

Iterative Soft Decision Based Complex K-best MIMO Decoder

Mehnaz Rahman

Department of ECE

Texas A&M University

College Station, Tx- 77840, USA

mehnaz@tamu.edu

Gwan S. Choi

Department of ECE

Texas A&M University

College Station, Tx- 77840, USA

gchoi@ece.tamu.edu

Abstract

This paper presents an iterative soft decision based complex multiple input multiple output (MIMO) decoding algorithm, which reduces the complexity of Maximum Likelihood (ML) detector. We develop a novel iterative complex K-best decoder exploiting the techniques of lattice reduction for 8×8 MIMO. Besides list size, a new adjustable variable has been introduced in order to control the on-demand child expansion. Following this method, we obtain 6.9 to 8.0 dB improvement over real domain K-best decoder and 1.4 to 2.5 dB better performance compared to iterative conventional complex decoder for 4th iteration and 64-QAM modulation scheme. We also demonstrate the significance of new parameter on bit error rate. The proposed decoder not only increases the performance, but also reduces the computational complexity to a certain level.

Keywords: Complex K-best Algorithm, MIMO, Lattice Reduction, Iterative Soft Decoding, SE Enumeration.

1. INTRODUCTION

With the advancement of wireless system, MIMO has been acclaimed by different wireless standards such as IEEE 802.11n, IEEE 802.16e to achieve high data rates and performance with ML or near-ML algorithms. Most of these standards have a specified minimum error rate to guarantee quality of service (QoS), which is either in bit error rate (BER) or packet error rate (PER) (e.g., 10^{-6} is specified as maximum tolerable BER according to IEEE 802.11n standard [1]).

The main challenge behind MIMO system is maintaining the performance of the receiver with low complexity. Several algorithms have been proposed to address the issue, offering different tradeoffs between complexity and performance. The ML detector minimizes BER performance through exhaustive search. However, with increased number of transmitting and receiving antennas, and bits in modulation, the complexity grows exponentially [2, 3]. In contrast, sub-optimal detectors with polynomial complexity such as zero forcing (ZF), minimum mean square error (MMSE) detectors etc. have been developed with significant performance loss.

Recently, lattice reduction (LR) has been proposed in order to achieve high performance, yielding much less complexity than the conventional K-best decoder [4, 5, 6]. LR-aided detector can achieve the same diversity as of ML at the cost of some performance loss [7, 8]. Later, it is implemented in complex domain [9]. All of these suboptimal detectors mentioned above were based on hard decision. Therefore, soft input-soft output (SISO) detectors, suitable for subsequent iterative decoding are introduced in [10].

Researchers further improved these SISO detectors with low density parity check (LDPC) decoder [10, 11]. The output of LDPC decoder is fed back to the detector for updating the soft value in order to achieve better performance. This is called iterative decoding. It can achieve near Shannon performance with less computational complexity compared to other near Shannon decoders [12].

This paper presents an iterative soft decision based complex K-best decoder, which enables the utility of lattice reduction and complex SE enumeration in MIMO decoder. For soft decoding, the log likelihood ratio (LLR) values for LDPC decoder are first computed from the K best candidates and then, they are fed back to LLR update unit as inputs to the next iteration. This process of iterations is continued until the gain of subsequent iteration becomes saturated. Then, the last updated LLR values are forwarded to the LDPC decoder for final detection. Besides list size K, a new tunable parameter Rlimit is introduced in order to enable adaption of the computation of on-demand child expansion for choosing the list candidates.

We compare the results of our proposed decoder with those of iterative conventional complex decoder in [11] and LR-aided real decoder in [13]. For 8×8 MIMO, it achieves 6.9 to 8.0 dB improvement over real domain K-best decoder and 1.4 to 2.5 dB better performance comparing to conventional complex K-best decoder for 4th iteration and 64 QAM modulation scheme. If we consider only 1st iteration, the gain increases to more than 9.0 dB and 2.9 dB comparing with iterative real and complex decoder respectively. This provides significant gain in terms of practical execution. The effect of Rlimit is also analyzed to achieve the maximum performance. The introduction of Rlimit also leads to complexity reduction significantly.

The rest of the paper is organized as follow. In Section II we introduce soft decision based complex MIMO decoding algorithm. Then, Section III presents the results of our studied cases and Section IV concludes this paper with a brief overview.

2. SYSTEM MODEL

Let us consider a MIMO system operating in M-QAM modulation scheme and having N_T transmit antenna and N_R receiving antenna as:

$$y = Hs + n, \quad (1)$$

where $s = [s_1, s_2, \dots, s_{N_T}]^T$ is the transmitted complex vector, H is complex channel matrix and $y = [y_1, y_2, \dots, y_{N_R}]^T$ is the symbol of N_R dimensional received complex vector. Noise is a N_R dimensional complex additive white Gaussian noise (AWGN) with variance and power σ^2 and N_0 respectively. Noise can be represented by $n = [n_1, n_2, \dots, n_{N_R}]^T$.

