Performance Analysis of MIMO-OFDM System Using QOSTBC Code Structure for M-PSK

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Abstract
MIMO-OFDM system has been currently recognized as one of the most competitive technology for 4G mobile wireless systems. MIMO-OFDM system can compensate for the lacks of MIMO systems and give play to the advantages of OFDM system. In this paper, a general Quasi orthogonal space time block code (QOSTBC) structure is proposed for multiple-input multiple-output–orthogonal frequency-division multiplexing (MIMO-OFDM) systems for 4X4 antenna configuration. The signal detection technology used in this paper for MIMO-OFDM system is Zero-Forcing Equalization (linear detection technique).

In this paper the analysis of high level of modulations (i.e. M-PSK for different values of M) on MIMO-OFDM system is presented. Here AWGN and Rayleigh channels have been used for analysis purpose and their effect on BER for high data rates have been presented. The proposed MIMO-OFDM system with QOSTBC using 4X4 antenna configuration has better performance in terms of BER vs SNR than the other systems.

Keywords: MIMO, OFDM, QOSTBC, M-PSK

1. INTRODUCTION
As the demand for high-data rate multimedia grows, several approaches such as increasing modulation order or employing multiple antennas at both transmitter and receiver have been studied to enhance the spectral efficiency. [1][2] In today’s communication systems Orthogonal Frequency Division Multiplexing (OFDM) is a widespread modulation technique. Its benefits are high spectral efficiency, robustness against inter-symbol interference, ease of implementation using the fast Fourier transform (FFT) and simple equalization techniques. Recently, there have been a lot of interests in combining the OFDM systems with the multiple-input multiple-output (MIMO) technique. These systems are known as MIMO OFDM systems.

Spatially multiplexed MIMO is known to boost the throughput, on the other hand, when much higher throughputs are aimed at, the multipath character of the environment causes the MIMO channel to be frequency-selective. OFDM can transform such a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels and also increase the frequency
efficiency. Therefore, MIMO-OFM technology has been researched as the infrastructure for next
generation wireless networks. [3]

Therefore, MIMO-OFDM, produced by employing multiple transmit and receive antennas in an
OFDM system has becoming a practical alternative to single carrier and Single Input Single
Output (SISO) transmission.[4] However, channel estimation becomes computationally more
complex compared to the SISO systems due to the increased number of channels to be
estimated. This complexity problem is further compounded when the channel from the ik transmit
antenna to the mh receive antenna is frequency-selective. Using OFDM, information symbols are
transmitted over several parallel independent sub-carriers using the computationally efficient
IFFT/FFT modulation/demodulation vectors. [5]-[8]

These MIMO wireless systems, combined with OFDM, have allowed for the easy transmission of
symbols in time, space and frequency. In order to extract diversity from the channel, different
coding schemes have been developed. The seminal example is the Alamouti Space Time Block
(STB) code [9] which could extract spatial and temporal diversity. Many other codes have also
been proposed [10]–[12] which have been able to achieve some or all of the available diversity in
the channel at various transmission rates.

In open-loop schemes, there are generally two approaches to implement MIMO systems. One is
to increase the spatial transmit diversity (STD) by means of space-time coding and space-
frequency coding. Another is to raise the channel capacity by employing spatial division
multiplexing (SDM) that simultaneously transmits independent data symbols through multiple
transmit antennas. STD mitigates impairments of channel fading and noise, whereas SDM
increases the spectral efficiency. [13][14]

In section 2, general theory of OFDM and the necessary condition for orthogonality is discussed.
In section 2.1, the signal model of OFDM system with SISO configuration is discussed in detail
with the help of block diagram. In section 2.2, M-PSK (M-Phase Shift Keying) modulation
technique is discussed in detail. In section 2.3, different channels used for analyses purpose are
discussed namely AWGN and Rayleigh channel. In section 3, general theory about the MIMO
system is presented. In section 4, MIMO-OFDM system with QOSTBC is discussed. In section
4.1, general theory about QOSTBC and the proposed QOSTBC code structure for 8x8 antenna
configuration is presented. In section 4.2, idea about the linear detection technique i.e. Zero
Forcing equalization for MIMO-OFDM system is presented. Finally in section 5, the simulated
results based on the performance of MIMO-OFDM system in AWGN and Rayleigh channels have
been shown in the form of plots of BER vs SNR for M-PSK modulation and for different antenna
configurations.

2. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

OFDM is a multi-carrier modulation technique where data symbols modulate a sub-carrier which
is taken from orthogonally separated sub-carriers with a separation of ‘f_k’ within each sub-carrier.
Here, the spectra of sub-carrier is overlapping; but the sub-carrier signals are mutually
orthogonal, which is utilizing the bandwidth very efficiently. To maintain the orthogonality, the
minimum separation between the sub-carriers should be ‘f_k’ to avoid ICI (Inter Carrier
Interference).

By choosing the sub-carrier spacing properly in relation to the channel coherence bandwidth,
OFDM can be used to convert a frequency selective channel into a parallel collection of
frequency flat sub-channels. Techniques that are appropriate for flat fading channels can then be
applied in a straight forward fashion.
2.1 OFDM Signal Model

Figure 1 shows the block diagram of a OFDM system with SISO configuration. Denote $X_l$ ($l = 1, 2, \ldots, N - 1$) as the modulated symbols on the $l_{th}$ transmitting subcarrier of OFDM symbol at transmitter, which are assumed independent, zero-mean random variables, with average power $\sigma_l^2$.

The complex baseband OFDM signal at output of the IFFT can be written as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{l=1}^{N-1} x_l e^{j \frac{2\pi}{N} ln}$$  \hspace{1cm} (1)

where $N$ is the total number of subcarriers and the OFDM symbol duration is $T$ seconds.

At the receiver, the received OFDM signal is mixed with local oscillator signal, with the frequency offset deviated from $\Delta f$ the carrier frequency of the received signal owing to frequency estimation error or Doppler velocity, the received signal is given by:

$$z_n = (x_n \otimes h_n) e^{j \frac{2\pi}{N} \Delta f n T} + z_n$$  \hspace{1cm} (2)

where $h_n$, $e^{j \frac{2\pi}{N} \Delta f n T}$, and $z_n$ represent the channel impulse response, the corresponding frequency offset of received signal at the sampling instants: $\Delta f T$ is the frequency offset to subcarrier frequency spacing ratio, and the AWGN respectively, while $\otimes$ denotes the circular convolution.
Assuming that a cyclic prefix is employed; the receiver have a perfect time synchronization. Note that a discrete Fourier transform (DFT) of the convolution of two signals in time domain is equivalent to the multiplication of the corresponding signals in the frequency domain.

Then the output of the FFT in frequency domain signal on the \( k \)th receiving subcarrier becomes:

\[
X_k = \sum_{n=0}^{N-1} X_n H_k Y_{1-k} + Z_k, \quad k = 0, \ldots, N-1
\]

\[
= X_0 H_k U_0 + \sum_{k=1}^{N-1} X_n H_k Y_{1-k} + Z_k \quad (3)
\]

The first term of (3) is a desired transmitted data symbol \( X_0 \). The second term represents the ICI from the undesired data symbols on other subcarriers in OFDM symbol. \( H_k \) is the channel frequency response and \( Z_k \) denotes the frequency domain of \( Z_n \). The term \( Y_{1-k} \) is the coefficient of FFT (IFFT), is given by:

\[
Y_{1-k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j(2\pi k/N)(-n+2))} \quad (4)
\]

when the channel is flat, \( Y_{1-k} \) can be considered as a complex weighting function of the transmitted data symbols in frequency domain. [15]

### 2.2 Different Modulations Techniques Used in OFDM System

Modulation is the process of mapping the digital information to analog form so it can be transmitted over the channel. Consequently every digital communication system has a modulator that performs this task. Closely related to modulation is the inverse process, called demodulation, done by the receiver to recover the transmitted digital information. [16]

Modulation of a signal changes binary bits into an analog waveform. Modulation can be done by changing the amplitude, phase, and frequency of a sinusoidal carrier. There are several digital modulation techniques used for data transmission. The nature of OFDM only allows the signal to modulate in amplitude and phase.

