, mainly because it provides a greater amount of information respect to grayscale images. Grayscale image segmentation methods are categorized recurring to the two properties utilized for partitioning: discontinuity and similarity, the most common are the region-based and edge-based methods. In more details, region-based methods are founded on similarity, they segment an image recurring to low-level features of it, such as intensity, texture or color, assuming that pixels belong to the same region if they have similar characteristics. Hence they can work for colored images as well. On the contrary, color image edge-detection techniques are extension of monochromatic methods, they operate on the three color channels independently combining their final results. More explicitly, the gradient operators proposed for grayscale segmentation approaches, are extended to color images by taking the vector sum of gradients for each component [1]. The paradigms of gray level image segmentation, pixel-based, region-based or edge-based can be also used for color images. Besides, we must include the so called physics-based methods which take into account information about the exposure to light during image acquisition processes as well. In summary, color image segmentation methods may be classified into three different groups depending on the technique employed: 1) feature-space based techniques; 2) image domain based techniques; and 3) physics based techniques. The feature-space methods work in a certain color space, assuming that color is a constant property of the surface of each object within an image. Thus these methods map pixels into regions having homogenous characteristics or create some ad hoc histograms upon color features, such as hue. Generally, objects will appear as clusters or in correspondence of peaks within histograms, in the latter case. The

image-domain based techniques, such as split and merge or region growing methods, perform a spatial grouping of pixels according to similarity or dissimilarity criteria to guarantee spatial compactness. Lastly, in the physics-based approach, all algorithms examine the objects portrayed in color images taking into account light exposure, highlights, and shadows. These phenomena frequently cause appearance of color variations of uniformly colored surfaces. To avoid over segmented regions, these algorithms analyze how light interacts with colored materials [2].

2. PRELIMINARIES

A color is perceived by human beings as a combination of the tri-stimuli captured by three different kinds of photoreceptors in the retina, called cones, whose spectral responses are centered on the wavelengths of red, green, blue. Therefore, a digital color image is represented by a two dimensional array of three dimensional vectors which represent the pixel's red, green and blue values, the three primary colors which form the color space. Although RGB image representation is the most often used, it is not adequate for performing color recognition in image analysis since this space has three independent channels, so any color can be created by combination of these three color bases. RGB color space is suitable for color display but it is not good for color scene segmentation and analysis because of the high correlation among the R, G, B components and the great sensitivity to variations in illumination [3]. For this reason, we have adopted HSV color space (Hue, Saturation, Value) given that it structures any color in the same way in which it is perceived by human eyes [4]. More precisely, hue is the color tone, i.e. red or yellow, saturation defines the amount of each color, i.e. dark green or pale green, finally the value, also called intensity or lightness, allows to distinguish between a dark or a light color. An accurate comparison study between different color spaces revealed that the HSV space provides good results in image segmentation [4].

3. METHODOLOGY

In this paper we propose a segmentation method based on pixel intensity without recurring to thresholding techniques. Firstly, the proposed segmentation algorithm performs the Principal Component Analysis on color images. PCA is a standard tool of modern data analysis that transforms a number of possible correlated variables into a smaller number of uncorrelated ones, preserving the most relevant information of the original data set. Then we proceeded to a histogram-based research of cluster centroids applying a meta-heuristic algorithm, the Firefly Algorithm (FA), belonging to a novel class of non-deterministic optimization methods.

Over the last few decades, researchers started to use metaheuristic algorithms for clustering to overcome the weakness of the existing conventional methodologies, generally deterministic [5]. Metaheuristic algorithms are a class of approximate methods that allow to discover possible solutions exploring the search space in order to find near-optimal solutions. Shortly, metaheuristic algorithms are iteratively processes developed to search a solution that is good enough in a time that is small enough [6]. These algorithms are frequently nature-inspired, they have the advantages of finding global optima due to the action of multiple search agents, randomly generated [7].

