Investigating Multifractality of Solar Irradiance Data through Wavelet Based Multifractal Spectral Analysis

K. Mofazzal Hossain email: hossainkm_1976@yahoo.co.in
Assistant Professor,
Department of Electronics & Instrumentation Engineering
Dr.B.C.Roy Engineering College, Durgapur
Durgapur-713206,West Bengal, India

Dipendra N. Ghosh email: ghoshdipen2003@yahoo.co.in
Associate Professor, Department of Mathematics
Dr.B.C.Roy Engineering College, Durgapur
Durgapur-713206, West Bengal, India

Koushik Ghosh email: koushikg123@yahoo.co
Lecturer, Department of Mathematics
University Institute of Technology, University of Burdwan
Burdwan-713104, West Bengal, India

Abstract

It has been already revealed that the daily Solar Irradiance Data during the time period from October, 1984 to October, 2003 obtained by Earth Radiation Budget Satellite (ERBS) exhibits an Anti-persistent trend having multi-periodic phenomena. The solar irradiance time series data being a complex non linear signal in this paper we have tried to detect the irregularity and multifractality in the signal using continuous wavelet transform modulus maxima (WTMM) algorithm. Singularity spectrum of the signal has been obtained to measure the degree of multifractality of the Solar Irradiance signal. The qualitative measure of the degree of multifractality of the Solar Irradiance signal will help us to decide the nature of the signal processing tools that can be used to extract the features of the signal in our future work. This may also give an input to the research work of researchers on the solar physics and geophysics.

Keywords: ERBS, Wavelet transform, WTMM, scaling exponent, multifractal dimension, Hölder exponent, singularity spectrum

1. INTRODUCTION

Total solar irradiance describes the electromagnetic radiant energy emitted by the sun over all wavelengths that falls each second on 1 square meter outside the earth's atmosphere. Solar refers to electromagnetic radiation in the spectral range of approximately 1–9ft (0.3–3m), where the shortest wavelengths are in the ultraviolet region of the spectrum, the intermediate wavelengths in the visible region, and the longer wavelengths are in the near infrared. Total solar irradiance means that the solar flux has been integrated over all wavelengths to include the contributions from ultraviolet, visible, and infrared radiation. The solar irradiance had been monitored with absolute radiometers since November 1978, on board six spacecraft (Nimbus-7, SMM, UARS, ERBS, EURECA, and SOHO), outside the terrestrial atmosphere (Fröhlich and Lean, 1998). Before measuring it from space, this quantity was thought to be constant, because the precision of the ground-based instruments at that time was not high enough to detect such a small variation. It consequently got the name of "solar constant", which had a value of only 1,353 W/m^2, as a part of the solar radiation is absorbed by the Earth's atmosphere. But from the data sent by the mentioned spacecraft it reveals that the solar irradiance varies about a small fraction of 0.1% over solar cycle being higher during maximum solar activity conditions. [1]

It is suggested that the solar variability is due to the perturbed nature of the solar core and this variability is provided by the variability of the solar neutrino flux from the solar neutrino detectors i.e., Homestake, Superkamiokande, SAGE and GALLEX-GNO. A major part of the Solar Irradiance variation is explained as a combined effect of the sunspots blocking and the intensification due to bright faculae and plages, with a slight dominance of the bright features effect during the 11-year solar cycle maximum. Solar Irradiance variation within solar cycle is thought to be due to the changing emission of bright magnetic elements, including faculae and the magnetic network. [2]

It has been revealed that the variation of the solar irradiance is anti-persistent and shows multi-periodicity. [3] The periods of the solar irradiance variation detected are 9.08-9.35, 13.53-14.03, 27.50-28.17, 30.26, 35.99-36.37, 51.14-51.52, 68.27-68.60, 101.15, 124.85, 150.63-153.98, 659.90, 729.37, 1259.82, 3464.50 and 4619.33 days.[4]. In this paper we would like to characterize the complex behaviour of the solar irradiance fluctuation by i) tracing the existence of multifractality and ii) scanning the singularities of the time series signal. Here we have computed the signal parameters like scaling exponents τ (q), multifractal scaling exponents h(q) and generalized multifractal dimensions D(q) which quantifies the multifractality of the signal. For tracking the singularities in the time series signal we have computed the singularity strength or Hölder exponent (α) and obtained the Hausdorff dimension or singularity spectrum f (α). The use of monofractal methods to extract quantitative information from signals is well known. Monofractals are homogeneous objects, in the sense that they have the same scaling properties, characterized by a single singularity exponent. Generally, there exist many observational signals which do not present a simple monofractal scaling behaviour. The need for more than one scaling exponent can derive from the existence of a crossover timescale, which separates regimes with different scaling behaviours. Different scaling exponents could be required for different segments of the same time series, indicating a time variation of the scaling behaviour. Furthermore, different scaling exponents can be revealed for many interwoven fractal subsets of the time series; in this case the process is not a monofractal but multifractal. Thus, multifractals are intrinsically more complex and inhomogeneous than monofractals and characterize systems featured by very irregular dynamics, with sudden and intense bursts of high-frequency fluctuations. The simplest type of multifractal analysis is given by the standard partition function multifractal formalism, developed to characterize multifractality in stationary measures. This method does not correctly estimate the multifractal behaviour of signal affected by trends or non-stationarities. But the solar irradiance time series signal is non stationary in nature. To analyze non-stationary signal wavelet transform based tool are more suitable compared to the traditional Fourier based tools [5]. Hence to characterize the multifractality of non-stationary signals another multifractal method based on the wavelet analysis named as Wavelet Transform Modulus Maxima (WTMM) method is being used in this paper. [6, 7] This method involves tracing the maxima lines in the continuous wavelet transform over all scales. WTMM allows one to detect scaling by means of the maxima lines of the continuous wavelet transform on different scales.