There can be coherent or non-coherent modulation techniques. Unlike non-coherent modulation, coherent modulation uses a reference phase between the transmitter and the receiver which brings accurate demodulation together with receiver complexity. [17]

#### 2.2.1 Phase Shift Keying

Phase-shift keying (M-PSK) for which the signal set is:

\[
S_i(t) = \sqrt{\frac{E_s}{T_s}} \cdot (\cos(2\pi f_c t + 2 \pi i \pi / M)) \quad (5)
\]

\[
i = 1, 2, \ldots, M \quad & \quad 0 < t < T_s
\]

where \( E_s \) the signal energy per symbol \( T_s \) is the symbol duration, and \( f_c \) is the carrier frequency.

This phase of the carrier takes on one of the \( M \) possible values, namely

\[
\theta_i = 2(i-1)\pi / M \quad \text{where} \quad i = 1, 2, \ldots, M
\]

An example of signal-space diagram for 8-PSK is shown in figure 2
2.3 CHANNELS

Wireless transmission uses air or space for its transmission medium. The radio propagation is not as smooth as in wire transmission since the received signal is not only coming directly from the transmitter, but the combination of reflected, diffracted, and scattered copies of the transmitted signal.

Reflection occurs when the signal hits a surface where partial energy is reflected and the remaining is transmitted into the surface. Reflection coefficient, the coefficient that determines the ratio of reflection and transmission, depends on the material properties.

Diffraction occurs when the signal is obstructed by a sharp object which derives secondary waves. Scattering occurs when the signal impinges upon rough surfaces, or small objects. Received signal is sometimes stronger than the reflected and diffracted signal since scattering spreads out the energy in all directions and consequently provides additional energy for the receiver which can receive more than one copies of the signal in multiple paths with different phases and powers. Reflection, diffraction and scattering in combination give birth to multipath fading. [18]

2.3.1 AWGN Channel

Additive white Gaussian noise (AWGN) channel is a universal channel model for analyzing modulation schemes. In this model, the channel does nothing but add a white Gaussian noise to the signal passing through it. This implies that the channel’s amplitude frequency response is flat (thus with unlimited or infinite bandwidth) and phase frequency response is linear for all frequencies so that modulated signals pass through it without any amplitude loss and phase distortion of frequency components. Fading does not exist. The only distortion is introduced by the AWGN. AWGN channel is a theoretical channel used for analysis purpose only. The received signal is simplified to:

\[ r(t) = s(t) + n(t) \] (6)

where \( n(t) \) is the additive white Gaussian noise. [18]

2.3.2 Rayleigh Fading Channel

Constructive and destructive nature of multipath components in flat fading channels can be approximated by Rayleigh distribution if there is no line of sight which means when there is no direct path between transmitter and receiver. The received signal can be simplified to:
where \( r(t) \) is the random channel matrix having Rayleigh distribution and \( n(t) \) is the additive white Gaussian noise. The Rayleigh distribution is basically the magnitude of the sum of two equal independent orthogonal Gaussian random variables and the probability density function (pdf) given by:

\[
p(r) = \frac{2}{\sigma^2} r e^{-\frac{r^2}{\sigma^2}} \quad 0 \leq r \leq \infty
\]

where \( \sigma^2 \) is the time-average power of the received signal. \([19][20]\)

### 3. MULTI INPUT MULTI OUTPUT (MIMO) SYSTEMS

Multi-antenna systems can be classified into three main categories. Multiple antennas at the transmitter side are usually applicable for beam forming purposes. Transmitter or receiver side multiple antennas for realizing different (frequency, space) diversity schemes. The third class includes systems with multiple transmitter and receiver antennas realizing spatial multiplexing (often referred as MIMO by itself).

