Inverse Kinematics Analysis for Manipulator Robot With Wrist Offset Based On the Closed-Form Algorithm

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Abstract

This paper presents an algorithm to solve the inverse kinematics for a six degree of freedom (6 DOF) manipulator robot with wrist offset. This type of robot has a complex inverse kinematics, which needs a long time for such calculation. The proposed algorithm starts from find the wrist point by vectors computation then compute the first three joint angles and after that compute the wrist angles by analytic solution. This algorithm is tested for the TQ MA2000 manipulator robot as case study. The obtained results was compared with results of rotational vector algorithm where both algorithms have the same accuracy but the proposed algorithm saving round about 99.6% of the computation time required by the rotational vector algorithm, which leads to used this algorithm in real time robot control.

Keyword: Manipulator Robot, Inverse Kinematics, DOF.

1. INTRODUCTION

A robotic manipulator (arm) consists of a chain of links interconnected by joints. There are typically two types of joints; revolute joint (rotation joint) and prismatic joint (sliding). It would be desirable to control both the position and orientation of a end-effecter or work piece, located at the tip of the manipulator, in its three-dimensional workspace. The end-effecter can be programmed to follow a planned trajectory, provided relationships between joint variables and position and the orientation of the end-effecter are formulated. This task is called the direct kinematics problem [1].

There are some difficulties to solve the inverse kinematics (IK) problem when the kinematics equations are coupled, multiple solutions and singularities exist. There are mainly two types of IK solution: analytical solution and numerical solution. In the first type, the joint variables are solved analytically according to given configuration data. In the second type of solution, the joint variables are obtained based on the numerical techniques. However, they are slow in practical applications [2]. An Artificial Neural Network (ANN) using backpropagation algorithm is applied to solve inverse kinematics problems of industrial robot manipulator. 6R robot manipulator with offset wrist was chosen as industrial robot manipulator because geometric feature of this robot does not allow solving inverse kinematics problems analytically [3].

2. DENAVIT-HARTENBERG (DH) PARAMETERS

A general arm equation that represents the kinematic motion of the manipulator can be obtained by systematically assigning coordinate frames for each link. The parameters associated with joint k are defined with respect to z_{k-1}, which is aligned with the axis of joint k. The first joint parameter, θ_k is called the joint angle. It is the rotation about z_{k-1} needed to make x_{k-1} parallel to x_k. The second joint parameter, d_k is called the joint distance. It is the translation along z_{k-1} needed to make x_{k-1} intersect with x_k see Fig.1. Note that for each joint, it will always be the case that one of these parameters is fixed and the other is variable. The variable joint parameter depends on the type of joint.

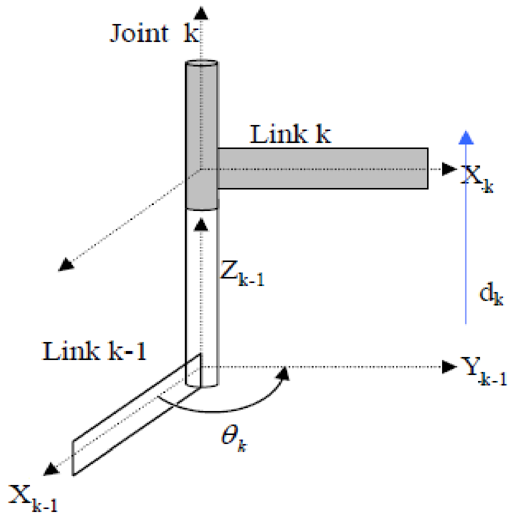

FIGURE 1: Joint angle θ and joint distance d

For instance, for a revolute joint, the joint angle θ_k is variable while the joint distance d_k is fixed while for a prismatic joint, the joint distance d_k is variable and the joint angle θ_k is fixed [1]. See Fig.2.