Nature inspired metaheuristic algorithms are applied for multilevel thresholding image segmentation in a comparative study between Particle Swarm Optimization (PSO), Artificial Bee Colony Optimization (ABC), Ant Colony Optimization (ACO) and Cuckoo Search (CS) algorithms, that are investigated in partitioning gray-scale images [8]. The well-known K-means clustering algorithm is applied by an optimization technique based on FA algorithm, with the squared error as objective function. The performance of the FA is compared with other two nature-inspired methods, ACO and PSO, and other nine traditional approaches [9]. A bio-inspired segmentation algorithm is proposed for solving crop type classification problems using a multispectral satellite image [10]. A colour image partitioning procedure is implemented applying a multi-level thresholding approach on the three R, G and B histograms, analyzed independently [11]. The predefined number of optimal thresholds for each colour component are attained by maximizing

Otsu's between-class variance function, recurring to the Firefly Algorithm. The maximum entropy thresholding method is widely used in literature, this approach has been performed in partitioning multi-channels images employing Firefly Algorithm [12], in this context entropy operators yield high values in regions characterized by drastic chromaticity variations. For an exhaustive review of the more conventional segmentation methods refer to [13], [14], [15].

In the present work we started with a pre-processing step, during which the original image has been de-correlated in order to improve its appearance and to enhance the contrast between pixel colors. Then applying PCA, the image dimensionality has been reduced from the three-dimensional HSV color space to the one-dimensional gray representation [16]. To this aim, given a color image

$$\vec{I}(i, j), \quad i = 1, \dots, m; \quad j = 1, \dots, n$$

with components $\vec{I} = (I_1, I_2, I_3)$, let be $\bar{\vec{I}}$ the vector mean, expressed by:

$$\bar{\vec{I}} = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n \vec{I}(i, j)$$

the covariance matrix 3×3 C is evaluated as:

$$C = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n \left(\vec{I}(i, j) - \bar{\vec{I}} \right) \cdot \left(\vec{I}(i, j) - \bar{\vec{I}} \right)^T$$

So, if we denote with $\lambda_1 \geq \lambda_2 \geq \lambda_3$ and $\vec{v}_1, \vec{v}_2, \vec{v}_3$, respectively, the eigenvalues and eigenvectors of C, the components of de-correlated image $\vec{I}^D = (I_1^D, I_2^D, I_3^D)$ may be computed as:

$$I_k^D = \vec{v}_k^T \cdot \vec{I} \quad k = 1, 2, 3$$

Through PCA, the de-correlated image \vec{I}^D is projected along the direction of the principal component \vec{v}_1 in which it has been registered the highest variability, therefore the intensity image I is generated by:

$$I = \vec{v}_1^T \cdot \vec{I}^D$$

Once the gray image has been obtained, we proceed in evaluating automatically the number of clusters and the associated representative levels through a histogram-based segmentation approach by means the Firefly Algorithm.

The Firefly Algorithm, developed by Yang X.S. [17], employs fireflies as search agents making use of their idealized flashing characteristics. In describing the Firefly Algorithm, three fundamental rules may be delineated [18],[19],[20],[21]:

- (1) All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;
- (2) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to brightness, decreasing as their distance increases. If there is no brighter one, a generic firefly will move randomly in the search space;
- (3) The brightness of a firefly is determined by an objective function. For a maximization problem, the brightness can simply be proportional to the objective function.

The attractiveness $\beta(r)$ of a firefly is proportional to the brightness and decreases with distance r , because the light is absorbed in the media, as a consequence luminosity becomes weaker with the distance from source, so we have:

$$\beta(r) = \beta_0 \cdot e^{-\gamma r^2}$$

where β_0 is the attractiveness at $r = 0$ and γ is the light absorption coefficient. As a firefly i is attracted to another brighter j , its movement is described at time t by:

$$x_i^{t+1} = x_i^t + \beta_0 \cdot e^{-\gamma r_{ij}^2} \cdot (x_j^t - x_i^t) + \alpha_t \cdot \epsilon_i^t \quad (1)$$