In radio communications MIMO means multiple antennas both on transmitter and receiver side of a specific radio link. In case of spatial multiplexing different data symbols are transmitted on the radio link by different antennas on the same frequency within the same time interval. Multipath propagation is assumed in order to ensure the correct operation of spatial multiplexing, since MIMO is performing better in terms of channel capacity in a rich scatter multipath environment than in case of environment with LOS (line of sight). This fact was spectacularly shown in [21].

MIMO transmission can be characterized by the time variant channel matrix:

\[
H(\tau, t) = \begin{bmatrix}
R_{1:1}(\tau, t) & R_{1:2}(\tau, t) & \cdots & R_{1:N_r}(\tau, t) \\
R_{2:1}(\tau, t) & R_{2:2}(\tau, t) & \cdots & R_{2:N_r}(\tau, t) \\
\vdots & \vdots & \ddots & \vdots \\
R_{N_t:1}(\tau, t) & R_{N_t:2}(\tau, t) & \cdots & R_{N_t:N_r}(\tau, t)
\end{bmatrix}
\]  

(9)

where the general element, \( h_{nt, nr} (\tau, t) \) represents the complex time-variant channel transfer function at the path between the \( n_{th} \) transmitter antenna and the \( n_{th} \) receiver antenna. \( N_T \) and \( N_R \) represent the number of transmitter and receiver antennas respectively.

Derived from Shannon’s law, for the capacity of MIMO channel the following expression was proven in [21] and [22]:

\[
C = \frac{1}{N_T N_R} \log_2 \left( \frac{1}{|\text{det} (I + H R_s H^H)|} \right)
\]

(10)

where \( H \) denotes the channel matrix and \( H^H \) its transpose conjugate, \( I \) represents the identity matrix and \( R_s \) the covariance matrix of the transmitted signal \( s \).
4. MIMO-OFDM WITH QUASSI ORTHOGONAL SPACE TIME BLOCK CODING (QOSTBC)

MIMO-OFDM systems with orthogonal space–time block coding (O-STBC) [12] are particularly attractive due to the fact that they require a relatively simple linear decoding scheme while still providing full diversity gain. Unfortunately, they suffer from a lower code rate when a complex signal constellation and the complexity that more than two transmit antennas are used. To overcome the disadvantages of O-STBC, quasi-orthogonal space–time block coding (QO-STBC) was proposed in the literature [23]-[24] and the existing works have shown that QO-STBC offers a higher data rate and partial diversity gain.

To design a QO-STBC with full diversity gain, an improved QO-STBC through constellation rotation was proposed in [25] and [26]. Maximum-likelihood (ML) decoding in QO-STBC works with pairs of transmitted symbols, leading to an increase in decoding complexity with modulation level M^2. This subsequently increases transmission delay when a high-level modulation scheme or multiple antennas are employed. Sung et al. [27] proposed a method to improve the QO-STBC performance with iterative decoding, which of course achieves higher reliability but increases decoding complexity. In [28]–[32], some new decoding methods were proposed to reduce the computational complexity.

4.1 Quassi Orthogonal Space Time Block Codes

Consider a system with eight transmit antennas (i.e. M = 4) and 4 receive antennas (i.e. N=4). In what follows, assume that perfect channel state information (CSI) is available at the receiver but unavailable at the transmitter. Also assume that the channel is quasi-static, i.e. the channel coefficients are constant within one block of code transmission and independently realized from block to block. Let A_{12} and A_{34} be Alamouti code as in [9]

\[
A_{12} = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_1 \end{bmatrix} \quad \text{and} \quad A_{34} = \begin{bmatrix} s_3 & s_4 \\ s_4 & s_3 \end{bmatrix}
\]

