A Subspace Method for Blind Channel Estimation in CP-free OFDM Systems

Xiaodong Yue yue@ucmo.edu
Department of Mathematics and Computer Science
University of Central Missouri
Warrensburg, MO 64093, USA

Xuefu Zhou xuefu.zhou@uc.edu
School of Electronics and Computing Systems
University of Cincinnati
Cincinnati, OH 45221, USA

Abstract

In this paper, a subspace method is proposed for blind channel estimation in orthogonal frequency-division multiplexing (OFDM) systems over time-dispersive channel. The proposed method does not require a cyclic prefix (CP) and thus leading to higher spectral efficiency. By exploiting the block Toeplitz structure of the channel matrix, the proposed blind estimation method performs satisfactorily with very few received OFDM blocks. Numerical simulations demonstrate the superior performance of the proposed algorithm over methods reported earlier in the literature.

Keywords: OFDM, Channel Estimation, Dispersive Channel, Wireless Communications.

1. INTRODUCTION

Due to its high spectral efficiency, robustness to frequency selective fading as well as the low cost of tranceiver implementations, orthogonal frequency-division multiplexing (OFDM) has been receiving considerable interest as a promising candidate for high speed transmission over wired and wireless channels [1][2].

Several channel identification methods for the OFDM systems with or without CP have been proposed in [3]-[13]. In [3][4], blind subspace methods were proposed for OFDM systems with CP. In practical OFDM systems operating over a dispersive channel, a cyclic prefix longer than the anticipated multipath channel spread is usually inserted in the transmitted sequence. And, in the IEEE 802.11a standard, CP's length is 25% of an OFDM symbol duration, indicating a significant loss in bandwidth efficiency. In order to improve the spectral efficiency, In [5]-[13], channel identification methods were introduced for OFDM systems without CP. Specifically, blind subspace based methods were introduced in [10]-[13]. The methods in [10][11] ignored the inter-block interference (IBI) in the received OFDM blocks by assuming that the number of sub-carriers is much larger than the channel length. In [12][13], IBI was not discarded. Furthermore, a block subspace method [13] was proposed to increase equivalent sample vectors which resulted in better performance than those of [10]-[12]. Unlike the method in [13], we propose a new blind channel estimation method without using sub-vectors of the received OFDM blocks. As a result, the rank of the noise subspace in our proposed method is larger than that of the block subspace method which results in lower channel estimation error. Computer simulations are presented to verify the effectiveness of the proposed method.

2. SYSTEM MODELS

Consider an OFDM system with N sub-carriers. The information symbols s(n) are transmitted through a linear time invariant baseband equivalent channel h(t). Assume h(t) has finite support

[0, (L-1)T] where T is the information symbol interval. Denote u(n) and $\bar{r}(n)$ to be the nth transmitted and received symbol respectively. The symbol stream s(n) is first serial to parallel converted into a vector $\mathbf{s}(i) = [s(iN), \cdots, s(iN+N-1)]^T$, where i is the block index. After the IDFT transform, we obtain $\mathbf{u}(i) = [u(iN), \cdots, u(iN+N-1)]^T$ which is then parallel to serial converted into u(n) and transmitted. The discrete-time received sequence is then given by

$$\bar{r}(n) = \sum_{l=0}^{L-1} h(l)u(n-l) + \bar{v}(n) \tag{1}$$

Where $h(\cdot)$ represents the corresponding discrete-time baseband multipath channel, and $\bar{\mathbf{v}}(n)$ is the additive white Gaussian noise. We can write $\mathbf{u}(i) = \mathbf{W}_N \mathbf{s}(i)$, where $\mathbf{W}_N$ is the $N \times N$ dimensional IDFT matrix with its (m,n)th element as $\frac{1}{\sqrt{N}} e^{j2\pi(m-1)(n-1)/N}$, and $\mathbf{s}(i)$ is the ith transmitted symbol block. The ith received signal block is grouped as $\bar{\mathbf{r}}(i) = [\bar{r}(iN), \cdots, \bar{r}(iN+N-1)]^T$. Using (1), we obtain $\bar{\mathbf{r}}(i) = \mathbf{H}_0\mathbf{u}(i) + \mathbf{H}_1\mathbf{u}(i-1) + \bar{\mathbf{v}}(i)$, where $\bar{\mathbf{v}}(i) = [\bar{v}(iN), \cdots, \bar{v}(iN+N-1)]^T$ is the corresponding noise vector, and