2. THEORY

CONTINUOUS WAVELET TRANSFORM

The continuous wavelet transform (WT) is a mathematical technique introduced in signal analysis in the early 1980s. Since then, it has been the subject of considerable theoretical developments and practical applications in a wide variety of fields. The WT has been early recognized as a mathematical microscope that is well adapted to reveal the hierarchy that governs the spatial distribution of singularities of multifractal measures. The wavelet transform is a convolution product of the data sequence (a function f(x), where x, referred to as "position", is usually a time or space variable. In this study x is referred as time (t) and hence the data sequence is time series) with the scaled and translated version of the mother wavelet, ψ(x). The scaling and translation are performed by two parameters; the scale parameter s stretches (or compresses) the mother wavelet to the required resolution, while the translation parameter *b* shifts the analyzing wavelet to the desired location:

$$Wf(s,b) = \frac{1}{s}\int_{-\infty}^{\infty} f(x)\, \psi\left(\frac{x-b}{s}\right)dx, \qquad (1)$$

where *s*, *b* are real, *s > 0* for the continuous version (CWT). $Wf(s,b)$ are the wavelet transform coefficients .The wavelet transform acts as a microscope: it reveals more and more details while going towards smaller scales, i.e. towards smaller *s* values [8].

The mother wavelet ψ(x) is generally chosen to be well localized in space (or time) and frequency. Usually, ψ(x) is only required to be of zero mean, but for the particular purpose of multifractal analysis ψ(x) is also required to be orthogonal to some lower order polynomials, up to the degree n:

$$\int x^m \psi(x)dx = 0, \quad \forall m, \quad 0 \le m < n \qquad (2)$$

Thus, while filtering out the trends, the wavelet transform can reveal the local characteristics of a signal, and more precisely its singularities. The Hölder exponent can be understood as a global indicator of the local differentiability of a function.

By preserving both scale and location (time, space) information, the *CWT* is an excellent tool for mapping the changing properties of non-stationary signals. A class of commonly used real-valued analyzing wavelets, which satisfies the above condition (2), is given by the successive derivatives of the *Gaussian* function:

$$\psi^{(n)}(x) = \frac{d^n}{dx^n} e^{-x^2/2} \qquad (3)$$

Note that the WT of a signal $f(x)$ with $\psi^{(n)}(x)$ in Eq. (3) takes the following simple expression:

$$\begin{aligned} Wf\ (s,b) &= \frac{1}{s}\int_{-\infty}^{\infty} f(x)\, \psi^{(n)}\left(\frac{x-b}{s}\right)dx\ , \\ &= s^n \frac{d^n}{dx^n} Wf\ (s,b) \qquad (4) \end{aligned}$$

Equation (4) shows that the WT computed with $\psi^{(n)}(x)$ at scale s is nothing but the n^{th} derivative of the signal $f(x)$ smoothed by a dilated version $\psi^{(0)}(x/s)$ of the Gaussian function. This property is at the heart of various applications of the WT microscope as a very efficient multi-scale singularity tracking technique. Thus, the higher derivatives, the more vanishing moments, that is, the local polynomial trends of higher order would be eliminated. We choose the third derivative of a Gaussian

$$\psi^{(3)}(x) = \frac{d^3}{dx^3} e^{-x^2/2} \qquad (5)$$

which is insensitive to trends up to a quadratic one.

WAVELET TRANSFORM MODULUS MAXIMA (WTMM)

The WTMM method inherits the advantages of the wavelet transform analysis and was developed to deal with strongly non-stationary data. It has an important ability to reveal hierarchical structure of singularities and therefore proves useful in analyzing self-similar structures like fractals. In small-scale levels s of wavelet transform, sharp hidden transitions (singularities) in Solar Irradiance dynamics would be extracted.

The continuous wavelet transform described in Eq. (1) is an extremely redundant representation, too expensive for most practical applications. To characterize the singular behaviour of functions, it is sufficient to consider the values and position of the Wavelet Transform Modulus Maxima (*WTMM*). The wavelet modulus maxima is a point (s_0, x_0) on the scale-position (or time) plane, *(s, x)*, where *|Wf(s0, x)|* is locally maximum for *x* in the neighborhood of x_0. These maxima are disposed on connected curves in the scale position *(s, x)* (or scale-time) half-plane, called *maxima lines*. An important feature of these maxima lines, when analyzing singular functions, is that there is at least one maxima line pointing towards each singularity The *WTMM* representation has been used for defining the partition function based multifractal formalism.

Let {*un(s)*}, where *n* is an integer, be the position (time) of all local maxima at a fixed scale *s*. By summing up the *q*'s power of all these *WTMM*, we obtain the partition function *Z*:[9]

$$Z(q,s) = \sum_n |Wf(s,u_n)|^q \qquad (6)$$

where q can be any real value except zero.

TRACING SINGULARITIES

The rapid changes in a time series f(x) are called singularities and a characterization of their strength is obtained with the Hölder exponents. The strength of the singularity of a function $f(x)$ at point x_0 is given by the Hölder exponent α, *i.e.*, the largest exponent such that $f(x)$ is Lipchitz at x_0 .There exists a polynomial $P_n(x-x_0)$ of order n and a constant C, so that for any point x in a neighborhood of x_0, one has:

$$|f(x) - P_n(x-x_0)| \le C|x-x_0|^\alpha \qquad (7)$$

where is $n \le \alpha(x_0)$ and $C > 0$.

