Homomorphic Filtering of Speckle Noise From Computerized Tomography (CT) Images Using Adaptive Centre-Pixel-Weighed Exponential Filter

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Abstract

Adaptive filters are needed to accurately remove noise from noisy images when the variance of noise present varies. Linear filter such as Exponential filter becomes effective in removing speckle noise when homomorphic filtering technique is used. In this paper, an Adaptive Centre-Pixel-Weighed Exponential Filter for removing speckle noise from CT images was developed. The new filter is based on varying the centre-pixel of the filter kernel based on the estimated speckle noise variance present in a noisy CT image. Ten samples of 85x73 CT images corrupted by speckle noise level ranging from 10% to 30% were considered and the new technique gave a reasonably accurate speckle noise filtering performance with an average Peak Signal to Noise Ratio (PSNR) of 70.2839dB compared to 69.0658dB for Wiener filter and 64.3711dB for the Binomial filter. The simulation software used in the paper is Matrix Laboratory (Matlab).

Keywords: Adaptive Filter, Exponential Filter, Speckle Noise, Homomorphic Filtering, CT Image, Centre-pixel, Centre-pixel.weighed.
based on the noise variance present in the noisy image. Adaptive filters are divided into adaptive linear filters and adaptive nonlinear filters. Adaptive nonlinear filters are effective in removing both additive and multiplicative noise from noisy images unlike adaptive linear filters that performs effectively for additive noise [4].

To effectively filter multiplicative noise using linear filters, homomorphic filtering technique is applied. Homomorphic filtering involves transforming multiplicative noise into additive noise and then applying a linear operation [5]. For a linear filter to be applied to images corrupted by speckle noise, the speckle noise needs to be transformed to additive noise using a logarithmic transformation [6] before the convolution operation is applied [7]. The logarithmic transformation is used to transform speckle noise. For non-speckle noises, other transform techniques such as Ascombe transform (for transforming Poisson noise to additive noise) and Ascombe-like transform (for transforming a combined additive and Poisson-like noise) exist [8],[9]. The process of transforming speckle noise to additive noise is known as logarithmic transformation. The logarithmic transformation is applied to speckle noise so that linear filtering is used to enhance the noisy image.

The removal of speckle noise from image is more complex compared to the removal of additive noise. Some researchers in the past have developed and used different filters to remove speckle noise from medical images. Medical images that has been given much attention are images captured using the CT machine because such images are very popular and also of high quality [10]. In a work done by [11], ε-Neighbourhood median filter was used to remove speckle noise from CT lung images. In another work done by [12], the effectiveness of wiener filter in removing speckle noise from CT images was investigated. In the work done by [13], Genetic-Neuro-Fussy technique was developed and used to remove speckle noise from ultrasound images. In a closely related work done by [14], a Bayesshrink Wavelet Threshold technique was used to remove speckle noise from ultrasound images. In a related work done by [15], Mathematical Morphology technique was developed and used in removing speckle noise from images.

In this paper, an Adaptive Centre-Pixel-Weighed Exponential Filter is developed. This technique is an example of an adaptive linear filter. The new technique uses the variation in the magnitude of filter centre pixel with the noise variance present in the image to achieve high performance.

2. METHODOLOGY
The filter technique developed in this work is an Adaptive Centre-Pixel-Weighed Exponential Filter. The developed technique uses the estimated speckle noise variance present in an image to vary the centre pixel of the filter for effective filtering. The variable weight technique is developed in this work because it is simpler than variable window technique and causes less blurring.