$$\mathbf{H}_0 = \begin{bmatrix} h(0) & & 0 & \cdots & 0 \\ \vdots & \ddots & & & \\ h(L-1) & \cdots & h(0) & & \\ 0 & & & & \\ \vdots & \ddots & & \cdots & \\ 0 & & h(L-1) & \cdots & h(0) \end{bmatrix}, \mathbf{H}_1 = \begin{bmatrix} 0 & \cdots & 0 & h(L-1) & \cdots & h(1) \\ & & & & \ddots & \vdots \\ & & & & & h(L-1) \\ \vdots & & & & & 0 \\ & & & & & \vdots \\ 0 & & & & & 0 \end{bmatrix}$$

By combining $\mathbf{H}_0$ and $\mathbf{H}_1$ and the corresponding $\mathbf{u}(i-1)$ and $\mathbf{u}(i)$ into a larger matrix/vector, we obtain the following signal model

$$\bar{\mathbf{r}}(i) = \bar{\mathbf{H}}\bar{\mathbf{u}}(i) + \mathbf{v}(i) \tag{2}$$

Where $\bar{\mathbf{H}} = \begin{bmatrix} h(L-1) & & \cdots & h(0) & & 0 \\ & \ddots & & & \ddots & \\ 0 & & h(L-1) & & \cdots & h(0) \end{bmatrix}$, $\bar{\mathbf{u}}(i) \triangleq \begin{bmatrix} \hat{\mathbf{u}}(i-1) \\ \mathbf{u}(i) \end{bmatrix}$, $\hat{\mathbf{u}}(i-1)$ is the inter-block interference and is composed by the last L-1 elements of $\mathbf{u}(i-1)$, $\bar{\mathbf{r}}(i) = [\bar{r}(iN), \cdots, \bar{r}(iN+N-1)]^T$. Since $\bar{\mathbf{H}}$ is not of full column rank, the signal model in (2) cannot satisfy the identifiability condition (full column rank) which is well known in the blind channel identification literature [14]. Fortunately, this problem can be easily solved by adopting multi-antenna in the receive end [9]-[13]. Assuming the number of receive antenna is M, (2) can be rewritten as

$$\mathbf{r}(i) = \mathbf{H}\bar{\mathbf{u}}(i) + \mathbf{v}(i) \tag{3}$$

Where $\mathbf{r}(i) \triangleq [\bar{r}^1(iN), \cdots, \bar{r}^M(iN), \cdots, \bar{r}^1(iN+N-1), \cdots, \bar{r}^M(iN+N-1)]^T$, $\bar{r}^j(\cdot)$ is the received signal from the jth receive antenna.

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(L-1) & & \cdots & \mathbf{h}(0) & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \mathbf{h}(L-1) & & \cdots & \mathbf{h}(0) \end{bmatrix}$$

$\mathbf{h}(l) \triangleq [h^1(l), \cdots, h^M(l)]^T$, $0 \le l \le L-1$, and $i = 1, \cdots, N_b$ (N_b denotes the number of received OFDM blocks). $h^j(l)$ is the channel associated with the jth receive antenna, $\bar{\mathbf{u}}(i) = [u(iN-L+1), u(iN-L+2), \cdots, u(iN+N-1)]^T$ and

$\mathbf{v}(i) = [\bar{v}^1(iN), \cdots, \bar{v}^M(iN), \cdots, \bar{v}^1(iN+N-1), \cdots, \bar{v}^M(iN+N-1)]^T$ is the corresponding noise vector.

3. SUBSPACE METHOD

Since sub-vectors of $\mathbf{r}(i)$ with length GM were used in the block subspace method [13], the rank of its noise subspace is MG-G-L+1, where $G \leq N$ is the sub-block size. It is well known that the higher the rank of the noise subspace, the lower the channel estimation error is in the least square minimization problem. If G is chosen to be the same or very close as N in the block subspace method to increase the rank of its noise subspace, the number of received signal vectors used to obtain the noise subspace is significantly reduced which offsets the gain from the increasing rank of the noise subspace. In fact, when G=N, the block subspace method [13] degenerates to the subspace method in [12] and there are only N_b received OFDM blocks can be used to calculate the noise subspace. Motivated by this fact, we propose a subspace method which has large rank of noise subspace and more sample vectors at the same time.

