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## EDITORIAL PREFACE

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IJBRM editors understand that how much it is important for authors and researchers to have their work published with a minimum delay after submission of their papers. They also strongly believe that the direct communication between the editors and authors are important for the welfare, quality and wellbeing of the Journal and its readers. Therefore, all activities from paper submission to paper publication are controlled through electronic systems that include electronic submission, editorial panel and review system that ensures rapid decision with least delays in the publication processes.

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## Propose a Model for Customer Purchase Decision in B2C Websites Using Adaptive Neuro-Fuzzy Inference System

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### Abstract

If companies are to enjoy long-term success in the Internet marketplace, they must effectively manage the complex, multidimensional process of building online consumer trust. The online environment and the quality and usability of websites help the browser and consumer to be attracted and accessible to the information and the product and services available online. In this Paper a new model would be suggested based on neuro-fuzzy System which depicts some of the hidden relationships between the critical factors such as security, familiarity, and designing in a B2C commercial website on other hand, and the competitive factor to other competitors on other hand. Then, the impacts of these factors on purchasing decision of consumers in B2C commercial websites are extracted. We are going to find the impact of these factors on the decision-making process of people to buy through the B2C commercial websites, and we also will analyze how these factors influence the results of the B2C trading. The study also provides a device for sellers to improve their commercial websites. Two questionnaires were used in this study. The first questionnaire was developed for e-commerce experts, and the second one was designed for the customers of commercial websites. Also, Expert Choice is used to determine the priority of factors in the first questionnaire, and MATLAB and Excel are used for developing the Fuzzy rules. Finally, the Fuzzy logical kit was use to analyze the generated factors in the model.

**Keywords:** Anfis, Clustering, E-commerce, Trust, Rules.

---

## 1. INTRODUCTION

The lack of trust in different components of the most e-commerce applications is known as one of the main reasons which may lead to some e-commerce companies to fail [3, 4].

If companies are to enjoy long-term success in the online market place, they must effectively manage the complex, multidimensional process of building online consumer trust. Trust exists in many forms, across multiple domains, and at variety of levels [8].

Trust, according to Fukuyama, is the lubricant of trade and, this way, determines the wealth of nations. The relative paucity of regulations and customs on the internet makes consumer familiarity and trust especially important in the case of e-commerce [9]. Yet the lack of an interpersonal exchange and the one-time nature of the typical business transaction on the internet make the unique conditions on this kind of consumer trust, because trust relates to the other people and this nourished through interactions with them [10].

Trust is an interpersonal determinant of behavior that deals with beliefs about the integrity, benevolence, ability and predictability of other people [11]. However, in contrast of face-to-face commerce and other applications of trust in the literature, there are typically no interpersonal interactions in e-commerce directly or implied. Such interactions, or even cues relating to them, are notably missing from e-commerce website [12].

The detailed information on trust and the security system that is implemented by companies for secure transactions are important attributes in B2C e-commerce. According to Lightner the rewards of B2C e-commerce are realized partially through well-designed websites, since they act as the primary contact with customers. There are some factors for consumers trust in online buying when they plan to buy, and also there are more factors and matter of interactions when they are on the buying process in online and after one[13].

Website designers must consider factors, in a website allowing the emergence of confidence between an online seller and a customer. A website is the first factor that influences on the reliability of the seller into the mind of customer, and this strongly effect can affect on the initial trust of customers [5, 7].

Lightner states, B2C websites allow companies to present their unique advantages, as long as they provide the necessary services for customers. While there are many factors for determining the success or failure of an e-commerce website, the service level provided for customers may serve as an indication of user satisfaction in transaction with a website. The aim is to specify what factors in the online environment really we should take in our main consideration and how online customer services help to build trust step-by-step [13]. Singh says that it is also necessary for customers to have trust in electronic commerce infrastructure and environment [14].

Marsh and Mitch in their plan, entitled as the Call to arms, have challenged the website designers and asked them to think of this point that how they can easily make the trust possible between an online website and its customers in the early stages of their partnership. They claim that websites can be designed in a special fashion so that the trust not only becomes the indivisible part of the plan, but also it can be considered as a further thought.

Furthermore, the decision about online purchase must be available based on the accurate and correct information rather than focusing on the partial insight, general concepts and individual experiences [1, 2].

Moreover, a lot of trust models have been presented. Most of them are mentally active, effect as unclear and ambiguous confidence in e-commerce websites, and don't involve the experience and understanding of customers during performing online transactions [6].

Therefore, in summary, the following objectives are achievable:

- To identify trust factors between the customer and sale agents.
- To measure the effect of a factor on confidence in contrast to other factors by fuzzy logic.
- To identify factors or security components that considerably affects the customer confidentiality upon the online purchase.
- To help customers when shopping online.
- To help designers of e-commerce website to use important factors in designing commercial websites.
- To help online enterprises for finding the customer needs.
- To detect the level of transaction on a commercial website by the sale agency.

## 2. ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

In 1993 Roger Jang, suggested Adaptive Neuro Fuzzy Inference system (ANFIS). ANFIS can serve as a basis for constructing a set of fuzzy 'if-then' rules with appropriate membership functions to generate the stipulated input-output pairs. Here, the membership functions are tuned to the input-output data and excellent results are possible.

Fundamentally, ANFIS is about taking an initial fuzzy inference (FIS) system and tuning it with a back propagation algorithm based on the collection of input-output data. The basic structure of a fuzzy inference system consists of three conceptual components: A rule base, which contains a selection of fuzzy rules; a database, which defines the membership functions used in the fuzzy rules; and a reasoning mechanism, which performs the inference procedure upon the rules and the given facts to derive a reasonable output or conclusion [15].

These intelligent systems combine knowledge, techniques and methodologies from various sources. They possess human-like expertise within a specific domain - adapt themselves and learn to do better in changing environments. In ANFIS, neural networks recognize patterns, and help adaptation to environments. Fuzzy inference systems incorporate human knowledge and perform interfacing and decision-making. ANFIS is tuned with a back propagation algorithm based on the collection of input-output data [15].

The adaptive network based fuzzy inference system (ANFIS) is a useful neural network approach for the solution of function approximation problems [17]. An ANFIS gives the mapping relation between the input and output data by using hybrid learning method to determine the optimal distribution of membership functions [19]. Both artificial neural network (ANN) and fuzzy logic (FL) are used in ANFIS architecture [18]. Such framework makes the ANFIS modeling more systematic and less reliant on expert knowledge [16]. Basically, five layers are used to construct this inference system. Each ANFIS layer consists of several nodes described by the node function. The inputs of present layers are obtained from the nodes in the previous layers. To illustrate the procedures of an ANFIS, for simplicity, it is assumed those two inputs ( $x, y$ ) and one output ( $f_i$ ) are used in this system. The rule base of ANFIS contains fuzzy if-then rules of Sugeno type. For a first order two-rule Sugeno fuzzy inference system, the two rules may be stated as:

- Rule 1: If  $x$  is  $A_1$  and  $y$  is  $B_1$  then  $z$  is  $f_1(x, y)$
- Rule 2: If  $x$  is  $A_2$  and  $y$  is  $B_2$  then  $z$  is  $f_2(x, y)$

Where  $x$  and  $y$  are the inputs of ANFIS,  $A$  and  $B$  are the fuzzy sets  $f_i(x, y)$  is a first order polynomial and represents the outputs of the first order Sugeno fuzzy inference system[17].The ANFIS architecture is shown in figure1. The circular nodes represent nodes that are fixed whereas the square nodes are nodes that have parameters to be learnt [17].

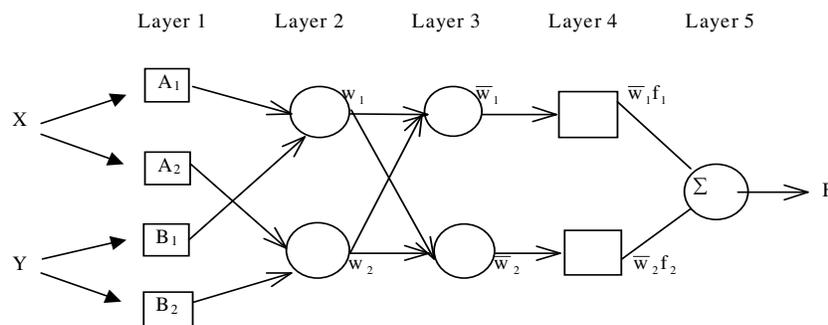


FIGURE 1: An ANFIS architecture for a two rule Sugeno system

A Two Rule Sugeno ANFIS has rules of the form:

$$\begin{aligned} \text{If } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \quad \text{THEN } f_1 = p_1x + q_1y + r_1 \\ \text{If } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \quad \text{THEN } f_2 = p_2x + q_2y + r_2 \end{aligned}$$

For the training of the network, there is a forward pass and a backward pass. We now look at each layer in turn for the forward pass. The forward pass propagates the input vector through the network layer by layer. In the backward pass, the error is sent back through the network in a similar manner to back propagation [20].

Layer 1:

The output of each node shown in equation 1:

$$\begin{aligned} O_{1,i} = \mu_{A_i}(x) \quad \text{for } i = 1,2 \\ O_{1,i} = \mu_{B_{i-2}}(y) \quad \text{for } i = 3,4 \end{aligned} \quad (1)$$

So, the  $O_{1,i}(x)$  is essentially the membership grade for  $x$  and  $y$ .

The membership functions could be anything but for illustration purposes we will use the bell shaped function given by equation 2:

$$\mu_{A_i}(x) = \frac{1}{1 + \left| \frac{x - c_i}{a_i} \right|^{2b_i}} \quad (2)$$

Where  $a_i, b_i, c_i$  are parameters to be learnt. These are the premise parameters.

Layer 2:

Every node in this layer is fixed. This is where the t-norm is used to 'AND' the membership grades - for example the product shown in equation 3:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \quad i = 1,2 \quad (3)$$

Layer 3:

Layer 3 contains fixed nodes which calculate the ratio of the firing strengths of the rules shown in equation 4:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2} \quad (4)$$

Layer 4:

The nodes in this layer are adaptive and perform the consequent of the rules shown in equation 5:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \quad (5)$$

The parameters in this layer ( $P_i, q_i, R_i$ ) are to be determined and are referred to as the consequent parameters

Layer 5

There is a single node here that computes the overall output shown in equation 6:

$$O_{s,i} = \sum_i \overline{w_i} f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \quad (6)$$

This then is how, typically, the input vector is fed through the network layer by layer. We now consider how the ANFIS learns the premise and consequent parameters for the membership functions and the rules.

There are a number of possible approaches but we will discuss the hybrid learning algorithm proposed by Jang, Sun and Mizutani which uses a combination of Steepest Descent and Least Squares Estimation (LSE).

It can be shown that for the network described if the premise parameters are fixed the output is linear in the consequent parameters.

We split the total parameter set into three:

$S$  = set of total parameters  $S_1$  = set of premise (nonlinear) parameters  $S_2$  = set of consequent (linear) parameters

So, ANFIS uses a two pass learning algorithm:

Forward Pass: Here  $S_1$  is unmodified and  $S_2$  is computed using a LSE algorithm.

Backward Pass:

Here  $S_2$  is unmodified and  $S_1$  is computed using a gradient descent algorithm such as backpropagation.

So, the hybrid learning algorithm uses a combination of steepest descent and least squares to adapt the parameters in the adaptive network [17].

The summary of the process is given below:

The Forward Pass:

1. Present the input vector.
2. Calculate the node outputs layer by layer.
3. Repeat for all data  $\rightarrow A$  and  $y$  formed.
4. Identify parameters in  $S_2$  using Least Squares Compute the error measure for each training pair.

Backward Pass:

1. Use steepest descent algorithm to update parameters in  $S_1$  (back propagation).
2. For given fixed values of  $S_1$  the parameters in  $S_2$  found by this approach are guaranteed to be the global optimum point.

### 3. RESEARCH METHODOLOGY

The proposed model has been established based on this principle that each real level of transactions in B2C websites includes two factors as follows:

1. Trust (T) level in B2C web site.
2. Competitive (C) in b2c web site for purchasing purposes.

Therefore, we propose to investigate into the truthfulness of the equations 7:

$$\begin{aligned} T &= H(S, F, D) \\ L_{B2C} &= G(T, C) \end{aligned} \tag{7}$$

The first part of equation 7 ( $T = H(S, F, D)$ ) that has three inputs, S is as the level of security, F is as the level of familiarity and D is as the level of design and Level of trust obtains of these three parameters performance.

The second part of equation 7 ( $L_{B2C} = G(T, C)$ ) that has two inputs, T is as the level of trust, and D is as the level of design and Level of B2C obtains of these two parameters performance. Figure 2 shows the structure of trust model.

