

An OFDM System Based on Dual Tree Complex Wavelet Transform (DT-CWT)

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ABSTRACT

In this paper, a novel orthogonal frequency division multiplexing (OFDM) system based on dual-tree complex wavelet transform (DT-CWT), is presented. In the proposed system, the DT-CWT is used to replace the fast Fourier transform (FFT) in the conventional OFDM. This results in considerable improvement in terms of bit error rate (BER) over not only the conventional OFDM but also to the wavelet packet modulation (WPM) based OFDM system. Moreover, the proposed system achieves better peak-to-average power ratio (PAPR) performance at acceptable increase in computational complexity. The complementary cumulative distribution function of PAPR for the proposed system shows ≈ 3 dB improvement over the conventional and the WPM based OFDM systems.

Keywords: OFDM, WPT, DT-CWT, FFT, Multicarrier Modulation.

1. INTRODUCTION

Traditionally, orthogonal frequency division multiplexing (OFDM) is implemented using fast Fourier transform (FFT). However, FFT has a major drawback arising from using rectangular window, which creates sidelobes. Moreover, the pulse shaping function used to modulate each subcarrier extends to infinity in the frequency domain. This leads to high interference and lower performance levels. Intercarrier interference (ICI) and intersymbol interference (ISI) can be avoided by adding a cyclic prefix (CP) to the head of OFDM symbol. But, this reduces the spectrum efficiency.

The wavelet packet modulation (WPM) system has a higher spectral efficiency and better robustness towards inter-channel interference than the conventional OFDM system. Moreover, WPM is capable of decomposing the time frequency plane in flexible way through filter bank (FB) constructions [1].

WPM system does not require CP, and according to the IEEE broadband wireless standard IEEE 802.16.3, avoiding CP gives wavelet OFDM an advantage of roughly 20% in bandwidth efficiency. Moreover, pilot tones are not needed with WPM system. This gives wavelet based OFDM system another 8% advantage over typical OFDM implementations [2].

However, the major problem with the common discrete wavelet packet transform (DWPT) is its lack of shift invariance; which means that on shifts of the input signal, the wavelet coefficients vary substantially. The signal information may not be stationary in the sub-bands so that the energy distribution across the sub-bands may change [3]. To overcome the problem of shift dependence, one possible approach is to simply omit the sub-sampling causing the shift dependence. Techniques that omit or partially omit sub-sampling are cycle spinning, oversampled FBs or undecimated wavelet transform (UWT). However, these transforms are redundant [4], which is not desirable in multicarrier modulation, as the complexity of redundant transform is high. In this paper we propose an OFDM system based on dual-tree complex wavelet transform (DT-CWT). This system is non-redundant achieves approximate shift invariance and is also inherits all the advantages of WPM system [5].

This paper is organized as follows: In section 2, we discuss the dual-tree complex wavelet transform (DT-CWT); in section 3, the multicarrier modulation systems (MCM) are discussed; the simulation results are presented in section 4; and we conclude this paper in section 5.

2. The Dual Tree Complex Wavelet Transform (DT-CWT)

Since early 1990s the WT and WPT have been receiving an increased attention in modern wireless communications [6]. A number of modulation schemes based on wavelets have been proposed [7], [8], - [15]. The DT-CWT was introduced by Kingsbury [16] – [20] as two real discrete WT (DWT). The upper part of the FB gives the real part of the transform while the lower one gives the imaginary part. This transform uses the pair of the filters ($h_0(n)$, $h_1(n)$ the low-pass/high-pass filter pair for the upper FB respectively) and ($g_0(n)$, $g_1(n)$ the low-pass/high-pass filter pair for the lower FB respectively) that are used to define the sequence of wavelet function

$\frac{\psi_h(n)}{s}$ and scaling function $\frac{\phi_h(n)}{s}$ as follows

$$\psi_h(t) = \sqrt{2} \sum_n h_1(n) \phi_h(2t - n) \tag{1}$$

$$\phi_h(t) = \sqrt{2} \sum_n h_0(n) \phi_h(2t - n) \tag{2}$$

where $h_1(n) = (-1)^n h_0(d - n)$. The wavelet function $\psi_g(t)$, the scaling function $\phi_g(t)$ and the high-pass filter for the imaginary part $g_1(n)$ are defined in similar way. The two real wavelets associated with each of the two real transform are $\psi_h(t)$ and $\psi_g(t)$. To satisfy the perfect reconstruction (PR) conditions, the filters are designed so that the complex wavelet $\psi(t) = \psi_h(t) + j\psi_g(t)$ is approximately analytic. Equivalently, they are designed so that $\psi_g(t)$ is approximately the Hilbert transform of $\psi_h(t)$.