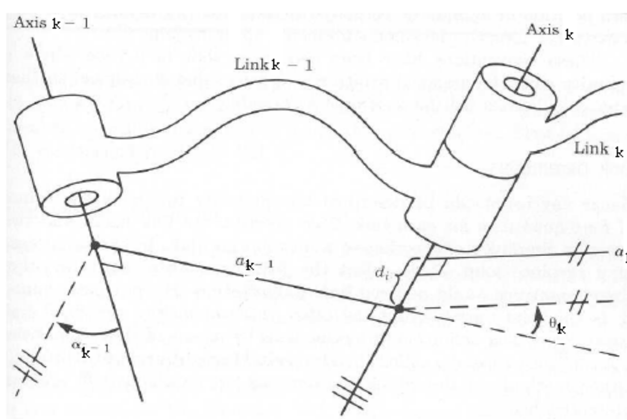

FIGURE 2: DH parameters defined for two links of a serial manipulator

3. FORWARD KINEMATIC ANALYSIS

These industrial robots are basically composed by rigid links, connected in series by joints (normally six joints), having one end fixed (base) and another free to move and perform useful work when properly end-effecter. As with the human arm, robot manipulators use the first three joints (arm) to position the structure and the remaining joints (wrist, composed of three joints in the case of the industrial manipulators) are used to orient the end-effecter. There are five types of arms commonly used by actual industrial robot manipulators: Cartesian, cylindrical, polar, SCARA and revolution [4]. The TQ MA2000 is considered in this work, it has 6 degree of freedom (DOF), one per each joint which means that each joint can be represented by a independent variable θ_n given that each joint is actuated, i.e. in this case each link has its own servo motor acting on it. Robot manipulator is an open kinematic chain whose joint positions can be defined by a vector of six single variables θ_n and the number of joints equals the number of degrees of freedom. Recall that the number of DOF is the number of independent position variables that should be specified in order to completely define a pose for the manipulator. It defines in how many ways the robot is able to move. In practice, to cover the entire 3D workspace with a 3-component orientation vector, six degrees of freedom are sufficient. Table I shows the DH parameters whole set of Kinematic parameters for the TQ MA2000 Robot. See Fig.2 The graphic representation of the TQ MA2000 robot is shown in Fig.3

Link i	Length a_i(cm)	Twist Angle α_i	Offset d_i (cm)	Joint angles θ_i
1	0	π/2	26	θ 1
2	23	0	0	θ2
3	24	0	0	θ3
4	0	π/2	5	θ4
5	0	π/2	4.4	θ5
6	0	0	8	θ6

TABLE 1: Complete DH parameter table for the TQ MA2000 6-DOF manipulator shown in Fig. 2

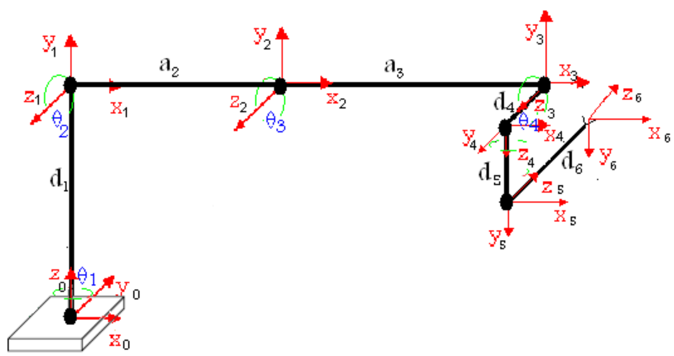

FIGURE 3: Graphic Represent of TQ MA2000 robot

The arm motion can be represented by transformation matrix as follow

Arm motion = $T_{base}^{wrist}(\theta_1,\theta_2,\theta_3)$ (1)

Where, T_i^j represent the transformation matrix from i[th] location to j[th] location. And T_{base}^{wrist} (T_0^3) is the orientation and position of the arm with respect to the base coordinate frame, depend on the first three joint angles $(\theta_1,\theta_2,\theta_3)$. And the wrist motion can be represented by the transformation matrix from wrist to end-effecter as follow.

