Optimum Algorithm for Computing the Standardized Moments Using MATLAB 7.10(R2010a)

K.A.Fayed *karamfayed_1@hotmail.com*
Ph.D.From Dept. of applied Mathematics and Computing, Cranfield University, UK.
Faculty of commerce/Dept. of applied Statistics and Computing, Port Said University, Port Fouad, Egypt.

Abstract

A fundamental task in many statistical analyses is to characterize the location and variability of a data set. A further characterization of the data includes skewness and kurtosis. This paper emphasizes the real time computational problem for generally the r^{th} standardized moments and specially for both skewness and kurtosis. It has therefore been important to derive an optimum computational technique for the standardized moments. A new algorithm has been designed for the evaluation of the standardized moments. The evaluation of error analysis has been discussed. The new algorithm saved computational energy by approximately 99.95%than that of the previously published algorithms.

Keywords: Statistical Toolbox**,** Mathematics, MATLAB Programming

1. INTRODUCTION

The formula used for Z –score appears in two virtually identical forms, recognizing the fact that we may be dealing with sample statistics or population parameters. These formulae are as follow:

$$z_i = \frac{x_i - \bar{x}}{s} \quad \text{Sample statistics} \tag{1}$$

$$Z_i = \frac{x_i - \mu}{\sigma} \quad \text{Population statistics} \tag{2}$$

Where:

x_i a row score to be standardized
n sample size
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ Sample mean
μ Population mean
s Sample standard deviation
σ Population standard deviation
z Sample z score
Z Populationz score.

Subtracting the mean centers the distribution and dividing by the standard normalizes the distribution. The interesting properties of Z score are that they have a zero mean (effect of centering) and a variance and standard of one (effect of normalizing). We can use Z score to compare samples coming from different distributions [1].

The most common and useful measure of dispersion is the standard deviation. The formula for sample standard deviation is as follow:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \text{Sample standard deviation} \qquad (3)$$

The population standard deviation is as follow:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} \qquad \text{Population standard deviation} \quad (4)$$

2. MOMENTS

In statistics, the moments are a method of estimation of population parameters such as mean, variance, skewness, and kurtosis from the sample moments.

a) Central Moments

Central moment is called moment about the mean. The central moments provide quantitative indices for deviations of empirical distributions. The r th central is given by:

$$m_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^r$$

$$or:$$

$$m_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^r \qquad (5)$$

Where:

m_r r th Sample and population central moments

b) Standardized Moment

The r th standardized moment in statistics is the r th central moment divided by σ^r (standard deviation raised to power r) as follow:

$$\alpha_r = \frac{m_r}{\sigma^r} \qquad (6)$$

Where:

α_r r th standardized moment

From Eq.(4), Eq.(5), & Eq.(6), We have:

$$\alpha_r = \frac{1}{n}\frac{\sum (x-\mu)^r}{\sigma^r} = \frac{1}{n}\Sigma(Z)^r$$

$$= \frac{(1/n)\sum (x-\mu)^r}{\left(\sqrt{(1/n)\sum (x-\mu)^2}\right)^r} = \frac{m_r}{\left(\sqrt{m_2}\right)^r}$$

Therefore:

$$\alpha_r = \frac{1}{n}\Sigma(Z)^r = \frac{m_r}{\left(\sqrt{m_2}\right)^r} \qquad (7)$$

Where:

m_2 Second central moments

c) Computing Population Standardized Moments From Sample z Score

In the real world, finding the standard deviation of an entire population is unrealistic except in certain cases such as standardized testing, where every element of a population is sampled. In most cases, the standard deviation is estimated by examining a random sample taken from the population as defined by eq.(3).

From eq.(5) & eq.(7), We have:

$$\alpha_r = \frac{1}{n}\Sigma(Z)^r = \frac{m_r}{\left(\sqrt{m_2}\right)^r}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{\left(\sqrt{(1/n)\sum (x-\bar{x})^2}\right)^r}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{(1/n)^{r/2}(n-1)^{r/2}\left(\sum (x-\bar{x})^2/(n-1)\right)^{r/2}}$$

$$= \frac{(1/n)\sum (x-\bar{x})^r}{((n-1)/n)^{r/2}\left(S^2\right)^{r/2}}$$

$$= \frac{1}{n}\left(\frac{n}{n-1}\right)^{r/2}\sum (x-\bar{x})^r / S^r$$

$$= \frac{1}{n}\left(\frac{n}{n-1}\right)^{r/2}\sum z^r$$

Therefore:

$$\alpha_r = \frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}\sum z^r \qquad (8)$$

Equation(8) represents the general equation for computing the r^{th} standardized moments of sample z-score.

d) Simplified Standardized Moments

From eq.(8), the term $\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}$ can be simplified using binomial theorem, since it can obtain the binomial series which is valid for any real number $k \text{ if } |x| < 1$ as follow:

$$(1+x)^k = 1 + k\,x + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots] \qquad (9)$$

The term $\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2}$ can be rewritten in the following form:

$$\frac{1}{n}\left(\frac{n}{n-1}\right)^{rl2} = \frac{1}{n}\left(\frac{n-1}{n}\right)^{-rl2} = \frac{1}{n}\left(1-\frac{1}{n}\right)^{-rl2}$$

(10)