The detector solves for the transmitted signal by solving non-deterministic hard problem:

$$\hat{s} = \arg_{\tilde{s} \in S^{N_T}} \min \|y - H\tilde{s}\|^2. \quad (2)$$

Here, \tilde{s} is the candidate complex vector, and \hat{s} is the estimated transmitted vector [8]. In the expression, $\|\cdot\|$ denotes 2-norm. This MIMO detection problem can be represented as the closest point problem in [14], which is an exhaustive tree search through all the set of all possible lattice points in $\tilde{s} \in S^{N_T}$ for the global best in terms of Euclidean distance between y and $H\tilde{s}$. Each transmit antenna offers two level of search for real-domain MIMO detection: one for real and the other for imaginary part. However, in complex domain detection method, only one level of search is required for each antenna.

ML detector performs a tree search through the set of all possible branches from root to node, thereby achieves the best performance. However, its complexity increases exponentially with the number of antennas and constellation bits. Therefore, suboptimal detectors such as LR-aided detector comes into consideration.

2.1 LR-aided Decoder

Lattice reduction provides more orthogonal basis with short basis vector from a given integer lattice. Hence, it effectively reduces the effects of noise and mitigates error propagation in MIMO detection. Since lattice reduction is most effective for unconstrained boundary, the following change is made to (2) to obtain a relaxed search.

$$\hat{s} = \arg_{\tilde{s} \in \mathcal{U}^{N_T}} \min \|y - H\tilde{s}\|^2, \quad (3)$$

where \mathcal{U} is unconstrained complex constellation set $\{\dots, -3 + j, -1 - j, -1 + j, 1 - j, \dots\}$. Hence, \hat{s} may not be a valid constellation point. This is resolved by quantizing $\hat{s} = Q(\hat{s})$, where $Q(\cdot)$ is the symbol wise quantizer to the constellation set S .

However, this type of naive lattice reduction (NLD) does not acquire good diversity multiplexing tradeoff (DMT) optimality. Hence, MMSE regularization is employed as proposed in [15, 16], where the channel matrix and received vector are extended as \bar{H} and \bar{y} :

$$\bar{H} = \begin{bmatrix} H \\ \sqrt{\frac{N_0}{2\sigma_2^2}} I_{N_T} \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} y \\ 0_{N_T \times 1} \end{bmatrix}, \quad (4)$$

where $0_{N_T \times 1}$ is a $N_T \times 1$ zero matrix and I_{N_T} is a $N_T \times N_T$ complex identity matrix [17, 18]. Then, Eq. (3) can be represented as:

$$\hat{s} = \arg_{\tilde{s} \in \mathcal{U}^{N_T}} \min \|\bar{y} - \bar{H}\tilde{s}\|^2. \quad (5)$$

Hence, lattice reduction is applied to \bar{H} to obtain $\tilde{H} = \bar{H}T$, where T is a unimodular matrix. Eq. (5) then become:

$$\hat{s} = T \arg \min_{\tilde{z} \in \mathcal{U}^{N_T}} \left(\|\tilde{y} - \tilde{H}\tilde{z}\|^2 + (1 + j)_{N_T \times 1} \right), \quad (6)$$

where $\tilde{y} = (\bar{y} - \bar{H}(1 + j)_{N_T \times 1})/2$ is the complex received signal vector and $1_{2N_T \times 1}$ is a $2N_T \times 1$ one matrix. After shifting and scaling, (6) became the following one.

$$\hat{s} = T\tilde{z} + (1 + j)_{N_T \times 1}. \quad (7)$$

Lattice reduction is an NP complete problem. However, polynomial time algorithms such as Lenstra-Lenstra-Lovasz (LLL) algorithm in [19] can find near orthogonal short basis vectors.

2.2 Complex K-Best LR-Aided MIMO Detection

Complex K-best LR-aided detection offers a breadth first tree search algorithm, which is performed sequentially starting at N_{th} -level. First, it requires QR decomposition on $\tilde{H} = QR$,

where Q is a $(N_R + N_T) \times (N_R + N_T)$ orthonormal matrix and R is a $(N_R + N_T) \times N_T$ upper triangular matrix. Then (6) is reformulated as

$$\hat{s} = \text{Arg} \min_{\tilde{z} \in \mathcal{U}^{N_T}} (\|\tilde{y} - R\tilde{z}\|^2 + (1 + j)_{2N_T \times 1}), \quad (8)$$

where $\tilde{y} = Q^T \tilde{y}$. The error at each step is measured by the partial Euclidean distance (PED), which is an accumulated error at a given level of the tree. For each level, the K best nodes are selected and passed to the next level for consideration. At the end, all the K paths through the tree are evaluated to find the one with minimum PED. The number of valid children for each parent in LR-aided K -best algorithm is infinite. Hence, in our proposed algorithm, the infinite children issue is addressed by calculating K best candidates using complex on-demand child expansion.

2.3 Complex On-demand Expansion

Complex on-demand expansion exploits the principle of Schnorr-Euchner (SE) enumeration [9, 20]. The strategy employs expanding of a node (child) if and only if all of its better siblings have already been expanded and chosen as the partial candidates [21, 22]. Hence, in an order of strict non-decreasing error, K candidates are selected. In conventional complex SE enumeration, expansion of a child can be of two types: Type I, where the expanded child has same imaginary part as its parent, i.e. enumerating along the real axis; and Type II for all other cases. The example of conventional complex on-demand SE enumeration is shown in Fig. 1.