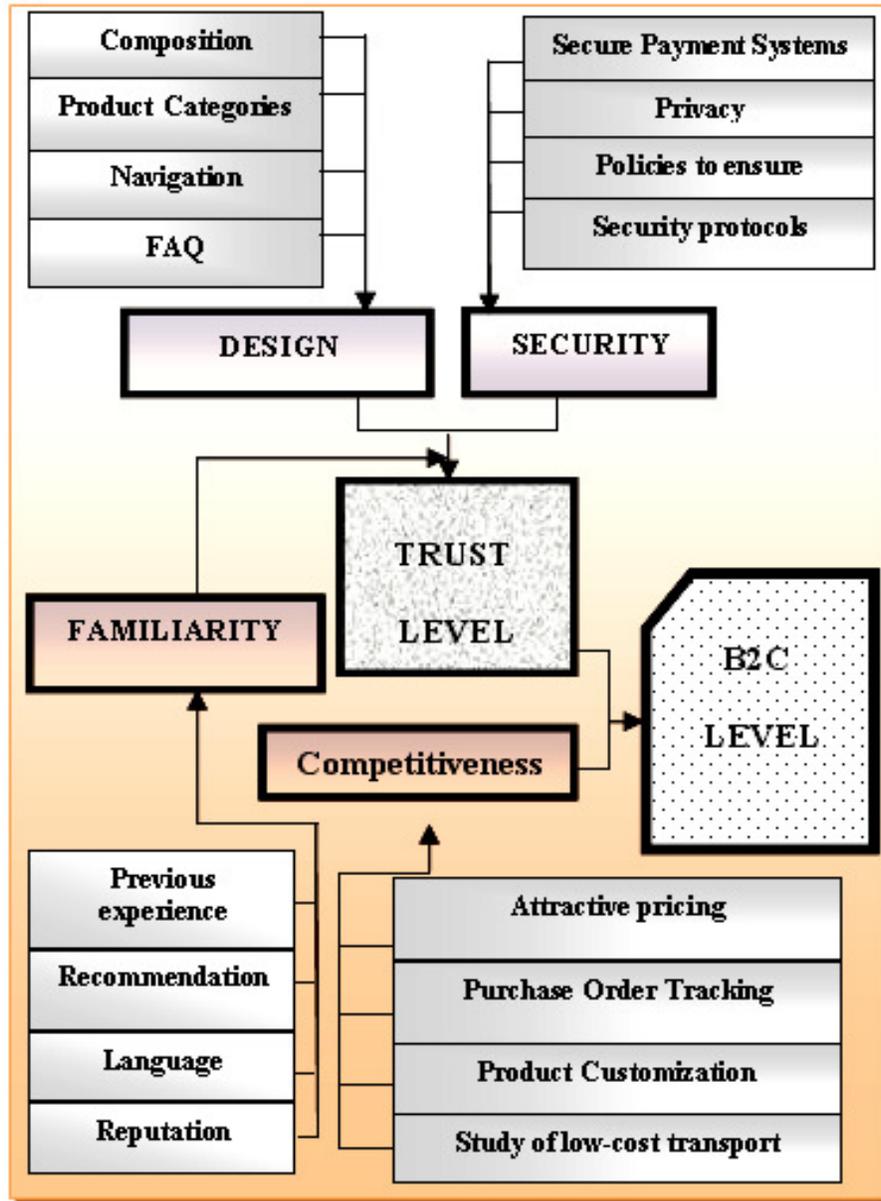
### 4. DATA COLLECTION AND ANALYSIS

This study used a web-based survey because of its advantages such as convenience; viable, effective way to access difficult-to-reach respondents [7]. The selected population in this study was included in two groups. The first group was included ten experts in the field of e-commerce and the Second group was included 150 numbers of E-Commerce and IT students. The first group completed the first questionnaire and after obtaining results from the first questionnaire and the second group completed the second. After collecting answer of first questionnaire and finding factors with higher priority, the second questionnaire was designed .it involves 4 major groups, too the method of scoring was chosen based on the likert scale of 5 degrees and 18 given questions in questionnaire were scored like 5 selections and in order of intensity of factor in each group from 0 to 4, like (0) very low (1) low (2) moderate (3) high (4) very high. In this questionnaire 16 questions are relative to 4 major groups, and 2 questions have been observed relative to the trust level and b2c level of website. Determining the credit of questionnaire has been done by counting kronbach's Alpha which has credit coefficient in accordance with table 1.

kronbach's alpha coefficient for each website		
Irshop.ir	Tobuy.ir	Parsim.com
0.85	0.83	0.83

TABLE 1: The counted credit coefficient of second questionnaire

The inserted credit coefficient in table 1 shows the acceptance of second questionnaire credit. After that customers referred to special website for experimental buying, it was asked the respondents to analyze 3 websites as parsim.com, tobuy.ir, irshop.ir.



**FIGURE 2:** The structure of trust model.

During this process, they should answer some questions in security groups, familiarity groups, design groups, competitiveness groups, and trust and b2c level.

Finally respondents should determine the trust level competitiveness and b2c level of website after analyzing website and answering the questions.

The order of answering the questions is that first of all the respondents should analyze the website and answer the questions in security groups, familiarity, design and then they were asked

to count the trust level and after that the it was weighted respondents were asked to evaluate the selective website for b2c dealings based on their expectation level of trust and competitiveness.

**Counting the Level of Security**

To count the level of security one sheet was created in EXCEL (security sheet) and linguistic values questionnaire were changed to numerical values. Actually it was related numerical value to each linguistic value (0, 1, 2, 3 and 4) in order to count the level of security the counted level of security is made by adding these values for each factor whose maximum for four factors is number 16. Also, its percent for level of security was counted that has been in table 2, and in general second equation has been used for level of security .the decided levels of design factor and familiarity are like table 2 too.

$$AccumulatedSecurityLevel = \sum_{i=1}^4 x_i \tag{8}$$

$$PercentageOfMaximum = (AccumulatedSecurityLevel / 16) * 100 \tag{9}$$

domain of values percent	Linguistic value
0-33	low
34-66	moderate
66-100	high

**TABLE 2:** linguistic and numeric values for security level

Some Pseudo code for counting the level of factors was written with vba programming in excel software. The sample Pseudo code for counting the level of security is shown in following:

```

Sub Security_Ind_Prot_Auth ()
Dim i as Integer
Dim col As Integer
Dim x as String
Worksheets ("security").Activate
For col = 2 To 6 Step 2
For i = 4 To 153 Step 1
Cells (i, col).Select
ActiveCell.Value = Trim ((ActiveCell.Value))
If StrComp (ActiveCell.Value, "very low",
vbTextCompare) = 0 Then
ActiveCell.Offset (0, 1).Value = 0
End If
If StrComp (ActiveCell.Value, "low", vbTextCompare) = 0 Then
ActiveCell.Offset (0, 1).Value = 1
End If
If StrComp (ActiveCell.Value, "moderate", vbTextCompare) = 0
Then
ActiveCell.Offset (0, 1).Value = 2
End If
If StrComp (ActiveCell.Value, "high", vbTextCompare) = 0 Then
ActiveCell.Offset (0, 1).Value = 3
End If
If StrComp (ActiveCell.Value, "very high", vbTextCompare) = 0
Then
ActiveCell.Offset (0, 1).Value = 4
End If
Next i
Next col
End Sub
    
```

**The Prioritize Factors Resulted From AHP Method**

The priority of counted factors in security groups, design, familiarity, and competitiveness has been noted in figure 2.

**Refining and Relative to Rule of the Level of Trust and B2C and Creating Member Functions.**

For refining and finding the rules of fuzzy model, it has been the clustering technique, and the kind of clustering has been chosen the fuzzy C-means (FCM) clustering in MATLAB software.

Equation 10 is as a major function in clustering k-means.

$$J_m(U, V) = \sum_{j=1}^n \sum_{i=1}^c u_{ij}^m \|X_j - V_i\|^2, \quad 1 \leq m < \infty \quad (10)$$

Where m is any real number greater than 1, u<sub>ij</sub> is the degree of membership of X<sub>j</sub> in the cluster i, X<sub>j</sub> is the jth of d-dimensional measured data, V<sub>i</sub> is the d-dimension center of the cluster, and ||\*|| is any norm expressed the similarity between any measured data and the center.

Fuzzy partition is carried out through an iterative optimization of Equation 8 with the update of membership u<sub>ij</sub> and the cluster centers V<sub>i</sub> by Equation 11 and 12:

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left( \frac{d_{ij}}{d_{ik}} \right)^{\frac{2}{m-1}}} \quad (11)$$

$$V_i = \frac{\sum_{j=1}^n u_{ij}^m X_j}{\sum_{j=1}^n u_{ij}^m} \quad (12)$$

The criteria in this iteration will stop when  $\max_{ij} \left[ \left| u_{ij} - \bar{u}_{ij} \right| \right] < \varepsilon$  , where ε is a termination criterion between 0 and 1 [21].All clustering activities were done in MATLAB software.

For example 27 centers of clusters for counting the level of trust is shown in figure 3 .after obtain the centers of clusters and save into DAT files, the centers of cluster were loaded into ANFIS. In fact this data is used as training data in ANFIS model.

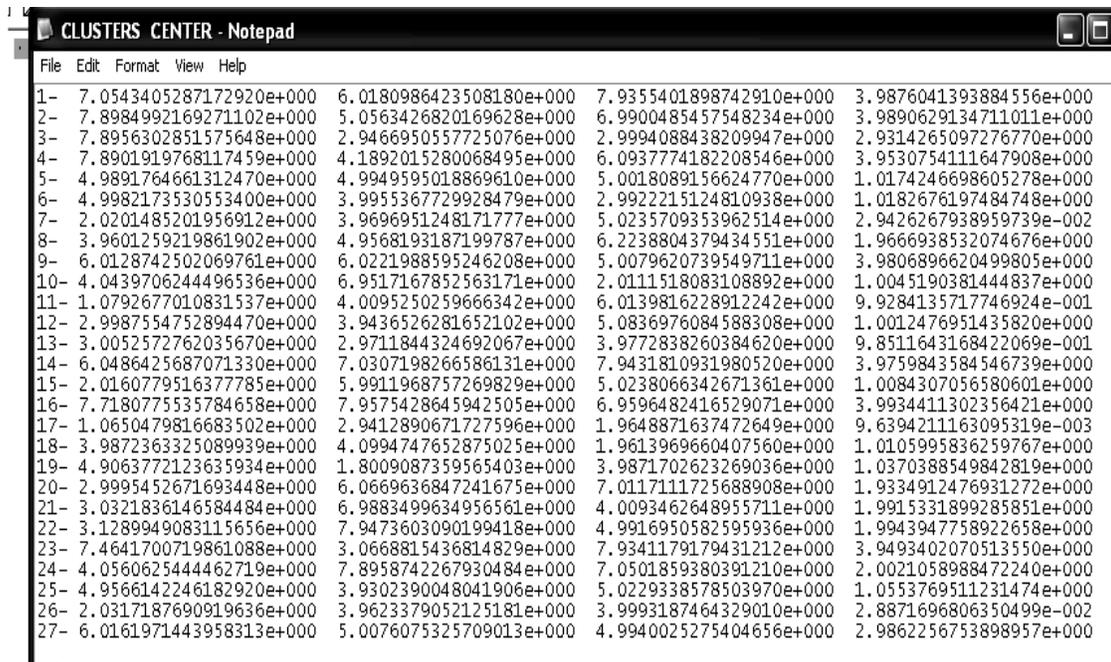


FIGURE 3: 27 centers of clusters for counting the level of security

**Expert ANFIS System**

The ANFIS system based on Expert knowledge contains 27 rules, 3 inputs and one single output for trust level. The structure of expert ANFIS is shown in figure 4. The fuzzy logic toolbox using the MATLAB software is employed to create the ANFIS model. In fuzzy logic tool box, relevant fis for trust model is created. In this model type of fis is selected Sugeno type.

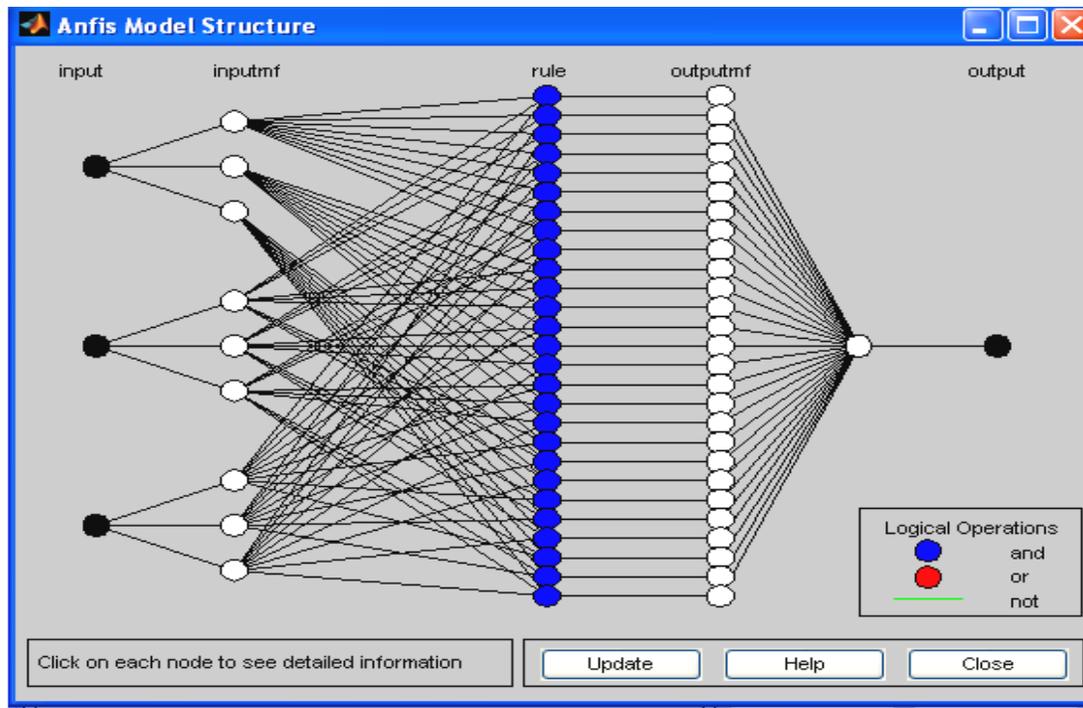
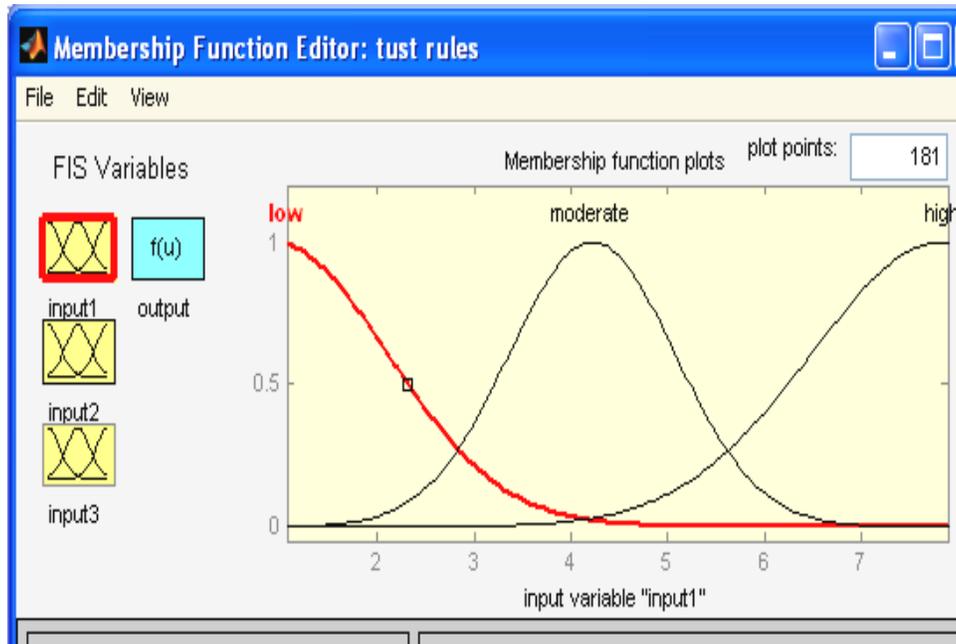


FIGURE 4: Expert ANFIS structure.

**Membership Function for Expert ANFIS**

Gaussmf are used to build the expert ANFIS model. The shape of membership functions after training the ANFIS for 100 epochs is shown figure 5.



**FIGURE 5:** Membership Function for ANFIS

ANFIS was proposed in an effort to formalize a systematic approach to generating fuzzy rules from an input-output data set. In ANFIS model 27 rules for trust level and 15 rules for B2C level is used. Table 3 shows number of rules of trust and B2C model.