The Hölder exponent measures the degree of irregularity of $f(x)$ at the point x_0.When a broad range of exponents is found, signals are considered as multifractal. A narrow range implies monofractality. Let us assume that according to Eq.(7), $f(x)$ has, at the point x_0, a local scaling (Hölder) exponent $\alpha(x_0)$; then, assuming that the singularity is not oscillating, one can easily prove that the local behaviour of $f(x)$ is mirrored by the WT which locally behaves as per the power law:

$$Wf(s,x_0) \sim s^{\alpha(x_0)}, \qquad (8)$$

Taking the log-log plot on both sides of the Eq. (8) Hölder exponent α can be estimated. A very important point (at least for practical purpose) rose by Mallat and Hwang is that the local scaling exponent $\alpha(x_0)$ can be equally estimated by looking at the value of the WT modulus along a maxima line converging towards the point x_0. Indeed one can prove that Eqs. (8) still holds when following a maxima line from large down to small scales. Depending on the value of $\alpha(x_0)$ at every x_0 we can scan the points of irregularity (opposite of regularity) or singularity.

If $\alpha(x_0)$ is	Regularity of $f(x)$ at x_0	Singularity of $f(x)$ at x_0
Higher	More	Less
Lower	Less	More

MULTIFRACTAL ANALYSIS

A natural way of performing a multifractal analysis of a function lies in generalizing the multifractal formalism using wavelets. From the deep analogy that links the multifractal formalism to thermodynamics [10], one can define the scaling exponent $\tau(q)$ from the power-law behavior of the partition function as given in Eqs (6):

$$Z(q,s) \sim s^{\tau(q)} \qquad (9)$$

Here we have varied the value of q from -20 to 20 with an increment of 0.2.Taking the log of the Eq.(9), $\tau(q)$ is being estimated for each value of q. The singularity spectrum $f(\alpha)$ is related to $\tau(q)$ by Legendre Transform as follows: a) from the plot of $\tau(q)$ vs. q the Hölder exponents α as a function of q can be determined from the relationship:

$$\alpha(q) = \frac{d\tau(q)}{dq} \qquad (10)$$

b) Singularity spectrum $f(\alpha)$ is calculated from the equation

$$f(\alpha) = q\alpha - \tau(q) \qquad (11)$$

From the properties of the Legendre transform, it is easy to see that *homogeneous* mono-fractal functions that involve singularities of unique Hölder exponent $\alpha(q)$ are characterized by a $\tau(q)$ spectrum which is a *linear* function of q. On the contrary, a *nonlinear* $\tau(q)$ curve is the signature of non-homogeneous functions that exhibit *multifractal* properties, in the sense that the Hölder exponent $\alpha(q)$ is a fluctuating quantity. The singularity spectrum $f(\alpha)$ of a multifractal function displays a single humped shape that characterizes intermittent fluctuations corresponding to Hölder exponent values spanning a whole interval $[\alpha_{\min}, \alpha_{\max}]$, where $\alpha_{\min}$ and $\alpha_{\max}$ are the Hölder exponents of the strongest and weakest singularities respectively.

Other than the signal parameters like scaling exponent $\tau(q)$, Hölder exponents $\alpha(q)$ and singularity spectrum $f(\alpha)$ as described above, multifractality can also be detected from the *multifractal scaling exponent or generalized Hurst exponent* $h(q)$ and the *generalized multifractal dimension* $D(q)$. Both $h(q)$ and $D(q)$ can be calculated from the scaling exponent $\tau(q)$ as below:

$$h(q) = \frac{1+\tau(q)}{q} \quad , q \neq 0 \qquad (12)$$

and

$$D(q) = \frac{\tau(q)}{q-1} = \frac{qh(q)-1}{q-1} \quad , q \neq 1 \qquad (13)$$

For monofractal time series $h(q)$ is independent of q whereas $D(q)$ depends on q. But for multifractal time series there is significant dependence of $h(q)$ on q. If q is positive, large fluctuations are characterized by a smaller values of $h(q)$) for multifractal time series. And, for negative q values, small fluctuations are usually characterized by larger values of $h(q)$.

From Eq.10, 11 and 12 Hölder exponent $\alpha(q)$ and Singularity spectrum $f(\alpha)$ can also be expressed in terms of the multifractal scaling exponent $h(q)$ as follows:

$$\alpha = h(q) + q\frac{dh(q)}{dq} \qquad (14)$$

and

$$f(\alpha) = q[\alpha - h(q)] + 1 \qquad (15)$$

Here we like to mention that multifractal scaling exponent or generalized Hurst exponent $h(q)$ is related to Hurst exponent H by the equation

$$H = h(q = 2) - 1 \qquad (16)$$

3. RESULTS

Fig.1 represents the original signal of the daily Solar Irradiance from October, 1984 to October, 2003 obtained by ERBS after simple exponential smoothing which is being denoised using DWT thresholding and the denoised signal is obtained as in fig.2.[3].