where r_{ij} represents the distance between the two fireflies. The second term is due to the attraction, while the third term defines the random component through the casual values ϵ_i , drawn from a Gaussian distribution at a given iteration. Thus, $\beta_0 = 0$ describes a simple random walk. The value of β_0 expresses the variation of attractiveness, $\gamma = 0$ corresponds to constant attractiveness and, conversely, as $\gamma \rightarrow \infty$ the attractiveness tends to zero, which is equivalent to a totally random search. Given the relevant level of variability of gray intensity histograms for real images, in this unsupervised clustering method, we proposed the Firefly Algorithm because it fulfils the aim of searching local and global maxima simultaneously, due to the decreasing of attractiveness $\beta(r)$ with distance. In order to capture the most of significant histogram peaks, the objective function is defined as the relative change of pixel frequency, therefore a firefly is attracted towards the others close by it but having meaningfully greater values of pixel frequency. In the following applications, the fireflies used as search agents are 30, the average scale L is set equal to 256 which is the number of gray levels, and the light absorption coefficient γ is set equal to $1/\sqrt{L}$. In order to avoid an excessive erratic motion in the search space, the random component is reduced during iterations, as $\alpha_t = \alpha_0 \delta^{-t}$ with $\alpha_0 = 0.1 \cdot L$ and $0 < \delta < 1$. After evaluating automatically the number K of clusters through the global maxima derived by the FA algorithm, the corresponding gray-levels intensities have been assigned as initial means μ_k , $k = 1, \dots, K$ to the Gaussian Mixture Model [22], [23]. The initial standard deviations σ_k are approximately computed through the differences respect to the nearest centroids.

A Gaussian Mixture Model (GMM) is a parametric probability density function represented as a weighted sum of Gaussian components. In this work GMM parameters are estimated from data using the iterative Expectation-Maximization (EM) algorithm. Let us recall that a univariate Gaussian density distribution is expressed by:

$$p(x | \mu, \sigma^2) = N(x | \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where $N(x | \mu, \sigma^2)$ represents a Gaussian distribution of mean μ and standard deviation σ . A Gaussian mixture model is a weighted sum of K components of Gaussian densities, analytically given by:

$$p(x) = \sum_{k=1}^K \pi_k \cdot N(x | \mu_k, \sigma_k^2) \quad (2)$$

Equation (2) represents a linear superposition of probability densities in which the mixing coefficients π_k indicate the weight of each distribution. According to the first and second axiom of probability theory, these coefficients must satisfy the following relations:

$$0 \leq \pi_k \leq 1 \quad \sum_{k=1}^K \pi_k = 1$$

Thus, a Gaussian mixture model is parametrized by means μ_k , variances σ_k^2 and the mixture weights π_k of all components. We can think of the mixing coefficients as prior probabilities corresponding to each component:

$$\pi_k = \frac{N_k}{N}$$

where N is the number of image points and N_k $k = 1, \dots, K$ are the numbers of data belonging to the k -th cluster. The GMM parameters have been calculated using the Expectation Maximization (EM) algorithm. This algorithm is an iterative optimization technique that starts from some initial estimation of parameters and then proceeds to iteratively update them until convergence is detected. In short it consists of four basic steps:

- 1) The Initialization Step: define initial guesses for parameters, in this context they have been derived by the FA algorithm
- 2) The Expectation Step: compute the responsibilities γ_k Eq. (3), namely the posterior probabilities of a given gray intensity x to belong to the k -th cluster, according to the Bayes rule we have:

$$\gamma_k(x) = p(k|x) = \frac{\pi_k \cdot N(x|\mu_k, \sigma_k^2)}{\sum_{i=1}^K \pi_i \cdot N(x|\mu_i, \sigma_i^2)} \quad (3)$$

- 3) The Maximization Step: re-estimate the parameters using Eq. (4) and the current responsibilities, given by Eq. (3):

$$\hat{\mu}_k = \frac{\sum_{i=1}^N \gamma_k(x_i) \cdot x_i}{\sum_{i=1}^N \gamma_k(x_i)} \quad \hat{\sigma}_k^2 = \frac{\sum_{i=1}^N \gamma_k(x_i) \cdot (x_i - \hat{\mu}_k)^2}{\sum_{i=1}^N \gamma_k(x_i)} \quad \hat{\pi}_k = \frac{\sum_{i=1}^N \gamma_k(x_i)}{N} \quad (4)$$

- 4) Compute convergence by the value of log likelihood after each iteration, halting if it does not change in a significant manner from one iteration to the next:

$$\ln(p(X|\mu, \sigma, \pi)) = \sum_{i=1}^N \ln \left(\sum_{k=1}^K \pi_k \cdot N(x_i|\mu_k, \sigma_k^2) \right)$$

Iterate Expectation Step (2) and Maximization Step (3) until convergence.