$$\psi_g(t) = H\{\psi_h(t)\} \tag{3}$$

The analysis (decomposition or demodulation) and the synthesis (reconstruction or modulation) FBs used to implement the DT-CWT and their inverses are illustrated in fig. 1 and fig. 2 respectively. The inverse of DT-CWT is as simple as the forward transform. To invert the transform, the real and the imaginary parts are inverted.

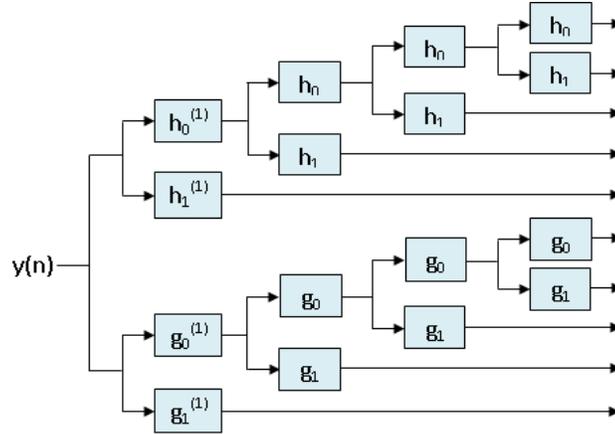


FIGURE 1: The dual tree discrete CWT (DT-DCWT) Analysis (demodulation) FB.

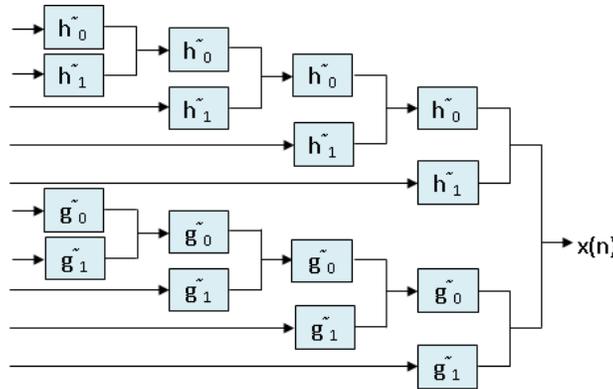


FIGURE 2: The Inverse dual tree discrete CWT (IDT-DCWT) Synthesis (modulation) FB.

The two low pass filters should satisfy the condition of one of them being approximately a half-sample shift of the other [21]

$$g_0(n) \approx h_0(n - 0.5) \Rightarrow \psi_g(t) \approx H[\psi_h(t)] \tag{4}$$

Since $g_0(n)$ and $h_0(n)$ are defined only on the integers, this statement is somewhat informal. However, we can make the statement rigorous using FT. In [4] it is shown that, if $G_0(e^{j\omega}) = e^{-j0.5\omega} H_0(e^{j\omega})$, then $\psi_g(t) = H[\psi_h(t)]$. The converse has been proven in [22] [23], making the condition necessary and sufficient. The necessary and sufficient conditions for biorthogonal case were proven in [24]. The half-sample delay condition given in terms of magnitude and phase function is given as

$$|G_0(e^{j\omega})| = |H_0(e^{j\omega})| \tag{5}$$

$$\angle(G_0(e^{j\omega})) = \angle H_0(e^{j\omega}) - 0.5\omega \tag{6}$$

In practical implementation of the DT-CWT, the delay condition (5) and (6) are approximately satisfied and the wavelets $\psi_h(t)$ and $\psi_g(t)$ are approximately Hilbert pairs, thus the complex wavelet $\psi_h(t) + j\psi_g(t)$ are approximately analytic. On the other hand, the FT is based on a complex valued oscillating cosine and sine components that form complete Hilbert transform

pairs; i.e., they are 90° out of phase of each other. Together they constitute an analytic signal, $e^{j\theta t}$, that is supported on one-half of the frequency axis ($\theta > 0$) [16].