TRUST	B2C LEVEL
NUMBER OF RULES	
27	15

**TABLE 3:** number of rules of the fuzzy system model obtained from FCM clustering

The rules describing the trust level are based on the degree of security, familiarity, and design that these degrees have been formulated like linguistic variable .similarly, the degree for trust level has been graded from very low to very high in 5 distinctive fuzzy, collections .these rules have been reached from the users ' answers after ordering, analyzing, and clustering .One of the collection rules of confidence level can be like following:

If (security = high and familiarity = low and design = moderate) then (trust =moderate).

Trust has been shown like a five fuzzy collections, while competitiveness and B2C level has been shown as 3 linguistic variables for fuzzy collection. One rule of the collection of b2c level rules can be like following:

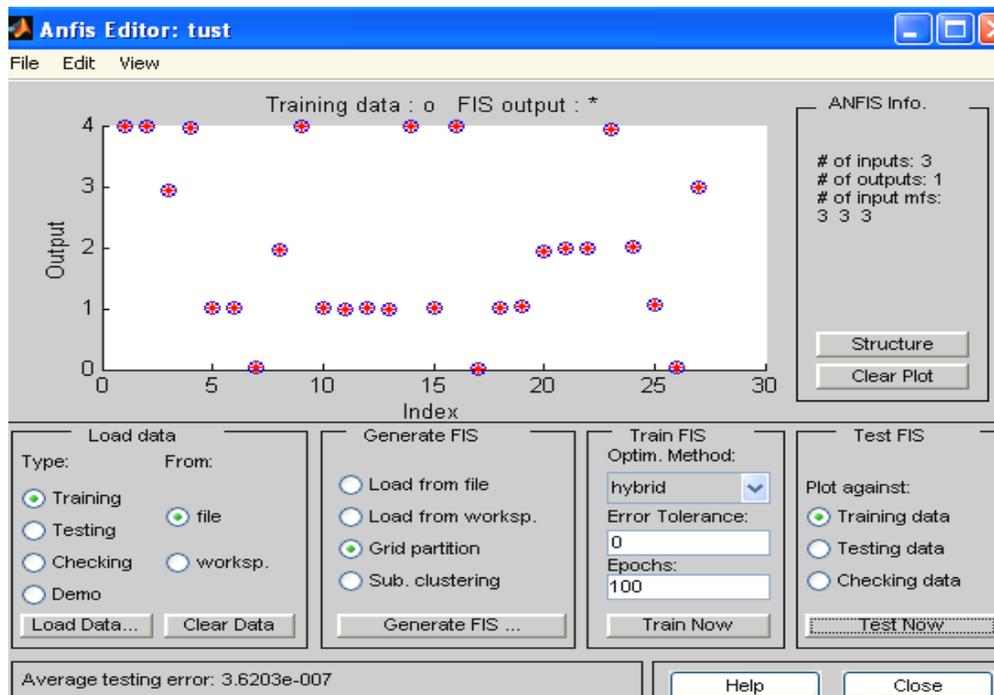
If (trust = low and competitiveness = highly) then (b2c level = moderate).

Totally there are 27 rules for trust that has been created by clustering the user's answers.

Also, the rules of the b2c level are 15 numbers. The rules from base of resultant system that consist of 2 separate and relative systems in order that the b2c level is gained despite the security inputs, familiarity inputs, design inputs, and competitiveness level of website.

### Training Data for Trust Level

The entire data set of trust level is 27 samples. They are referred to as training data, testing data and checking data. Upon training, the ANFIS shows the training error which reflects the how good the mapping function is. To validate the model, we further apply the testing data to see how the ANFIS behaves for known data. ANFIS maps the function onto the testing data as per the training. Having created the data set the next step is to train the network. This means we create a new FIS to fit the data into membership functions. Using the grid partitioning method, the ANFIS automatically selects the membership function and also generates the new FIS. Figure 6 shows training and testing data in ANFIS network that is loaded.



**FIGURE 6:** training and testing data in ANFIS network.

In figure 7, the course of error during the training of adaptive network is shown.

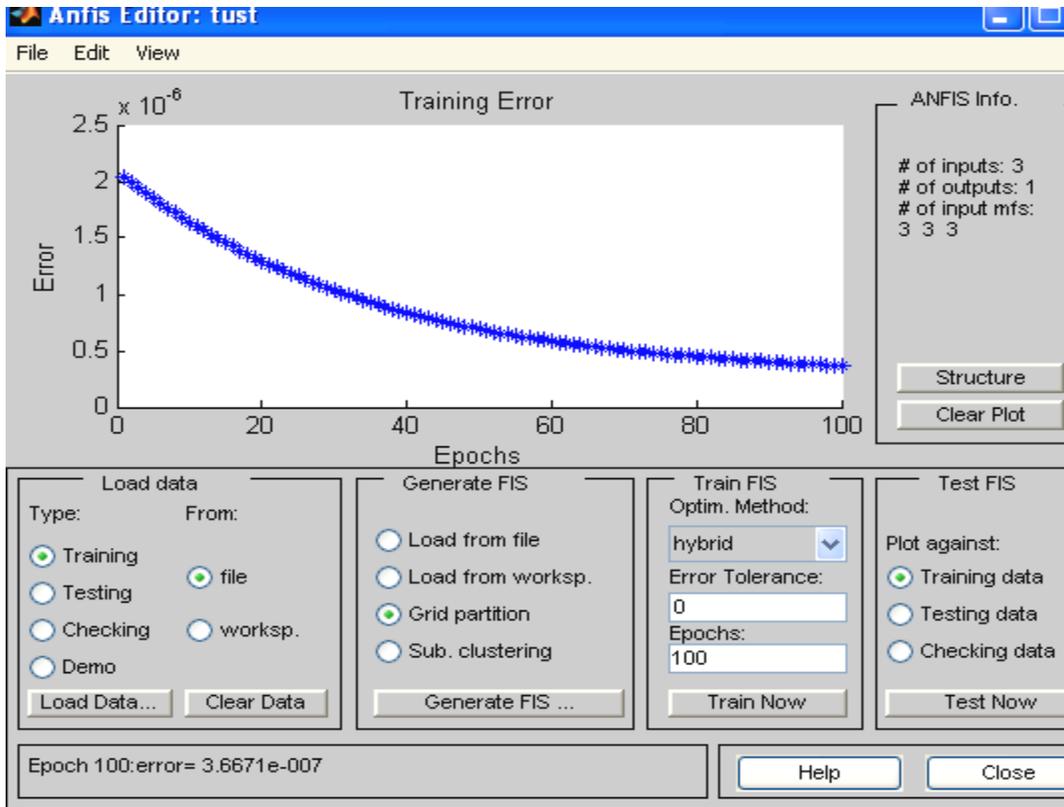


FIGURE 7: course of error during the training of adaptive network.

At the end of 100 training epochs, the network error (mean square error) convergence course of each ANFIS was derived. From the curve, the final convergence value is 3.6671e-007.

### 5. B2C AND TRUST LEVELS IN THE DEVELOPED FUZZY SYSTEM

After discovering the rules related to trust level, relevant inputs and outputs for earning trust level in fuzzy tool box to be organized and were created relevant membership for input and output figure 8 shows the fuzzy system that can be used to derive the trust level.

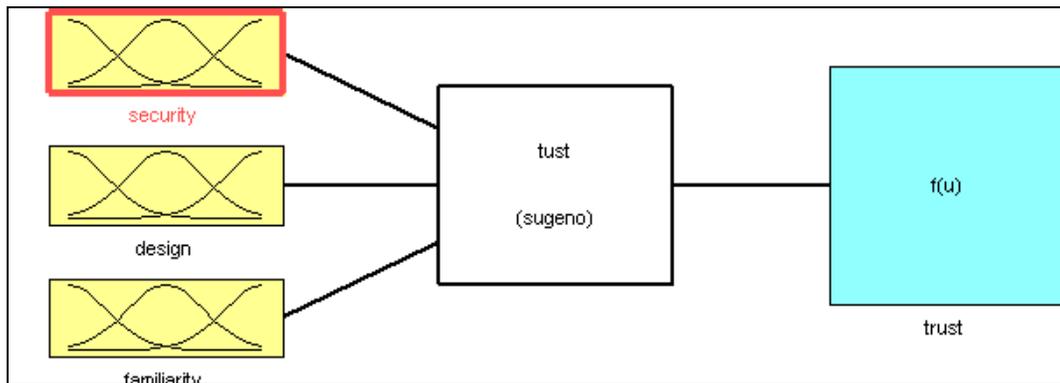
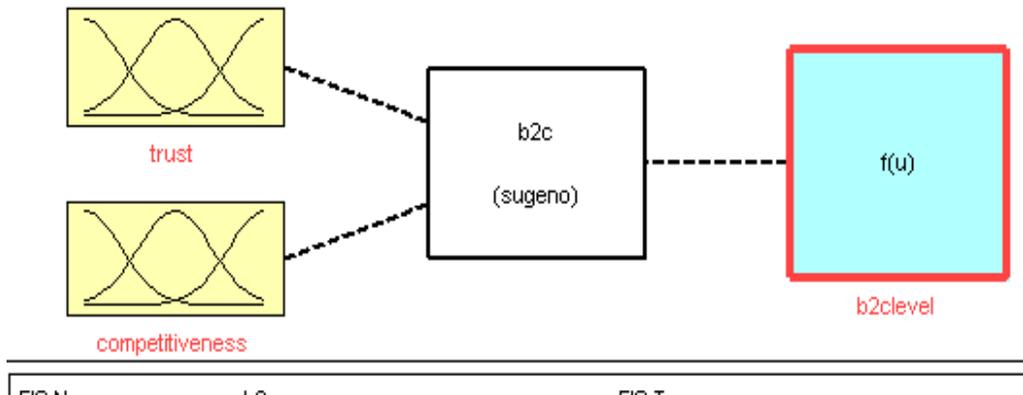


FIGURE 8: Fuzzy system to obtain trust level based on security, familiarity and design inputs

Also after that discovering the rules related to b2c level, relevant inputs and outputs for earning b2c level in fuzzy tool box were organized and were created relevant membership function for input and output. Figure 9 shows the fuzzy system that can be used to derive the b2c level.

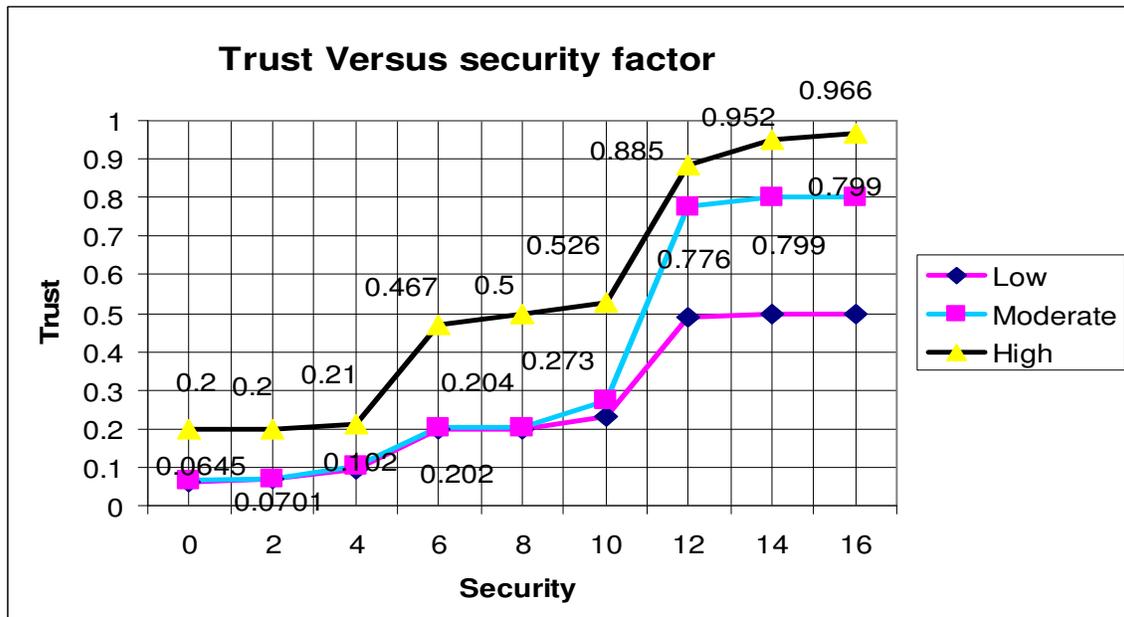


**FIGURE 9:** Fuzzy system to obtain B2C level based on trust, competitiveness inputs

### 6. ANALYSIS OF TRUST VERSUS SECURITY FACTOR

For complete understanding of participation needed in trust level, it is necessary to separately test the participation of each factor.

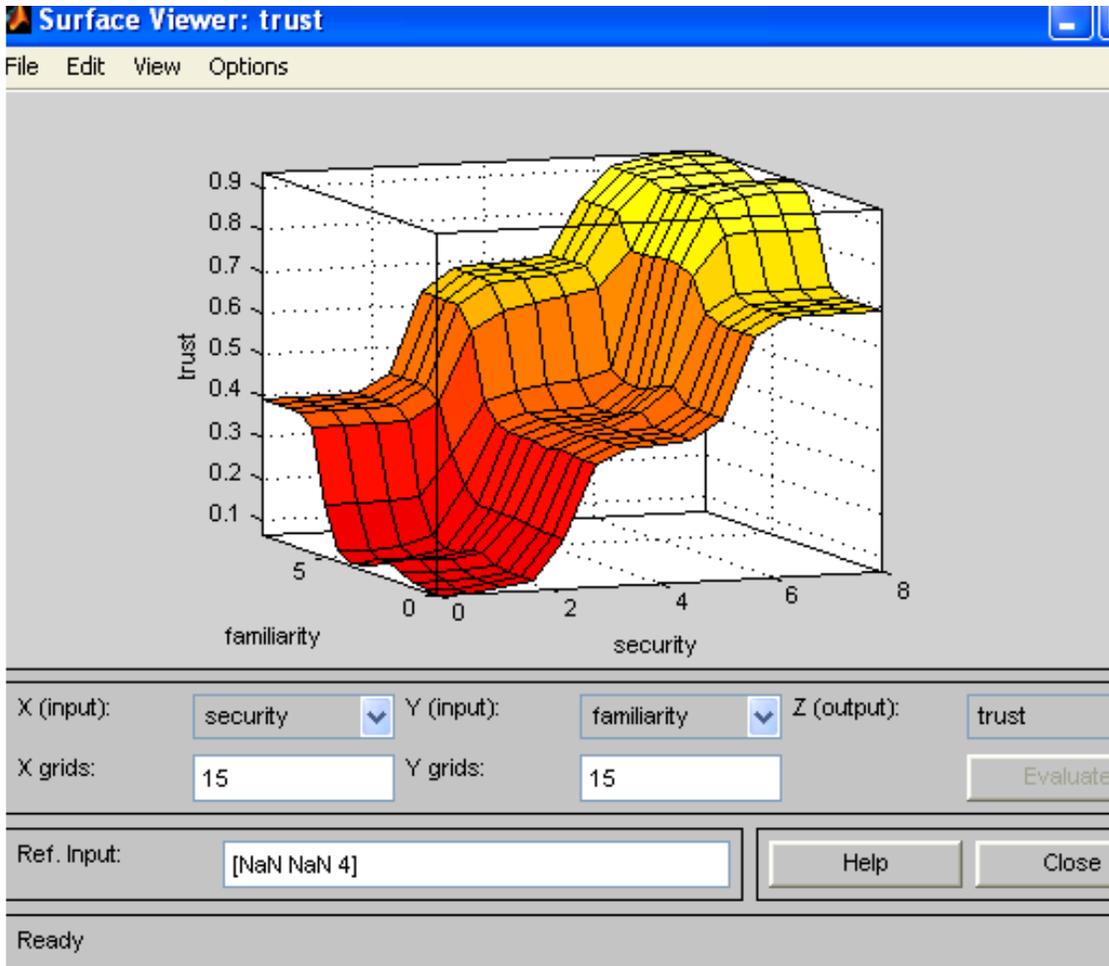
The Figure 10 shows contribution to Trust of a given Website originating from the Security. Therefore, the contribution from Familiarity and Design has been kept constant at three levels, namely: low, moderate and high corresponding to numeric values for Familiarity and Design of (1–7 and 15). Figure 4 shows that Trust level is monotonically increasing for increasing perceived security of a website for any given level of Familiarity and Design. However when both F and D is 'High' (numeric value of 15) the Trust level is at its maximum for maximum Security. The three curves have one common feature that they exhibit a 'staircase shaped' curvature.