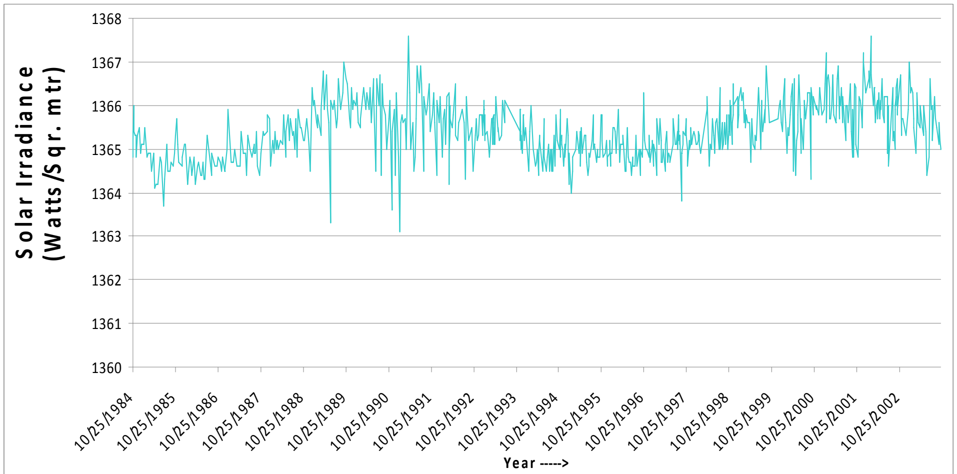

Fig.1:Daily TSI data from October 1984 to October 2003

Fig.2:Signal obtained after DWT Threshold Denoising

CWT, $Wf(s,b)$ of this data is being taken. The absolute values of the coefficients i.e. $|Wf(s,b)|$ is plotted with color coding, independently at each scale s, using 128 colors from deep brown ($|Wf(s,b)|=0$) to white ($\max|Wf(s,b)|$) as shown in fig.3. Scale and time are on the vertical and horizontal axis, respectively. The plot was obtained by using the "Wavelet toolbox" of Matlab software.

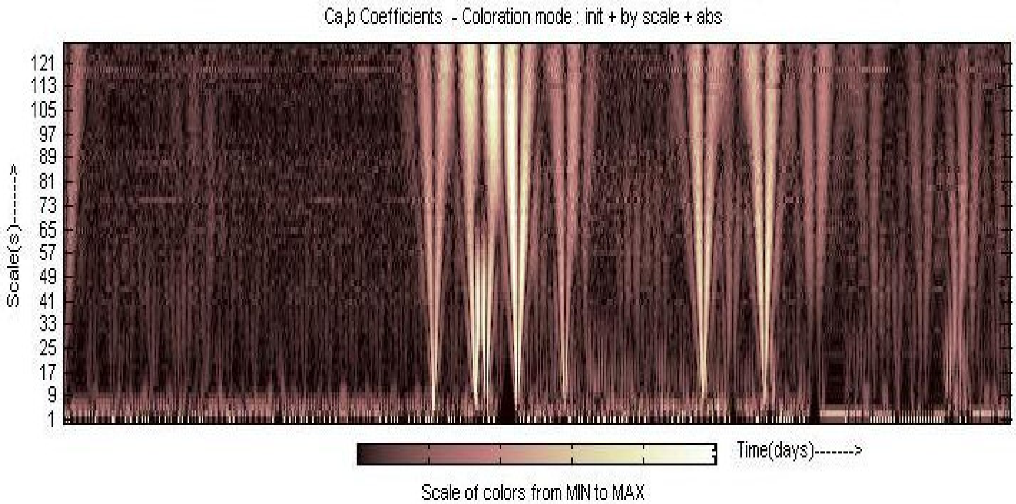

Fig.3: Colour coded CWT coefficient plot (Scale(s) Vs. Time)

Fig.4 represents the WT skeleton defined by the set of all maxima lines.