3. DT-CWT Based OFDM

In the baseband equivalent conventional OFDM transmitter with m^{th} frame of N QAM or PSK symbols, a_k^m , $k = 0, 1, \dots, N - 1$, the OFDM frame is given by:

$$x^m[n] = \sum_{k=0}^{N-1} a_k^m e^{j2\pi nk/N} \tag{7}$$

for AWGN channel. The received signal is given as

$$y(n) = \alpha x(n) + w(n). \tag{8}$$

where α is the attenuation factor per block of data and $w(n)$ is the AWGN noise. At receiver side the transmitted data is recovered.

$$\hat{a}_k^m = \langle y(n), e^{-j2\pi nk/N} \rangle - \langle w(n), e^{-j2\pi nk/N} \rangle. \tag{9}$$

With WPM system [4], the transmitted signal $x[n]$ is constructed as the sum of M wavelet packet function $\phi_j[n]$ individually modulated with the QAM or PSK symbols.

$$x[n] = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} a_{i,j} \phi_j[n - iM] \tag{10}$$

And at receiver side, the transmitted data is recovered from the received signal $y(n)$.

$$\hat{a}_{i,j} = \langle y(n), \phi_j[n - iM] \rangle - \langle w(n), \phi_j[n - iM] \rangle. \tag{11}$$

Similar to the conventional OFDM and WPM systems, a functional block diagram of OFDM based on DT-CWT is shown in fig. (3). At the transmitter an inverse DT-CWT (IDT-CWT) block is used in place of inverse FFT (IFFT) block in conventional OFDM system or in place of the inverse DWPT (IDWPT) block in WPM system. At the receiver side a DT-CWT is used in place of FFT block in conventional OFDM system or in place of DWPT block in WPM system.

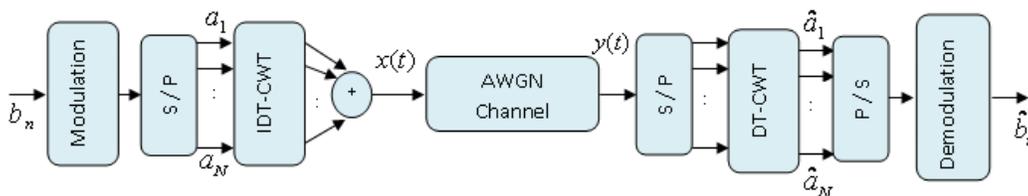


FIGURE 3: DT-CWT modulation (DT-CWTM) functional block diagram.

Data to be transmitted is typically in the form of a serial data stream. PSK or QAM modulations can be implemented in the proposed system and the choice depends on various factors, like the

bit rate and sensitivity to errors. The transmitter accepts modulated data (in this paper we use 16 and 64QAM). At transmitter, the data stream is first passed through a serial to parallel (S/P) converter, giving N lower bit rate data streams. The data is then modulated using an IDT-CWT matrix realized by an N-band synthesis FB.

IDT-CWT works in a similar fashion to an IFFT or IDWPT. It takes as the input QAM symbols and outputs them in parallel time-frequency “subcarriers”. In fig. (2) as the synthesis process, it can be shown that the transmitted signal, $x[n]$ can be written as follows:

Let $\varphi(n)$ be the scaling function, $\psi(n)$ be the wavelet function, and a_i is k^{th} symbol, $i = 1, 2, \dots, N$

$$x(n) = R_s [x(n)] + I_m [x(n)]. \tag{12}$$

$$R_s [x(n)] = a_{1,k} \varphi_{1,k}(n) + \sum_{j=2}^{\frac{N}{2}} a_{j,k} \psi_{j,k}(n). \tag{13}$$

$$I_m [x(n)] = a_{\frac{N}{2}+1,k} \varphi_{\frac{N}{2}+1,k}(n) + \sum_{j=\frac{N}{2}+2}^N a_{j,k} \psi_{j,k}(n). \tag{14}$$