Reconstruct the received signal vector $\mathbf{r}(i)$ and define $\mathbf{y}(i,j) = [\bar{r}^1(iN+j), \cdots, \bar{r}^M(iN+j), \cdots, \bar{r}^1(iN+N-1+j), \cdots, \bar{r}^M(iN+N-1+j)]^T$ where $i = 1, \cdots, N_b$ and $j = 0, \cdots, N-1$. Due to the block Toeplitz structure of the channel matrix. It is straightforward to show that

$$\mathbf{y}(i,j) = \mathbf{H}\tilde{\mathbf{u}}(i,j) + \tilde{\mathbf{v}}(i,j) \tag{4}$$

Where $\tilde{\mathbf{u}}(i,j) = [u(iN-L+1+j), u(iN-L+2+j), \cdots, u(iN+N-1+j)]^T$ and $\tilde{\mathbf{v}}(i,j)$ is the corresponding noise vector. Under signal model (4), there are $(N_b-1)N+1$ received signal vectors can be used to estimate the channel as comparing to N_b vectors in [4] [10]-[12]. As a result, the proposed method is expected to have better performance than those of [4] [10]-[12]. Since the received signal vector $\mathbf{y}(i,j)$ has length NM, the rank of its noise subspace is MN-N-L+1 which is larger than that of the block subspace method. As a result, the performance of the proposed subspace method is expected to be better than that of the block subspace method [13]. Performing the singular value decomposition (SVD) on the covariance matrix of the received signal vector $\mathbf{y}(i,j)$ to obtain the noise subspace $\mathbf{U}_n$. Let $\mathbf{U}_n(i)$ be the ith column of $\mathbf{U}_n$ and partition $\mathbf{U}_n(i)$ into block vector $\mathbf{U}_n(i) = [\mathbf{u}_{n,i}^T(0), \cdots, \mathbf{u}_{n,i}^T(N-1)]^T$, where each $\mathbf{u}_{n,i}$ is an $M \times 1$ vector. Since $\mathbf{u}_n^H\mathbf{H} = \mathbf{0}$, then $\mathbf{u}_n^H(i)\mathbf{H} = \mathbf{0}$ can be expressed alternatively as $\mathbf{g}_i^H\mathbf{h} = \mathbf{0}$, where $\mathbf{h} = [\mathbf{h}^T(0), \cdots, \mathbf{h}^T(L-1)]^T$ and

$$\mathbf{g}_i = \begin{bmatrix} \mathbf{u}_{n,i}(N-1) & \cdots & \mathbf{u}_{n,i}(0) & & \\ & \ddots & & \ddots & \\ & & \mathbf{u}_{n,i}(N-1) & \cdots & \mathbf{u}_{n,i}(0) \end{bmatrix} \tag{5}$$

The channel estimation can be obtained by the constrained least square optimization criterion

$$\hat{\mathbf{h}} = \operatorname{argmin}_{\|\mathbf{h}\|=1} \sum_{i=1}^{MN-N-L+1} \left\| \mathbf{g}_i^H \mathbf{h} \right\|^2 = \operatorname{argmin}_{\|\mathbf{h}\|=1} \mathbf{h}^H \mathbf{G}\mathbf{G}^H \mathbf{h} \tag{6}$$

Where $\mathbf{G} = [\mathbf{g}_1, \cdots, \mathbf{g}_{MN-N-L+1}]$.

Finally, it is worth mentioning that the proposed subspace method differs the subspace methods [4] [10] in that 1.) the proposed method processes the received signal vector with IBI; 2.) the rank of the noise subspace in our proposed method is larger than those of the subspace methods (e.g. MN-N-L+1 comparing with M(N-L+1)-N); 3.) the proposed method uses $(N_b-1)N+1$ vectors instead of $\mathbf{N_b}$ vectors to calculate the noise subspace, which is also a distinguishing factor from the subspace method [12]. Similarly, our proposed subspace method differs the block subspace method [13] in that 1.) the rank of the noise subspace in our proposed method is larger than that of the block subspace method; 2.) our method uses $(\mathbf{N_b}-1)\mathbf{N}+1$ vectors instead of $\mathbf{N_b}(\mathbf{N}-\mathbf{G}+1)$ vectors to calculate the noise subspace.