**FIGURE 10:** Trust versus security factor

## 7. ANALYSIS OF TRUST VERSUS SECURITY AND DESIGN FACTOR

In this section Trust level is depicted as a continuous function of its input parameters as security and familiarity. Figure 10 intends to depict variation of Trust as encapsulated in the rules for Trust. The highest gradient for Trust is when Familiarity is 'moderate' and Security is 'moderate' to 'high'. This suggest that when people are somewhat familiar with a website then a small increase in security levels from between moderate to high security will boost their trust in a significant way. Looking at Figure 11 diagonally from (low, low) to (high, high) levels of Security and Familiarity one observes three plateaus where the last one is around 0.966, and remains at that level even when the input factors are increased further. This result is somehow unexpected and may be due to the fuzzy nature of the expert system where a 'Trust' level of 100% is unrealistic.



**FIGURE 11:** Trust level is positively related to levels of security and familiarity.

## 8. SIMULATE FUZZY TRUST MODEL THROUGH MATLAB SOFTWARE

Simulink is a tool used to visually program a dynamic system and look at results. Any logic circuit or a control system for a dynamic system can be built by using standard BUILDING BLOCKS available in Simulink Libraries. Various toolboxes for different techniques, such as Fuzzy Logic, Neural Networks, DSP, Statistics etc. are available with Simulink, which enhance the processing power of the tool. The main advantage is the availability of templates / building blocks, which avoid the necessity of typing code for small mathematical processes.

The model equations were implemented in MATLAB/ SIMULINK with standard blocks in a sub model. SIMULINK offers a block to include a Fuzzy controller designed by the MATLAB Fuzzy Toolbox.

After obtain of trust FIS, fuzzy trust model was simulated in MATLAB software. Figure 12 shows the simulation fuzzy trust model. A Simulink model shown in figure 12 is developed which has 2 fuzzy logic controllers with a rule viewer, a process, 2 multiplexer, difference element, 4 constant blocks, and a display window.

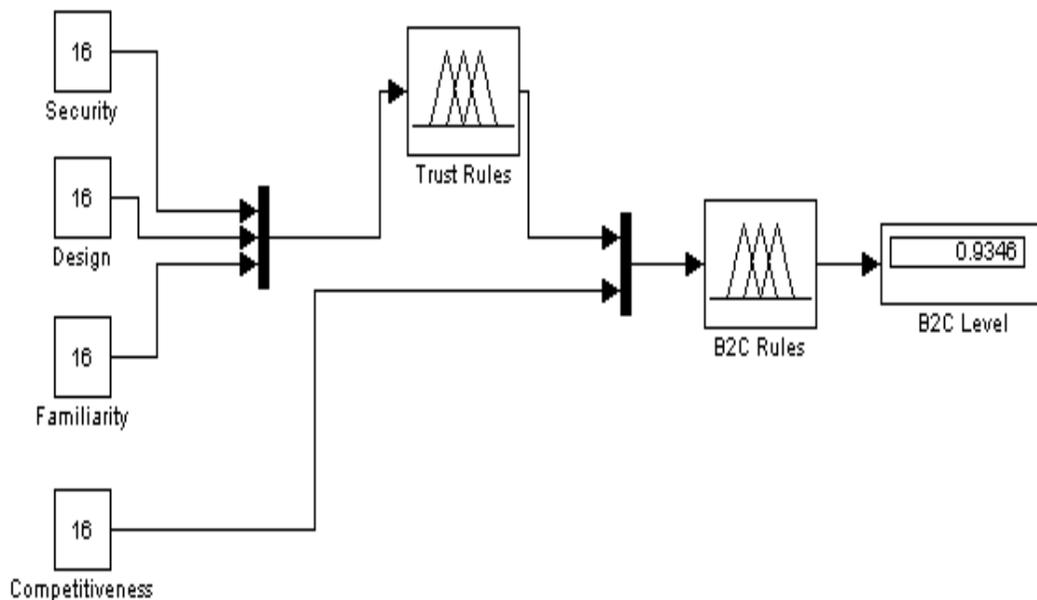


FIGURE 12: Block Diagram of fuzzy trust model.

## 9. CONCLUSION

Trust to B2C website depends on different factors. From our observation we have found that the easiest way to be reliable and trusted to the Customers is to maintain an easy and simple image in online environment.

The results of this research will be useful to online companies who are investing huge amounts of money on developing ecommerce web sites. Results show that trustworthiness plays a moderately important role in success of B2C e-commerce web sites. In addition, we used the AHP method to analyze the gathered data. By using AHP method and completing questionnaire through face to face meeting we finally were able to provide ranking of all factors and sub-factors. This ranking shows the relative importance of success factors compared to each other. This will

provide a clear map to webmasters and marketing managers of online companies who are trying to improve their web site's performance and reach customer satisfaction.

In addition the vendor can use the survey data to ascertain the Trust level of the site as per user's perception and rectify if needed if this is not obvious or is having a negative impact on the Trust level. Furthermore a measure of the competitiveness is directly deductible from this survey and could be used to retain or increase market share. Lastly as the usage of the survey procedure matures (possibly by providing incentives as discounts on a completed transaction) the Fuzzy Inference Systems could be modified and adjusted where necessary.

The results of this study will help businesses understand consumer online shopping for the trust factor. Although the model of this study can not include "trust" of all possible factors, but levels of "security", "design", "familiarity" and "competitiveness" are detected.

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# Measurement and Comparison of Productivity Performance Under Fuzzy Imprecise Data

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## Abstract

The creation of goods and services requires changing the expended resources into the output goods and services. How efficiently we transform these input resources into goods and services depends on the productivity of the transformation process. However, it has been observed there is always a vagueness or imprecision associated with the values of inputs and outputs. Therefore, it becomes hard for a productivity measurement expert to specify the amount of resources and the outputs as exact scalar numbers. The present paper, applies fuzzy set theory to measure and compare productivity performance of transformation processes when numerical data cannot be specified in exact terms. The approach makes it possible to measure and compare productivity of organizational units (including non-government and non-profit entities) when the expert inputs can not be specified as exact scalar quantities. The model has been applied to compare productivity of different branches of a company.

**Keywords:** Productivity; Performance Measure; Efficiency; Fuzzy Set Theory.

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## 1. INTRODUCTION

Any for-profit or non-profit organization requires a set of input resources in order to operate and survive. In return, it provides goods or value-adding services for its clients or stake-holders. The efficiency with which it consumes the resources to provide those services, is measured by the productivity of the organization. The notion of productivity therefore, focuses on exploring the relationship between the results achieved and the resources expended to achieve those results. In its basic form, the productivity is measured by the ratio of outputs (often goods or services) to the input resources (such as labor, capital, management, materials, energy etc).

Two most common measures of productivity are total measure and partial measure. Total measure includes all the input resources used in achieving the desired outputs whereas partial measure focuses on an incomplete list of input factors. If a partial measure focuses on one factor only (e.g. output per labor hour), it is referred to as single factor productivity measure whereas including more than one factor gives multi factor productivity. Sometimes, the use of single factor productivity can be misleading when there is a tradeoff involved among multiple inputs. For example, an organization may procure a better and more expensive software or technology that require less manual processing by its staff. Thus, it is possible to increase labor productivity but at the expense of increased technological costs. Therefore, if an improvement in the single factor productivity has been achieved, it is important to carefully examine the factors responsible for it or alternatively, have a more holistic approach towards productivity in terms of multi or total productivity measure. The main reasons for process productivity measurement are to monitor and control the organizational performance, judge the effectiveness of our decisions and to create a metric that causes behavioral change among the employees leading towards a productive unit.

Measuring productivity is not an easy task mainly because both output as well inputs are difficult to measure or count in a meaningful way. At first instance, the determination of the output seems quiet straightforward but due to problems in measuring the quality in service sector and the prohibitive costs of surveys; it becomes difficult to specify the exact amount of satisfactory output. Frequent service offerings, price and fees fluctuations, service aggregation

are some of the other issues that further add to the problem. Personnel, capital and management are considered to be the critical inputs to enhance productivity. Inappropriate time standards, disparity in employee skills and motivation levels, flexibility in over and underutilization of budgets, technological changes, economies of scale, unaccounted hidden costs and the difficulties in measuring the efforts of management, all these factors make it increasingly more difficult to ascertain the systems inputs in precise numerical terms. Thus determining both the system inputs as well as the results achieved is an onerous task and it is highly unlikely that an expert would be able to specify them in precise quantities. The fundamental flaw in the traditional approaches is that the imprecision of parameters is ignored. If such imprecision has not been incorporated into the productivity measurement model, it may result in misrepresentation of a situation which further leads to erroneous results. A model that explicitly incorporates the effects of such vagueness may be appropriate under these conditions. Fuzzy set theory has proved to be a very valuable tool to handle this type of imprecision or vagueness in data.

## 2. LITERATURE REVIEW

Miller and Rao [1] analyzed profit-linked productivity models at the firm level. The issue of productivity measurement under multiple criteria has been explored in Ray and Sahu [2] and the sensitivity of productivity measurement in a multi-product setting has been discussed in Ray and Sahu [3]. Garrigosa and Tatje [4] performed comparative study between profits and productivity as the two measures for performance. Sudit [5] discussed various productivity measures applicable in diverse settings. Chiou et al. [6] utilized quality function deployment in an approach that measures the productivity of technology in product development process. Agrell and West [7] critically examined a set of relevant properties that a productivity index must satisfy in order to assess the performance of a decision-making unit. Neely et al. [8] and Singh et al. [9] provided fairly detailed reviews of the previous research on productivity measurement. Suwignjo et al. [10] made use of tools such as cognitive maps, cause-effect diagrams, tree diagrams as well as analytical hierarchical process to quantify the effects of performance factors. Odeck [11] analyzed the efficiency and productivity growth of vehicle inspection services using DEA piecewise linear function and Malmquist indices. Ylvinger [12] presented multi-input and multi-output generalized structural efficiency measures based on linear programming DEA models to estimate the relative performance of an industry. Hannula [13] mentioned the trade-off between validity and practicality of productivity measures, and presented a practical method expressing total productivity as a function of partial productivity ratios with acceptable validity at an organizational unit level. Raa [14] presented an approach to quantify the inconsistency in aggregating the firm productivities through allocative efficiency and excess marginal productivities. Chavas and Mechemache [15] investigated the measures for technical, efficiency, allocative efficiency and price efficiency which can be conveniently summed into an overall efficiency measure. Cooper et al. [16] provided a fairly comprehensive account of applications of data envelop analysis (DEA) in performance measures. Majority of these publications do not address the vagueness or imprecision in data.

There are quite a few publications that explore the imprecise nature of the input-output data in productivity and efficiency measures. Chen et al. [17] applied fuzzy pattern recognition clustering techniques to determine productivity characters and a business unit is diagnosed through these characters. Joro et al. [18] showed that the DEA formulation to identify efficient units is similar to the multi-objective linear programming model based on the reference point approach to generate efficient solutions. Triantis and Girod [19] proposed a three stage approach to measure the technical efficiency in a fuzzy parametric programming environment by expressing input and output variables in terms of their risk-free and impossible bounds. Girod and Triantis [20] illustrated the implementation of a fuzzy set-based methodology that can be used to accommodate the measurement inaccuracies using risk-free and impossible bounds to represent the extremes for fuzzy input and output. Triantis and Eeckaut [21] used fuzzy pairwise dominance to measure the distance of a production plan from a frontier. Cooper et al. [22] provided imprecise data envelop analysis (IDEA) that permits a mixture of imprecise and exact data. Cooper et al. [23] further extended it for assurance region and cone-ratio concepts by placing bounds on variables rather than data values. The approach is applicable to bounded data and data sets satisfying ordinal relations and has been illustrated through an application to branch offices of a telecommunication company in Korea. Cooper et al. [24] removed a limitation of IDEA and assurance region IDEA which required access to actually attained maximum

values in the data, by introducing a dummy variable for normalization of maximal values. Despotis and Smirlis [25] developed an approach to transform a non-linear DEA model to a linear programming equivalent, on the basis of the original data set, by applying transformations only on the variables. Despotis and Smirlis [25] model allows post-DEA discriminating among the efficient units by endurance indices and is an alternative to Cooper et al. [22]. Zhu [26] reviewed and compared two different approaches dealing with imprecise DEA; one using scale transformations and the second using variable alterations through an efficiency analysis. Zhu [26] presented these two approaches as improvements over Cooper et al. [23]. Triantis [27] proposed a fuzzy DEA approach to compute fuzzy non-radial technical efficiency measures. Kao and Liu [28] provided a fuzzy DEA procedure by transforming it into a crisp DEA model using the  $\alpha$ -cut concept of fuzzy set theory and the resulting efficiency measures are provided in terms of fuzzy sets. Kao and Liu [29] applied a maximizing–minimizing set method for fuzzy efficiency ranking of 24 university libraries in Taiwan. Lertworasirikul [30] and Lertworasirikul et al. [31] proposed two main approaches; a possibility approach and a credibility approach to resolve the problem of ranking fuzzy sets in fuzzy DEA models. León et al. [32] developed fuzzy versions of the classical BCC-DEA model by using ranking methods based on the comparison of  $\alpha$ -cuts. Entani et al. [33] and Wang et al. [34] changed fuzzy input - output data into intervals using  $\alpha$ -level sets and suggested two interval-DEA models. Dia [35] fuzzy-DEA model requires the decision maker to specify an aspiration level and a safety  $\alpha$ -level in order to transform it into a crisp DEA model. Kao and Liu [36] transformed fuzzy input and output data into intervals by using  $\alpha$ -level sets and fuzzy extension principle and built a family of crisp DEA models for the intervals. Soleimani-damaneh et al. [37] addressed some computational and theoretical pitfalls of the fuzzy DEA models and provided a fuzzy DEA model to produce crisp efficiencies for DMUs with fuzzy input and output data. You et al. [38] presented a fuzzy multiple objective programming approach to imprecise data envelopment analysis (IDEA) with an increased discriminating power than available from Cooper et al. [22]. Wang et al. [39] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic and the models are applied to evaluate the performances of eight manufacturing enterprises in China.