The received signal can be written in the following form:

$$y(n) = a_{1,k} \hat{\varphi}_{1,k}(n) + \sum_{j=2}^{\frac{N}{2}} a_{j,k} \hat{\psi}_{j,k}(n) + \left\{ a_{\frac{N}{2}+1,k} \hat{\varphi}_{\frac{N}{2}+1,k}(n) + \sum_{j=\frac{N}{2}+2}^N a_{j,k} \hat{\psi}_{j,k}(n) \right\} + w(n). \tag{15}$$

where $\hat{\varphi}_{m,n}(n) = \alpha \varphi_{m,n}(n)$ and $\hat{\psi}_{m,n}(n) = \alpha \psi_{m,n}(n)$,

$$\langle y(n), [\varphi]_{1(1,k)}(n) \rangle = a_{1(k)} \langle [\varphi]_{1(1,k)}(n), [\varphi]_{1(1,k)}(n) \rangle + \sum_{j=2}^{\frac{N}{2}} a_{j(k)} \langle [\psi]_{j(1,k)}(n), [\varphi]_{1(1,k)}(n) \rangle + \sum_{j=\frac{N}{2}+2}^N a_{j(k)} \langle [\psi]_{j(1,k)}(n), [\varphi]_{1(1,k)}(n) \rangle + \langle w(n), [\varphi]_{1(1,k)}(n) \rangle$$

For perfect synchronization and orthogonality between subcarrier, to recover the transmitted data in each symbol we match the transmitted waveform with the carrier j according to the following formula:

$$\langle \varphi_{j,k}(n), \varphi_{m,n}(n) \rangle = \langle \psi_{j,k}(n), \psi_{m,n}(n) \rangle = \begin{cases} 1, & \text{if } j = m \text{ and } k = n \\ 0, & \text{otherwise} \end{cases}. \tag{17}$$

and

$$\langle \psi_{j,k}(n), \varphi_{m,n}(n) \rangle = 0 \tag{18}$$

Equation (17), (18) indicate that the wavelet function and scaling function are orthogonal to each other, thus we will be able to separate the subcarriers at receiver. Thus,

$$\hat{a}_{1,k} = \langle y(n), \varphi_{1,k}(n) \rangle - \langle w(n), \varphi_{1,k}(n) \rangle. \tag{19}$$

$$\hat{a}_{\frac{N}{2}+1,k} = \langle y(n), \varphi_{\frac{N}{2}+1,k}(n) \rangle - \langle w(n), \varphi_{\frac{N}{2}+1,k}(n) \rangle. \tag{20}$$

$$\hat{a}_{j,k}_{j=1-\frac{N}{2}} = \langle y(n), \psi_{j,k}(n) \rangle - \langle w(n), \psi_{j,k}(n) \rangle \tag{21}$$

$$\hat{a}_{j,k}^{j,k} = (y(n), \psi_{j,k}(n)) - (w(n), \psi_{j,k}(n))$$

4. Results and Analysis

In this section we show the results for BER, PAPR and PSD respectively. The simulation parameters are documented as follows: Modulation type is 16-QAM and 64 QAM; the number of subcarriers is 64, 128, 256, 512, and 1024 subcarriers; a wavelet packet base is Daubechies-1 (DAUB-1); maximum tree depth (D = 7); PAPR threshold is 2dB; shaping filter is Raised Cosine (rolloff factor $\alpha = 0.001$, upsampler = 4); DT-CWT using different filters (LeGall 5,3 tap filters (leg), Antonini 9,7 tap filters (anto), Near Symmetric 5,7 tap filters (n-sym-a), and Near Symmetric 13,19 tap filters (n-sym-b)) for the first stage of the FB and (Quarter Sample Shift Orthogonal 10,10 tap filters (q-sh-06) only 6,6 non-zero taps, Quarter Sample Shift Orthogonal 10,10 tap filters (q-sh-a) with 10,10 non-zero taps, unlike q-sh-06, Quarter Sample Shift Orthogonal 14,14 tap filters (q-sh-b), Quarter Sample Shift Orthogonal 16,16 tap filters (q-sh-c), and Quarter Sample Shift Orthogonal 18,18 tap filters (q-sh-d) for the succeeding stages of the FB.

In the first experiment, the BER performance of the DT-CWT-based OFDM system is tested and compared with the conventional OFDM and WPM systems. The Daubechies-1 wavelet packet bases are used to construct the wavelet packet trees in WPM system with maximum tree depth (D = 7). PAPR threshold is 2dB, shaping filter is Raised Cosine (rolloff factor $\alpha = 0.001$, upsampler = 4). 16 QAM and 64 QAM are used with 64 subcarriers. The results are shown in Figure (4).