2.1 Proposed Model
In general, noiseless image, Y(m,n) corrupted by a multiplicative noise, N(m,n) is given as shown in (1) where X(m,n) is the noisy image [16].

\[ X(m,n) = Y(m,n)N(m,n) \]  \hspace{1cm} (1)

The expression in (1) is modified by adding a correcting factor (\( \beta \)) to both sides as given in (2).

\[ X(m,n) + \beta = Y(m,n)N(m,n) + \beta \]  \hspace{1cm} (2)

The parameter, \( \beta \) in (2) is a real number such that \( \beta \leq 10^{-7} \). Transforming the modified noisy image in (2) into image corrupted by additive noise using a logarithmic transformation according to [16], yields (3).
The correcting factor ($\beta$) is added to (1) before logarithmic transformation because the pixels with zero values give $-\infty$ after logarithm transformation and therefore, zero pixels need modification to ensure real value after transformation [17]. The value of $\beta$ is made very negligible to avoid excessive change in pixel values. The expression in (3) is enhanced by linear filter, $h(m, n)$ to obtain the filtered image, $\bar{Y}(m, n)$ using a convolution operation as given in (4).

$$
\bar{Y}(m, n) = \text{Exp} \left( \sum_{j=-R_f}^{R_f} \sum_{k=-C_f}^{C_f} h(j, k) \log_e(X(m-j, n-k) + \beta) \right)
$$

Where $R_f = \frac{M_f-1}{2}$ and $C_f = \frac{N_f-1}{2}$ and $M_f$ and $N_f$ are the row and column sizes of the filter respectively. After filtering, the mean square error (MSE) in removing noise from noisy image using a linear filter is given in (5).

$$
\text{MSE} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} [Y(m, n) - \bar{Y}(m, n)]^2
$$

The parameters, $M$ and $N$ in (5) are the image row and column length respectively. The parameter, $h(m, n)$ in (4) that gives the minimum value of MSE (5) is the proposed Adaptive Centre-Pixel-Weighted Exponential Filter. The general expression for the proposed filter is given in (6) where $Z$ is the tuning factor of the filter.

$$
h(m, n) = \frac{2Z}{\pi} e^{-2(m^2+n^2)}
$$

The value of $Z$ that minimizes the expression in (5) as established in this work is given in (7) for 3x3 filter kernel.

$$
Z = \begin{cases} 
1.5 : |m| + |n| \neq 0 \\
Z_1 : |m| + |n| = 0 
\end{cases}
$$

Substituting (7) into (6), (6) is rewritten as given in (8). The expression in (8) is the proposed Adaptive Centre-Pixel-Weighted Exponential Filter developed in this paper.

$$
h_{opt}(m, n) = \begin{cases} 
\frac{3}{\pi} e^{-2(m^2+n^2)} : |m| + |n| \neq 0 \\
\frac{2Z_1}{\pi} : |m| + |n| = 0
\end{cases}
$$

The variation of the value of the centre-pixel method was applied in this paper because it is simpler and retains the symmetric property of the filter. The filtering technique developed in this work is effective for speckle noise variance ranging from 0% to 30%.

The variable, $Z_1$ in (8) is dependent on the estimated noise variance present in the noisy image and is given by (9) where $\hat{\delta}$ is the estimated speckle noise.

$$
Z_1 = \frac{1}{39.79(\hat{\delta}^2)^2 - 0.5564\hat{\delta}^2 + 1.587}
$$
The parameter, \( \delta^2 \) in (9) is the estimated speckle noise variance in a noisy image being filtered. The denominator in the (9) is obtained by applying the Least Squares Method to speckle noise variance estimated using speckle noise variance estimation technique such as the technique developed in [18]. The speckle noise estimation technique developed in this paper is as given in (10). The technique in (9) is the Shifted Mean of Averaging Absolute Derivation technique.

\[
\delta^2 = 0.05804(12.3673 - \bar{M})(\bar{M} - 0.1275) \tag{10}
\]

The parameter \( \bar{M} \) in (10) is the mean of the speckle noise present in the noisy image and is given in (11).

\[
\bar{M} = \frac{1}{MN} \sum_{k=1}^{M} \sum_{l=1}^{N} abs(log_e W(m, n)) \tag{11}
\]

The parameter \( log_e W(m, n) \) in (11) is the natural logarithm of the speckle noise present in the image and it is given by (12).

\[
log_e W(m, n) = log_e((X(m, n) + \beta) - \frac{1}{M_f N_f} \sum_{j=-R_f}^{R_f} \sum_{k=-C_f}^{C_f} log_e(X(m, n) + \beta)) \tag{12}
\]

### 2.2 Developed Filter’s Response to Noise Variance

The response of the developed filter to change in the noise variance is determined by the parameter, \( Z_1 \). The actual and estimated values of \( Z_1 \) as obtained using the Shifted Mean of Averaging Absolute Derivation technique in (10) is given in table 1.