As evident from this literature survey, most of the existing approaches that deal with imprecise nature of data, present several variations of the DEA approach in a fuzzy environment. DEA-based approaches are optimization approaches in the sense that they identify the best set of weights to identify the maximum achievable efficiency for an organizational unit, rather than identifying its true efficiency. Secondly, DEA-based approaches provide a relative measure of efficiency amongst a set of decision making units (DMU's). These approaches compare the DMU's input and output against a composite input and output. If a particular DMU uses more inputs than the composite, it is termed as inefficient and vice-versa. As a potential drawback, if one DMU has substantially higher performance than others, most of the DMU's (except the one with exceptional performance) are likely to be termed as inefficient. Similarly, a DMU with an exceptionally low performance may render other DMU's as efficient, not because of their own performances but due to the relative nature of the measurement. Furthermore, when new DMU's enter or leave the system (e.g. a new member joining or leaving a supply chain), efficiencies need to be re-evaluated. This establishes the need to have an approach that measures real productivity of a system in an absolute sense and in an environment involving imprecision and vagueness of data. This is one area where the present paper intends to contribute.

The next section deals with some basic concepts of fuzzy set theory that have been used to develop the proposed framework to model productivity. The subsequent section presents a fuzzy set theoretic model for multi factor productivity. The proposed model is illustrated through an application to 13 branches of a credit union. The computational experience and some important observations drawn from this experience are discussed. Finally, concluding remarks and some directions for further research are presented.

### 3. BASIC FUZZY CONCEPTS

Since its inception by Lofti Zadeh [40], fuzzy logic has revolutionized the business world with its ability to model the imprecise decision making situations. This section presents some basic concepts in fuzzy set methodology that have been utilized to develop the proposed model in this paper. For details of these concepts, the reader is referred to Kaufmann and Gupta [41] and Zimmermann [42].

#### 3.1 Fuzzy Set and Membership Function

A fuzzy set A in X is characterized by a membership function,  $\mu_A(x)$  which associates with each element in X, a real number in the interval [0,1] with the value of  $\mu_A(x)$  at x representing the “grade of membership” of x in A.

#### 3.2 Triangular Fuzzy Number (TFN)

A triangular fuzzy number is a linear approximation for a normal and convex fuzzy set and is represented by the triplet  $(a_1, a_2, a_3)$ . Under this linear approximation, the membership function for triangular fuzzy number can be expressed as follows:

$$\mu_A = \begin{cases} 0, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2 \\ \frac{x - a_3}{a_2 - a_3}, & a_2 \leq x < a_3 \\ 0, & x > a_3 \end{cases}$$

(1)

#### 3.3 $\alpha$ -cut of a Fuzzy Set

As shown in Figure 1,  $\alpha$ -cut of a fuzzy set denoted by  $A_\alpha$ , is a subset of its domain that allows us to represent a fuzzy set in a confidence interval form, as  $A_\alpha = [a_1^\alpha, a_3^\alpha]$ . Using equation (1), the triangular fuzzy number approximation in its  $\alpha$ -cut confidence interval form can be expressed as follows:

$$A_\alpha = [a_1^\alpha, a_3^\alpha] = [a_1 + (a_2 - a_1)\alpha, a_3 + (a_2 - a_3)\alpha]$$

(2)

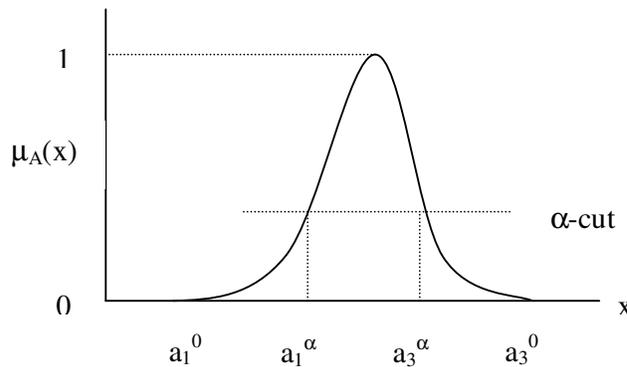


FIGURE 1:  $\alpha$ -cut of a fuzzy set

#### 4. FUZZY SET MODEL FOR PRODUCTIVITY

Having stated the fuzzy concepts to develop this methodology, we now present the proposed fuzzy model for multiple factor productivity. The present appear does not specifically deal with the issue of aggregating the inputs and outputs. The issue of heterogeneity of outputs and inputs and their aggregation through appropriate weights has been addressed by various authors. For more details, the reader is referred to [43], [44], [45], [46], [47]. The present paper assumes that these conversion weights are input to the model and therefore, they are known.

Most transformation processes have positive inputs and outputs, but without any loss of generality, it is possible to define fuzzy sets over negative real number domains. Although, in order to avoid a zero denominator in productivity measure, we make the assumption that aggregate sum of inputs is non-zero which is a fairly standard assumption.

##### 4.1 Notation

$P^k$  = Fuzzy measure of the productivity of the  $k^{th}$  unit.

$O_i^k$  = Triangular fuzzy number representing the  $i^{th}$  output of the  $k^{th}$  unit.

(in

case of single output measure,  $O^k$  could be used).

$I_j^k$  = Triangular fuzzy number representing the  $j^{th}$  input of the  $k^{th}$  unit.

$w_j^k$  = Unit conversion factor or weight for the  $j^{th}$  input of the  $k^{th}$  unit.

$z_i^k$  = Unit conversion factor or weight for the  $i^{th}$  output of the  $k^{th}$  unit.

$k = 1, 2, 3, \dots, m$ .  $i = 1, 2, 3, \dots, p$ .  $j = 1, 2, 3, \dots, n$ .

Triangular fuzzy numbers  $O^k$  and  $I_j^k$  can be specified in its triplet form as

$$O_i^k = (z_i^k * o_{i1}^k, z_i^k * o_{i2}^k, z_i^k * o_{i3}^k)$$

$$I_j^k = (w_j^k * i_{j1}^k, w_j^k * i_{j2}^k, w_j^k * i_{j3}^k),$$

where  $o_{i1}^k, o_{i2}^k, o_{i3}^k, i_{j1}^k, i_{j2}^k, i_{j3}^k \in R^+$

##### 4.2 The Model

The fuzzy measure of multifactor productivity can be expressed by the following equation

$$P^k = \frac{O^k}{\sum_{j=1}^n I_j^k} \quad \text{for } k = 1, 2, 3, \dots, m$$

(3)

Following equation (2), we can express the triangular fuzzy numbers  $O^k$  and  $I_j^k$  in their confidence interval form as follows:

$$O^k = [o_1^k + \alpha(o_2^k - o_1^k), o_3^k + \alpha(o_2^k - o_3^k)]$$

(4)

$$I_j^k = [i_{j1}^k + \alpha(i_{j2}^k - i_{j1}^k), i_{j3}^k + \alpha(i_{j2}^k - i_{j3}^k)] \quad \text{for } j = 1, 2, \dots, n$$

(5)

For two fuzzy sets A and B,  $A=[a_1, a_3]$  and  $B=[b_1, b_3]$  with  $a_1, a_3, b_1, b_3 \in R$ , the addition of these sets,  $A (+) B$ , is given as (Kaufmann and Gupta 1988):

$$A (+) B = [a_1, a_3] (+) [b_1, b_3] = [a_1 + b_1, a_3 + b_3]$$

(6)

This addition operator can be conveniently extended for three or more sets.

Making use of the summation result (6) for fuzzy sets, the summation of the process inputs of equation (5) is expressed as follows:

$$\sum_{j=1}^n I_j^k = \left[ \sum_{j=1}^n (i_{j1}^k + \alpha(i_{j2}^k - i_{j1}^k)), \sum_{j=1}^n (i_{j3}^k + \alpha(i_{j2}^k - i_{j3}^k)) \right] \quad (7)$$

Substituting equations (4) and (7) into equation (3), we obtain the productivity measure for the  $k^{\text{th}}$  process as follows:

$$P^k = \frac{[o_1^k + \alpha(o_2^k - o_1^k), o_3^k + \alpha(o_2^k - o_3^k)]}{\left[ \sum_{j=1}^n (i_{j1}^k + \alpha(i_{j2}^k - i_{j1}^k)), \sum_{j=1}^n (i_{j3}^k + \alpha(i_{j2}^k - i_{j3}^k)) \right]}, \text{ for } k = 1, 2, \dots, m \quad (8)$$

For two fuzzy sets A and B, where  $A=[a_1, a_3]$  and  $B=[b_1, b_3]$  with  $a_1, a_3, b_1, b_3 \in \mathbb{R}$ , the division of these sets,  $A/B$ , is given as

$$\begin{aligned} A/B &= [a_1, a_3] / [b_1, b_3] \\ &= [\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)] \end{aligned}$$

This general formula is applicable when inputs and outputs are not necessarily positive real numbers, however, if the sets A and B have been defined over  $\mathbb{R}^+$ , as is the case in most physical transformation processes, then it can be shown that the division operator simplifies to

$$A/B = [a_1, a_3] / [b_1, b_3] = [a_1/b_3, a_3/b_1] \quad (9)$$

Since all the output and input resource estimates belong to  $\mathbb{R}^+$ , we apply the simplified division operator (9) to equation (8) to compute:

$$P^k = \left[ \frac{o_1^k + \alpha(o_2^k - o_1^k)}{\sum_{j=1}^n (i_{j3}^k + \alpha(i_{j2}^k - i_{j3}^k))}, \frac{o_3^k + \alpha(o_2^k - o_3^k)}{\sum_{j=1}^n (i_{j1}^k + \alpha(i_{j2}^k - i_{j1}^k))} \right] \quad (10)$$

Equation (10) expresses multifactor productivity measure in its interval form. Next, we express this productivity in its triplet form by finding its  $\alpha$ -cut's at  $\alpha=0$  and  $\alpha=1$  levels. The above equation (10) reduces to its triplet form as follows:

$$P^k = \left[ \frac{o_1^k}{\sum_{j=1}^n (i_{j3}^k)}, \frac{o_2^k}{\sum_{j=1}^n (i_{j2}^k)}, \frac{o_3^k}{\sum_{j=1}^n (i_{j1}^k)} \right] \tag{11}$$

It is worth noting that equation (11) is a more general approach for computing multifactor productivity. The traditional approach can be deduced as a special case of this more general approach.

**4.3 Traditional Approach as a Special Case**

If there were no imprecision involved in specifying the model parameters, then the three triplets for inputs and outputs will coincide as follows:

$$o_1^k = o_2^k = o_3^k = o^k \text{ and } i_{j1}^k = i_{j2}^k = i_{j3}^k = i_j^k$$

Putting this into equation (11) implies

$$P^k = \left( \frac{o^k}{\sum_{j=1}^n (i_j^k)} \right), \text{ which is the traditional crisp measure of productivity.}$$

**4. COMPARISON OF PRODUCTIVITY**

The productivity measure in itself has a limited usefulness unless it is compared with the productivity of another organizational unit or industry average. This could be modeled through the removal concept of a fuzzy set at reference point  $z = 0$ . The removal of a fuzzy set that follows a possibility distribution and it amounts to finding an expected value representation of a fuzzy set. Such a transformation is necessary because we often work in a fuzzy decision environment where we have to deal with vague information and data, but we are often required to take crisp decisions. As defined in Kauffman and Gupta (1988), the removal of a fuzzy number is the average of its LHS removal  $R_l(A, z)$  and its RHS removal  $R_r(A, z)$ , where  $z \in R$ . Therefore, removal essentially measures the average distance of a fuzzy number from a reference point  $z$ . It can be shown that at reference point  $z=0$ , the removal of fuzzy productivity measure is given by:

$$R(P^k, z = 0) = \left( \frac{1}{4} \right) \left( \frac{o_1^k}{\sum_{j=1}^n (i_{j3}^k)} + \frac{2 \cdot o_2^k}{\sum_{j=1}^n (i_{j2}^k)} + \frac{o_3^k}{\sum_{j=1}^n (i_{j1}^k)} \right) \tag{12}$$

If we have two organizational units with their respective removals as  $R(P^1, z = 0)$  and  $R(P^2, z = 0)$ . Unit 1 will be more productive than unit 2 if  $R(P^1, z = 0) > R(P^2, z = 0)$ . This concept can be used to compare the productivities of a number of processes or business units by ranking them in the descending order of removals.

## 5. MODEL ILLUSTRATION

We illustrate the usefulness of our model by evaluating thirteen branches of a credit-union ([48]) in terms of their productivity performance. Without loss of generality, we consider each branch to have four relevant inputs: number of personnel or staff, number of computers or IT infrastructure, area of the branch in square footage and the administrative costs. The output measure is daily transactional volume. For unit and time aggregation purposes, these inputs are converted to common monetary unit by applying the following conversion weights from the problem situation: \$50,000 per staff member including perks and benefits, \$7,000 per computer or other technological equipment, \$2,500 per square foot and on an average 300 business days in a year. For each input and output, the decision maker assigns his best estimate of the value i.e. an estimate in which he has maximum belief. Furthermore, the decision maker also provides a range outside which the input or output values are not likely to lie. For the credit union branches, there is no ambiguity involved in specifying the number of IT infrastructure and branch areas, so these variables are treated as non-fuzzy but the proposed fuzzy model is general enough to handle both fuzzy and non-fuzzy inputs. This estimation of inputs in terms of triangular fuzzy numbers is presented in Table 1.