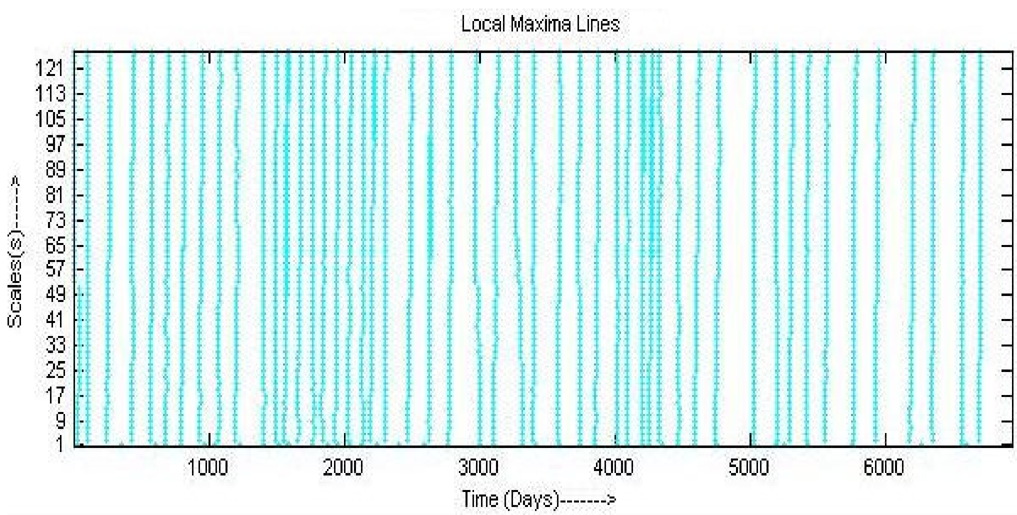

Fig.4: WTMM Skeleton

The plot of $\tau(q)$ vs q for the scale, s=3, 65,127 are being shown in fig.5

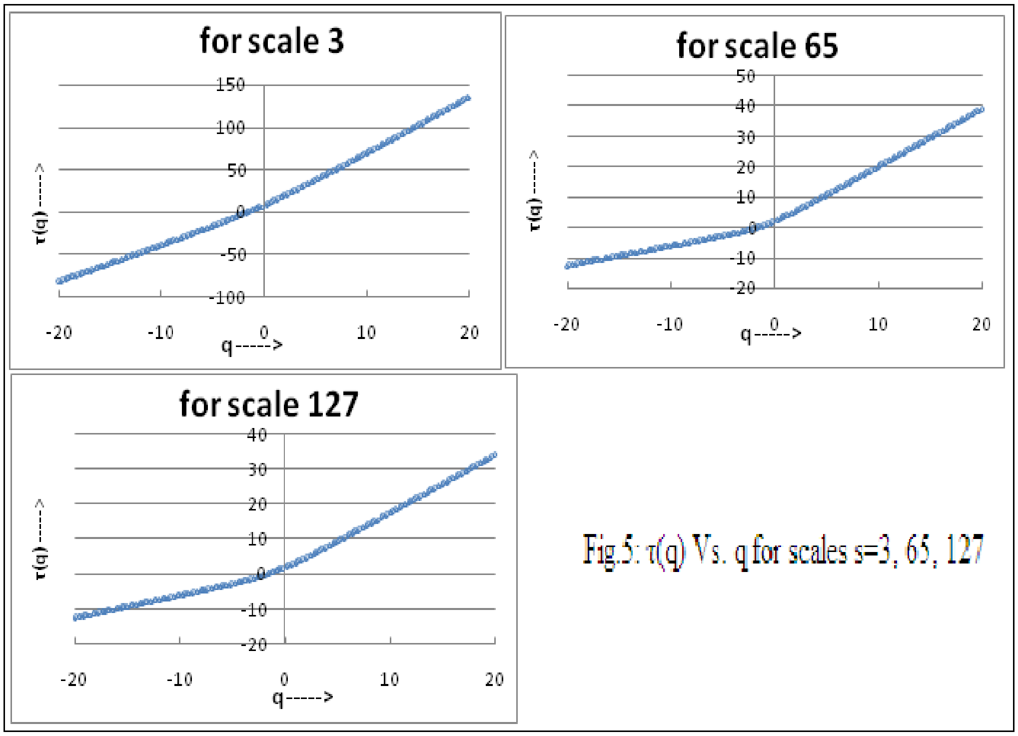

Fig.5: τ(q) Vs. q for scales s=3, 65, 127

The singularity spectrum i.e $f(\alpha)\ vs.\ \alpha$ for the scales s=3, 65, 127 is represented in fig.6.

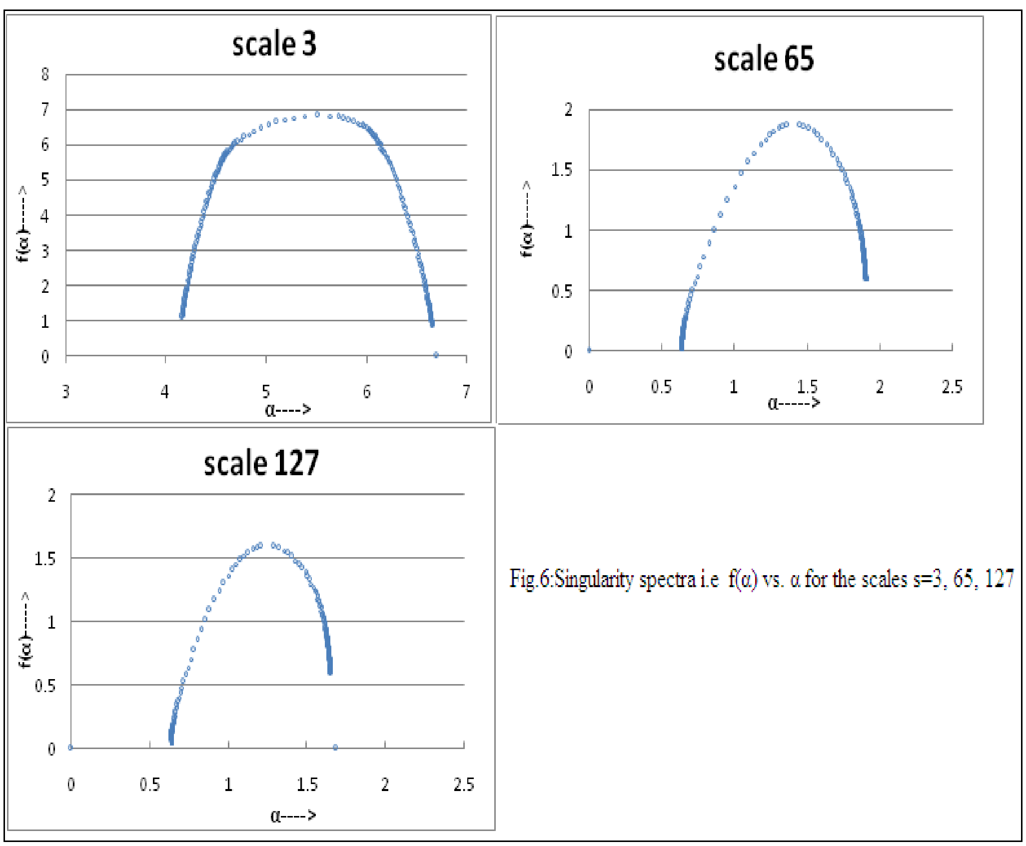

Fig.6:Singularity spectra i.e f(α) vs. α for the scales s=3, 65, 127

Fig.7 represents the D(q) vs.q curve for the scales s=3,65 and 127 as shown below.