<table>
<thead>
<tr>
<th>Actual noise variance</th>
<th>Actual values of ( Z_1 )</th>
<th>Estimated noise variance</th>
<th>Estimated values of ( Z_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100</td>
<td>0.5183</td>
<td>0.1199</td>
<td>0.4779</td>
</tr>
<tr>
<td>0.150</td>
<td>0.4169</td>
<td>0.1635</td>
<td>0.3907</td>
</tr>
<tr>
<td>0.200</td>
<td>0.3260</td>
<td>0.2042</td>
<td>0.3192</td>
</tr>
<tr>
<td>0.250</td>
<td>0.2541</td>
<td>0.2509</td>
<td>0.2530</td>
</tr>
<tr>
<td>0.300</td>
<td>0.2000</td>
<td>0.3079</td>
<td>0.1928</td>
</tr>
</tbody>
</table>

**TABLE 1:** The actual and the estimated values of \( Z_1 \).

Based on table 1, it is observed that the values of the \( Z_1 \) decrease as the values of noise variance present increases. This shows that the filter responds strongly when the noise variance is low and lightly when the noise variance is high. This property of the developed filter can also be seen in figure 1. From the figure, it is seen that \( Z_1 \) falls as the noise variance increases.
The figure also shows that the noise estimation technique used in this paper has good performance. This is shown by the closeness of the actual and estimated values of $Z_1$ which implies minimal tuning error.

### 2.3 Performance Metrics

The effectiveness of the proposed Adaptive Centre-Pixel-Weighed Exponential Filter is estimated using Peak Signal-to-Noise Ratio (PSNR) and mean square error (MSE). The PSNR of a noisy image is the ratio of the maximum power of the signal to the maximum power of the noise distorting the image [19]. For a normalized image, the PSNR can be rewritten as shown in (13) to ensure that PSNR is always positive [20].

$$PSNR = 63 + 10\log_{10}\left(\frac{1}{MSE}\right)$$  \hspace{1cm} (13)

The PSNR is measured in decibel.

On the other hand, MSE is the average of the squared intensity differences between the filtered image pixels and reference image pixels and is given in (14) [21].

$$MSE = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} [(Y(m,n) - \hat{Y}(m,n))^2]$$  \hspace{1cm} (14)

MSE assumes that the reduction in perceptual quality of an image is directly related to the visibility of the error signal [19].

The performances of the noise variance estimation techniques are quantified using estimation error ($\xi$) is shown in (15) [22].

$$\xi = \frac{abs(\sigma_n - \hat{\sigma}_n)}{\sigma_n} \times 100$$  \hspace{1cm} (15)

The parameter, $\sigma_n$ is the actual standard deviation of speckle noise present in the noisy image while $\hat{\sigma}_n$ is the estimated standard deviation of speckle noise present in the same image.

### 2.4. Steps in applying the proposed Algorithm

The steps needed in applying the algorithm developed in this work are as follows:

**Step 1:** Normalize the noisy image
Step 2: Compute the natural logarithm of the normalized noisy image in step 1.
Step 2: Estimate the speckle noise variance present in the noisy image in step 2.
Step 3: Use the estimated noise variance to compute the value of Z1 and filter kernel.
Step 4: Compute the convolution operation between the natural logarithm of the normalized noisy image obtained in step 2 and the computed filter kernel from step 3.
Step 5: Compute the inverse natural logarithm of the output from step 4 to obtain the filtered image.

3. RESULTS AND DISCUSSION

To effectively analyze the performance of the new technique, the performance of the developed filter and the speckle noise variance estimation technique developed is discussed.