Computational Complexity: Next we analyze the computational complexity of the proposed subspace method with the subspace method in [10], the subspace method in [12] and the block subspace method in [13]. The main computational complexity of the proposed subspace method is from 1) the singular value decomposition to obtain the noise subspace and 2) EVD used to estimate the channel vector. These operations are shown in [15] [16] of order $O(N^3M^3+L^3M^3)$. Similarly, the SVD-related operations for the subspace method [10], the subspace method [12] and the block subspace method [13] are of order $O((N-L+1)^3M^3+L^3M^3)$, $O(N^3M^3+L^3M^3)$ and $O(G^3M^3+L^3M^3)$, respectively. Furthermore, the proposed subspace method, the subspace method [10], the subspace method [12] and the block subspace method [13] have complexity $O(((N_b-1)N+1)M^2N^2$, $O(N_bM^2(N-L+1)^2)$, $O(N_bM^2N^2)$ and $O(N_b(N-G+1)M^2G^2)$, respectively, for the computation of the covariance matrix. Since $\mathbf{G}\leq\mathbf{N},\mathbf{L}\ll\mathbf{N}$ in practice and the subspace methods [10] [12] require much more OFDM blocks to achieve satisfactory performance than that of the proposed method, the computational complexity of our subspace method is slightly higher than those of the subspace methods [10] [12] and the block subspace method [13]. This is the trade off between computational complexity and estimation accuracy.

4. SIMULATIONS

In our simulation, the number of receive antenna M=2. Information sequence $\mathbf{s}(\cdot)$ is QPSK modulated. A multipath channel [10] $\mathbf{h}^1=[(-0.3825,0.0010),(0.5117,0.2478),(-0.3621,0.3320),$ $(-0.4106,0.3408),(0.0087,0.0546)]$, $\mathbf{h}^2=[(-0.2328,0.1332),$ $(-0.3780,-0.3794),(-0.0320,-0.4532),(0.5081,-0.0125),(0.4195,0.0220)]$ is used in all simulations. The Normalized Root Mean Square Error (NRMSE) $\frac{1}{\|\mathbf{h}\|}\sqrt{\frac{1}{\mathrm{N_m ML}}\sum_{\mathrm{p=1}}^{\mathrm{N_m}}\left\|\hat{\mathbf{h}}_{\mathrm{p}}-\mathbf{h}\right\|^2}$ is adopted, where the subscript p refers to the pth Monte Carlo run and $\mathbf{N_m}$ denotes the total number of runs which is 100 in all simulations. $\hat{\mathbf{h}}_{\mathbf{p}}$ is the estimated channel vector of the pth run, and $\mathbf{h}$ is the actual channel vector. Channel estimators including the proposed subspace method, the subspace method [10], the subspace method [12] and the block subspace method [13] are compared. All methods eliminate the CP and training, but require multiple receive antenna. 3 OFDM blocks are used for channel estimation for all methods. The number of sub-carriers N=16.

Example 1: In this experiment, the normalized root mean square error is examined as a function of SNR. Note that similar as in [10], there is a complex scalar ambiguity in the proposed blind channel estimator. In our simulation, the same method as in [10] is adopted to determine the phase ambiguity and compensate the channel estimate prior to the sample MSE computations. It can be seen from Fig. 1 that the estimator error of the proposed subspace method is consistently better than [10] [12] [13]. In addition, the subspace methods [10] [12] cannot achieve satisfactory performance with 3 OFDM blocks.