By replacing $x \text{ by } \frac{-1}{n}$ and $k \text{ by } \frac{r}{2}$ we have:

$$\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}} = \frac{1}{n}\left[1-\frac{1}{n}\right]^{-\frac{r}{2}}$$

$$= \frac{1}{n}\left[1 + \frac{\left(-\frac{r}{2}\right)}{1!}\left(-\frac{1}{n}\right) + \frac{\left(-\frac{r}{2}\right)\left(-\frac{r}{2}-1\right)}{2!}\left(-\frac{1}{n}\right)^2 + \cdots\right]$$

$$= \frac{1}{n}\left[1 + \frac{r}{2n} + \frac{r}{4n^2} + \frac{r^2}{8n^2} + \cdots\right]$$

$$= \left[\frac{1}{n} + \frac{r}{2n^2} + \frac{r}{4n^3} + \frac{r^2}{8n^3} + \cdots\right] \qquad (11)$$

For large values of n ,we get:

$$\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}} \cong \left[\frac{1}{n}+\frac{r}{2n^2}\right] \qquad (12)$$

Substituting Eq.(12) in eq.(8), we get:

$$\alpha_{sr} = \left[\frac{1}{n}+\frac{r}{2n^2}\right]\sum_{i=1}^{n} z_i^r$$

$$= \left[\frac{2n+r}{2n^2}\right]\sum_{i=1}^{n} z_i^r \qquad (13)$$

Where:

α_{sr} r^{th} simplified standardized moments.

e) Mathematical Formulae of Standardized and Simplified Moments

Using Eq.(8) & Eq.(13), we can get the following formulae:

Name	r^{th}	Standardized moments	Simplified moments
Mean	1	$\alpha_1 = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{1}{2}}\sum_{i=1}^{n} z_i = 0$	$\alpha_{1s} = \left[\frac{2n+1}{2n^2}\right]\sum_{i=1}^{n} z_i = 0$
Variance	2	$\alpha_2 = \left[\frac{1}{n-1}\right]\sum_{i=1}^{n} z_i^2 = 1$	$\alpha_{2s} = \left[\frac{n+1}{n^2}\right]\sum_{i=1}^{n} z_i^2 = 1$
Skewness	3	$\alpha_3 = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{3}{2}}\sum_{i=1}^{n} z_i^3$	$\alpha_{3s} = \left[\frac{2n+3}{2n^2}\right]\sum_{i=1}^{n} z_i^3$
Kurtosis	4	$\alpha_4 = \frac{n}{(n-1)^2}\sum_{i=1}^{n} z_i^4$	$\alpha_{4s} = \left[\frac{n+2}{n^2}\right]\sum_{i=1}^{n} z_i^4$

f) Ratio Between Population and Sample z-Score

From Eq.(7) & Eq.(8), we can get the exact and simplified ratio of population and sample z-score as follow:

Since:

$$\frac{1}{n}\sum_{i=1}^{n} Z_i^r = \frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r$$

We get:

$$\frac{\sum_{i=1}^{n} Z_i^r}{\sum_{i=1}^{n} z_i^r} = \left[\frac{n}{n-1}\right]^{\frac{r}{2}} \qquad \textit{exact ratio} \qquad (14)$$

And from Eq.(7) & Eq.(13), we can get:

$$\frac{1}{n}\sum_{i=1}^{n} Z_i^r = \left[\frac{2n+r}{2n^2}\right]\sum_{i=1}^{n} z_i^r$$

$$\frac{1}{n}\sum_{i-1}^{n} Z_i^r = \frac{1}{n}\left[\frac{2n+r}{2n}\right]\sum_{i-1}^{n} z_i^r$$

$$\frac{\sum_{i=1}^{n} Z_i^r}{\sum_{i=1}^{n} z_i^r} = \left[1+\frac{r}{2n}\right] \qquad \textit{simplified ratio} \qquad (15)$$

Eq.(14) and Eq.(15) appear to be very dependent on the sample size. Therefore the ratio between population and sample z-score(required for computing ther^{th} standardized moments) depends on the sample size as given in Table_1. This table shows the variation. Figure_1 shows that the sample z score gets closer to population Z score. Therefore, computing standardized moments using simplified technique is recommended for small sample size.

g) Formulae of Skewness and Kurtosis Applied in Statistical Packages
The usual estimators of the population skewness and kurtosis used in Minitab, SAS, SPSS, and Excel are defined as follow [2], [3],[4]:

$$G_1 = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s}\right)^3 \qquad (16)$$

$$G_2 = \left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)} \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{s^4}\right\} - 3\frac{(n-1)^2}{(n-2)(n-3)} + 3 \qquad (17)$$

Where:
$s$is the sample standard deviation.
$G_1$is the usual estimator of population skewness.
$G_2$is known as the excess kurtosis(without adding 3).

Sample size(n)	Exact ratio		Simplified ratio	
	r=3	r=4	r=3	r=4
20	1.07998	1.10803	1.07500	1.10000
30	1.05217	1.07015	1.05000	1.06667
50	1.03077	1.04123	1.03000	1.04000
100	1.01519	1.02030	1.01500	1.02000
200	1.00755	1.01008	1.00750	1.01000
400	1.00376	1.00502	1.00375	1.00500
600	1.00251	1.00334	1.00250	1.00333
1000	1.00150	1.00200	1.00150	1.00200
1400	1.00107	1.00143	1.00107	1.00143
2000	1.00075	1.00100	1.00075	1.00100
2600	1.00058	1.00077	1.00058	1.00077
3000	1.00050	1.00067	1.00050	1.00066
3600	1.00042	1.00056	1.00042	1.00055
4000	1.00038	1.00050	1.00038	1.00050
4500	1.00033	1.00044	1.00033	1.00044
5000	1.00030	1.00040	1.00030	1.00040
5500	1.00027	1.00036	1.00027	1.00036
6000	1.00025	1.00033	1.00025	1.00033
8000	1.00019	1.00025	1.00019	1.00025
10000	1.00015	1.00020	1.00015	1.00020