After determining the parameter values with EM technique, we have proceeded to the assignment of pixels of a given gray level x_i to the cluster \tilde{k} by means the evaluation of the maximum value of responsibilities:

$$\gamma_{\tilde{k}}(x_i) = \max_{k \in K} \{ \gamma_k(x_i) \}$$

all the pixels with gray level x_i are attributed to the \tilde{k} -th cluster, given that, by definition, $\gamma_k(x_i)$ represents the probability of the \tilde{k} -th GMM's component to have generated the value x_i . The proposed segmentation method performs classification of pixels in a straightforward and effective way reducing greatly the computational costs [24].

4. EXPERIMENTAL RESULTS

In this section we present some experimental results obtained basically with segmentation of test images extracted from Berkeley Segmentation Dataset BSDS300 [25]. Firstly, we applied our method on Iris, the test image shown in Fig.1. A de-correlated image \vec{I}^D has been produced on HSV space, as we can see in Fig.1 (on the left) the outcome gives prominence to pixel colors, increasing the level of contrast.



FIGURE 1: Original and De-correlated Images.

Afterwards, the de-correlated image has been projected on the principal component \vec{v}_1 , the derived image is represented in Fig.2. Given the relevant level of variability of gray intensity histograms for real images (see the enlarged histogram in Fig.3), in this unsupervised clustering method we propose the use of a meta-heuristic algorithm, more specifically the Firefly Algorithm (FA). The FA fulfils the aim of searching simultaneously local and global maxima, due to the decreasing of attractiveness $\beta(r)$ with distance. In order to capture the most of significant histogram peaks, the objective function is defined as the relative change of pixel frequency, therefore a firefly is attracted towards the others close by it but having meaningfully greater values of pixel frequency. In the present application, the fireflies used as search agents are 30, the average scale L is set equal to 256 which is the number of gray levels, the light absorption coefficient γ is defined as $1/\sqrt{L}$. In order to avoid an excessive erratic motion in the search space, the random component may be reduced during iterations, so α_t decreases as $\alpha_t = \alpha_0 \delta^{-t}$ with $\alpha_0 = 0.1 \cdot L$ and $0 < \delta < 1$.

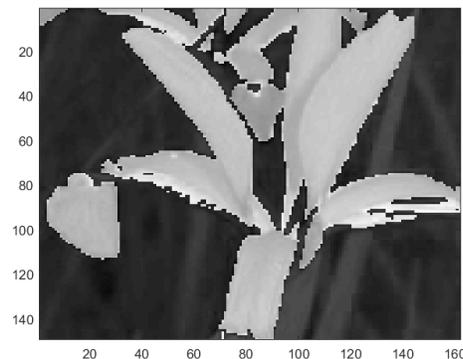


FIGURE 2: Iris Grayscale Image with PCA.

After evaluating automatically the number K of clusters through the maxima derived by FA (Fig.3 on then left), the corresponding gray-levels intensities have been assigned as initial means μ_k , $k = 1, \dots, K$ to the Gaussian Mixture Model [26], [27]. The initial standard deviations σ_k are approximately computed through the differences respect to the nearest centroids, on the contrary the initial centroids μ_k of clusters are issued by FA algorithm.

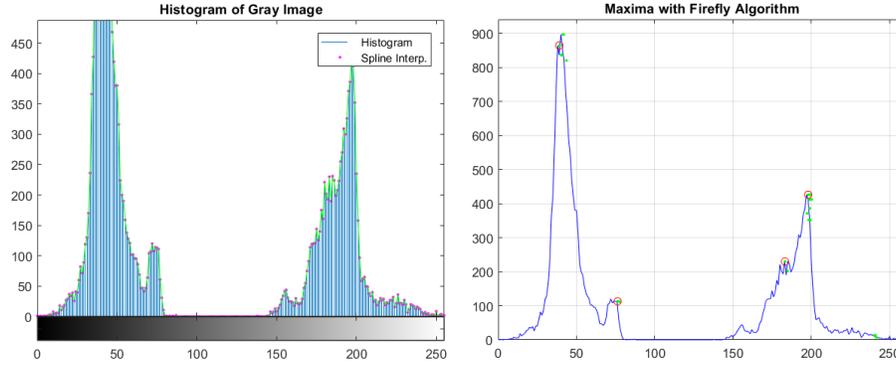


FIGURE 3: Grayscale Image Histogram and Maxima Computed with FA.