Branch #	Input 1 (Personnel)	Input 2 (IT)	Input 3 (Space) (1000's sq ft)	Input 4 (Expenses) (\$100,000)
1	(4,6,8)	(8,8,8)	(4,4,4)	(147.30, 628.95, 1036.56)
2	(6,7,8)	(8,8,8)	(2.56, 2.56, 2.56)	(411.44, 716.50, 957.01)
3	(7,9,11)	(10,10,10)	(1.34,1.34,1.34)	(287.92,428.13,603.01)
4	(8,10,12)	(12,12,12)	(1.5,1.5,1.5)	(242.77,722.95,945.69)
5	(4,6,8)	(9,9,9)	(1.68,1.68,1.68)	(433.55,692.00, 807.29)
6	(5,7,9)	(7,7,7)	(3.75,3.75,3.75)	(455.71,777.62,1162.68)
7	(7,9,11)	(10,10,10)	(3.31,3.31,3.31)	(780.11,1145.37,1711.52)
8	(6,8,10)	(7,7,7)	(1.5,1.5,1.5)	(149.69,755.97,1358.58)
9	(8,10,12)	(8,8,8)	(1.6,1.6,1.6)	(610.24,1019.76,1829.51)
10	(6,8,10)	(9,9,9)	(1.72,1.72,1.72)	(216.25,712.23,945.11)
11	(9,10,11)	(7,7,7)	(1.92,1.92,1.92)	(396.64,905.15,1249.84)
12	(7,9,11)	(8,8,8)	(4.43,4.43,4.43)	(231.06,749.94,1398.88)
13	(6,8,10)	(10,10,10)	(2.5,2.5,2.5)	(177.56,778.15,1444.62)

**TABLE 1:** Input Data for the 13 Credit Union Branches

The model given in equations (11, 12) was used to compute the fuzzy productivity and the fuzzy removal of productivity (i.e. its crisp equivalent) for the thirteen branches. The relevant output, fuzzy and crisp productivities for this data set are provided in Table 2.

Branch #	Output (Transaction Volume)	Fuzzy productivity	Crisp of productivity
1	(55830,56570,57318)	(0.147, 0.232,0.688)	0.325
2	(36740,36800,36852)	(0.107,0.141,0.231)	0.155
3	(38004,38446,38783)	(0.177,0.247,0.357)	0.257
4	(35469,35685,36017)	(0.107,0.140,0.379)	0.191
5	(52927,53869,54817)	(0.186,0.219,0.344)	0.242
6	(70254,72446,78574)	(0.167,0.248,0.427)	0.273

7	(32585,35856,37443)	(0.054,0.087,0.130)	0.090
8	(42900,45027,47270)	(0.092,0.169,0.744)	0.294
9	(85399,86221,87220)	(0.137,0.243,0.400)	0.255
10	(46924,47142,47316)	(0.142,0.186,0.540)	0.263
11	(36652,40482,44298)	(0.084,0.127,0.296)	0.158
12	(39582,39594,39620)	(0.078,0.137,0.344)	0.174
13	(56144,57484,58816)	(0.111,0.204,0.724)	0.31

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**TABLE 2:** Output Data and Productivity Performance Measurement Results

## 6. EXPERIMENTAL RESULTS

The main objective in this experimental analysis was to test the sensitivity of productivity measure w.r.t. right hand side fuzziness, left hand side fuzziness and both types of fuzziness. We first reduced the left triplet range for a sample productivity data set in decrements of 5%, then its right triplet range in decrements of 5% and finally reduced both triplet ranges simultaneously in decrements of 5%. The results of experimental analysis are summarized in the following Table 3.

Percent Decrement	Percentage change in productivity w.r.t. crisp measure while reducing			
	LHS triplet	LHS triplet	Both triplets	
5		4.91	5.345	4.880
10		4.46	5.315	4.445
15		4.04	5.300	4.010
20		3.635	5.285	3.620
25		3.245	5.270	3.245
30		2.870	5.270	2.885
35		2.510	5.255	2.555
40		2.150	5.255	2.240
45		1.820	5.240	1.955
50		1.490	5.240	1.685
55		1.175	5.240	1.430
60		0.875	5.255	1.205
65		0.590	5.255	0.995
70		0.305	5.270	0.800
75		0.035	5.285	0.620
80		-0.235	5.300	0.470
85		-0.490	5.315	0.320
90		-0.730	5.345	0.200
95		-0.970	5.360	0.095
100		-1.210	5.390	0.005

**TABLE 3:** Results of Experimental Analysis

A number of interesting observations can be drawn from the experimental experience.

- First of all, the fuzzy measure of productivity performance is robust in the sense that errors in specifying the triangular fuzzy numbers do not result in a large change in productivity measure. The maximum change found was 5.39% for the experimental case.
- Changes in the RHS triplet range does not give a large change in the productivity suggesting that productivity change is not significantly affected by the RHS fuzziness. LHS triplet fuzziness was found to have a more profound effect on productivity measures. Furthermore, LHS triplet ranges can result in both underestimation and overestimation in productivity measure w.r.t. crisp measure. Therefore, much more careful attention should be given in selecting the LHS triplet ranges.
- Finally, as we change both the ranges together, the fuzzy measure of productivity approaches the traditional crisp measure asymptotically. At the point of maximum belief, productivity measure from the fuzzy formula becomes the same as the crisp measure from the traditional formula. This further reinforces the point that

traditional measure of productivity is a special case of a more general fuzzy measure of productivity given in the present paper.

## 7. CONCLUDING REMARKS

The present paper recognizes that measurements of system inputs and outputs for productivity measurement is a difficult task resulting in vagueness or imprecision in data. The paper proposes an approach based on fuzzy set theory to model this type of vagueness. The proposed approach provides a general model for productivity measurement. The traditional single and multi factor productivity measures can be deduced as its special cases. The paper further presents a method to compare the productivity performance across different organizations using the fuzzy removal concept. It may be noted that the approach is equally applicable for non-profit and non-government organizations. The approach is illustrated with the help of a case example from a credit union, and the computational experience and important observations are also discussed.

The literature dealing with the imprecise nature of data mainly consists of variations of DEA model under fuzzy logics. These approaches are relative in nature and identify the maximum achievable efficiency for an organizational unit amongst a set of similar decision making units within an organization. An absolute and real treatment of efficiency is needed for organizations that consist of seemingly unrelated organization units or entities. The relative approach is an intra-organization approach but for competitive reasons, an inter-organization approach is preferable. That is one area where approaches such as the current one, could be potentially useful.

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# A Supply Chain Design Approach to Petroleum Distribution

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## Abstract

Product distribution account for a significant portion of the logistical costs of a product. Distribution activities are repetitive in nature and they impact the delivery lead time to customers. A well designed supply chain network can substantially improve these costs and lead times. This paper presents a supply chain network design approach for distribution of petroleum products of a retailer by identifying the depot locations and gas station allocations. A heuristic procedure to solve large sized problems is also recommended. Finally, concluding remarks and recommendations for further research are presented.

**Keywords:** Supply Chain; Petroleum Distribution; Mathematical Programming.

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## 1. INTRODUCTION

Supply chain managers frequently come across location and allocation problems at the design phase of a supply chain that involves determining the number of warehouses and assigning retail allocations. It appears imperative to treat the location and allocation decisions simultaneously. But due to the complexity of the problem, a breakdown into two stages i.e. location and allocation, helps to manage the complexity of large sized problems. The location decision involves substantial investment. Since it can't be changed frequently, therefore, it has long term implications. The warehouse location acts as a prelude to the overall process of supply chain design with far reaching effects on the performance of the logistics and distribution system. On the other hand, the allocation decision is more dynamic in nature as these assignments need to reviewed and changed from time to time as the supply chain grows. This is especially true for the natural gas and petroleum products distribution in developing countries where the retail outlets and gasoline fuel stations for such products are mushrooming at an increasing rate.

The present paper examines the supply chain network structure of a petroleum retail distributor in the Sultanate of Oman. The company called Al-Maha Distribution Company (AMC), is a national distributor of petroleum products. The company was founded in 1993 by extending the capabilities of Oman Refinery to engage in distribution and marketing of petroleum products. The company started operations in 1994 by opening a gas station in Al-Khuwair and now has more than 100 gas stations to cover most of the Sultanate. The company's head office is located in Authaibah with branch offices in Khasab, Salalah, Mina Al Fahal and Seeb. The company faces a stiff competition from two other major players: British Petroleum (BP) and Shell Select. Due to substantial investment in refinery and distribution operations, it is hard to exit the petroleum industry. Therefore, being a good competitor and adding value through its supply chain is the way to survive in this competition.

The company sells its products through direct sales at gas stations besides catering to the shipping and fishing industries through its marina fuel stations. In the past, AMC has strategically dominated in the capital region Muscat while serving other geographic areas as much as possible. The AMC product line consists of four main categories of products: petroleum, diesel, kerosene and jet fuels, in addition to fuel oils and lubricants. Petroleum products have further

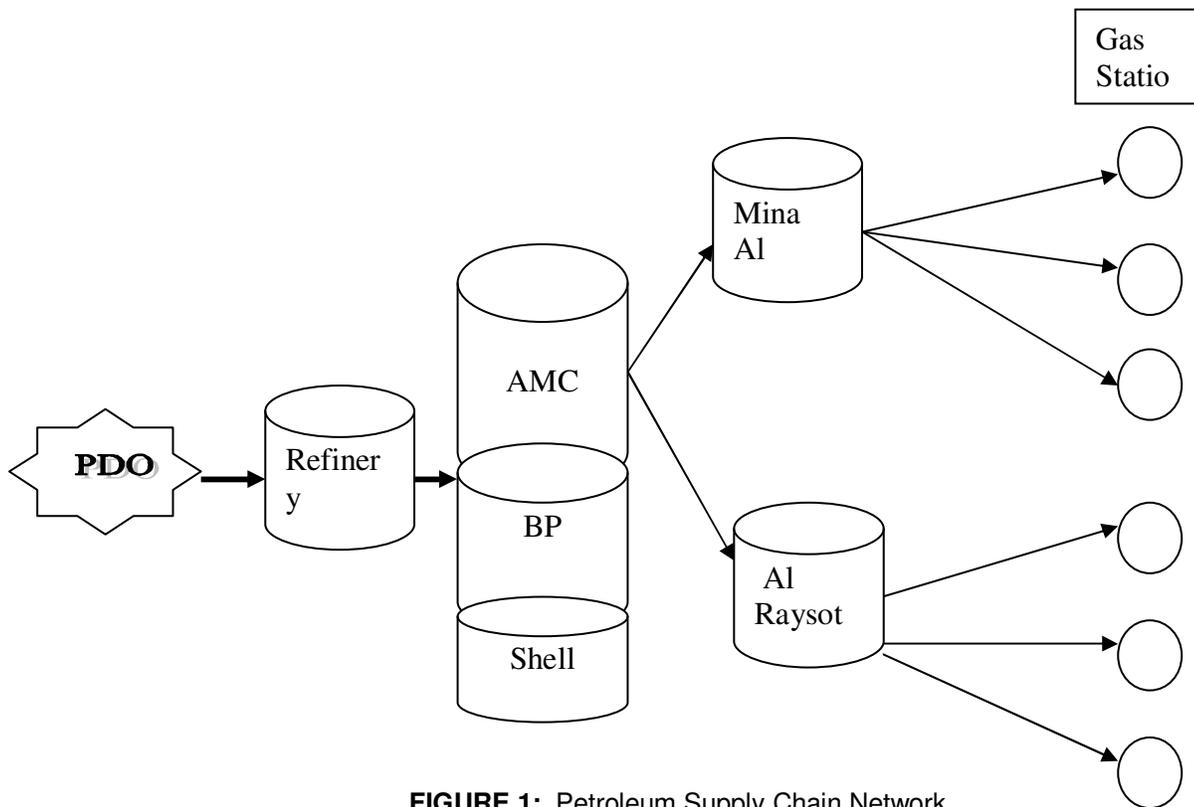
product proliferation into 90 Octane and 95 Octane, popularly known as regular and super brands of gasoline. Kerosene & Jet fuels benefit the aviation industry. They consist of higher specifications fuels and require separate trucks for delivery to different airports. The present paper will only focus on the retail segment of petroleum distribution, which is by far the most important operation of the company. It was estimated that 60% of the company sales are from petroleum sales through gas stations, 20% from Government and power companies and the remaining 20% from other buyers. AMC gas stations epitomize retail, operational and engineering efficiency. Different services offered at these gas stations include filling fuel, car-care, car wash and a stop-by shopping facility called "souk". Apart from its retail business, AMC is also a prominent supplier of fuel and lubricants to a number of Ministries and institutional buyers. Its long client list includes the names of the Ministry of Electricity and Water, the Ministry of Defense, the Royal Air Force of Oman, the Royal Omani Police and Petroleum Development of Oman (PDO).

Given the fact that all the three major players in this industry carry good image for their products, possess state-of-the-art technology, meet the required standards, and the price structure that is centrally controlled by the Ministry of Oil; it is evident that the battlefield to gain higher market share would be in the supply chain and distribution area. Therefore, the company is convinced that a sound distribution strategy and an effective supply chain structure holds a great promise for the future and would be a key element of their plans to enjoy a superior market performance.

## **2. PETROLEUM SUPPLY CHAIN NETWORK**

Supply chain managers frequently come across location and allocation problems at the design phase of a supply chain that involves determining the number of warehouses and assigning retail allocations. It appears imperative to treat the location and allocation decisions simultaneously. But due to the complexity of the problem, a breakdown into two stages i.e. location and allocation, helps to manage the complexity of large sized problems. The location decision involves substantial investment. Since it can't be changed frequently, therefore, it has long term implications. The warehouse location acts as a prelude to the overall process of supply chain design with far reaching effects on the performance of the logistics and distribution system. On the other hand, the allocation decision is more dynamic in nature as these assignments need to be reviewed and changed from time to time as the supply chain grows. This is especially true for the natural gas and petroleum products distribution in developing countries where the retail outlets and gasoline fuel stations for such products are mushrooming at an increasing rate.

Most of the oil wells are situated in Al Fahood and Murmul areas. The crude oil is extracted from the wells by Petroleum Development of Oman (PDO) and is shipped to Government owned refinery. The refinery processes this crude oil into a number of distinct petroleum products. As shown in Figure 1, the Oman Refinery ships these petroleum products to two depots, one located in Mina Al-Fahal near Muscat and the other located in Mina Al-Raysot near Salalah. The third depot in Khasab is not operational yet. AMC has ownership rights for the Raysot depot. The Mina Al Fahal depot has two terminals, one owned by Shell and the other jointly owned by AMC and British Petroleum. While the shipment to the Raysot depot is through sea transportation using a large ship, a pipeline mode of transportation is used for the shipment to Mina Al-Fahal depot.



**FIGURE 1:** Petroleum Supply Chain Network

From the terminals, the secondary shipments to various gas stations are through the road transportation. Typically, two different types of trucks are used for this movement: standard four compartment 36,400 liters truck tractors with 9100 liters capacity for each compartment, and the rigid chassis 22,500 liters trucks with three compartments of 9000, 9000 and 4500 liters capacities. Although most of the dispatching decisions are taken by AMC, the actual transportation aspect is contracted out to trucking fleet companies and individual truck owners. This policy decision besides giving Omani businessmen opportunities in the competitive trucking industry, also provides excellent sources of income for individual truck owners and drivers.