TABLE 1: Ratio between population and sample z-score

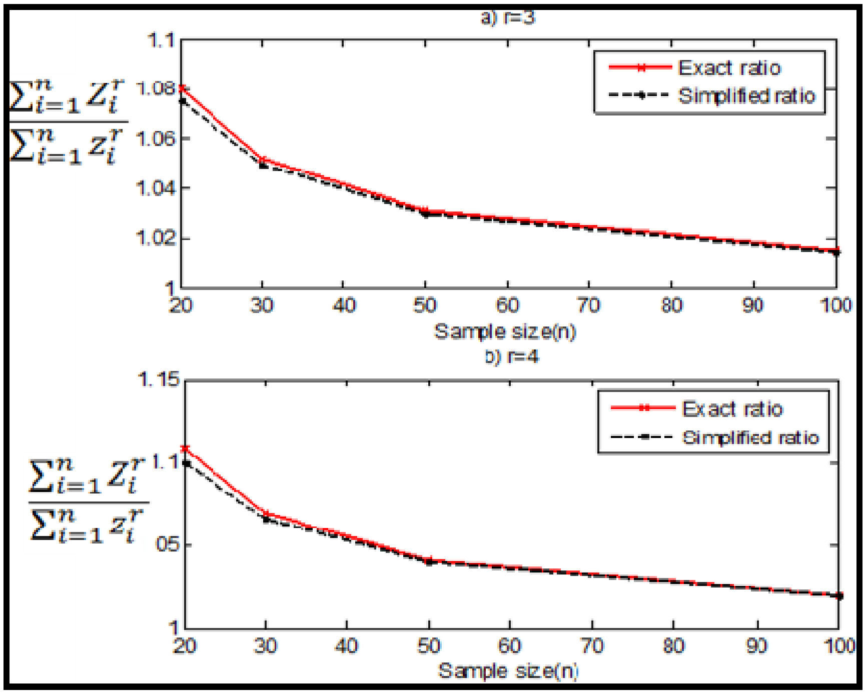

FIGURE 1: Ratio between population and sample z-score.

h) Error Analysis of Standardized Moments

The absolute relative error(ARE) between the standardized and simplified moments is given by:

$$ARE = \left|\frac{standardised - simplified}{standardised}\right| = \left|\frac{\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r - \frac{1}{n}\left[1+\frac{r}{2n}\right]\sum_{i=1}^{n} z_i^r}{\frac{1}{n}\left[\frac{n}{n-1}\right]^{\frac{r}{2}}\sum_{i=1}^{n} z_i^r}\right|$$

$$= \left|\frac{\left[\frac{n}{n-1}\right]^{\frac{r}{2}} - \left[1+\frac{r}{2n}\right]}{\left[\frac{n}{n-1}\right]^{\frac{r}{2}}}\right| \qquad (18)$$

Therefore, the Absolute Relative Error(ARE) appears to be very dependent on the sample size in regardless with the sample z-score as given in Table_2. This table indicates that the error associated with the standardized moments(Skewness and Kurtosis) of the statistical packages technique is very large compared to the simplified one especially when the sample size is less than 300.Figure_2 shows the variation. Therefore, computing standardized moments using simplified technique is recommended especially when the sample size is less than 600.

Sample size(n)	Skewness			Absolute Relative Error (ARE)`		
	Exact	Simplified	Statistical package	Simplified		Statistical Package
				Pract.	Comp.	
20	-0.37911	-0.37736	-0.41057	0.4608	0.4608	8.2977
30	-0.24103	-0.24053	-0.25390	0.2060	0.2060	5.3420
50	-0.81154	-0.81094	-0.83686	0.0744	0.0744	3.1197
100	0.19241	0.19237	0.19535	0.0186	0.0186	1.5293
200	-0.11239	-0.11239	-0.11324	0.0046	0.0046	0.7572
400	0.21474	0.21474	0.21555	0.0011	0.0011	0.3768
600	0.01677	0.01677	0.01682	0.0005	0.0005	0.2508
1000	0.05781	0.05781	0.05790	0.0001	0.0001	0.1502
1400	-0.12846	-0.12846	-0.12860	9.5e-5	9.5e-5	0.10728
2000	-0.02750	-0.02750	-0.02753	4.6e-5	4.6e-5	0.07507
2600	0.03271	0.03271	0.03273	2.7e-5	2.7e-5	0.05773
3000	-0.01793	-0.01793	-0.01795	2.1e-5	2.1e-5	0.05003
3600	-0.02616	-0.02616	-0.02617	1.4e-5	1.4e-5	0.041688
4000	-0.01818	-0.01818	-0.01819	1.1e-5	1.1e-5	0.037517
4500	0.005310	0.005310	0.005312	9.2e-6	9.2e-6	0.033347
5000	-0.04197	-0.04197	-0.04198	7.5e-6	7.5e-6	0.030011
5500	-0.04199	-0.04199	-0.04200	6.2e-6	6.2e-6	0.027282
6000	0.033432	0.033432	0.033440	5.2e-6	5.2e-6	0.025007
8000	-0.00851	-0.00851	-0.00851	2.9e-6	2.9e-6	0.018754
10000	-0.00057	-0.00057	-0.00057	1.8e-6	1.8e-6	0.015002