Wrist motion =$T_{wrist}^{end-effecter}(\theta_4,\theta_5,\theta_6)$ (2)

And $T_{wrist}^{end-effecter}$ (T_3^6) is the orientation and position of the grip of the wrist with respect to the third link coordinate frame, depend on the last three joint angles $(\theta_4,\theta_5,\theta_6)$. Note that $T_5^6(\theta_6)$ maps tool-tip coordinates into roll coordinates, $T_4^5(\theta_5)$ maps roll coordinates into yaw coordinates, and $T_3^4(\theta_4)$ maps yaw coordinates into wrist pitch coordinates. Thus the composite transformation $T_3^6(\theta_4,\theta_5,\theta_6)$ maps end-effecter coordinates into wrist coordinates. Similarly $T_2^3(\theta_3)$ maps wrist coordinates into elbow coordinates, $T_1^2(\theta_2)$ maps elbow coordinates into shoulder coordinates, and $T_0^1(\theta_1)$ maps shoulder coordinates into base coordinates. Thus the composite transformation $T_0^3(\theta_1,\theta_2,\theta_3)$ maps wrist coordinates into base coordinates. The general solution can be expressed as:

[Total motion] = [arm motion] [wrist motion] (3)

Using the D-H method, the end-effector position can be written as: [1].

$$T_{base}^{end-effecter}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = T_0^1(\theta_1)T_1^2(\theta_2)T_2^3(\theta_3)\,T_3^4(\theta_4)T_4^5(\theta_5)T_5^6(\theta_6) \quad (4)$$

The homogenous transformation matrix of the T_0^6 which include the rotation matrix (3x3) and position vector (3x1) which given by equation (5) may be given as

$$T_{base}^{end-effecter} = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \quad (5)$$

The upper left 3x3 matrix, R, specifies the orientation of the end-effecter, while the 3 x 1 upper right sub matrix P specifies the position of the end-effecter. Thus, in the direct kinematic solution, for any given value of the joint angles θ, the arm matrix $T_{base}^{end-effecter}$ can be evaluated.

4. INVERSE KINEMATIC ANALYSIS

One of the most fundamental and ever present problems in robotics and computer animation is the inverse kinematics (IK). This problem may be stated as follows: given a desired hand position and orientation (posture), and the forward kinematics map, find the set of all configurations (joint angle vectors) of the robot or animation character that satisfy the forward kinematics map. The IK mapping is in general one to many involves complex inverse trigonometric functions, and for most manipulators and animation figures has no closed form solutions. Extremely fast IK computation is required in computer animation, for real-time applications in fast moving manipulator. Inverse kinematics deals with the problem of finding the required joint angles to produce a certain desired position and orientation of the end-effecter. Finding the inverse kinematics solution for a general manipulator can be a very tricky task. Generally they are non-linear equations. Close-form solutions may not be possible and multiple, infinity, or impossible solutions can arise. Nevertheless, special cases have a closed-form solution and can be solved [5].

5. PROPOSED ALGORITHM FOR IK

The proposed algorithm can be applied directly to robots with shoulder and elbow offset. However, wrist offset makes the hand position dependent on all joint angles rather than just the first four. This requires dealing with position and orientation simultaneously and entails the creation of 6-dimensional workspace which although possible, would substantially increase the off-line processing and storage requirements. Manipulators can often be considered to be composed of two parts. The first 3 joints form a regional structure whose primary purpose is to position the wrist in space. The last 3 joints form the orienting structure whose purpose is to orient the hand or grasped object.

5.1 Decompose the Manipulator at the Wrist

This is the single most important step of the procedure. The spherical wrist makes this possible. Find the position of the wrist point, which subsequently is used to solve for the first 3 joint angles. The wrist joint angles are then solved to give the correct orientation.

Fig.3 show the robot at reset position (all the joint angles equal zero), the proposed algorithm start with specify the required position in (x, y, z) coordinate and orientation angles of pitch, yaw, and roll.(φ_p, φ_y, and φ_r) angles respectively. Then change the wrist joint angles (θ_4, θ_5, and θ_6) from the zero (reset state) to new values that equal the required orientation angles of pitch, yaw, and roll.(φ_p, φ_y, and φ_r) angles respectively, the arm joint angles remain unchanged (at reset state, i.e. equal zero), then compute the foreword kinematics to compute the resulted position of the arm end point (O_4) and position of end-effecter (O_6) for the robot, find the vector between these two points as in equation (6)

$$V = O_6 - O_4 \qquad (6)$$

To compute the suitable position of the arm end point to reach the required position use equation (6) with that the vector from the origin point O_0 to the required position O_r is equal to the vector from origin base frame O_0 to the arm end point position O_4 sum with the vector V which is from O_4 to the required position as in equation (7). as shown in Fig. 4

$$V_{04} = V_{or} - V \qquad (7)$$

Where V is the vector compute by equation (6), and V_{or} is the vector from the origin O_0 to the required position. The position of the arm end point O_4 is the end of the vector V_{04}. A planar two-link manipulator often makes up the last two links of the regional structure.