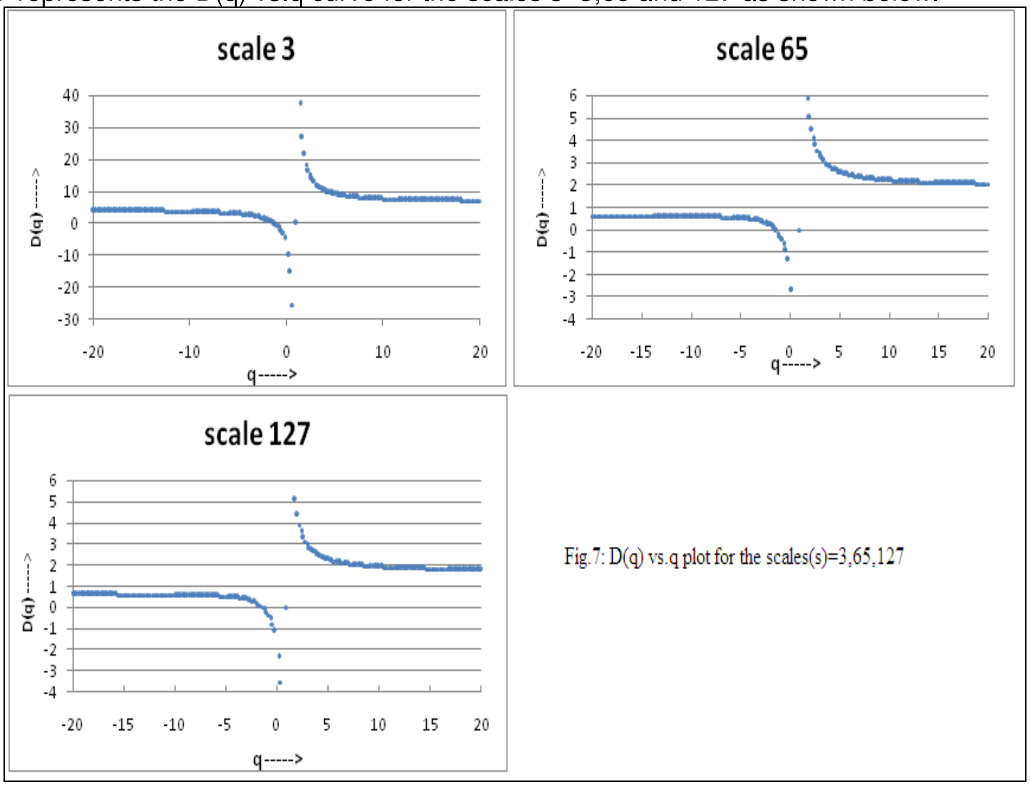

Fig.7: D(q) vs.q plot for the scales(s)=3,65,127

Fig.8 represents the h(q) vs.q curve for the scales s=3,65 and 127 as shown below.

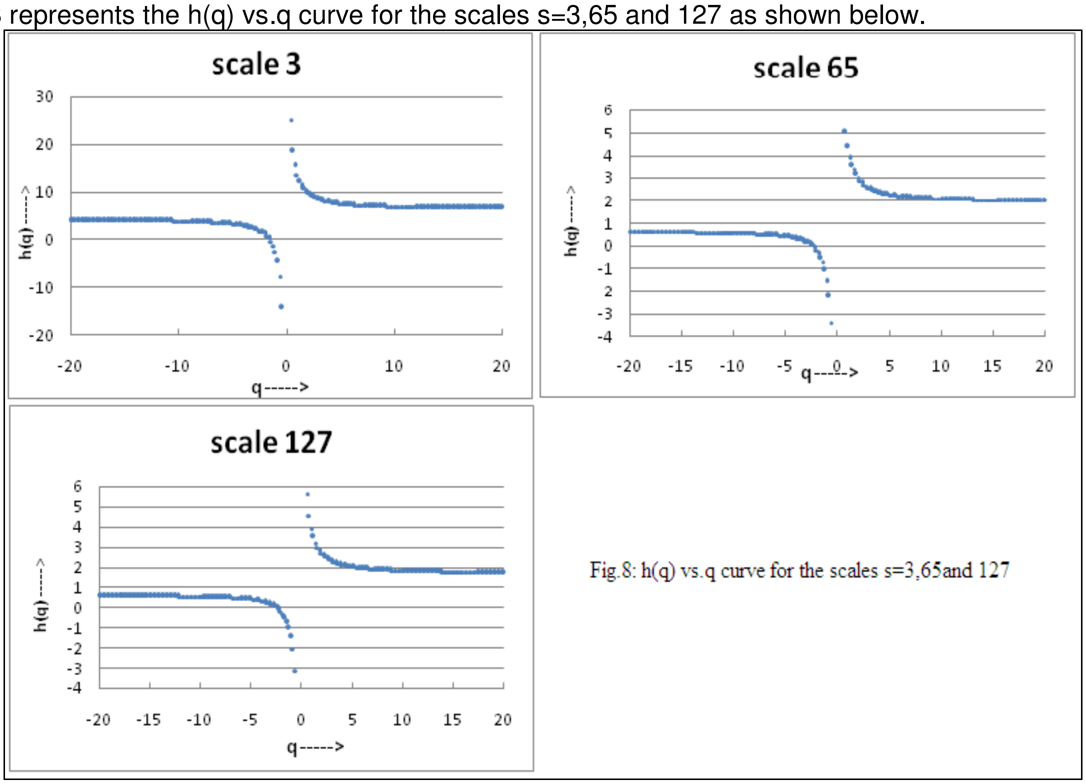

Fig.8: h(q) vs.q curve for the scales s=3,65and 127

The plot of D (h) vs. h for the scale, s=3, 65,127 are being shown in fig.9.