TABLE 2: a)Skewness (ARE)

Sample size(n)	CPU time (Second)		
	Exact	Simplified	Statistical package
20	0.000185	0.000126	0.000146
30	0.000189	0.000133	0.000145
50	0.000200	0.000156	0.000154
100	0.000213	0.000153	0.000163
200	0.000234	0.000181	0.000185
400	0.000301	0.000257	0.000239
600	0.000394	0.000302	0.000357
1000	0.000487	0.000407	0.000457
1400	0.000584	0.000513	0.000518
2000	0.000746	0.000676	0.000690
2600	0.000989	0.000883	0.000909
3000	0.001096	0.001046	0.001002
3600	0.001233	0.001164	0.001162
4000	0.001396	0.001329	0.001375
4500	0.001489	0.001018	0.001355
5000	0.001594	0.001559	0.001574
5500	0.001785	0.001250	0.001683
6000	0.001891	0.001286	0.001916
8000	0.002164	0.001500	0.002286
10000	0.002318	0.001802	0.003122

TABLE 2: a)Skewness(CPU)

Sample size(n)	Kurtosis			Absolute Relative Error(ARE)		
	Exact	Simplified	Statistical Package	Simplified		Statistical Package
				Pract.	Comp.	
20	3.0034	2.9816	3.377	0.725	0.725	12.439
30	2.897	2.8876	3.1077	0.32593	0.32593	7.2722
50	2.628	2.6249	2.7182	0.1184	0.11840	3.4341
100	2.6438	2.643	2.6878	0.0298	0.0298	1.6648
200	3.0312	3.031	3.0626	0.00747	0.00747	1.0361
400	2.8435	2.8434	2.8566	0.00187	0.00187	0.4634
600	2.967	2.967	2.9768	0.00083	0.00083	0.32998
1000	2.932	2.932	2.938	0.00029	0.00029	0.19384
1400	3.066	3.066	3.071	0.00015	0.00015	0.14793
2000	3.023	3.023	3.027	7.5e-5	7.5e-5	0.1014
2600	2.947	2.947	2.949	4.4e-5	4.4e-5	0.0749
3000	2.963	2.963	2.965	3.3e-5	3.3e-5	0.06553
3600	2.882	2.882	2.884	2.3e-5	2.3e-5	0.05220
4000	2.976	2.976	2.978	1.8e-5	1.8e-5	0.04946
4500	2.952	2.952	2.954	1.4e-5	1.4e-5	0.04341
5000	3.010	3.010	3.011	1.1e-5	1.1e-5	0.04024
5500	2.998	2.998	2.999	9.9e-6	9.9e-6	0.03636
6000	3.073	3.073	3.074	8.3e-6	8.3e-6	0.03454
8000	3.041	3.041	3.042	4.6e-6	4.6e-6	0.02552
10000	2.911	2.911	2.912	2.9e-6	2.9e-6	0.01909

TABLE 2: b) Kurtosis(ARE)

Sample size(n)	CPU time (Second)		
	Exact	Simplified	Statistical Package
20	0.000164	0.000125	0.000187
30	0.000169	0.000127	0.000196
50	0.000170	0.000153	0.000214
100	0.000192	0.000151	0.000216
200	0.000216	0.000176	0.000248
400	0.000290	0.000256	0.000302
600	0.000327	0.000295	0.000363
1000	0.000547	0.000430	0.000457
1400	0.000563	0.000543	0.000554
2000	0.000737	0.000832	0.001109
2600	0.000967	0.000914	0.000962
3000	0.001020	0.000989	0.001173
3600	0.001187	0.001181	0.001231
4000	0.001338	0.000944	0.001073
4500	0.001426	0.001482	0.001643
5000	0.001128	0.001598	0.001619
5500	0.001138	0.001732	0.001868
6000	0.001826	0.001290	0.001893
8000	0.002628	0.001832	0.002436
10000	0.002571	0.001830	0.002922

TABLE 2: b) Kurtosis(CPU)