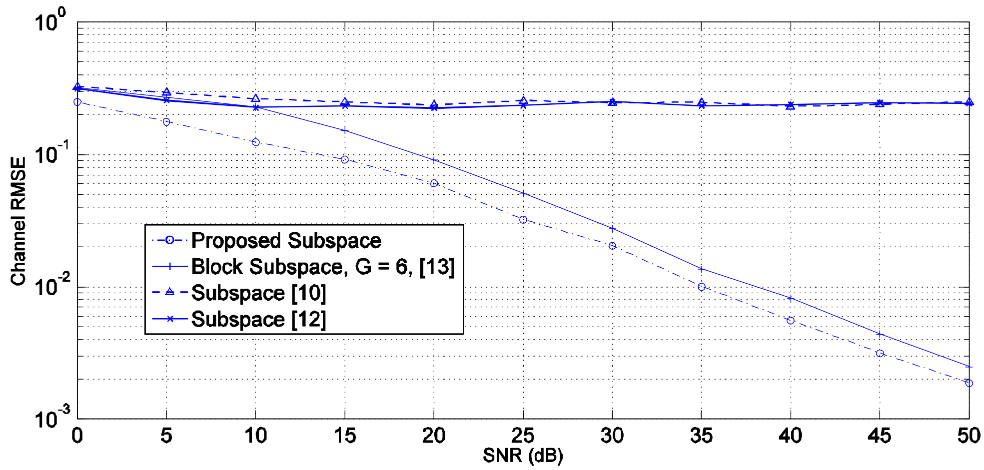

FIGURE 1: NRMSE vs SNR

Example 2: The effect of the number of sub-carriers N is shown in Fig. 2 with SNR=35dB and $N_b = 3$. Larger N indicates larger rank of the noise subspace for the proposed method, yielding more constraints on the channel vector (6) and thus, leads to improvement in the channel estimation. Finally, it worth mentioning that under current simulation setting, when N>16, more sample vectors are available in the block subspace method than that of the proposed subspace method. Therefore, the performance gap between two methods is closing. Overall, the estimator error of the proposed subspace method is consistently better than [10] [12] [13].

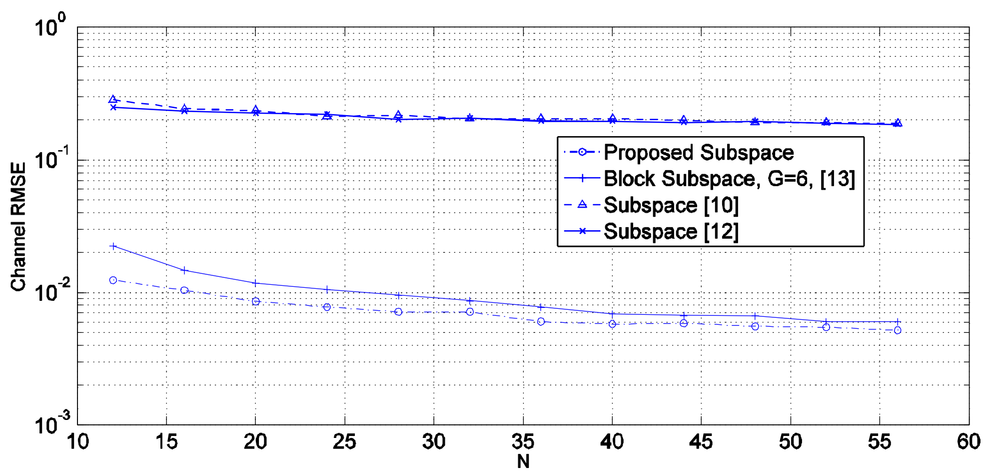

FIGURE 2: NRMSE vs N

Example 3: The effect of OFDM block N_b is presented in Fig. 3 when SNR=35dB and N=16. It can be seen from Fig. 3 that the estimation error decreases for all methods when the number of received OFDM blocks increases. Also notable is that under the simulation setting, when $N_b > 3$, more sample vectors are available in the proposed subspace method than that of the block subspace method. Therefore, the performance gap between two methods is expanding. Again, the estimator error of the proposed subspace method is consistently better than [10] [12] [13].

5. CONCLUSION

By exploiting the block Toeplitz structure of the channel matrix, a blind subspace method for OFDM systems without CP is proposed in this paper. The strength of the proposed method lies in high data and spectral efficiency, thus being attractive for channel estimation under fast changing channel environment. Comparison of the proposed method with several existing blind subspace channel estimation methods illustrates the good performance of the proposed method.