3.1. Discussion of the performance of the developed filter

The performance of the developed model in effectively removing speckle noise from noisy images was compared with other filtering techniques using ten (10) samples of 85x73 CT lung images corrupted by speckle noise of variance ranging from 10% to 30%. The PSNR and MSE values calculated in this paper using (13) and (14) respectively for the three filtering techniques are shown in tables 2 and 3.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Noise Variance (%)</th>
<th>Average PSNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>3X3 Binomial filter</td>
<td>68.7851</td>
<td>66.0713</td>
</tr>
<tr>
<td>3X3 Wiener filter</td>
<td>72.9188</td>
<td>70.2405</td>
</tr>
<tr>
<td>3X3 Proposed filter</td>
<td>73.0741</td>
<td>71.3014</td>
</tr>
</tbody>
</table>

Table 2: The PSNR values for the three filtering techniques.

Table 2 shows how the PSNR value for the three filters vary with the speckle noise variance present in the noisy images. From table 2, it is observed that the proposed filter gives the highest average PSNR (70.2839dB) compared to Wiener filter (69.0658dB) and Binomial filter (64.3711dB).

The trend in table 2 is clearly shown in figure 2. From figure 2, it is observed that for all noise levels considered, the proposed filter gives the largest values of PSNR. Figure 2 shows that the difference between the PSNR of the proposed adaptive filter and the PSNRs of wiener and binomial filters widens as the noise variance increases. However, as noise variance decreases, the differences between PSNR of the proposed adaptive filter and the PSNRs of wiener and binomial filters tend to diminish. This is because the proposed filter is the most stable, though, all the filters used give strong filtering at low noise variance and light filtering at high noise variance [23].
FIGURE 2: Plot PSNR against Speckle Noise Variance for the three filtering techniques.

This shows that the proposed filter is more stable, gives the best result and therefore is the best filter for the preprocessing among the three filters considered.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Noise Variance (%)</th>
<th>Average MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>3X3 Binomial filter</td>
<td>0.3106</td>
<td>0.5502</td>
</tr>
<tr>
<td>3X3 Wiener filter</td>
<td>0.1493</td>
<td>0.2489</td>
</tr>
<tr>
<td>3X3 Proposed filter</td>
<td>0.1339</td>
<td>0.2025</td>
</tr>
</tbody>
</table>

TABLE 3: The PSNR values for the three filtering techniques.

Considering table 3, it is also observed that the proposed filter gives the lowest value of MSE (0.28232) compared to Wiener filter (0.36744) and Binomial filter (0.97394). The trend in table 3 is clearly shown in figure3. Figure 3 shows that the difference between the MSE of the proposed adaptive filter and the MSEs of wiener and binomial filters widens as the noise variance increases. However, as noise variance decreases, the differences between MSE of the proposed adaptive filter and the MSEs of wiener and binomial filters tend to diminish. This is because the proposed filter is the most stable, though, all the filters used give strong filtering at low noise variance and light filtering at high noise variance [23].

FIGURE 3: Plot MSE against Speckle Noise Variance for the three filtering techniques.
From figure 3, it is observed that for all noise levels considered, the proposed filter gives the lowest values of MSE and is more stable compared to wiener and binomial filters.

3.2. Results of noise estimation technique
The performance of the speckle noise variance estimation technique developed in this paper was compared with Median of Median Absolute Derivative Technique developed in [18] using ten (10) samples of 85x73 CT lung images corrupted by speckle noise of variance ranging from 10% to 30%. The average estimated speckle noise variances obtained in the analysis using the two (2) techniques are shown in table 4. The average speckle noise estimation errors calculated from table 1 using (15) for the two (2) techniques are as shown in table 5.

<table>
<thead>
<tr>
<th>Noise Estimation Techniques</th>
<th>Actual Noise Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td>Developed Estimation Technique</td>
<td>0.1199</td>
</tr>
<tr>
<td>Median of Median Absolute Derivative</td>
<td>0.0828</td>
</tr>
<tr>
<td>Technique [18]</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4: Average estimated speckle noise variance for the two noise estimation techniques.