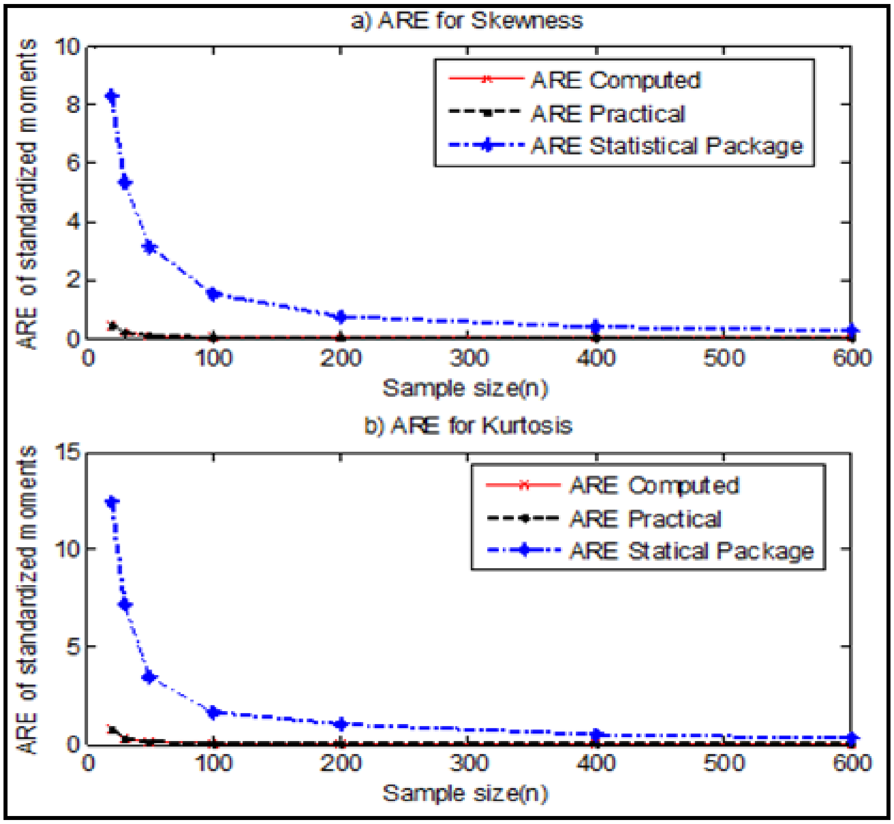

FIGURE 2: Absolute Relative Error of standardized moments

The percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one of the standardized moment is given by:

$$\delta E = \left| \frac{ARE\ of statistical\ package - ARE\ of simplified}{ARE\ of statistical\ package} \right| * 100 \qquad (19)$$

Where:

δE isthe percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one.

Table_3 shows the percentage reduction in Absolute Relative Error between the statistical packages technique and the simplified one for different sample size. This table indicates that the simplified technique of the standardized moments gives reduction in ARE by approximately 96.7% compared to the statistical package technique especially when the sample size is less than 100.Figure_3 shows the variation.

The squared error(E_r) between the standardized and simplified moments is given by:

$$E_r = (standardised - simplified\ )^2$$
$$= (ARE * standardised)^2 \qquad (20)$$

Sample size(n)	Skewness (r=3)		Kurtosis (r=4)	
	Error percentage(%)	Error reduction(%)	Error percentage(%)	Error reduction(%)
20	5.553	94.447	5.828	94.172
30	3.856	96.144	4.482	95.518
50	2.385	97.615	3.448	96.552
100	1.216	98.784	1.790	98.210
200	0.608	99.392	0.721	99.279
400	0.292	99.708	0.404	99.596
600	0.199	99.801	0.252	99.748
1000	0.067	99.933	0.150	99.850
1400	0.089	99.911	0.101	99.899
2000	0.061	99.939	0.074	99.926
2600	0.047	99.953	0.059	99.941
3000	0.042	99.958	0.050	99.950
3600	0.034	99.966	0.044	99.956
4000	0.029	99.971	0.036	99.964
4500	0.028	99.972	0.032	99.968
5000	0.025	99.975	0.027	99.973
5500	0.023	99.977	0.027	99.973
6000	0.021	99.979	0.024	99.976
8000	0.015	99.985	0.018	99.982
10000	0.012	99.988	0.015	99.985
Mean	0.73 %	99.27 %	0.879%	99.121%

TABLE 3: Error reduction of standardized moments

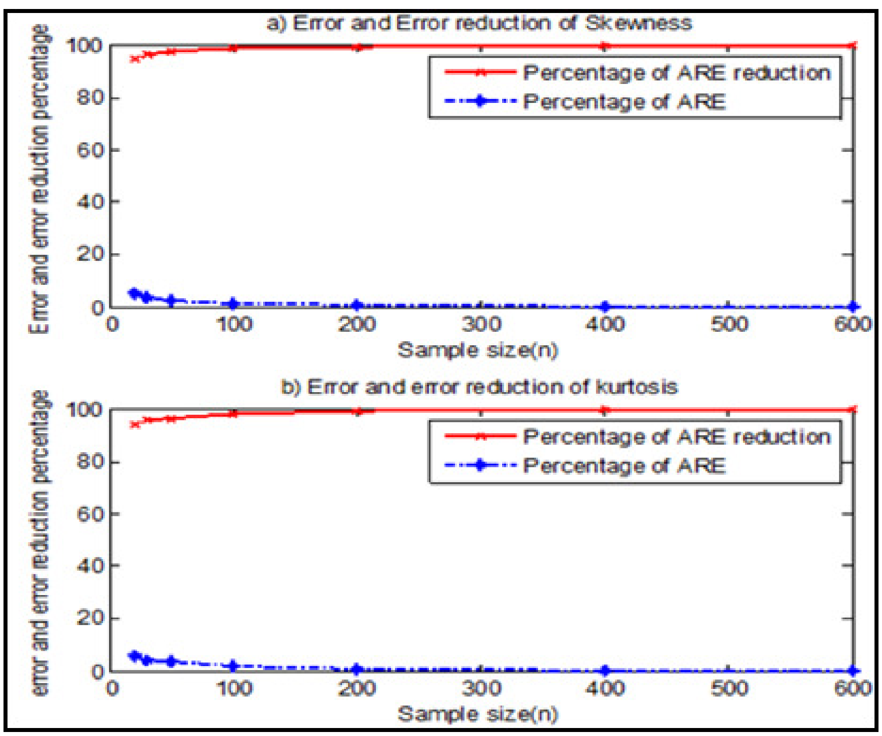

FIGURE 3: Error and error reduction of standardized moments

3. POPULATION EXAMPLE

A data set of 10000 points was randomly generated to have a mean of 100 and a standard deviation of 10. The histogram for this data is shown in figure_4 and looks fairly bell-shaped. A different sample size was randomly selected from the data set to calculate the two statistics(skewness and kurtosis).