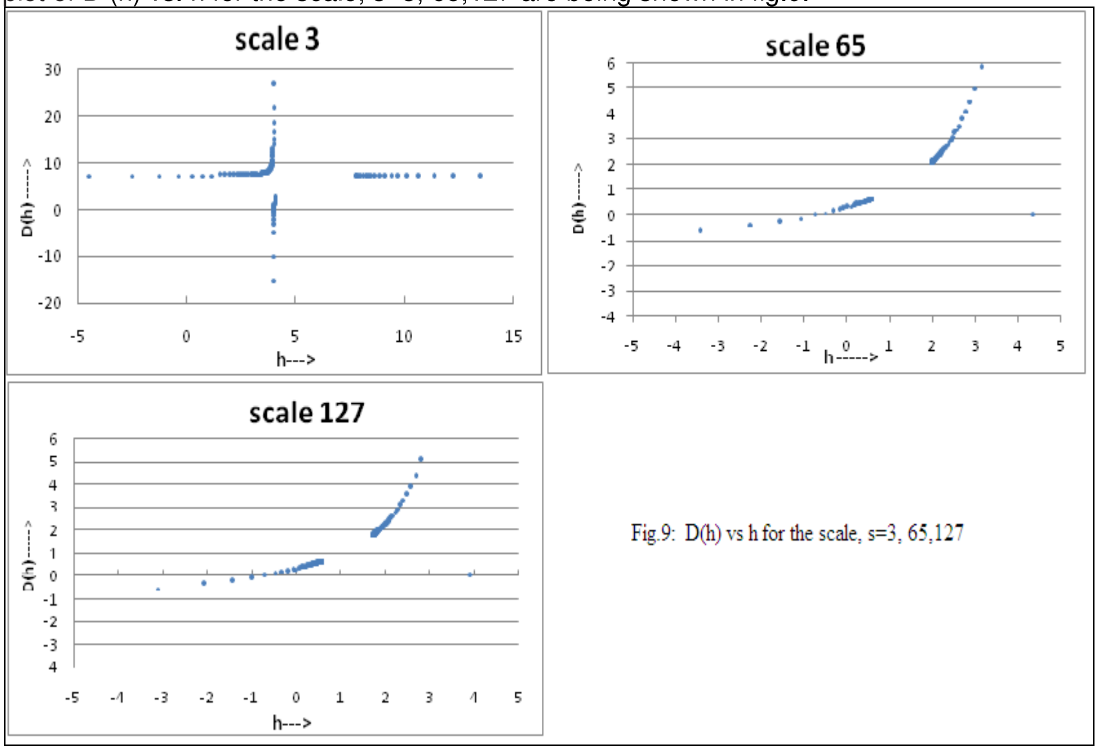

Fig.9: D(h) vs h for the scale, s=3, 65,127

Fig.10: $\alpha(q)$ vs.q curve for the scales s=3,65 and 127 as shown below

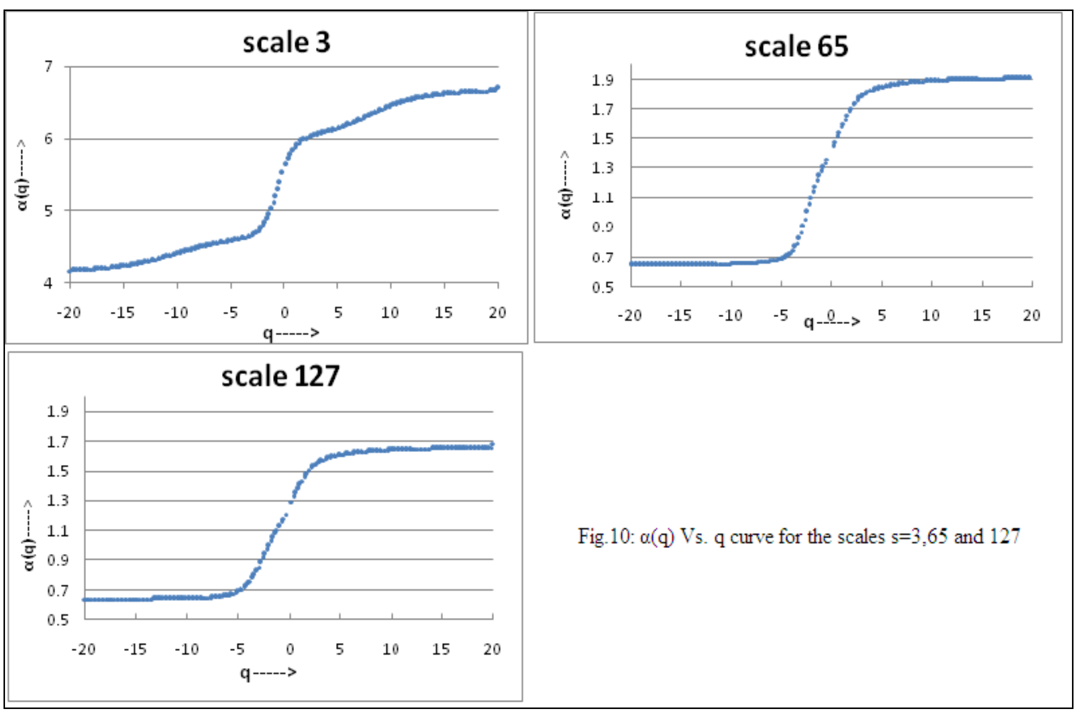

Fig.10: α(q) Vs. q curve for the scales s=3,65 and 127

4. CONCLUSIONS

WTMM method allows us to determine the multifractal characterization of the nonstationary solar irradiance time series. The concept of WTMM of the solar irradiance time series is used here to have a deeper insight into the process occurring in nonstationary dynamical system such as multi-periodic fluctuation in solar irradiance values. The dependency of the $\tau(q)$ and $h(q)$ on q as observed in fig.5 and fig.8, indicates that the solar irradiance variation has multifractal behavior. This behavior of exhibiting multifractal characteristics can be more established from the singularity spectrum as in fig.6. The multifractal analysis gives information about the relative importance of various fractal exponents present in the series. In particular, the width of the singularity spectrum indicates the range of present exponents. To get the quantitative characterization of multifractal spectra, the singularity spectrum is fitted to a quadratic function around the position of its maximum at α_0, i.e. $f(a) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C$. The coefficients can be obtained by an ordinary least-squares procedure. [11]In this fitting the additive constant $C = f(\alpha_0)$. With low α_0, the process becomes correlated; for example if the process had the tendency to move upward in the past, it will move upward with a probability larger than 1/2 in the next time step. Roughly speaking, a small value of α_0 means that the underlying process is more regular in appearance. From the fig.6 we observe that the value of α_0 is very high for lower scales and decreases with increase in the scale. It means that the signal is correlated at higher scales.

To obtain an estimate of the range of possible fractal exponents, we measured the width of the singularity spectrum, extrapolating the fitted curve to zero. The width of the spectrum was then defined as $W = \alpha_{\max} - \alpha_{\min}$ with $f(\alpha_{\max}) = f(\alpha_{\min}) = 0$. The width of the spectrum W is a measure of how wide the range of fractal exponents found in the signal and thus it measures the degree of multifractality of the series. The wider the range of possible fractal exponents, the `richer' is the process in structure. From the fig.6 we observe that W is decreasing with increase in the scale size i.e. solar irradiance signal is richer in structure at lower scales.