### 3. LITERATURE REVIEW

In this section, the literature on location-allocation models has been reviewed. Shycon and Maffei [1] proposed the use of simulation tools for better product distribution. Perl and Daskin [2] considered the integration of transportation planning and location models. Lee and Luebbe [3] presented warehouse location models under multiple criteria. Hall [4] discusses a program to find new multi-facility locations. Brandaun and Chiu [5] provide an extensive review of location research problems. Crainic et. al [6] developed a model for multi-mode multi-commodity location-allocation problems with balancing requirements. Cornuejols et al [7] discussed the un-capacitated facility location problem. Ho and Pearl [8] addressed the issue of facility location under service sensitive demand. Further work on multi-commodity location allocation problem is presented in Crainic [9] and Gendron and Crainic [10]. Application of tabu search methodology in emergency medical services locations can be found in Gendreau et. al [11]. Ingizio and Cavalier [12] discuss some heuristic approaches to solve set covering type location problems. Ballou [13] reviews and provides an introduction to some fundamental warehouse location models. Klose [14] uses a lagrangean relaxation approach for a two stage capacitated facility location model. Location decisions in light of demand probability have been considered in Berman and Krass [15]. Pal, Tardos and Wexler [16] considers hard capacities for facilities allowing demand splitting. Burkard and Dollani [17] present a restrictive optimal location problem on a tree network. Lin et al.

[18] analysis the location, routing and loading problem for bill delivery systems of a telecommunication company in Hong Kong. Han et al [19] discuss the integration issues at Wal-Mart supply chain in Korea. Zhou et al. [20] proposed a new balanced start spanning forest and genetic algorithm to solve location allocation problem. Syarif et al. [21] modeled and solved a facility location problem using a spanning tree based genetic algorithm. Ghosh [22] describes a tabu-search neighborhood heuristic for un-capacitated facility location model. Amiri [23] modeled the design of distribution network in a supply chain and solved it using a lagrangean relaxation method. Melo et al. [24] provided a mathematical modeling framework for dynamic facility location using decomposition techniques and meta-heuristics. The issue of supply chain design reliability has been addressed in Snyder [25]. Swamy and Kumar [26] propose primal-dual algorithm for multilevel hierarchical location problem. Ageev, Ye and Zhang [27] propose a combinatorial heuristic algorithm for k-level facility location. Gill and Bhatti [28] provided a set covering based model for identifying warehouse locations and a least distance allocation procedure.

#### 4. LOCATION-ALLOCATION SUPPLY CHAIN MODEL

The model minimizes the number of depot locations selected (and hence investment) while ensuring that each gas station is assigned to exactly one selected depot location and such an allocation satisfies the maximum permissible distance from a gas station to depot. The model is presented as follows:

Find matrix  $\mathbf{x}$  and vector  $\mathbf{y}$  so as to

$$\text{Minimize } \sum_{j=1}^n y_j$$

subject to:

$$d_{ij} \cdot x_{ij} \leq d_{\max} \quad \forall i=1,2,3,\dots,m; j=1,2,3,\dots,n$$

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i=1,2,3,\dots,m$$

$$(x_{ij} - y_j) \leq 0 \quad \forall i=1,2,3,\dots,m; j=1,2,3,\dots,n$$

$$x_{ij}, y_j \in \{0,1\} \quad \forall i, j$$

where,

$m$  = number of gas stations

$n$  = number of candidate depot locations.

$x_{ij}$  = 1 if gas station  $i$  is assigned to depot  $j$ , 0 otherwise

$y_j$  = 1 if depot  $j$  is selected, 0 otherwise

$d_{ij}$  = Distance of gas station  $i$  from depot location  $j$  expressed in kms.

$d_{\max}$  = Maximum threshold distance beyond which a gas station can not be assigned to a depot due to commuting distance, over-night costs etc.

For the reasonable sized problem with 50 gas stations and 10 candidate depot locations, it will give rise to 1050 constraints and 510 variables making it impractical to solve real problems using mathematical programming approach. Hence, the problem is broken down into location and allocation stages to manage the complexity and size and a heuristic procedure is suggested below which could be applied for large sized practical problems.

## 5. HEURISTIC SOLUTION APPROACH

The approach is two folds. First, the depot locations are chosen from the available set of locations which can cover the gas stations based on a pre-assigned maximum threshold distance. Then the gas stations are allocated to these depot locations. The steps of the procedure are as follows:

### 5.1 Location of Depots

The location of depots involves two steps. First to construct a binary coefficient matrix so as to identify the potential locations, then selecting the actual locations using a mathematical programming model.

#### Step 1. Construction of a binary coefficient matrix

Based on the maximum permissible distance,  $d_{\max}$ , a binary coefficient matrix  $[\alpha_{ij}]$  is prepared, which is to be used as an input to the mathematical programming model of step 2. The following relation can be used to construct this binary matrix:

$$\alpha_{ij} = 1 \text{ if } d_{ij} \leq d_{\max} \text{ or } 0 \text{ otherwise; } \forall i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n.$$

#### Step 2. Set covering mathematical model

Using the binary coefficient matrix in Step 1 as an input, the best depot locations to cover all the gas stations are selected based on the following set covering model (Gill & Bhatti [2007]):

Find vector  $\mathbf{y}$  so as to

$$\text{Minimize } \sum_{j=1}^n y_j$$

subject to:

$$\sum_{j=1}^n \alpha_{ij} \cdot y_j \geq 1 \quad \forall i = 1, 2, 3, \dots, m$$

$$y_j \in \{0, 1\} \quad \forall j = 1, 2, 3, \dots, n$$

The objective function above expresses the minimization of the number of depot locations while the constraint set ensures that each gas station is covered by at least one depot..

### 5.2 Allocation of Gas Stations to Depots

Gas station allocation is done according to the following procedure.

Step 1. Identify a set  $\theta = \{j\}$  such that  $y_j = 1$ .

Step 2. Consider a sub-matrix of distance matrix  $[d_{ij}]$  for those  $j \in \theta$ .

Step 3. Set  $i = 1$

Step 4. Find  $d_{ij}^* = \min(\text{vector } \mathbf{d}_{ij})$  for  $j \in \theta$ , i.e., find the minimum entry in the  $i^{\text{th}}$  row. If the column index for this minimum entry is  $j^*$ , assign  $i^{\text{th}}$  gas station to  $j^*$  depot.

Step 5. While  $i < m$ , set  $i = i + 1$  and repeat step 4, i.e., we continue to repeat step 4 until all the gas stations have been assigned.

## **6. APPLICATION TO PETROLEUM SUPPLY CHAIN DESIGN**

While analyzing the current distribution system, it became evident that catering to distant and newer gas station locations through a fewer depots results in longer lead time, uncertainties and lost sales. Looking at the tremendous growth in retail volumes over the past few years as well as the potential to grow in the coming years, it was felt that a larger number of inventory holding and forwarding points (depots) would be necessary to serve the interior regions of the Sultanate. Therefore, an important issue is to determine the number and locations of such depots. A major determinant of depot location is the distance to be covered. Apart from the depot locations, other issues that the company need to consider are consolidating demand of different gas stations on single trips, shipment sizes, dispatch rules and routes which are not a part of the current analysis.

The decision process consists of hierarchical decisions including:

- Deciding on the number and location of depots
- Allocation of the demand points to these depots

The problem is relatively complex because of the number of different but interrelated decisions that need to be made. The obvious choices are either to decompose the problem into different decision areas or alternately, to consider all the decision areas simultaneously in which case the decision-making process could be more accurate but less manageable. The scenario represents an application of the model presented in this paper.

### **6.1 Data Requirements**

The first step in evaluating the data requirements for this case, is to identify the depot candidate locations. Based on a number of factors such as proximity to major towns, communication facilities, infrastructural considerations, driver availability; Sohar, Suwayk, Muscat, Dank, Nizwa, Sur, Mahawt, Marmul and Salalah were selected to be good candidates for depot locations. The next step required choosing a maximum distance between a depot and its allocated gas stations. After much deliberations and considering the driver's comfort, a maximum one way distance of 400 kilometers (800 Km round trip) seemed reasonable. As a driver has to deliver to gas station and make a return trip to the depot, a maximum total distance of 800 kilometers on a trip was considered reasonable. The distances between depot locations and gas stations as well as distances between gas stations posed a problem. Considering the geographical structure of the Sultanate and its road network that is still evolving after Guno cyclone, the difficult desert as well as mountainous terrain, it becomes evident that the concepts of geographic coordinate distances were not directly applicable in this case. The Sultanate of Oman has nine geographic regions: The Governorate of Muscat, Al-Batinah, Governorate of Musandam, Al-Dhahirah, Al-Dakhliyah, Ash-Sharqiyah, Al-Wusta, and the Governorate of Dhofar. Unfortunately, these geographic divisions didn't help because within the same geographic division, the terrains could be quite non-

uniform making it impossible to apply the traditional approaches. Therefore, the entire region was divided into 18 different zones with the guiding principle that it was either possible to know the actual road distance between zonal centers from existing road maps or the intra-zonal distances were relatively easier to compute due to a uniform bed within a zone. Inter-zonal distances were calculated based on actual road distance. This effort resulted in the data regarding distances between potential depot locations and the gas stations and between the gas stations which is summarized in Tables 1 and Table 2. All the gas stations have been coded from 1 through 59.



GS	Sohar	Suwayq	Muscat	Dank	Nizwa	Sur	Mahawt	Marmul	Salalah
1	313	411	535	508	637	800	881	1391	1423
2	292	390	514	487	616	779	860	1370	1402
3	231.5	329.5	453.5	427	555.5	719	799.5	1309.5	1341.5
4	55.5	153.5	277.5	251	379.5	543	623.5	1133.5	1165.5
5	17	106	230	235	315	478	559	1069	1101
6	5	106	230	235	327	490	571	1081	1113
7	10	106	230	235	322	485	566	1076	1108
8	13.5	106	230	235	345.5	509	589.5	1099.5	1131.5
9	28	12	136	164	238	401	482	992	1024
10	34.5	18.5	142.5	171	244.5	408	488.5	998.5	1030.5
11	43	27	151	179	253	416	497	1007	1039
12	73	57	181	209	283	446	527	1037	1069
13	94.5	11.5	124.5	231	226.5	390	470.5	1043.5	1075.5
14	114	8	144	250	246	409	490	1063	1095
15	129.5	23.5	159.5	266	261.5	425	505.5	1078.5	1110.5
16	169	75	37	332	139	302	383	956	988
17	204	110	72	367	174	337	418	991	1023
18	205	111	73	368	175	338	419	992	1024
19	192	98	60	355	162	325	406	979	1011
20	197.5	103.5	65.5	361	167.5	331	411.5	984.5	1016.5
21	212.5	118.5	80.5	376	182.5	346	426.5	999.5	1031.5
22	215.5	121.5	83.5	379	185.5	349	429.5	1002.5	1034.5
23	203.5	109.5	26.5	321	147.5	311	391.5	964.5	996.5
24	216.5	122.5	13.5	334	160.5	324	404.5	977.5	1009.5
25	237	143	7	354	181	344	425	998	1030
26	318.5	224.5	100.5	456	262.5	426	506.5	1079.5	1111.5
27	169.5	75.5	165.5	306	267.5	431	511.5	1084.5	1116.5
28	186.5	193.5	318.5	48.5	318.5	444	460.5	899.5	931.5

**TABLE 2:** Distance Matrix Between Depot Locations and Gas Stations (GS)

Table 2. continued....

GS	Sohar	Suwayq	Muscat	Dank	Nizwa	Sur	Mahawt	Marmul	Salalah
29	227	234	359	8	359	484	501	940	972
30	275	282	407	40	407	532	549	988	1020
31	282	289	414	47	414	539	556	995	1027
32	325	238	167	360	7	292	309	824	856
33	307	238	149	342	25	274	291	806	838
34	303	238	145	338	29	270	287	802	834
35	282	238	124	317	50	249	266	781	813
36	361	238	203	396	29	328	345	860	892
37	386	292	228	319	126	353	370	943	975
38	313	219	155	246	53	280	297	870	902
39	359	265	201	292	99	326	343	916	948
40	382	288	224	315	122	349	366	939	971
41	502	408	344	499	306	7	453	1044	1076
42	572	478	414	569	376	77	523	1114	1146
43	554	460	396	551	358	59	505	1096	1128
44	488	394	330	485	292	7	439	1030	1062
45	579	485	421	576	383	84	530	1121	1153
46	398	304	240	395	202	97	349	940	972
47	576	482	418	509	316	543	0	886	1505
48	668	574	510	601	408	635	92	978	1597
49	626	595	531	488	371	577	426	440	472
50	1091	1060	996	953	836	1042	891	5	937
51	1145	1114	1050	1007	890	1096	945	59	991
52	1238	1101	1037	994	877	1083	1519	946	168
53	1332	1195	1131	1088	971	1177	1613	1040	262
54	984	953	889	846	729	935	1371	798	134
55	1044	1013	949	906	789	995	1431	858	74
56	1140	1109	1045	1002	885	1091	1527	954	22
57	1135	1104	1040	997	880	1086	1522	949	17
58	1101	1070	1006	963	846	1052	1488	915	17
59	1091	1060	996	953	836	1042	1478	905	27

## 6.2 Model Application

In this section, we apply the approach presented in section 5.

### Identifying Locations for Depots

#### *Step 1. Binary Coefficient Matrix*

Based on the maximum permissible distance of 400 KMS as discussed earlier, we prepared a binary coefficient matrix  $[\alpha_{ij}]$ , which is used as an input to the mathematical programming model.

#### *Step 2. Solving the Mathematical Model*

Using the binary coefficient matrix, the best depot locations to cover all the gas stations are selected based on the set covering model given in Gill and Bhatti (2007). Although such a model theoretically has  $m$  constraints and  $n$  variables but in the context of the present problem,

a number of redundant constraints can be eliminated. Solving the above mathematical model resulted in the following depot locations: Sohar, Mascut, Nizwa, Mahawat, Marmul and Salalah. Note that the Salalah and Muscat depots are already operating.

### Allocation of Gas Stations to Depots

- Step 1. Identify a set of candidate locations  $\theta$ . From solving the mathematical model,  $\theta = \{\text{Sohar, Mascut, Nizwa, Mahawat, Marmul, Salalah}\}$ .
- Step 2. Consider a sub-matrix of distance matrix  $[d_{ij}]$  for those  $j \in \theta$ . Therefore, we consider matrix  $[d_{ij}]$  of Table 1 relevant to Sohar, Mascut, Nizwa, Mahawat, Marmul and Salalah.
- Step 3. Set  $i = 1$
- Step 4. Find  $d_{ij}^* = \min(\text{vector } \mathbf{d}_{ij})$  for  $j \in \theta$  and assign  $i^{\text{th}}$  gas station to depot  $j^*$ .
- Step 5. While  $i < m$ , set  $i = i + 1$  and repeat step 4, i.e., we continue to repeat step 4 until all the gas stations have been assigned.

This procedure resulted in the following allocation of gas stations to the depots as given in Table 3.