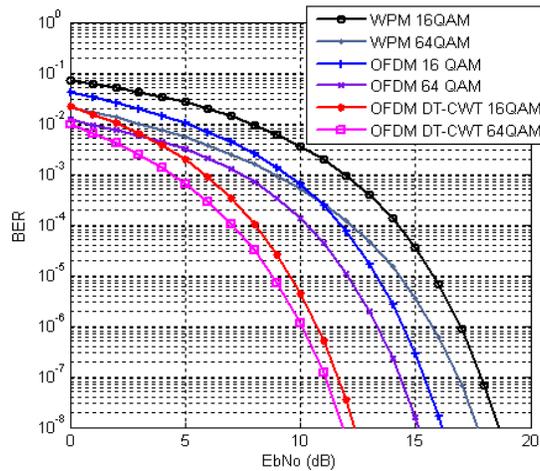


Figure 4: BER performance of OFDM based on DT-CWT using 16 QAM and 64 QAM.

It is quite clear from Fig. (4) that the proposed scheme is significantly outperforming the conventional OFDM and the WPM systems, in term of BER. It is also clear that the conventional OFDM is performing better than the WPM .

The above experiment is repeated for different set of filters. OFDM DT-CWT₁ represents the system when using near-symmetric (n-sym) 13,19 tap filters in the first stage of the FB and quarter sample shift orthogonal (q-sh) 14 tap filters in the succeeding stages. OFDM DT-CWT₂ represents the system when using (n-sym 13,19 with q-sh 10 (10 non zero taps) filters). OFDM DT-CWT₃ is the system when using (antonini (anto) 9,7 tap filters with q-sh0 10 (only 6 non zero taps) filters). OFDM DT-CWT₄ is the system when using (anto 9,7 with q-sh 14 filters). OFDM DT-CWT₅ is the system when using (n-sym 5,7 with q-sh 14 filters). OFDM DT-CWT₆ is the system

when using (LeGall (leg) 5,3 tap filters with q-sh 14 filters). OFDM DT-CWT₇ represents the system when using (n-sym 5,7 with q-sh 16 filters) and OFDM DT-CWT₈ is the system when using (leg 5,3 with q-sh 18 filters).

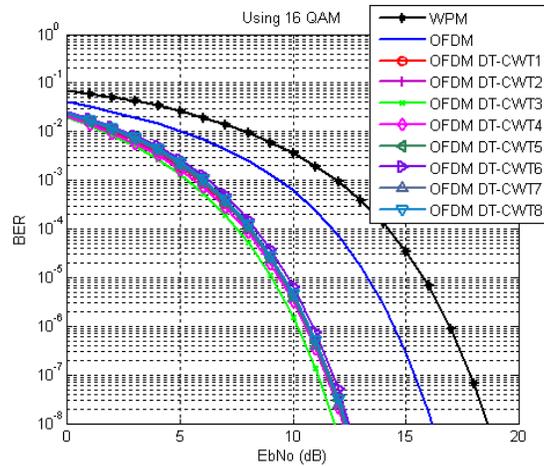


Figure 5: BER in 16QAM OFDM based on DT-CWT using different type of filters.

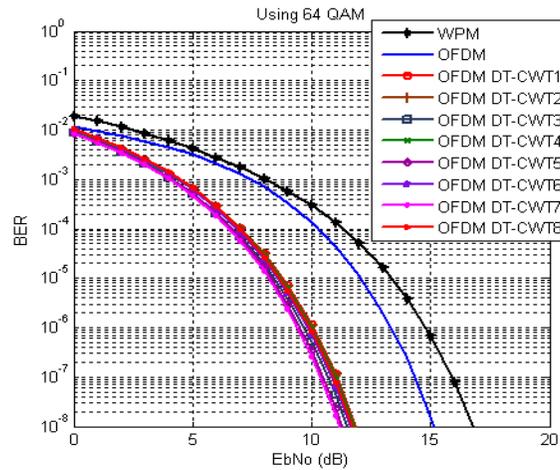


Figure 6: BER in 64QAM OFDM based on DT-CWT using different type of filters.

The results in fig. 5 (using 61 QAM) and fig. 6 (using 64 QAM) show that there is no degradation in the performance of the proposed system as a result of using different set of mismatching filters.

Next, the PAPR value of the proposed system is investigated and compared with the conventional OFDM and WPM systems in term of the complementary cumulative distribution function (CCDF).