Here the subscript 12 and 34 are used to represent the indeterminate s_1, s_2, s_3 and s_4 in the transmission matrix. Now consider the space time block code for M and N equals to 4 according the method given in [24], the matrix for 4X4 antenna configuration can also be constructed as follows :

\[
B = \begin{bmatrix} A_{12} & A_{14} \\ A_{32} & A_{34} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_4 & s_3 & s_1 & s_2 \\ s_3 & s_4 & s_2 & s_1 \end{bmatrix}
\]

(11)

Note that it has been proven in [33] maximum diversity of the order of 4*N for a rate one code is impossible in this case. Now, suppose V, i = 1,2,...,4 as the ith column of Q, it is easy to see that
\[ \begin{align*}
\langle V_1, V_2 \rangle &= 0 & \langle V_2, V_3 \rangle &= 0 \\
\langle V_1, V_3 \rangle &= 0 & \langle V_2, V_4 \rangle &= 0
\end{align*} \] (12)

Where \( \langle V_i, V_j \rangle = \sum_{n=1}^{K} \langle V_{i,n}, V_{j,n} \rangle \) is the inner product of vectors \( V_i \) and \( V_j \). Therefore, the subspace created by \( V_1 \) and \( V_4 \) is orthogonal to the subspace created by \( V_2 \) and \( V_3 \), and similar is true for other columns as given by equation (11).

### 4.2 Signal Detection of Mimo-ofdm System

Signal detection of MIMO-OFDM system can be carried out by various sub-carrier channel signal detection. Although the whole channel is a frequency-selective fading, but various sub-carriers channel divided can be regarded as flat fading, so that the flat fading MIMO signal detection algorithm for MIMO-OFDM system can be directly into the detection of all sub-channels, and signal detection algorithm of the corresponding MIMO-OFDM system can be obtained. Similarly, the other optimization algorithms used in flat fading MIMO signal detection can also be leaded into the MIMO-OFDM system. MIMO-OFDM detection methods consist of linear and nonlinear detection test.

#### 4.2.1 Zero Forcing Algorithm [34]

Zero Forcing algorithm is regard the signal of each transmitting antenna output as the desired signal, and regard the remaining part as a disturbance, so the mutual interference between the various transmitting antennas can be completely neglected. The specific algorithm is as follows: For \( k = 0, 1, 2, \ldots, K-1 \), so that,

\[ R(k) = [R_1(k), R_2(k), \ldots, R_M(k)]^T \] (13)

\[ S(k) = [S_1(k), S_2(k), \ldots, S_M(k)]^T \] (14)

\[ N(k) = [N_1(k), N_2(k), \ldots, N_M(k)]^T \] (15)

\[ H(k) = \begin{bmatrix}
H(k)_{11} & H(k)_{12} & \cdots & H(k)_{1M} \\
H(k)_{21} & H(k)_{22} & \cdots & H(k)_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
H(k)_{N1} & H(k)_{N2} & \cdots & H(k)_{NM}
\end{bmatrix} \] (16)

Here \( R(k), S(k), N(k) \) respectively express output signal, the input signal and noise vector of the \( k \) sub-channels in MIMO-OFDM system, for \( M \) transmitting antennas and \( N \) receiving antennas, \( H(k) \) expresses channel matrix of the \( k \) sub-channels, mathematical expression of sub-channel in the MIMO-OFDM system is as follows:
There is a linear relationship between input signal $S(k)$ and output signal $R(k)$, that is similar to the flat fading channel for each subcarrier channel in MIMO-OFDM system. Its equivalent block diagram is shown in Figure 4. Therefore, signal detection can be transformed into K sub-channels in their signal detection to complete in MIMO-OFDM system and each sub-channel detection of the above can be used flat fading MIMO channel to achieve the detection algorithm.