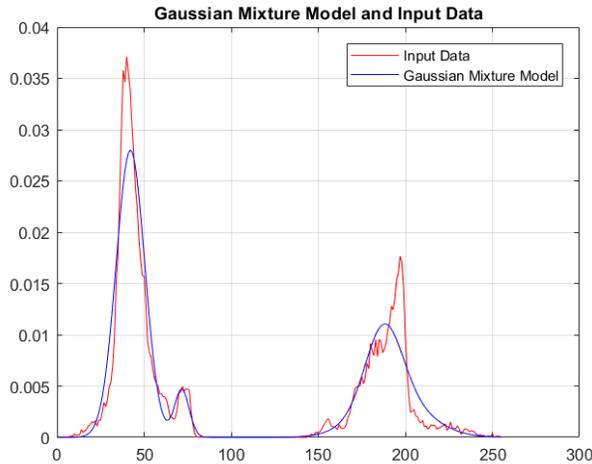


FIGURE 4: Grayscale Histogram and Gaussian Mixture Model.

To evaluate the consistency of this proposed clustering algorithm, the quality of final outcomes is estimated by using several validation measures, more precisely: the Root Mean Square Error (RMSE), the Normalized Correlation Coefficient (NK) and finally the Davies-Bouldin (DB) index [28], [29]. The RMSE is the simplest validity index, a large value of RMSE means that the image is of poor quality, given an image $I_{ij} \ i = 1, \dots, m \ j = 1, \dots, n$ and the segmented image $I_{s_{ij}} \ i = 1, \dots, m \ j = 1, \dots, n$, it is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n |I_{ij}|^2}{n \cdot m \sum_{i=1}^m \sum_{j=1}^n |I_{ij} - I_{s_{ij}}|^2}}$$

The normalized correlation coefficient (NK) quantifies in percentage the level of correlation between the two images and it is expressed as follows:

$$NK = \frac{\sum_{i=1}^m \sum_{j=1}^n I_{ij} \cdot I_{s_{ij}}}{\sum_{i=1}^m \sum_{j=1}^n |I_{ij}|^2}$$

Finally, for testing the consistency of the method, we have calculated the Davies-Bouldin index, that is based on similarity measures R_{ij} between clusters, obtained through the dispersion S_i of each cluster and the dissimilarity measures D_{ij} . Usually, given a partition of an initial dataset I in disjoint clusters $C_i, i = 1, \dots, k$ of cardinality n_i , R_{ij} is defined in the following way:

$$R_{ij} = \frac{S_i}{D_{ij}}$$

where $S_i = \left(\frac{1}{n_i} \cdot \sum_{I_{ij} \in C_i} |I_{ij} - \mu_i|^2\right)^{\frac{1}{2}}$ represents the intra-cluster standard deviation and $D_{ij} = \left(|\mu_i - \mu_j|^2\right)^{\frac{1}{2}}$ measures the distance between two different clusters C_i and C_j , through the distance of their centre points. The Davies-Bouldin index estimates the average of similarity between each cluster as:

$$DB = \frac{1}{k} \sum_{i=1}^k \max_{j=1, \dots, k; k \neq j} \frac{S_i + S_j}{D_{ij}}$$

even with this validation index a lower value implies a better clustering segmentation.

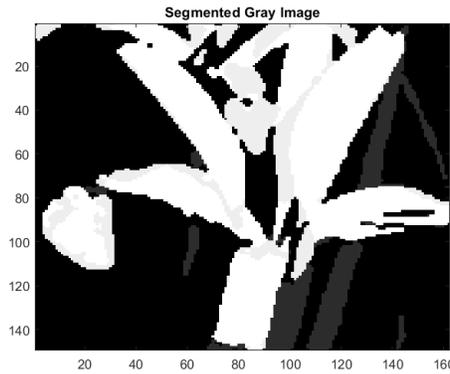


FIGURE 5: Gray segmented image of Iris.

The segmented image with four clusters is displayed in Fig.5, Table 1 reports the GMM parameters and the estimated validation indices, in the last row. We point out that the final value of standard deviations of some clusters are rather different respect to the initial ones. To this end we recall that, during the starting step, variances were evaluated approximately by minimizing the deviations from the initial cluster centroids.