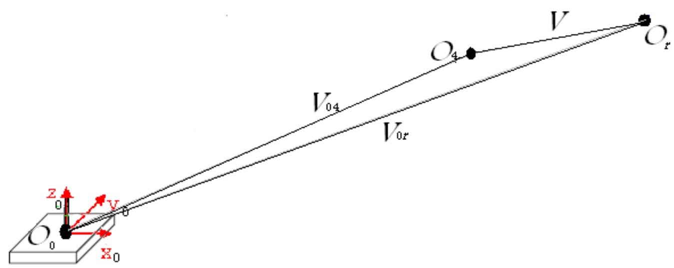

FIGURE 4: Wrist with setting its angles

The link 4 parameters have been included in the regional structure because they locate the wrist point O_4, which will be known relative to O_0. Equation (8) as shown in fig.(5).

$$V_{04} = O_4 - O_0 \qquad (8)$$

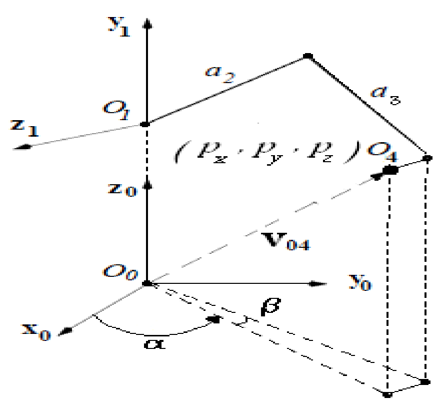

FIGURE 5: Vector from O_0 to O_4

Project the position of the wrist point into the x_0, y_0 plane, i.e., just select the p_x, p_y coordinates of O_4.

5.2 Solution for Θ_1

The wrist offset makes the calculation of θ_1 more complex, because the x_1 axis is no longer in the vertical plane of the upper arm and forearm Figure. When x_1 and the wrist point are projected onto the x_0, y_0 plane, then the angle θ_1 from x_0 to x_1 is no longer the same as the angle from x_0 to the projected wrist point. The projection on the x_0, y_0 plane is redrawn in Figure for clarity. The position of the arm end point at (p_x, p_y, p_z), see Fig.5. Then compute α and β as in equation (9) and (10).

$$\alpha = \text{atan2}(p_y, p_x) \tag{9}$$

$$\beta = \text{asin}\,(d_4/L) \tag{10}$$

Where $L = \sqrt{{p_x}^2 + {p_y}^2}$, and hence for right move then θ_1 will be as follow

$$\theta_1 = \alpha + \beta \tag{11}$$

And for lefty move

$$\theta_1 = (\alpha - \beta) - 180 \tag{12}$$

5.3 Solution For Θ_2 And Θ_3

For each solution of θ_1 find two set of solution for θ_2 and θ_3, elbow up and elbow down, therefore will compute four set of solution of the first three angles that is (left, right, elbow up, elbow down):

$$\theta = \text{atan}\,\left(\frac{pz_0}{r_0}\right) \tag{13}$$

Where $pz_0 = p_z - d_1$, and $r_0 = \sqrt{{p_x}^2 + {p_y}^2}$

The cosine law for link 2 and link 3:

$$\alpha 2 = \pm \text{acos}\,\left(\frac{r^2 + a_2^2 - a_3^2}{2ra_2}\right) \tag{14}$$

$$\alpha 3 = \pm \text{acos}\,\left(\frac{r^2 + a_3^2 - a_2^2}{2ra_3}\right) \tag{15}$$

Where $r = \sqrt{{p_x}^2 + {p_y}^2 + pz_0^2}$ (16)