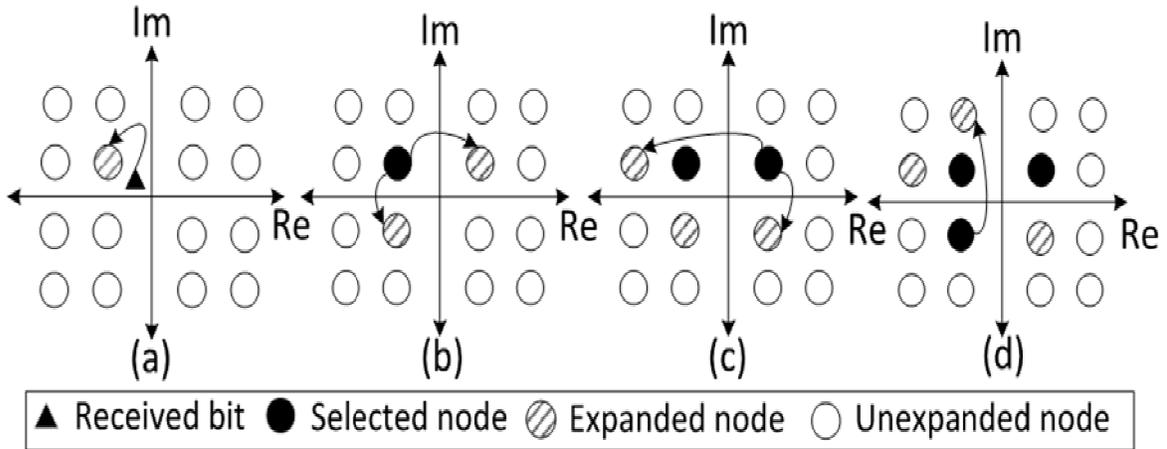


FIGURE 1: Complex SE Enumeration.

First received symbol is rounded to the nearest integer as shown in Fig. 1(a), which includes quantizing of both real and imaginary components of the signal to the nearest integer. Type-I candidate will be expanded two times along real and imaginary axis using SE enumeration, and the two expanded nodes are considered candidates, as demonstrated in Fig. 1(b). Then, the one with the minimum PED is chosen, and expanded for further calculation depending on the type. As in Fig. 1(c), the chosen node is of type I, it will be expanded to 2 more nodes. If the chosen node is of Type II, as shown in Fig. 1(d), it will be expanded only along imaginary axis.

The number of nodes needs expanded at any level of the tree is considered as measurement of complexity analysis. The worst case scenario will be if all the nodes chosen are of type I. Then, at an arbitrary level of tree, the number of expanded nodes is bounded by $K + 2(K - 1)$. Taken over the entire tree, the complexity for the search becomes $3N_T K - 2N_T$. Comparing with the real

domain detection algorithm in [13], the number of the expanded nodes is $4N_T K - 2N_T$. For instance, with K as 4 and N_T equal to 8, the number of expanded node is 80 and 112 considering complex and real decoder respectively. Hence, complex SE enumeration requires less calculation, thereby reduces hardware complexity.

In this paper, we introduce another parameter, R_{limit} while performing the complex on demand child expansion. In contrast with the conventional one, the type of a child is not considered for further expansion. The example of improved complex SE enumeration with R_{limit} as 3 is given in Fig. 2.

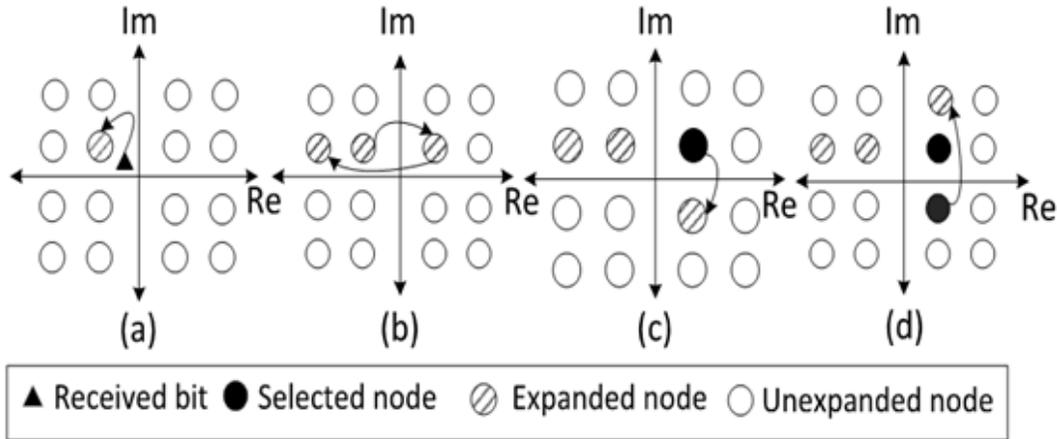


FIGURE 2: Improved Complex SE Enumeration with R_{limit} as 3.