K = 4	Data Clusters (N. iterations EM =10)			
	N.pixels: 162 x 149 = 24138			
μ_k initial	39.00	76.29	183.23	198.05
μ_k final	42.18	71.65	187.63	193.22
σ_k initial	5.66	2.86	4.57	5.32
σ_k final	8.18	4.19	10.43	19.47
N_k	13255	1804	4338	4741
π_k final	0.574	0.049	0.196	0.179
		RMSE 0.069	NK 0.981	DB 2.814

TABLE 1: Cluster Parameters Evaluated with GMM.

Finally, using the original data set as reference image, we have performed a colorization process on the segmented grayscale image, recurring to the means of the three colour channels computed on pixels belonging to the same cluster, the final result is shown in Fig.6.

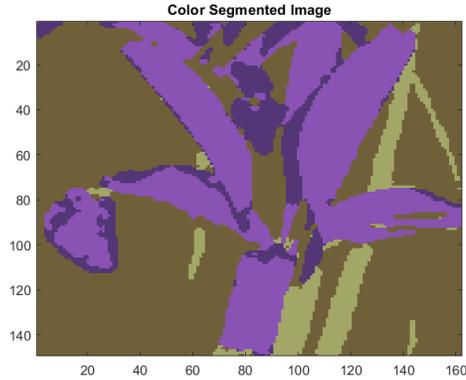


FIGURE 6: Color Segmented Image of Iris.

As a second test image let us consider the butterfly displayed in the training image 138 of Dataset BSDS300:



FIGURE 7: Butterfly original image and grayscale image with PCA.

Through the FA algorithm the number of derived maxima is $K = 5$ (Fig. 8) In Table 2 are explicitly specified the initial and final values of centroids. After ten iterations of the Expectation Maximization algorithm, the convergence to final values is reached within a predefined precision level. These values do not present significant differences respect to the initial ones. The validation indices confirm the reliability of this feature-space based method for image segmentation.

$K = 5$	Data Clusters (N. iterations EM =10)				
N.pixels: 321x481 = 154401					
μ_k initial	86.00	115.73	166.30	208.94	251.68
μ_k final	84.66	115.71	167.39	210.90	246.50
σ_k initial	9.15	4.93	4.77	4.86	2.04
σ_k final	17.91	13.74	12,45	12.81	4.18
N_k	63476	26001	29369	33012	2543
π_k final	0.408	0.170	0.192	0.213	0.015
		RMSE 0.023	NK 0.988	DB 0.198	

TABLE 2: Cluster Parameters Evaluated with GMM.

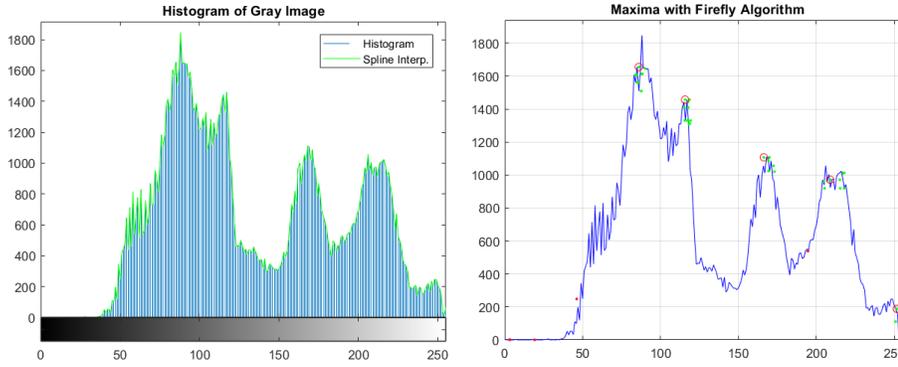


FIGURE 8: Grayscale Image Histogram and Maxima Computed with FA.