Zero-forcing (ZF) detection algorithm for MIMO detection algorithm is the most simple and basic algorithms, and the basic idea of zero forcing algorithm is get rid of MIMO-channel interference by multiplying received signal and the inverse matrix of channel matrix. Zero-Forcing solution of MIMO-OFDM system is as follows:

$$S_{ZF} = H^{-1} R = S + H^{-1} N$$  \hspace{1cm} (18)

In which $H^{-1}$ is the channel matrix for the generalized inverse matrix, the type is obtained for hard-decision demodulation after that to be the source signal estimates:

$$S_{ZF} = \mathbb{E}(S_{ZF})$$  \hspace{1cm} (19)

5. SIMULATION RESULTS DISCUSSIONS

The system discussed above has been designed and results are shown in the form of SNR vs BER plot for different modulations and different channels. Here different antenna configurations such as 1x1, 2x2 are used to show the advantage in term of SNR of using 4X4 antenna configuration over the other configurations. The analyses have been done for three channels AWGN and Rayleigh channel.
Figure 5 (a)-(e): SNR vs BER plots for PSK over AWGN channel for MIMO-OFDM system employing different antenna configurations (a) 32-PSK (b) 64-PSK (c) 128-PSK (d) 256-PSK (e) 512-PSK.

SNR vs BER plots for M-PSK over AWGN channel for MIMO-OFDM system employing different antenna configurations are presented in Figure 5. Here the graph depicts that in MIMO-OFDM system as we goes on increasing the no. of Transmitters and Receivers the BER keeps on decreasing due to space diversity and the proposed system provide better BER performance as compared to the other antenna configurations.
In Figure 6 SNR vs BER plots for M-PSK over Rayleigh channel for MIMO-OFDM system employing different antenna configurations are presented. It can be concluded from the graphs that in MIMO-OFDM system as we goes on increasing the no. of Transmitters and Receivers the BER keeps on decreasing due to space diversity and the proposed system provide better BER performance as compared to the other antenna configurations. But here BER is greater than the AWGN channel.
Table 1 shows the improvement in terms of decibels shown by proposed system employing QOSTBC code structure for 4X4 antenna configuration over the system employing QOSTBC code structure for 2X2 antenna configuration for different modulation schemes over different environments (channels).

<table>
<thead>
<tr>
<th>Different Modulations</th>
<th>For AWGN Channel</th>
<th>For Rayleigh Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>32-PSK</td>
<td>3.22 dB</td>
<td>3.85 dB</td>
</tr>
<tr>
<td>64-PSK</td>
<td>5.02 dB</td>
<td>3.42 dB</td>
</tr>
<tr>
<td>128-PSK</td>
<td>4.75 dB</td>
<td>3.88 dB</td>
</tr>
<tr>
<td>256-PSK</td>
<td>6.5 dB</td>
<td>4.98 dB</td>
</tr>
<tr>
<td>512-PSK</td>
<td>3.42 dB</td>
<td>3.05 dB</td>
</tr>
</tbody>
</table>

**TABLE 1:** Table showing the improvement in terms of dB, by using the proposed QOSTBC code structure (for 4X4 antenna configuration) for different Modulations and for different Channels.

### 7. CONCLUSION

In this paper, an idea about the performance of the MIMO-OFDM systems at higher modulation levels and for different antenna configurations is presented. MIMO-OFDM system can be implemented using higher order modulations to achieve large data capacity. But there is a problem of BER (bit error rate) which increases as the order of the modulation increases. The solution to this problem is to increase the value of the SNR so, that the effect of the distortions introduced by the channel will also go on decreasing, as a result of this, the BER will also decreases at higher values of the SNR for high order modulations.

The motive of using high order antenna configuration (4X4) is to increase the space diversity, which will automatically lower the BER at given SNR as compared to lower order Antenna configuration (1x1, 2x2). By doing so, higher data capacity at any given SNR can be achieved. The proposed MIMO-OFDM system with 4X4 antenna configuration provides better performance in terms of SNR as compared to the MIMO-OFDM system with 2X2 antenna configuration at a BER of $10^{-2}$.

### 8. REFERENCE