As shown in Fig. 2, after rounding the received symbol to the nearest integer, first real SE enumeration is performed to calculate R_{limit} candidates. Hence, it means that, all the calculated nodes up to R_{limit} will have same imaginary value, as demonstrated in Fig. 2(b). Then, the one with minimum PED is selected and expanded only along the imaginary axis using imaginary domain SE enumeration. This process is continued till K nodes are selected at that level of tree as presented in Fig. 2(c)-(d).

The complexity analysis of the improved child expansion proceeds as follows. At any level of tree search, first $K R_{limit}$ nodes need to be expanded. After that, only imaginary domain SE enumeration will be performed. Hence, considering the worst case, the total number of nodes calculated at each level is $K R_{limit} + (K - 1)$. For N_T levels, the complexity becomes $N_T K (R_{limit} + 1) - N_T$. Therefore, introduction of R_{limit} may increase the complexity as evidenced in result section, although offers better BER performance comparing to the conventional one. However, comparing with the real domain detection, the total complexity is still less. We have used improved complex on demand expansion to perform the list calculation and then the chosen K paths are passed to the iterative soft input soft output (SISO) decoder.

2.4 Iterative Soft Decoding

LDPC decoder in [12] calculates approximate LLR from the list of possible candidates using (9).

$$L_E(x_k|Y) \approx \frac{1}{2} \max_{x \in X_{k,+1}} \left\{ -\frac{1}{\sigma^2} \|y - Hs\|^2 + x_{[k]}^T \cdot L_{A,[k]} \right\} - \frac{1}{2} \max_{x \in X_{k,-1}} \left\{ -\frac{1}{\sigma^2} \|y - Hs\|^2 + x_{[k]}^T \cdot L_{A,[k]} \right\}, \quad (9)$$

here $x_{[k]}^T$ and $L_{A,[k]}$ are the candidates values $\{-1$ or $1\}$ and LLR values except k -th candidate respectively. In order to perform the soft decoding, the LLR values are first computed at the last

layer of K-best search. Then, the soft values are fed into the iterative decoder for the subsequent iteration. This process continues until the difference in error levels between the last two iterations becomes negligible. Lastly, the updated LLR values are used for hard decision.

From the perspective of hardware design as proposed in [16, 23], the LLR calculation unit takes one of the candidates at a given time and computes the LLR value. Then, the new LLR is compared to the maximum of previous LLRs. Hence, this unit has to keep track of 2 values for each LLR; one for those whose k-th bit of the candidate list is 1 (Lambda-ML), and the other for 0 (Lambda-ML-bar). After that, the LLR values are calculated as the subtraction of Lambda-ML and Lambda-ML-bar divided by 2.

3. SETUP AND RESULTS

This section demonstrates the performance of the proposed iterative soft decision based complex K-best decoder. The test and simulation environment has been implemented using IEEE 802.11n standard. All the simulations are for 8×8 MIMO with different modulation schemes. The ratio of the signal and noise power is considered as signal to noise ratio (SNR).

We first analyze the performance of four iterations of our proposed decoder for different modulation scheme. Then, the effect of Rlimit on BER performance is shown for 64QAM modulation scheme. Finally, we demonstrate the comparison of performance of our proposed work with that of iterative conventional complex decoder and real decoder for 64QAM modulation scheme.

The total number of the nodes expanded for 8×8 MIMO is considered as measurement of the complexity analysis. For iterative real decoder, as shown in [13] the improvement gained from 3rd to 4th iteration is limited and negligible for iterations beyond that. Hence, we consider BER versus SNR curve up to four iterations in order to perform comparison among maximum performance.

3.1 Simulation and Analysis

The performance of four iterations of our proposed soft decision based complex decoder for QPSK modulation scheme is presented in Fig. 3.

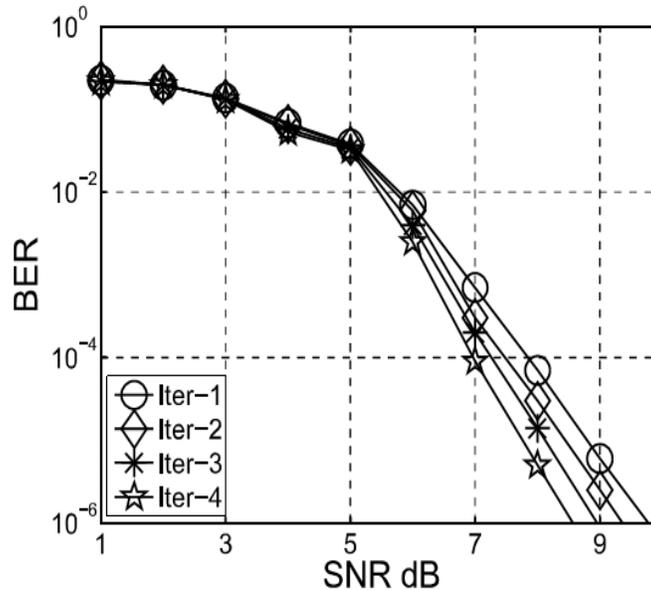


FIGURE 3: BER vs SNR curve of the first 4 iterations of iterative complex decoder for 8×8 MIMO system with K as 4 and QPSK modulation scheme.