Then compute θ_2 and θ_3

$$\theta_2 = \theta - \alpha 2 \tag{17}$$

$$\theta_3 = \alpha 2 + \alpha 3 \tag{18}$$

5.4 The Solution of the Last Three Joints $(\theta_4, \theta_5$ and $\theta_6)$

It is possible to obtain the following equations from the forward kinematics problem:

$$\mathbf{T}_0^6 = \mathrm{T}_0^3\, \mathrm{T}_3^6 \tag{19}$$

$$\mathrm{T}_0^6 = \begin{bmatrix} R_0^3 & P_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_3^6 & P_W \\ 0 & 1 \end{bmatrix} \tag{20}$$

$$T_0^6 = \begin{bmatrix} R_0^3 R_3^6 & R_0^3 P_w + P_a \\ 0 & 1 \end{bmatrix} \quad (21)$$

Where Pw is the wrist position and Pa position of arm end point then the rotation part of equation (5).

$$R = R_0^3 R_3^6 \quad (22)$$

where R is the required orientation of the robot. Translation part of equation (5)

$$P = R_0^3 P_w + P_a \quad (23)$$

Then $R_3^6 = (R_0^3)^{-1} R$ (24)

Where $R = \begin{bmatrix} N_x & O_x & A_x \\ N_y & O_y & A_y \\ N_z & O_z & A_z \end{bmatrix}$, and $R_3^6 = \begin{bmatrix} n_{wx} & o_{wx} & a_{wx} \\ n_{wy} & o_{wy} & a_{wy} \\ n_{wz} & o_{wz} & a_{wz} \end{bmatrix}$ (25)

$$R_3^6 = \begin{bmatrix} C_4 C_5 C_6 + S_4 S_6 & C_4 S_5 & C_4 C_5 S_6 - S_4 C_6 \\ S_4 C_5 C_6 - C_4 S_6 & S_4 S_5 & S_4 C_5 S_6 + C_4 C_6 \\ S_4 C_6 & -C_5 & S_5 S_6 \end{bmatrix} \quad (26)$$

Solving for orientation angles:

$\theta_4 = \operatorname{atan}\left(\frac{o_{wy}}{o_{wx}}\right)$, $\theta_5 = \operatorname{atan}\left(\frac{c_4 o_{wx} + s_4 o_{wy}}{-o_{wz}}\right)$, and $\theta_6 = \operatorname{atan}\left(\frac{s_4 n_{wx} - c_4 n_{wy}}{s_4 a_{wy} - c_4 a_{wx}}\right)$ (27)

5.5 Case Study

In [6] they use the relative orientation representation by the rotation vector are based on the Euler theorem which states that (a displacement of a rigid body with one fixed point can be described as a rotation about some axis). And applied this algorithm to solve the inverse kinematics for the TQ MA2000 manipulator robot to compute the sets of angles. Consider the required position is (-25, -10, 50) cm as (x, y, z) with respect to the base coordinate frame. The orientation of the gripper rotates (180o, 225o, 135o) degree as $(\varphi_p, \varphi_y, \varphi_r)$ the rotating angle of the gripper about pitch , yaw, and roll rotating axis respectively. There are four set of the joint variables (solution) of the inverse kinematics of the robot.

Set = [θ1 , θ2 , θ3 , θ4 , θ5 , θ6]
Set 1 = [-162.6436, -7.8977, 79.7788, 108.1189, 27.6436, 135]
Set 2 = [-162.6436, 75.8702, -79.7788, -176.0914, 27.6436, 135]
Set 3 = [-1.2296,187.8977, -79.7788, 71.8811, -133.7704, 135]
Set 4 = [-1.2296, 104.1298, 79.7788, 356.0914, -133.7704, 135]

The structure of the robot for all solution sets(1, 2, 3, 4) are shown in Fig.6 (a- right elbow down, b- right elbow up, c- lefty elbow down, and d- lefty elbow up) respectively.

The proposed algorithm is solving the same case steady and finds the same results with the same accuracy. By using DELL laptop computer with Central Processing Unit (CPU) Intel® Core™ 2 Duo with 2.2GHz; the computation time for the proposed algorithm is equal 0.016 second and the computation time for rotation vectors is equal 3.984 second these less computation time is suitable for used the proposed algorithm with real time manipulator robot control system.