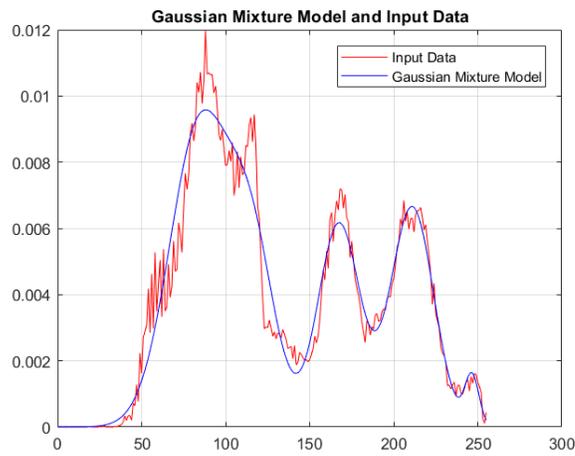


FIGURE 9: Grayscale Histogram and Gaussian Mixture Model.

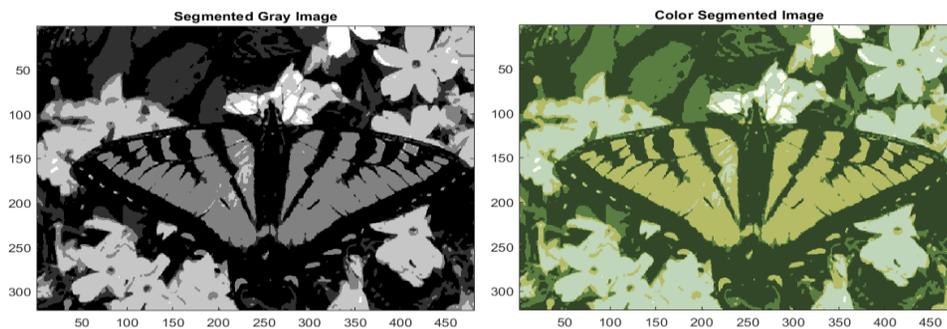


FIGURE 10: Gray and Color Segmented Images of Butterfly.

The colorization process performed on the segmented grayscale image of Fig.10 (on right) puts in evidence the five predominant colour features appearing in the original image.

5. CONCLUSIONS

This research has addressed on colour image segmentation, one of the fundamental task of image processing. To this aim, it has been developed a feature-space based approach recurring to PCA, the nature-inspired Firefly Algorithm and the Gaussian Mixture Model. This method, applied on several images extracted from Berkeley Segmentation Dataset BSDS300, is able to find out automatically the number of groups in which an image may be partitioned and their

centroids. Their initial estimates, computed by a metaheuristic algorithm, are used as initial values for evaluating the GMM parameters. The results appear fairly solid and reliable, as can be inferred by the validation analysis performed on the test image analysed in this paper. A noteworthy advantage of the proposed methodology derives from the use of the maxima of responsibilities for pixel assignment, that implies a consistent reduction of computational costs.

6. REFERENCES

- [1] S. Di Zenzo, "A note on the gradient of a multi-image", *Comput. Vision and Graph. Image Process.*, vol. 36, (1986).
- [2] W. Skarbek, A. Koschan, "Colour Image Segmentation: a Survey", Technical Report, Technical University Berlin, (1994).
- [3] D. Comaniciu and P. Meer, "Robust Analysis of Feature Spaces: Color Image Segmentation", *IEEE Conference on Computer Vision and Pattern Recognition*, (1997).
- [4] A. Jurio, M. Pagol, M. Galar, C. Lopez-Molina, D. Paternain, "A Comparative Study of Different Color Spaces in Clustering Based Image Segmentation", *Communications in Computer and Information Science*, Ed. Springer-Verlag, (2010).
- [5] I. H. Osman, G. Laporte, "Metaheuristics: a bibliography", *Annals of Operations Research*, Vol. 63, N. 5, pp. 511-623, (1996).
- [6] C. Blum, A. Roli, *Metaheuristics in combinatorial optimization: Overview and conceptual comparison*, *ACM Computing Surveys*, Vol. 35, N. 3, pp. 268-308 (2003).
- [7] F. Rothlauf, "Design of Modern Heuristics Principles and Application", Springer (2011).
- [8] C. Deepika, J. Nithya "Nature Inspired Metaheuristic Algorithms for Multilevel Thresholding Image Segmentation - A Survey", *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol.8, N.10 (2014).
- [9] J. Senthilnath, S.M. Okmar, V. Mani, "Clustering using a Firefly Algorithm: Performance Study", *Swarm and Evolutionary Computation*, Vol.1, pp 164-171, (2011).
- [10] J. Senthilnath, S. Kulkarni, J.A. Benediktsson, X.S. Yang, "A Novel Approach for Multispectral Satellite Image Classification Based on the Bat Algorithm", *IEEE Geoscience and Remote Sensing Letters*, (2016).
- [11] V. Rajinikantha, M. S. Couceiro, "RGB Histogram based Color Image Segmentation Using Firefly Algorithm", *Procedia Computer Science*, Ed Elsevier, Vol. 46, pp. 1449 –1457, (2015).
- [12] M.H. Horng, T.W. Jiang, "Multilevel Image Thresholding Selection Based on the Firefly Algorithm", *Proc. IEEE 7th Intern. Conf. on Ubiquitous Intelligence and Computing* (2010).
- [13] L. Lucchese, S. Mitra, "Color Image Segmentation: A State-of-the-Art Survey", *Image Processing, Vision, and Pattern Recognition*, *Proc. of the Indian National Science Academy (INSA-A)*, Vol. 67, N. 2, pp. 207–221, (2001).
- [14] A.P. Vartak, V. Mankar, "Color Image Segmentation: a Survey", *Intern. Journ. Emerging Technon. Advanced Engineering.*, Vol. 3, N. 2., (2013).
- [15] N.M. Zaitoun, J.A. Musbah, "Survey on Image Segmentation Techniques", *Proc. Int. Conf. on Communication, Management and Information Technology*, Ed. Elsevier, (2015).