As shown in Fig. 3, for QPSK modulation with list size, K of 4 and R_{limit} of 4, we observe 0.4 dB improvement in BER due to the 2nd iteration at the BER of 10^{-6} . When we compare the performance of 1st iteration with 3rd and 4th one, the improvement increases to 0.7 dB and 1.0 dB respectively.

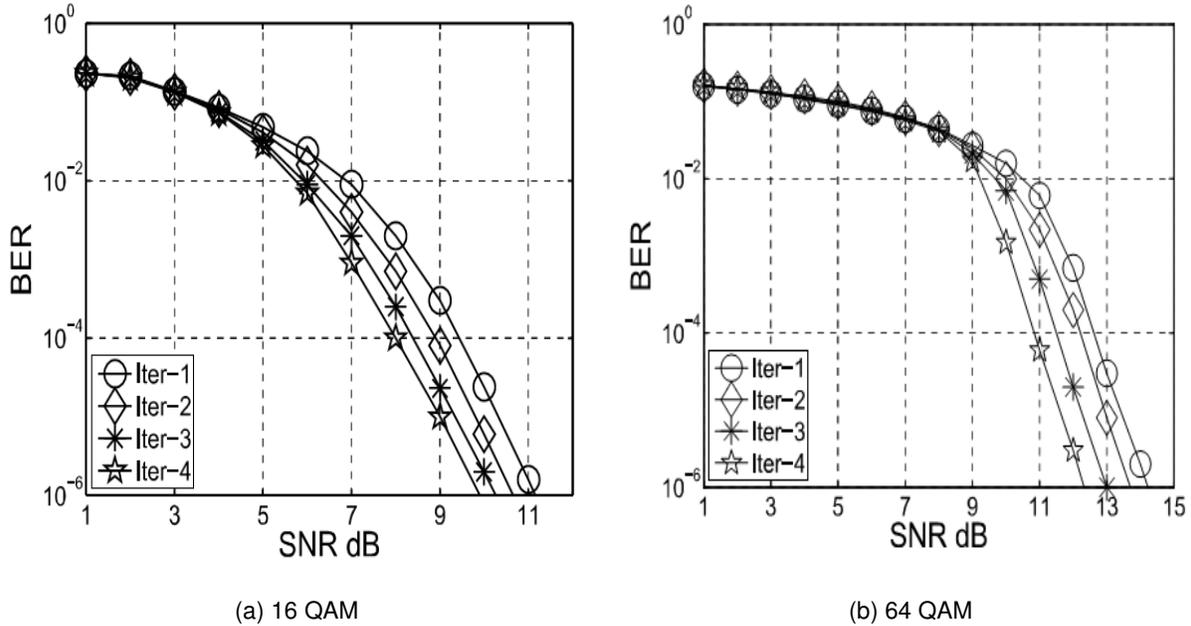


FIGURE 4: BER vs SNR curve of the first 4 iterations of iterative complex decoder for 8×8 MIMO system with K as 4.

Next, Fig. 4 represents the performance curve of 4th iteration for 16 QAM and 64 QAM modulation scheme. As demonstrated in Fig. 4(a), the performance of 2nd iteration is approximately 0.4 dB better than the 1st one with K as 4 and R_{limit} set to 4 for 16 QAM modulation scheme. When increasing the iteration, the performance improves by 0.8 dB for the 3rd and 1.1 dB for the 4th iteration compared to the 1st one.

For 64QAM having same K as 16QAM, the improvement due to the 2nd iteration is 0.4 dB, as shown in Fig. 4(b). If we then compare the 3rd and 4th iteration with respect to the 1st one, the improvements are 0.8 dB and 1.0 dB respectively. By extensive simulation, we observe that the performance does not improve beyond 4th iteration. Therefore, with iteration number, the performance between i -th and $(i+1)$ -th iteration gets saturated.

3.2 Effect of R_{limit} on BER

The effect of R_{limit} , as discussed in section 3.2 for proposed complex on demand child expansion is shown in Fig. 5. It represents BER performance for the 4th iteration over different SNR considering 8×8 MIMO and 64 QAM modulation scheme with list size, K as 4.