6. CONCLUSION

By camper this algorithm with the vector rotation algorithm that is used for solve the inverse kinematics of the same manipulator robot [6]. They have same accuracy but the computation time by proposed method is less than that for the first method to compute all the angles. The computation time for the solving of inverse kinematics of the manipulator robot by the proposed algorithm is about 0.4% of the computation time that required by use the rotational vector algorithm, therefore this algorithm is best to used for real time manipulator robot control system.

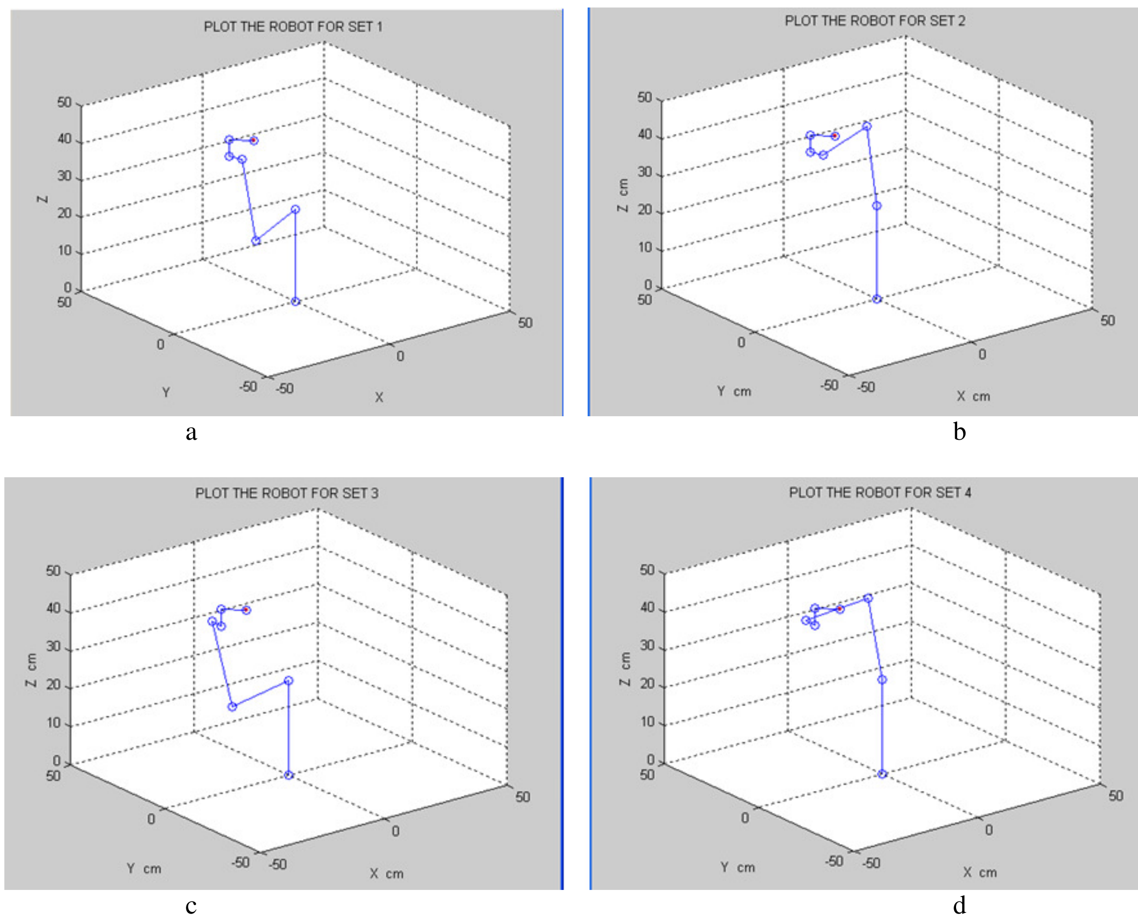

FIGURE 6: Drawing of the robot according to set of solution a- set 1 b-set 2 c- set 3 d- set 4

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