In order to analyze PAPR, we generate the transmitted waveforms using 16 QAM modulation with 64 subcarriers and investigate the PAPR values for each of the three considered systems. Fig. 7 shows, the dynamic range of the PAPR signals of the three systems over a span of time. It is

quite clear that the proposed system is showing better performance than the conventional OFDM and WPM systems in terms of the range of variations of PAPR.

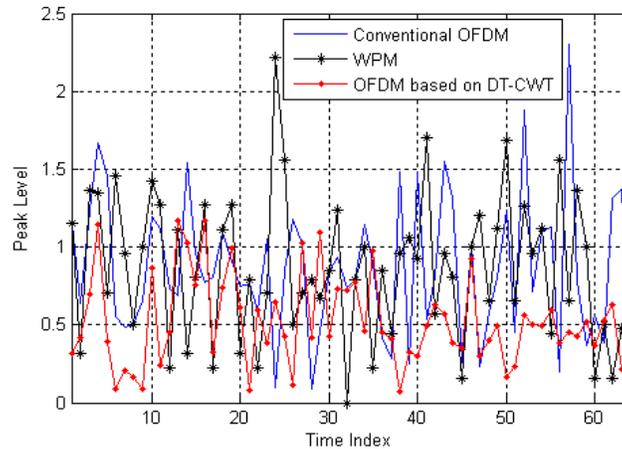


FIGURE 7: The Envelope of the Conventional OFDM, WPM and OFDM based on DT-CWT.

To quantify the PAPR values of the considered systems, the CCDF is obtained for each system. 16 QAM modulation scheme is used with 64 carriers. The results are shown in Figure 8. The figure shows that the DT-CWT based OFDM system achieves nearly 3 dB improvement over the conventional OFDM and WPM systems at 0.1% of CCDF.

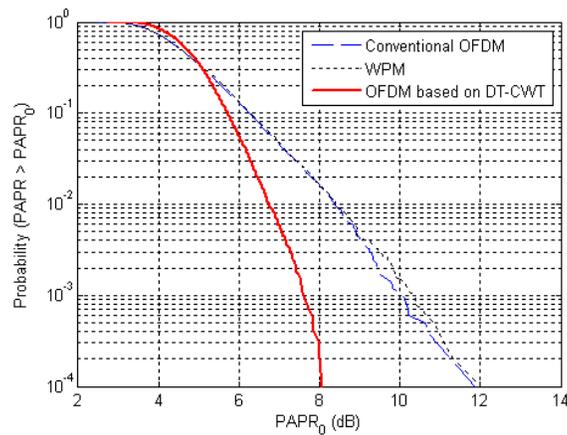


FIGURE 8: CCDF the Conventional OFDM, WPM and OFDM based on DT-CWT.

Next, the above experiment is repeated with the different set of filters as in the first experiment (16 QAM modulation scheme is used with 64 carriers). The results are shown in Figure 9.

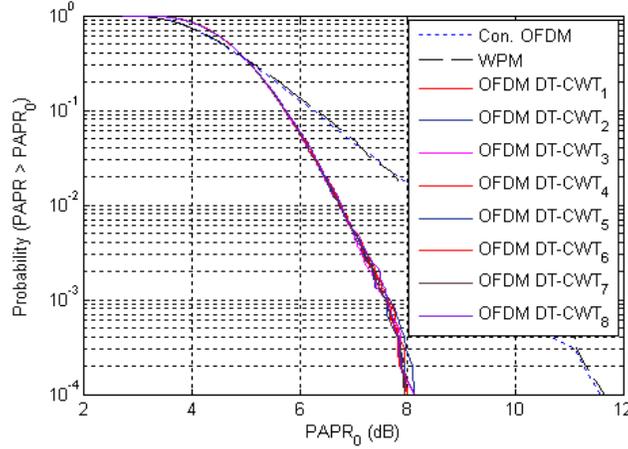


FIGURE 9: The effect of using different set of filters in design of the OFDM based on DT-CWT.

The results in fig. 9 show that there is no observed degradation in the performance of the DT-CWT based OFDM system as a result of using different set of mismatching filters.

In the third part of this experiment, the above experiments are repeated for conventional OFDM and OFDM based on (DT-CWT) systems using 16 QAM with different numbers of subcarriers (64, 128, 256, 512, and 1024). The results are shown in Figure 10 and Figure 11 respectively.