The estimated speckle noise variances using the two (2) techniques are shown in table 4. From this table, it is seen that the noise variance estimated using the developed estimation technique is closer to the actual noise variance present compared to median of median absolute derivative technique for all the noise levels considered. Looking at figure 4, it is observed that the developed estimation technique has the higher graph showing it has higher accuracy. The estimated variance of the developed estimation technique becomes closer to the actual noise variance as the actual noise variance present increases as shown in figure 4. However, at very low noise level, the estimated variance from each technique becomes closer. This is because the developed estimation technique is more stable, though, both estimation technique used give strong filtering at low noise variance and light filtering at high noise variance[23].

![FIGURE 4: Plot of average estimated speckle noise variance against actual noise variance for the two noise estimation techniques.](image)

Based on the plot in figure 4, it is concluded that the developed estimation technique has better average speckle noise variance estimation accuracy for all the noise variance range considered. Also, it can be observed that the estimated noise variance for the developed estimation technique becomes more accurate as the actual noise variance present increases.
<table>
<thead>
<tr>
<th>Noise Estimation Technique</th>
<th>Actual Noise Variance</th>
<th>Average Noise Estimation Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

| Developed Estimation Technique             | 9.5080                | 4.3999                             |
|                                            | 1.0322                | 0.1818                             |
|                                            | 1.3048                | 3.2853                             |
| Median of Median Absolute Derivative Technique [18] | 9.0000                | 11.2457                            |
|                                            | 14.2416               | 13.0005                            |
|                                            | 15.1570               | 12.5289                            |

**TABLE 5:** Average speckle noise estimation error for the four noise estimation techniques.

The difference in performances among the two estimation techniques considered is clearly seen in table 5. From table 5, it is observed that the developed estimation technique has lower estimation error except at 10% noise variance showing that it has better estimation accuracy.

![Plot of average speckle noise estimation error against actual noise variance for the four noise estimation techniques.](image)

**FIGURE 5:** Plot of average speckle noise estimation error against actual noise variance for the four noise estimation techniques.

Looking at figure 5, it is observed that the developed estimation technique has the lower graph showing it has higher accuracy. The estimation error of the developed estimation technique decreases as the variance of noise present increases as shown in figure 5 and it falls sharper than the other technique. However, at very low noise level, the estimation error of the two techniques becomes approximately the same. This shows that the developed estimation technique is more stable and has higher accuracy.

4. **CT LUNG IMAGES USED IN THE PAPER**

The CT images used in the paper are shown in figure 6. The images are of sizes 85X73. Figure 7 shows the images when each is corrupted by 30% speckle noise.
FIGURE 6: Noiseless images used in the paper.
Source: http://radiopaedia.org

FIGURE 7: Unfiltered images corrupted by 30% noise.

FIGURE 8: 30% noisy images filtered using Wiener Filter.
Figures 8, 9 and 10 show the noisy images in figure 7 filtered using Wiener filter, proposed filter and Binomial filter respectively. Based on visual observation, it is seen that images filtered using proposed filter has best visual details compared to others. This shows that the proposed filter gives the best filtering performance among the three filters used.

5. CONCLUSION
In this paper, an Adaptive Centre-Pixel-Weighed Exponential Filter was developed. The developed technique filters speckle in a noisy CT image with an average PSNR as high as 70.2839dB for the noise variance ranging from 10% to 30%. The new technique out performs other techniques used to verify its performance, including the Wiener filter which gave an average MSE of 69.0658dB and Binomial filter which gave an average MSE of 64.3711dB for the same noise range. Since the filter developed in this paper is linear, it blurs the edges of the filtered images.

In further research, an attention will be given to an Adaptive alpha-trimmed Centre-Pixel-Weighed Exponential Filter. This filter is proposed combines the properties of linear and nonlinear filters which ensure effective filtering without edge blurring.

6. ACKNOWLEDGEMENT
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7. REFERENCES