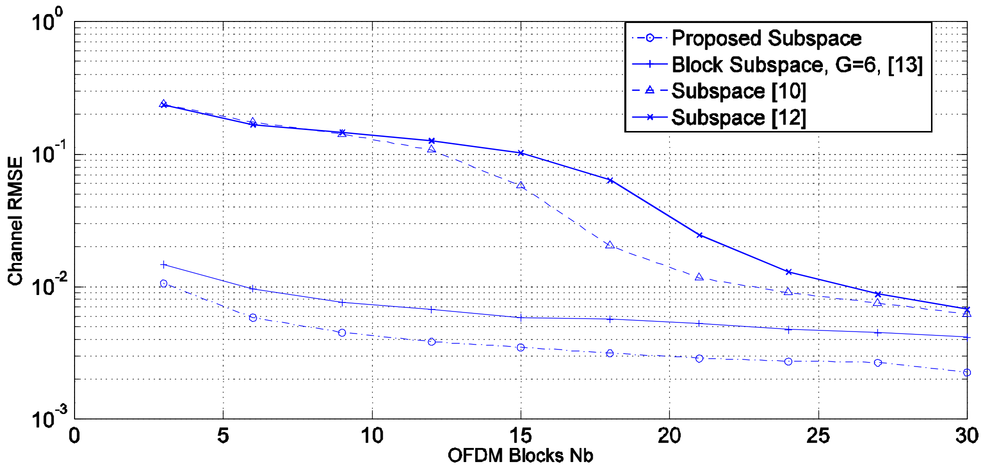

FIGURE 3: NRMSE vs N_b

6. REFERENCES

1. Z. Wang and G. Giannakis. *"Wireless Multicarrier Communications"*. IEEE Signal Processing Magazine, 29-48, 2000

2. R. van Nee and R. Prasad. *"OFDM for Wireless Multimedia Communications"*, Artech House Publishers, (2000)

3. B. Muquet, M. de Courville, P. Duhamel, and V. Buenac. *"Subspace based blind and semi-blind channel identification method for OFDM systems"*. In Proceedings of the IEEE-SP Workshop on Signal Processing Advances in Wireless Communications. Annapolis, MD, USA, 1999

4. X. Cai and A. Akansu. *"A subspace method for blind channel identification in OFDM systems"*. In Proceedings of the IEEE International Conference on Communications, 2000

5. C. Wang and Z. Zhou. *"A new detection algorithm for OFDM system without cyclic prefix"*. In Proceedings of the IEEE Mobile and Wireless Communications, 2004

6. C. Lin, J. Wu and T. Lee. *"GSC-based frequency-domain equalizer for Cp-free OFDM systems"*. In Proceedings of the IEEE International Conference on Communications, 2005

7. M. Teoltsch and A. Molisch. *"Efficient OFDM transmission without cyclic prefix over frequency-selective channels"*. In Proceedings of the IEEE PIMRC, 2000

8. H. Xuejun and B. Houjie. *"Blind channel identification and equalization in OFDM system without cyclic prefix"*. In Proceedings of the International Conference on Communication Technology, 2003

9. S. Wang and J.anton. *"Blind Channel Estimation for non-CP OFDM Systems using Multiple Receive Antenna"*. IEEE Signal Processing Letters, 299-302, 2009

10. S. Roy and C. Li. *"A Subspace Blind Channel Estimation Method for OFDM Systems without Cyclic Prefix"*. IEEE Trans. Wireless Communications, 572-579, 2002

11. H. Ali, J. Manton and Y. Hua. *"A SOS subspace method for blind channel identification and equalization in bandwidth efficient OFDM systems based on receive antenna diversity"*. In Proceedings of the IEEE Signal Processing Workshop on Statistical Signal processing, 2001

12. J. Yu, C. Chen and M. Lee. *"Blind channel estimation for SIMO OFDM systems without cyclic prefix"*. In Proceedings of the International Conference on Wireless Communications, Networking and Mobile Computing, 2006

13. J. Yu. *"Channel Estimation for SIMO OFDM Systems without Cyclic Prefix"*. Electronics Letters, 1369-1371, 2007

14. H. Liu. *"Signal Processing Applications in CDMS Communications"*, Artech House Publishers, (2000)

15. D. Johnson and D. Dudgeon. *"Array Signal Processing: Concepts and Techniques"*, Prentice Hall, (1993)

16. G. Golub and F. Van Loan. *"Matrix Computation"*, The Johns Hopkins University Press, (1996)