Finally, parameter *B* serves as an asymmetry parameter, which is zero for symmetric shapes, positive or negative for a left- or right-skewed (centered) shape, respectively. *B* captures the dominance of low- or high-fractal exponents with respect to the other. A right-skewed spectrum indicates relatively strongly weighted low-fractal exponents, and for left-skewed spectrum indicates relatively strongly weighted high-fractal exponents. From fig.6 we observe that for scale 65 and 127 the singularity spectrum is left skewed whereas for scale 3 the singularity spectrum is more or less symmetrical. Hence we can say that with increasing scales the signal is found to have high fractal exponents. The parameter scale(s) in the wavelet analysis also has a significant role. The high scales correspond to a non-detailed global view (of the signal), whereas the low scales correspond to a detailed view. Similarly, in terms of frequency, low frequencies (high scales) correspond to a global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal (that usually lasts a relatively short time).So the above discussion regarding the values of α_0, W, B at various scales give a measure of the detailed or non-detailed global view of the signal.

5. REFERENCES

1. A.Oncica, M. D.Popescu, M. Mierla, G. Maris, "*Solar Variability: From Core to Outer Frontiers*" In Proceedings of the 10th European Solar Physics Meeting Proc. ESA, SP-506, Prague, 193, 2002.
2. P. Raychaudhuri, "*Total Solar Irradiance Variability and the Solar Activity Cycle",* http://arxiv.org/ftp/astro-ph/papers/0601/0601335.pdf.
3. K. M. Hossain, D. N. Ghosh and K. Ghosh, "*Scaling Analysis by FVSM and DWT denoising of the measured values of solar irradiance*", accepted for publication in International Journal of Information and Computing Science(IJICS), Kolkata, Dec.2009
4. K. M. Hossain, D. N. Ghosh and K. Ghosh," *Search for Periodicities in Solar Irradiance data from ERBS* ", communicated to Solar Physics, US, June 2009

5. Othman O. Khalifa, Sering Habib Harding & Aisha-Hassan A. Hashim, "*Compression Using Wavelet Transform*", Signal Processing: An International Journal(SPIJ), Volume (2) : Issue (5),17-26,Malaysia,September/October,2008
6. J.F. Muzy, E. Bacry, A. Arneodo,"*Wavelets and multifractal formalism for singular signals: Application to turbulence data*", Physical Review. Lett. 67, 3515-3518, 1991.
7. A. Arneodo, E. Bacry, J.F. Muzy," *The thermodynamics of fractals revisited with wavelets"* Physica A 213, 232-275 ,1995
8. Mohamed Nerma, Nidal Kamel, and Varun Jeoti," *An OFDM System Based on Dual Tree Complex Wavelet Transform (DT-CWT)",* Signal Processing: An International Journal (SPIJ),Volume (3) : Issue (2),14-26,Malaysia, March/April,2009
9. Bogdan ENESCU, Kiyoshi ITO, and Zbigniew R. STRUZIK, "*Wavelet-Based Multifractal Analysis of real and simulated time series of earthquakes*", Annuals of Disas. Prev. Res. Inst., Kyoto Univ., No. 47 B,2004
10. H.Eugene Stanley, Paul Meakin,"*Multifractal Phenomena in Physics and Chemistry*", Nature Vol.335, page 405-409,1988
11. Luciano Telesca, Vincenzo Lapenna and Maria Macchiato, "*Multifractal fluctuations in earthquake-related geo-electrical signals*", New J. Phys. 7, 214,2005