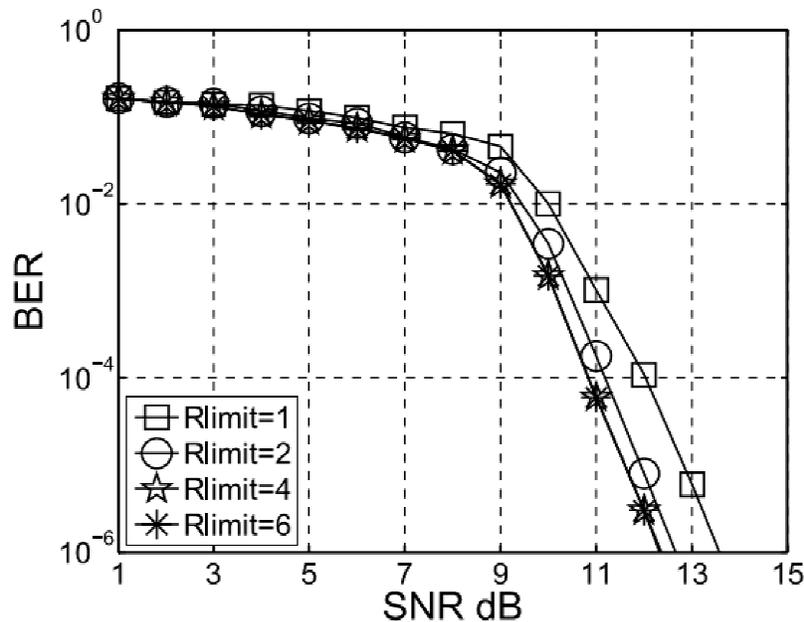


FIGURE 5: BER vs SNR curve of the 4th iteration of iterative complex decoder for 8 x 8 MIMO with 64QAM modulation scheme having K as 4.

It is evident that if the value of R_{limit} is increased, the performance improves and then, it saturates with R_{limit} . On the other hand, decreasing R_{limit} will degrade BER. Hence, as shown in Fig. 4, when R_{limit} increases from 4 to 6, the performance get saturated. However, decreasing the R_{limit} to 2 and then 1, degrades the performance 0.3 dB and 1.1 dB respectively.

Similar curves can be obtained considering 1st, 2nd and 3rd iteration of proposed iterative decoder for different R_{limit} . By extensive simulation, we also observe that, for QPSK and 16 QAM modulation schemes, R_{limit} set to 4 can obtain the maximum performance. Even if the value of R_{limit} is increased, the performance does not improve.

3.3 Comparison of Performance

The comparison of the performance of different iterations of our proposed work with those of iterative conventional complex decoder and real decoder for 64QAM modulation scheme with K as 4 is presented in Fig. 6.

For proposed iterative complex decoder, we have considered R_{limit} as 1, 2 and 4 for performance evaluation. Simulation with R_{limit} higher than 4 is not considered, since it is the minimum value required to achieve the maximum performance. We consider BER versus SNR curve up to four iterations in order to perform comparison among maximum performance, as shown in [14].

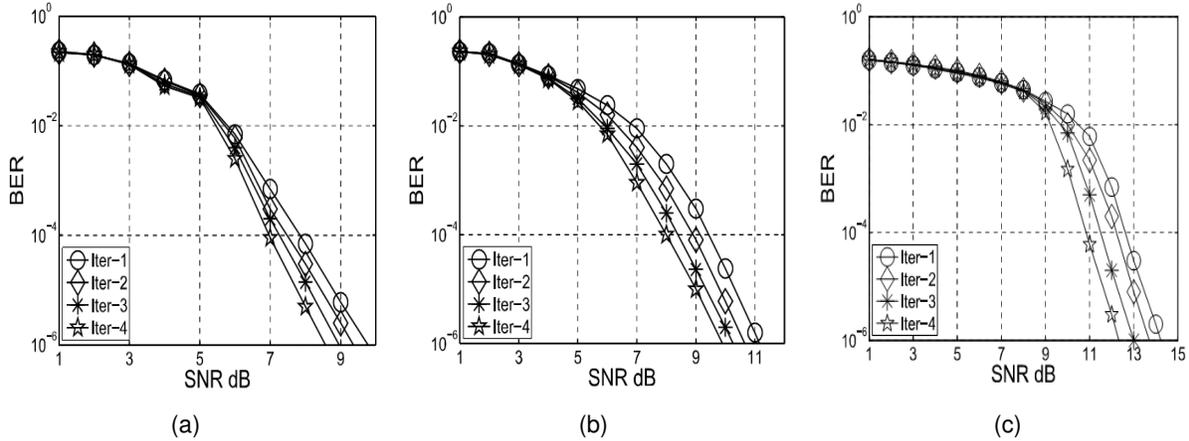


FIGURE 6: BER vs SNR curves of proposed iterative complex decoder, conventional complex and real decoder for 8 x 8 MIMO with 64QAM modulation scheme having K as 4. For proposed decoder, Rlimit is set to 1, 2, and 4.

As demonstrated in Fig. 6(a), a 3.4 dB improvement in performance can be achieved comparing the 1st iteration of proposed decoder with that of conventional iterative complex decoder with Rlimit as 4 at the BER of 10^{-6} . When Rlimit is changed to 2 and 1, the improvements become 3 dB and 2.9 dB respectively. We also compare the performance of proposed decoder with that of the iterative real decoder for 1st iteration [14]. As presented in Fig.6(a), 9.0 dB to 9.5 dB improvement can be achieved using Rlimit as 1 to 4.

Next, as shown in 6(b), a 1.5 dB improvement can be obtained if we consider the performance of 1st iteration of proposed decoder with the 4th iteration of conventional complex one using Rlimit as 4. Decreasing Rlimit to 2 and 1 results in 1 dB and 0.8 dB improvement. Comparing to the 4th iteration of iterative real decoder, 6.1 dB to 6.8 dB SNR gain can be achieved using Rlimit set to 1 to 4 respectively.