- [16] S. Dikbas, T. Arici, Y. Altunbasak, "Chrominance Edge preserving Grayscale Transformation with approximate First Principal Component for Color Edge Detection", Proc. IEEE Conf. Image Process. (ICIP'07), Vol. 9, pp. 497–500, (2007).
- [17] X. S. Yang, Nature-inspired Metaheuristic Algorithms, Luniver Press, United Kingdom, (2008).
- [18] X. S. Yang, Firefly Algorithm, Stochastic Test Functions and Design Optimization, International Journal of Bio-Inspired Computation, Vol. 2, pp. 78-84, (2010).
- [19] X.S. Yang, "Firefly Algorithm, Levy Flights and Global Optimization Research and Development", Intelligent Systems XXVI (Eds M. Bramer, R. Ellis, M. Petridis), Springer London, pp. 209-218, (2010).
- [20] J. Senthilnath, S. N. Omkar, V. Mani, "Clustering using Firefly Algorithm: Performance Study", Swarm and Evolutionary Computation, Elsevier, Vol.1, pp 164–171, (2011).
- [21] S. Fong, S. Deb, X.S. Yang, Y. Zhuang, "Towards Enhancement of Performance of K-Means Clustering Using Nature-Inspired Optimization Algorithms", The Scientific World Journal, Vol. 2014, (2014).
- [22] B.G. Lindsay, "Mixture Models: Theory, Geometry and Applications", NFS-CBMS Regional Conference Series in Probability and Statistics (1995).
- [23] G.J. McLachlan, K.E. Basford, Mixture Models: Inference and Applications to Clustering, Ed. Marcel Dekker, New York, (1988).
- [24] D. Giuliani, "A Grayscale Segmentation Approach using the Firefly Algorithm and the Gaussian Mixture Model", International Journal of Swarm Intelligence Research, Vol. 9, Issue 1, Ed. IGI Global (2017).
- [25] D. Martin, C. Fowlkes, D. Tal, J. Malik, "A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics," ICCV, (2001).
- [26] B.G. Lindsay, "Mixture Models: Theory, Geometry and Applications", NFS-CBMS Regional Conference Series in Probability and Statistics, (1995).
- [27] G.J. McLachlan, K.E. Basford, Mixture Models: Inference and Applications to Clustering, Ed. Marcel Dekker, New York, (1988).
- [28] K. Jaskirat, A. Sunil, V. Renu, "A comparative analysis of thresholding and edge detection segmentation techniques", International Journal of Computer Applications, Vol. 39, (2012).
- [29] R.N. Nihar, K.M. Bikram, K.R. Amiya, "A Time Efficient Clustering Algorithm for Gray Scale Image Segmentation", International Journal of Computer Vision and Image Processing, Vol. 3, N. 1, pp 22-32, (2013).