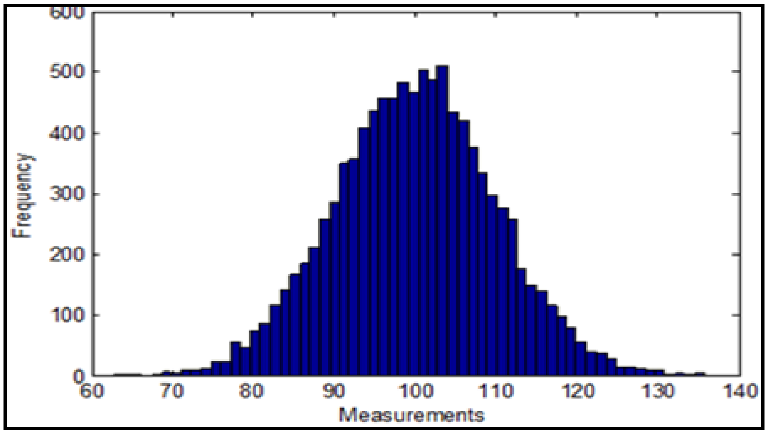

FIGURE 4: Histogram of 10000 points randomly generated(μ=100,σ =10)

4. IMPACT OF SAMPLE SIZE ON SKEWNESS AND KURTOSIS

The 10000 point data set above was used to explore what happens to skewness and kurtosis based on sample size. There appears to be a lot of variation in the results based on sample size. The results are shown in Table_2.Figure_5shows how the skewness and kurtosis changed with sample size.

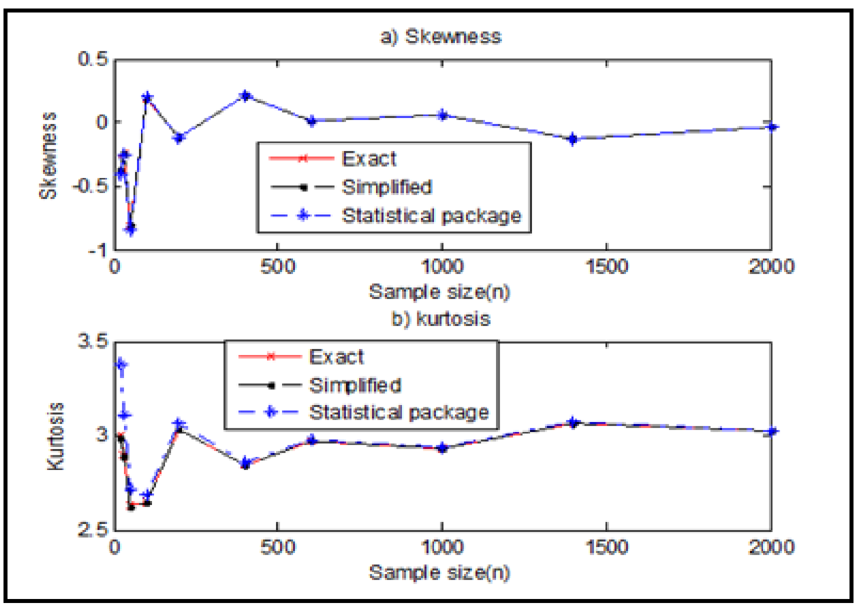

FIGURE 5: Impact of size sample on skewness and kurtosis

5. PROCESSING TIME OF STANDARDIZED MOMENTS

The processing time required for Computing the skewness and kurtosis is executed by LaptopDELL-inspiron-1520.Table_2 indicates that the processing time required for computing the skewnessusing the simplified technique is minimum than other especially when the sample size increases. Figure_6 shows the variation.