Fig. 6(c) presents the comparison curves considering the 4th iteration of iterative decoders. As demonstrated in the figure, a 2.4 dB improvement can be obtained using Rlimit as 4 at the BER of 10^{-6} comparing the conventional iterative complex decoder. In addition, when simulating for Rlimit as 2 and 1, the gain becomes 2.2 dB and 1.4 dB respectively. Similar analysis can be performed comparing to the 4th iteration of iterative real decoder. A gain of 6.9 dB to 8.0 dB can be achieved for Rlimit set to 1 to 4.

Then, we have performed the computational complexity analysis for the presented work. The total number of the nodes expanded for 8 x 8 MIMO is considered as measurement of the analysis. Complexity analysis of proposed and conventional complex decoder is shown in Table 1.

K	Proposed		Conv. Complex	Proposed vs Conv. (in dB)		
	Rlimit	Node	Node	1st vs 1st	4th vs 4th	1st vs 4th
4	1	56	80	2.9	1.4	0.8
4	2	88	80	3.0	2.2	1.0
4	4	152	80	3.4	2.5	1.5

TABLE 1: Complexity Analysis of Conventional and Proposed Complex Decoder.

As tabulated in Table 1, for iterative conventional complex decoder, we need to perform 80 calculations for K equal to 4. Although our proposed decoder calculates 56, 88 and 152 nodes using same list size and Rlimit set to 1, 2, and 4 respectively. Hence, with less computational complexity, the proposed decoder can achieve 1.4 dB better performance than that of conventional one for the 4th iteration. However, 2.2 to 2.5 dB gain can be achieved by tolerating higher computational complexity using proposed complex decoder. Considering 1st iteration with same level of complexity, 2.9 dB to 3.4 dB gain can be achieved using proposed decoder. Next, complexity analysis of proposed and iterative real decoder is presented in Table 2.

K	Proposed		Real	Proposed vs Real (in dB)		
	Rlimit	Node	Node	1st vs 1st	4th vs 4th	1st vs 4th
4	1	56	112	9.0	6.9	6.1
4	2	88	112	9.1	7.7	6.5
4	4	152	112	9.5	8.0	6.8

TABLE 2: Complexity Analysis of Iterative Real and Proposed Complex Decoder.

As shown in Table 2, the number of the nodes need to be expanded for LR-aided real decoder [14] for list size 4 is equal to 112. Considering the same list size, proposed complex decoder requires 56, 88 and 152 node expansion for Rlimit set to 1, 2 and 4 respectively. Hence, proposed decoder can achieve 6.9 dB to 7.7 dB better performance even with less computational complexity comparing with the iterative real one. Allowing more complexity, can increase the performance to 8.0 dB. If we consider the performance of only 1st iteration, with same level of complexity the proposed decoder can attain 9.0 to 9.5 dB improvement comparing with the real one.

Therefore, our iterative soft complex decoder with Rlimit offers a trade-off between performance and complexity for different iteration. It not only increases the performance, but also can reduce complexity to a certain level.

4. CONCLUSION

In this paper, an iterative soft decision based complex domain K-best decoder is proposed exploiting the improved complex on-demand child expansion. It includes the use of LR algorithm in order to achieve orthogonality among the constellation points reducing the effect of noise. An additional parameter, R_{limit} is introduced to tune the complexity of computation with improvement in BER performance. Reduction of computational complexity directly results to less power consumption of the decoder as well.

We also compare the result of 4th iteration of our proposed decoder with iterative conventional complex decoder and obtain 1.4 to 2.5 dB improvement at the BER of 10^{-6} for 8×8 MIMO and 64 QAM modulation scheme with comparable complexity. Comparing with iterative LR-aided real domain decoder, the improvement increases more than 7.0 dB with less computational complexity. Although more than 2.9 dB and 9.0 dB gain can be achieved with same level of complexity comparing 1st iteration of proposed decoder with that of conventional iterative complex and real decoder respectively.

Future work of this proposed decoder includes evaluating the detector performance using additional channel and simulation environment and also implementing the algorithm on FPGA and so on.