Depot	Allocated Gas stations Number
Sohar	1 – 15 and 28 - 31
Muscat	16-27
Nizwa	32-46, and 49
Mahawt	47 and 48
Marmul	50 and 51
Salalah	52 - 59

**TABLE 3:** Depot Locations and Gas Stations Allocations

## 7. CONCLUDING REMARKS

The present paper analyses petroleum products distribution strategy of a company with a view to improve its distribution network for a better area coverage, to identify its major depot locations and allocation of gas stations.

It is envisioned that the scope of the analysis could further include issues such as depot capacities. The capacity issues is important if the company has a practice of frequently reviewing its supply chain decisions. The capacity decision was omitted from current analysis based on the assumption that depots with sufficient capacities can be constructed. Secondly, for the existing two depots, capacity had never been a problem.

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# Attitude Formation of Benefits Satisfaction: Knowledge and Fit of Benefits

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### Abstract

Using the theoretical framework of the Theory of Reasoned Action [6], we examine benefits satisfaction as an attitude formed by the beliefs about benefits (i.e., benefits knowledge) and the perceived value of these benefits (i.e., fit of benefits to individual needs). We use questionnaires to gather data from a random sample of 591 employees in a large county agency in the South-eastern United States. The data support that knowledge of benefits is associated with enhanced benefits satisfaction and mediates the effect of explanations about benefits on satisfaction. The results provide strong evidence that benefits perceived to suit employee needs generate highest benefits satisfaction. Employees satisfied with their benefits are less likely to consider leaving the organization. The tested model is a starting point for future studies to apply the extended Theory of Reasoned Action [1] and incorporate perceived behavioural control and subjective norms (i.e., co-workers' attitudes) in forming benefits satisfaction. Understanding employees' affective and cognitive reactions to compensation, including benefits, can render better practices. Companies should use information campaigns to improve employee beliefs about benefits. Better attentiveness to individual needs and preferences can maximize the utility of a benefits plan and improve its acceptance. We replicate and extend past research in a parsimonious model of benefits satisfaction with a random sample of public sector employees.

**Keywords:** Benefits, Benefits Satisfaction, Turnover Intentions

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## 1. INTRODUCTION

As unique and crucial sources of creativity and competencies, employees can determine the success or failure of organizations. Thus, organizations take steps to assure both individual well-being and favourable work attitudes. One such step is providing employees with favourable work conditions, including benefits coverage. Anticipating that benefits can have a positive impact on employees and their attitudes, employing organizations allocate ample financial resources for employee benefits (Employee Benefits, 2002). However, the intended positive effects would occur when employees appreciate their benefits [2], [3]. Past research has identified some determinants of employee satisfaction with benefits, but recent increases in benefits costs and changes in workforce demographics demand revising the issue be revisited within the contexts of these contemporary problems.

Previous models use pay satisfaction theories to conceptualize benefits satisfaction [4], [3], [5], whereas we draw on the alternative framework of the Theory of Reasoned Action [6]. The Theory of Reasoned Action explains benefits satisfaction as an attitude that is formed by the beliefs and values associated with benefits and that can influence certain behavioural intentions. In the following paragraphs, we examine the role of explanations, knowledge, and fit of benefits in forming benefits satisfaction that is in turn expected to diminish employee intent to leave the organization.

## 2. BENEFITS SATISFACTION

Benefits satisfaction has been conceptualized as a discrepancy between expected and offered benefits [4], [7], but it can be also studied as an attitude formation as suggested by the Theory of Reasoned Action [6]. Essentially, the Theory of Reasoned Action claims that beliefs and values concerning a certain attribute determine attitudes towards the attribute and thus is associated behavioural intentions and behaviours. In the case of benefits, employees' attitudes towards benefits are likely to be determined by their beliefs about the benefits coverage and the values they place upon those benefits. Employees who believe they are well provided for by benefits and value their coverage are likely to develop a positive attitude or a sense of satisfaction with their benefits. Positive attitudes about benefits can lead to behavioural intentions to maintain the relationship with the benefits provider (i.e., employer).

Previous research has identified factors, including beliefs and values, which can impact attitudes towards benefits [5], [7]. For example, generous benefits and increased coverage have been found associated with employee satisfaction [7] whereas increased employee contributions and costs have negatively influence on employee satisfaction [4]. Rabin [8] suggested better communication as a predictor of benefits satisfaction and Barber, Dunham, and Formizano [9] advocate flexible benefits options to improve employee acceptance. Consistent with these findings, we delineate a model describing how beliefs and values of benefits can shape benefits satisfaction. The first factor in the model – benefits knowledge – is employees' belief that the employer provides a certain level and number of benefits. The second factor – the value of benefits – is employees' perception that benefits are congruent with their needs. Finally, we propose that benefits satisfaction is associated with less intent to leave the organization [2].

## 3. EXPLANATIONS AND KNOWLEDGE OF BENEFITS

Despite significant resources allocated to benefits, employees may still have little appreciation for their benefits and the effort of the organization. In practice, people often lack knowledge about the worth or cost of provided benefits [3]. The desired impact on work attitudes and behaviours is more likely when individuals are aware of the coverage [10]. Communication becomes critical in benefits management [11] because it offers employees information about the available coverage, its features, and various options. It allows employees not only to create an accurate belief about the actual coverage but also to form a more positive attitude about the employing company and the care provided.

If employees improve their awareness about the offered benefits, they are more likely to view their coverage favourably and appreciate its potential usefulness. Dreher, Ash, and Bretz [12] found a stronger relationship between improved benefits coverage and satisfaction for those employees who had more accurate information about benefits. Danehower, Celuch, and Lust [13] suggested that effective communication can also increase acceptance and satisfaction with benefits. Moreover, satisfaction is enhanced when benefit-related communication is personalized and tailored to the employee age group [14] because the average employee may understands little of the explanations about benefits provided in brochures or at workshops [15]. Therefore, we focus on individual beliefs rather than on actual information offered by employers about benefits. Offering more explanations of benefits would favour better knowledge of benefits and facilitate forming more positive attitudes towards those benefits. Thus, we hypothesize:

*Hypothesis 1.* Explanations and knowledge of benefits will be positively related to benefits satisfaction. Knowledge of benefits will mediate the relationship between explanations about benefits and benefits satisfaction.

## 4. BENEFITS FIT

Whereas organizations manifest their care for employees by providing benefits, employees look at benefits as satisfying essential needs for health care, recuperation, and comfort. Different employees, however, need or prefer different coverage. For example, individual preferences for health care were found to vary with age, gender, income level, and marital status [16]. Because of the available pension plans and child care, public agencies became an attractive workplace for elder and female office workers [17]. Individual needs and expectations are also likely to shape attitudes towards available benefits. [18]

found that financial needs were related to pay satisfaction [18]. Similarly, benefits matching employees' needs would be recognized as valuable. In other words, the value of benefits is derived from their fit with individual needs.

According to the Theory of Reasoned Actions, when an attribute is perceived as valuable, individuals form a positive attitude about it [6]. The value of benefits is the correspondence between employee needs and provided coverage and will be a factor in forming benefits satisfaction. Benefits fit is somewhat reflected in flexible benefits plans that allow employees to choose the types and levels of benefits. A field experiment by Barber and colleagues [9] demonstrates that of employees given a choice to enrol in a newly-introduced flexible benefits plan, approximately 90% chose to do so, and their benefits satisfaction significantly improved after the intervention. Thus, because employees seek and appreciate benefits that are relevant to their current needs, we hypothesize:

*Hypothesis 2.* Perceived benefits fit will be positively related to benefits satisfaction.

## 5. TURNOVER INTENTIONS

Benefits are oftentimes viewed as organizational care making the job more attractive and secure. Moreover, the Theory of Reasoned Action suggests that a positive attitude is associated with favourable behavioural intentions. In this way, employees who are satisfied with their benefits are less likely to consider leaving the organization [3]. Thus, in replication of previous research, we hypothesize:

*Hypothesis 3.* Benefits satisfaction will be negatively related to turnover intentions.

## 6. METHODS

### 6.1 Sample and Data Collection Procedure

A random sample of 591 employees in a large county agency in the South-eastern United States participated in the study with a net of 517 useful observations. Data were collected at the organizational site with questionnaires distributed and collected by one of the authors of the study. Of the final sample, 79.25% of respondents are Caucasian, 12.6% are African-American, and 51.0% are male. The average participant is 38.3 years old and has some college education, 3.2 years of job tenure, and 6.0 years organizational tenure.

### 6.2 Measures

All variables were measured on 5-point Likert-type scales. Benefits knowledge was captured with three items [19] such as "I have knowledge about my benefits package."

*Benefits explanations* variable was measured with two questions asking about "the amount of explanations received about benefits" and "the amount of explanations received about changes in benefits."

*Benefits fit* was measured by rating the agreement with the statement "My benefit package suits my needs."

*Benefits satisfaction* was measured with four items that form the subscale of the Pay Satisfaction Questionnaire [2] such as "I am satisfied with benefits that I receive."

*Turnover intentions* were measured with six items [19] that ask about respondents' intentions for the next two months to "leave the current employer for a different job" and "look for a similar job outside the current employer."

We also controlled for procedural benefits fairness, measured with four items [19], because previous research [19], [20] has found it associated with benefits satisfaction. Pay fairness, measured with 17 items [19], was also included because feelings of deprivation may foster employees to seek organizational exit [21]. Finally, we controlled for age and gender.

### 6.3 Analysis

Hierarchical multiple regression analyses with the standardized composite scores were used to test the hypothesized relationships and path analysis with manifested variables (SAS v.8) estimated the overall model fit. Because the variables were self-reports, principle component analysis tested for common method bias [22] and did not reveal one dominant factor.

## 7. Results

Table 1 shows zero-order correlations and internal consistencies of the study variables. The regression analyses results are presented in Table 2. Explanations about benefits is positively related to benefits knowledge ( $\beta=.56$ ,  $p<.001$ ,  $R^2 =.31$ ) and knowledge about benefits predicts satisfaction in the expected way ( $\beta =.171$ ,  $p<.01$ ). Explanations about benefits and benefits satisfaction also hold the predicted positive relationship ( $\beta =.15$ ,  $p<.01$ ) but the relationship diminishes after including benefits knowledge in the analysis ( $\beta =.07$ ,  $p>.05$ ). According to [23], this is evidence that benefits knowledge mediates the effect of explanations on benefits satisfaction. Thus, hypothesis 1 is supported.

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Insert Table 1 and 2 about here  
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As expected, fit of benefits to individual needs is a strong positive predictor of benefits satisfaction ( $\beta=.57$ ,  $p<.01$ ) and the largest factor in forming benefits satisfaction within this model ( $\Delta R^2=.196$ ,  $p<.00$ ). Furthermore, we find that individuals satisfied with their benefits package are less likely to report intentions to leave the organization ( $\beta=.143$ ,  $p<.01$ ). Thus, hypotheses 2 and 3 are supported. Finally, the overall path model with manifested variables has a moderate but acceptable fit to the theoretical model ( $\chi^2 = 37.16$ , d.f. = 8, GFI=.98, AGFI = .90, RMR = .03, NFI = .97). Thus, our theoretical model of benefits satisfaction has a satisfactory fit with the data.

## 8. DISCUSSION

Using the Theory of Reasoned Action, we examine benefits satisfaction as an attitude formed by beliefs about benefits (such as benefits knowledge and explanations) and the perceived value of those benefits (such as fit of benefits to individual needs). In particular, the data support that employees have higher benefits satisfaction when they perceive to have focused more explanations and more knowledge about their benefits. The results also provide strong evidence that benefits satisfaction is strongest when benefits are perceived to suit employee needs. Our study replicates previous findings that employee satisfied with provided benefits coverage are less likely to consider leaving the organization. Thus, it replicates and extends previous research in a parsimonious model of benefits satisfaction.

The findings should be interpreted within the study limitations. Self-reported data is susceptible to common method bias that may have inflated the effect sizes. The model tested here is a promising start, but future studies should apply the extended theory [1] by perceived behavioural control and subjective norms (i.e., co-workers' attitudes) in forming benefits satisfaction.

Previous research has related satisfaction with benefits to employee expectations or benefits amount. We extended the conceptualization of previous research to include knowledge of benefits and explanations about benefits as determinants of benefits satisfaction. We also explicitly tested the perceived fit between employee need and benefits provided by the organization and demonstrated a very strong relationship. Finally, a strong point of the paper is the use of sound theory that encompasses all of the new concepts that are linked to benefits satisfaction and turnover intentions.

Understanding employees' affective and cognitive reactions to compensation, including benefits, can render better practices as suggested by the findings here. First, benefits information campaigns should focus on modelling employee beliefs about benefits rather than offering abundant details about benefits. Second, attentiveness to individual needs and preferences can maximize the usefulness of a benefits

plan and improve its acceptance. By offering employees with choices, flexible benefits plans demonstrate the significance of benefits congruence to employee needs. Furthermore, for benefits plans to satisfy essential individual needs, employees and benefits managers should work together to adequately identify and address these needs. Finally, the results provide some evidence that success of benefits plans is determined not only by the financial cost but also by employees' beliefs about the available coverage and the value placed on it.

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Table 1.

Correlations of the Study Variables a

Variables	1	2	3	4	5	6	7	8	9
1. Age	-								
2. Gender	-.12*	-							
3. Procedural benefits fairness	.13*	.05	(.86) b						
4. Benefits explanations	.08	.12*	.44**	(.84)					
5. Benefits knowledge	.04	-.03	.42**	.56**	(.73)				
6. Benefits fit	.09	.02	.57**	.38**	.40**	-			
7. Benefits satisfaction	.22**	.08	.59**	.39**	.40**	.73**	(.85)		
8. Pay fairness	.14**	.04	.45**	.31**	.13*	.26**	.30**	(.91)	
9. Turnover intentions	-.17**	-.12*	-.27**	-.12*	-.07	-.15**	-.21**	-.33**	(.86)

<sup>a</sup> N = 517; b Reliabilities are shown on the diagonal; \*\* p < .01 \* p < .05

Table 2.

Results of Regression Analysis with Dependent Variables of Benefits Knowledge, Benefits Satisfaction, and Turnover Intentions a

Variables	Benefits knowledge Model 1	Benefits satisfaction Model 2		Model 3	Turnover Intentions Model 4	
Model 1.						
Explanations about benefits	.555**	.15**	.07	-.01		
Age		.14**	.14**	.14**		
Gender		.05	.06	.07*		
Procedural justice of benefits		.50**	.46**	.21**		
Model 2.						
Benefits knowledge			.17**	.10*		
Model 3.						
Fit of benefits to individual needs				.57**		
Model 4.						
Pay fairness						-.29**
Benefits satisfaction					-.33**	-.14**
F	179.28**	59.75**	48.86**	89.22**	48.91**	28.35**
R2	.31	.38	.39	.59	.11	.13
Adjusted R2	.31	.38	.38	.58	.11	.13

<sup>a</sup> N = 517; Standardized regression coefficients are shown; \*\* p < .01, \* p < .05

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