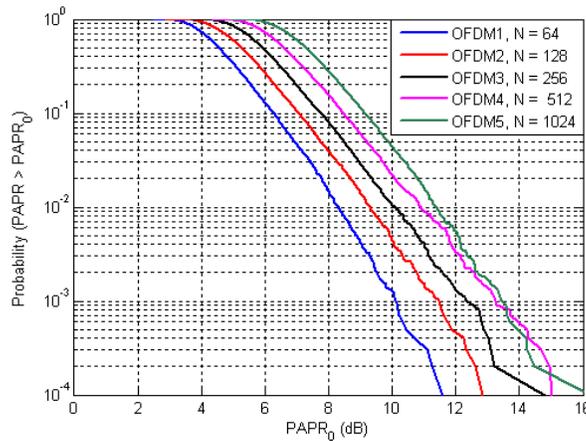


FIGURE 10: CCDF of PAPR for 16-QAM modulated conventional OFDM symbol with various values of subcarriers (N).

Theoretically, If $\mathbb{E}\{|\mathbf{x}[n]|^2\}$ is normalized to unity, then the CCDF of the PAPR is given by:

$$Pr[\lambda > \lambda_0] = 1 - \left(1 - e^{-\frac{\lambda_0}{N}}\right)^N \quad 23$$

Where N is the number of subcarriers.

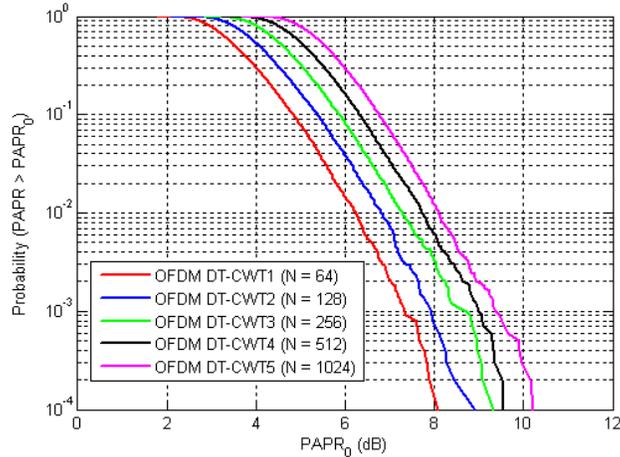


FIGURE 11: CCDF of PAPR for 16-QAM modulated OFDM based on DT-CWT symbol with various values of subcarriers (N).

It is quite clear from Fig. 10 and Fig. 11 that the PAPR increases with the number of the subcarriers (see equation 23), and this is shared property between the proposed and the conventional OFDM systems. But, as the figures indicate the proposed system is showing better robustness towards PAPR performance degradation with increased number of subcarriers, than the conventional OFDM.

In the third experiment we test the spectrum of the three systems in terms of their power spectrum density. The results are shown in Figure 12.

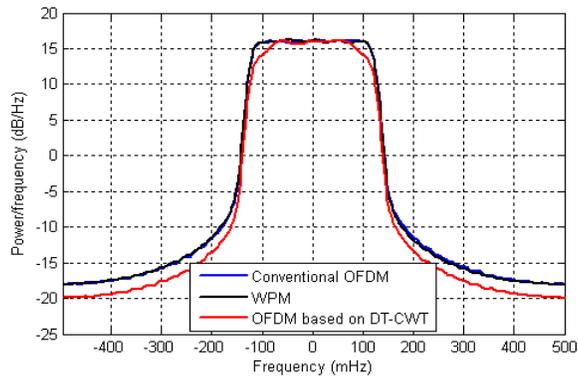


FIGURE 12: PSD of the Conventional OFDM, WPM and OFDM based on DT-CWT.

Figure 12 shows that, the proposed system is relatively showing better spectrum characteristics, in terms of more low out of band attenuation, than the conventional OFDM and the WPM systems.

5. Conclusion

In this paper a new OFDM system based on DT-CWT is proposed. The performance of the proposed OFDM system is compared with the conventional OFDM and the WPM, in terms of BER and PAPR. The results show better BER performance by the proposed system to the conventional OFDM and the WPT and better performance by the conventional OFDM to the WPT. of BER than WPM system. In term of PAPR, the proposed system is showing significantly better performance ($\approx 3\text{dB}$) to the conventional OFDM and WPT. The results also show that there is no observed BER and PAPR degradation as a result of using different set of mismatching filters with the proposed DT-CWT based system.

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