5. REFERENCES

- [1] "IEEE Standard for Information Technology- Local and Metropolitan Area Networks- Specific Requirements- Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 5: Enhancements for Higher Throughput." IEEE Standard 802.11n-2009 (Amendment to IEEE Standard 802.11-2007 as amended by IEEE Standard 802.11k-2008, IEEE Standard 802.11r-2008, IEEE Standard 802.11y-2008, and IEEE Standard 802.11w-2009), pp. 1-565, Oct. 2009.
- [2] J. Jalden and B. Otterston. "On the Complexity of Sphere Decoding in Digital Communications." IEEE Transaction on Signal Processing, vol. 53, no. 4, pp. 1474-1484, Apr. 2005.
- [3] I. Lai, G. Ascheid, H. Meyr and T.-D. Chiueh. "Low-Complexity Channel-Adaptive MIMO Detection with Just-Acceptable Error Rate." IEEE 69th Vehicular Technology Conference: VTC-2009 Spring, Apr. 2009, pp. 1-5.
- [4] M. Shabany and P. Glenn Gulak. "The Application of Lattice-Reduction to the K-Best Algorithm for Near-Optimal MIMO Detection." IEEE International Symposium on Circuits and Systems (ISCAS), May 2008, pp. 316-319.
- [5] C. Windpassinger and R. Fischer. "Low-Complexity Near-Maximum Likelihood Detection and Precoding for MIMO Systems Using Lattice Reduction." Proceeding IEEE Information Theory Workshop, Mar. 2003, pp. 345-348.
- [6] Q. Zhou and X. Ma. "An Improved LR-aided K-Best Algorithm for MIMO Detection." Proceeding IEEE International Conference on Wireless Communication and Signal Processing (WCSP), Oct. 2012, pp. 1-5.
- [7] X. Qi and K. Holt. "A Lattice-Reduction-Aided Soft Demapper for High-Rate Coded MIMO-OFDM Systems." IEEE Signal Processing Letters, vol. 14, no. 5, pp. 305-308, May 2007.
- [8] B. M. Hochwald and S. Ten Brink. "Achieving Near-Capacity on a Multiple-Antenna Channel." IEEE Transactions on Communications, vol. 51, no. 3, pp. 389-399, Mar. 2003.

- [9] F. Sheikh, E. Wexler, M. Rahman, W. Wang, B. Alexandrov, D. Yoon, A. Chun and A. Hossein. "Channel-Adaptive Complex K-Best MIMO Detection Using Lattice Reduction." IEEE Workshop on Signal Processing Systems (SiPS), pp. 1-6, Oct. 2014.
- [10] M. Rahman, E. Rohani and G. Choi. "An Iterative Soft Decision Based Adaptive K-Best Decoder Without SNR Estimation." Asilomer Conference on Signals, Systems and Computers, Nov. 2014, pp. 1016-1020.
- [11] Q. Wen, Q. Zhou, C. Zhao and X. Ma. "Fixed-Point Realization of Lattice-Reduction Aided MIMO Receivers With Complex K-Best Algorithm." IEEE International Conference on Acoustics, Speech and Signal Processing, May 2013, pp. 5031-5035.
- [12] K. Gunnam, G. Choi, W. Weihuang and M. Yeary. "Multi-Rate Layered Decoder Architecture for Block LDPC Codes of the IEEE 802.11n Wireless Standard." IEEE International Symposium on Circuits and Systems (ISCAS), May 2007, pp. 1645-1648.
- [13] M. Rahman, E. Rohani and G. Choi. "An Iterative LR-Aided MMSE Extended Soft MIMO Decoding Algorithm." International Conference on Computing, Networking and Communications, California, Feb. 2015.
- [14] E. Agrell, T. Eiriksson, A. Vardy and K. Zeger. "Closest Point Search in Lattices." IEEE Transaction on Information Theory, vol. 48, no. 8, pp. 2201-2214, Aug. 2002.
- [15] J. Jalden and P. Elia. "DMT Optimality of LR-Aided Linear Decoders for a General Class of Channels, Lattice Designs, and System Models." IEEE Transaction on Information Theory, vol. 56, no. 10, pp. 4765-4780, Oct 2010.
- [16] M. Rahman, E. Rohani, J. Xu and G. Choi. "An Improved Soft Decision Based MIMO Detection Using Lattice Reduction." International Journal of Computer and Communication Engineering, vol. 3, no. 4, pp. 264-268, Apr. 2014.
- [17] J. Jalden and P. Elia. "DMT Optimality of LR-Aided Linear Decoders for a General Class of Channels, Lattice Designs, and System Models." IEEE Transaction on Information Theory, vol. 56, no. 10, pp. 4765-4780, Oct. 2010.
- [18] M. Taherzadeh and A. Khandani. "On the Limitations of the Naive Lattice Decoding." IEEE Transaction on Information Theory, vol. 56, no. 10, pp. 4820-4826, Oct. 2010.
- [19] A. K. Lenstra, H. W. Lenstra and L. Lovasz. "Factoring Polynomials with Rational Coefficients." Mathematische Annalen, vol. 261, no. 4, pp. 515-534, Dec. 1982.
- [20] M. Mahdavi and M. Shabany. "Novel MIMO Detection Algorithm for High-Order Constellations in the Complex Domain." IEEE Transaction on VLSI Systems, vol. 21, no. 5, pp. 834-847, May 2013.
- [21] M. Shabany and P. Glenn Gulak. "The Application of Lattice-Reduction to the K-Best Algorithm for Near-Optimal MIMO Detection." IEEE International Symposium on Circuits and Systems (ISCAS), May 2008, pp. 316-319.
- [22] C. P. Schnorr and M. Euchner. "Lattice basis reduction: Improved practical algorithms and solving subset sum problems." Mathematical Programming, vol. 66, pp. 181-191, Aug. 1994.
- [23] M. Rahman and Gwan S. Choi. "Fixed Point Realization of Iterative LR-Aided Soft MIMO Decoding Algorithm." An International Journal on Signal Processing, vol. 9, issue 2, pp. 14-24, May 2015.