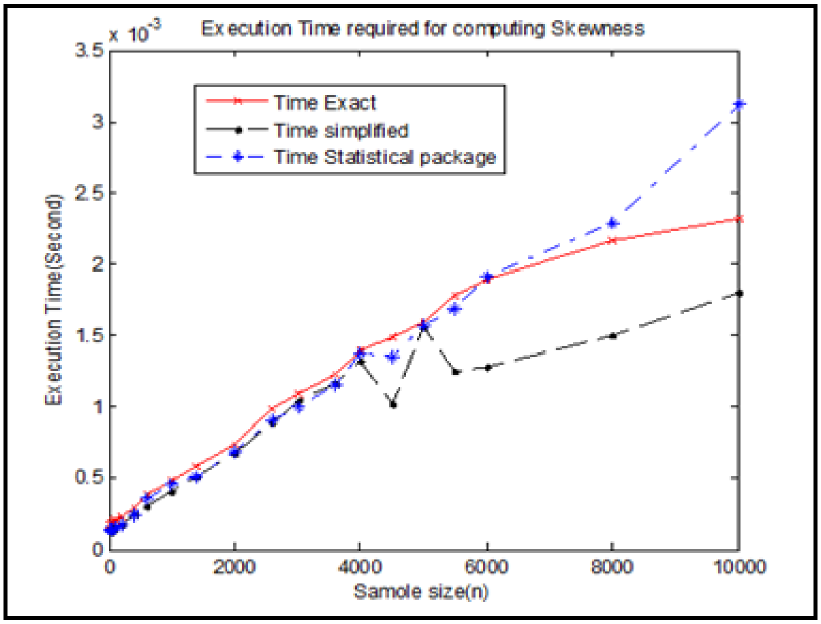

FIGURE 6:a) Execution time required for computing skewness

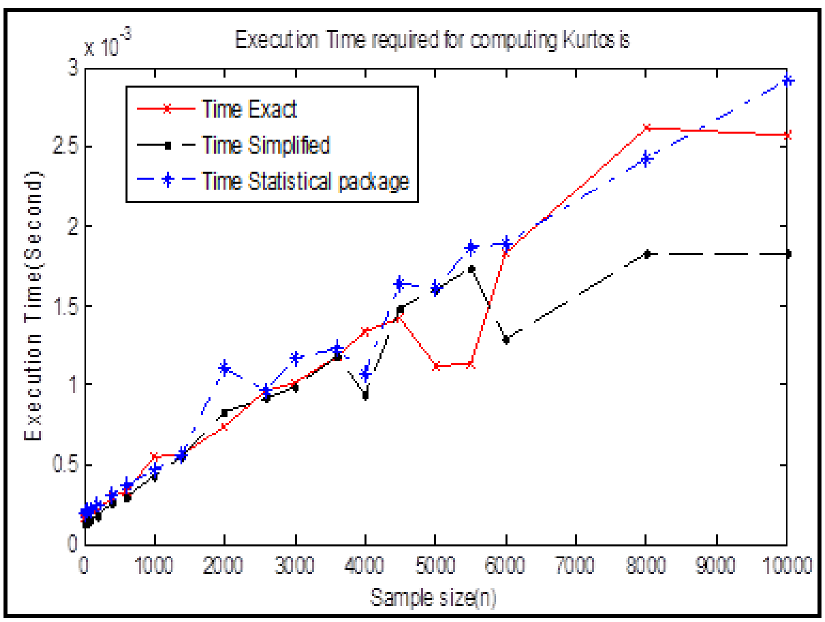

FIGURE 6: b) Execution time required for computing kurtosis

6. COMPUTATIONAL ENERGY OF STANDARDIZED MOMENTS

Computing the computational energy for standardized moments (skewness and kurtosis) requires the determination of the sample size(n), the square error(E_r), and the central processing time(CPU time). Therefore, consider the sample size(n) represents the resistance, the square error is measured in [volts]^2, and the CPU time in second. Then, the computational energy per sample size is given by:

$$CE = \frac{E_r * t_r}{n} \qquad (21)$$

Where:

CE is the computational energy per sample size.
E_r is the rth square error.
t_r is r^{th} CPU time.
n is the sample size.

The computational energy saved by the simplified technique compared to the exact one is given by:

$$\delta CE = \frac{CE_e - CE_s}{CE_e} * 100 \quad (22)$$

Where:

δCE is the relative computational energy saved by the simplified technique.
CE_e is the computational energy for the exact technique.
CE_s is the computational energy for the simplified technique.

Table_4 shows the computational energy(CE) for each technique. This table indicates that the simplified technique saved computational energy by approximately 96.7% compared to the statistical package technique. Figure_7 shows the variation.

Sample size(n)	**CE-Skewness (r=3)**			
	CE Exact	**CE Simplified**	**CE Statistical Package**	**CE saved by simplified (%)**
20	2.82e-07	1.92e-07	7.22e-05	99.73
30	1.55e-08	1.09e-08	8.01e-06	99.86
50	1.45e-08	1.13e-08	1.97e-05	99.94
100	2.72e-11	1.95e-11	1.41e-07	99.98
200	3.12e-13	2.41e-13	6.69e-09	99.99
400	4.19e-14	3.58e-14	3.91e-09	99.99
600	4.61e-17	3.53e-17	1.05e-11	99.99
1000	1.62e-17	1.36e-17	3.44e-11	99.99
1400	6.21e-17	5.45e-17	7.02e-11	99.99
2000	5.96e-19	5.40e-19	1.47e-12	99.99
Mean	----	----	----	99.95%

TABLE 4: Computational Energy of standardized moments(a:Skewness)

Sample size(n)	**CE-Kurtosis (r=4)**			
	CE Exact	**CE Simplified**	**CE Statistical Package**	**CE saved by simplified (%)**
20	3.88e-05	2.96e-05	0.01304	99.77
30	5.02e-06	3.77e-06	2.89e-3	99.86
50	3.29e-07	2.96e-07	3.48e-4	99.91
100	1.19e-08	9.37e-09	4.18e-05	99.97
200	5.53e-10	4.51e-10	1.22e-05	99.99
400	2.04e-11	1.80e-11	1.31e-06	99.99
600	3.30e-12	2.98e-12	5.79e-07	99.99
1000	3.95e-13	3.10e-13	1.47e-07	99.99
1400	8.50e-14	8.20e-14	8.14e-08	99.99
2000	1.89e-14	2.13e-14	5.2e-08	99.99
Mean	----	-----	------	99.95%

TABLE 4: Computational Energy of standardized moments(b:Kurtosis)

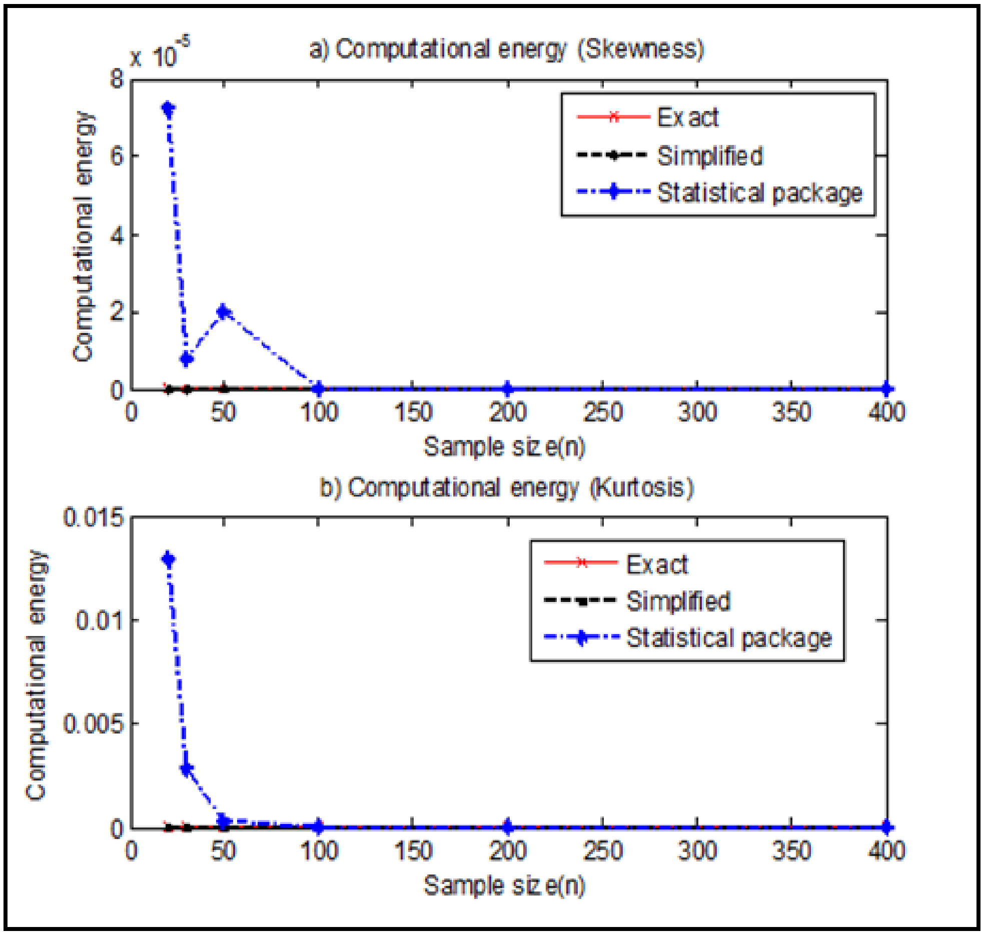

FIGURE 7: Computational Energy for standardized moments

7. MATLAB PROGRAMMING

A complete program can be obtained by writing to the author[4]. There is a part of MATLAB program shown here:

```
% Grenrate random data set of size (n) points with mean (mu) and a standard
% deviation (segma)and returns: (1) skewness and kurtosis,(2) cpu time,(3) ARE &
% SquareError,(4) Computational Energy(CE),(5) computational energy saved
% by the simplified technique compared to the exact one
options.Interpreter='tex';
prompt = {'Enter Sample size:','Enter mean(\mu) :','Enter std.dev.(\sigma) :'};
dlg_title = 'Generate random data set';
num_lines = 1;
def = {'','',''};
options.Resize='on';
options.WindowStyle='normal';
answer = inputdlg(prompt,dlg_title,num_lines,def,options);
ifisempty(answer)
error('No inputs were found!')
end
n=str2num(answer{1})
mu= str2num(answer{2})
sigma = str2num(answer{3})
if n< 3 || isempty(n)
error('n must be integer &>=2')
```

```
end
// Part of the program is omitted //
tic
            S_SP=(n/((n-1)*(n-2)))*sum(((s-mean(s))./std(s)).^r);
t_SP= toc;
tic
            S_E=(1/n)*(n/(n-1))^(r/2)*sum((zscore(s)).^r);
t_E = toc;
tic
            S_S=(1/n+r/(2*n^2))*sum((zscore(s)).^r);
t_S = toc;
   A_E=abs(((S_E-S_S)/S_E)*100);
  A_S=abs((((n/(n-1))^(r/2)-(1+r/(2*n)))/(n/(n-1))^(r/2))*100) ;
  A_SP=abs(((S_E-S_SP)/S_E)*100);
  SK=dataset({ S_E,'Exact'},{ S_S,'Simplified'},{ S_SP,'Stat_Package'} )
 ARE=dataset({ A_E,'Practical'},{ A_S,'Computed'}, {A_SP,'Stat_Package'} )
        // Part of the program is omitted //
```

8. CONCLUSIONS

Computer algorithms for fast implementation of standardized moments are an important continuing area of research.A new algorithm has been designed for the evaluation of the standardized moments. As a result the new technique offered four advantages over the current technique:

(1) It drastically reduces the CPU time for calculating the standardized moments especially when the sample size increases.
(2) It drastically reduces the absolute relative error(ARE) for calculating the standardized moments(Skewness and Kurtosis) by 99.27% compared to the current one.
(3) It gives minimum square error compared to the current algorithm.
(4) It has lowest computational energy.

The aforementioned features are combined in a mathematical formula to describe the system performance. This formula is called the computational energy. A quantitative study has been carried out to compute the computational energy for each technique. The results show that the simplified technique saved computational energy by 96.7% compared to the current one.

8. REFERENCES

[1] Neil Salkind, “Encyclopedia of measurement and statistics”, 2007.

[2] D.N.Joanes&C.A.Gill,”Comparing measures of sample skewness and kurtosis”, *Journal of the royal statistical society* (series D), Vol.47, No.1,page 183-189, March,1998.

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[4] The Mathworks, Inc., MATLAB, the Language of Technical Computing, Version 7.10.0.499 (R2010a), February 5, 2010.

[5] Email: karamfayed_1@hotmail.com