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EDITORIAL PREFACE

The International Journal of Scientific and Statistical Computing (IJSSC) is an effective medium for interchange of high quality theoretical and applied research in Scientific and Statistical Computing from theoretical research to application development. This is the *First* Issue of *Fourth* Volume of IJSSC. International Journal of Scientific and Statistical Computing (IJSSC) aims to publish research articles on numerical methods and techniques for scientific and statistical computation. IJSSC publish original and high-quality articles that recognize statistical modeling as the general framework for the application of statistical ideas.

The initial efforts helped to shape the editorial policy and to sharpen the focus of the journal. Started with Volume 4, 2013, IJSSC appears with more focused issues. Besides normal publications, IJSSC intend to organized special issues on more focused topics. Each special issue will have a designated editor (editors) – either member of the editorial board or another recognized specialist in the respective field.

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Abstract

The paper presents a probabilistic analysis of an evaporator of a desalination plant. Multi stage flash desalination process is being used for water purification. The desalination plant operates round the clock with seven evaporators and during normal operation; six of these evaporators will be in service while one is under maintenance and works as standby. Any major failure/annual maintenance brings the evaporator to a complete halt and stops the water production. The priority is given to repair over maintenance. For the present analysis, seven years maintenance data has been extracted from the operations and maintenance reports of the plant.Measures of the plant effectiveness have been obtained probabilistically. Semi-Markov processes and regenerative point techniques are used in the entire analysis.

Keywords: Desalination plant, Maintenance, Failures, Semi- Markov, Regenerative process.

1. NOTATIONS

U_{ms} Under Maintenance during summer

U_{mwb} Under Maintenance during winter before service

U_{mwa} Under Maintenance during winter after service

W_{ms} Waiting for Maintenance during summer

W_{mwb} Waiting for Maintenance during winter before service

W_{mwa} Waiting for Maintenance during winter after service

F_{rs} Failed unit is under repair during summer

$F_{rwb}F_{r}$ Failed unit is under repair during winter before service

$_{wa}$ Failed unit is under repair during winter after service

β_1 Rate of summer to winter change

β_2 Rate of winter to summer change

λ Rate of failure of any component of the unit

γ Rate of Maintenance

γ_1 Rate of shutting down

γ_2 Rate of recovery after shut down

α Rate ofrepair

© Symbol for Laplace Convolution

Ⓢ Symbol for Stieltje's convolution

* Symbol for Laplace transforms

** Symbol for Laplace Stieltje's transforms

$\varphi_i(t)$ c.d.f. of first passage time from a regenerative state i to a failed state j

$p_{ij}(t)$, $Q_{ij}(t)$ p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in (0, t]

$g_m(t)$, $G_m(t)$ p.d.f. and c.d.f. of maintenance rate

$g_{sr}(t)$, $G_{sr}(t)$ p.d.f. and c.d.f. of recovery rate after shutdown

g(t), G(t) p.d.f. and c.d.f. of repair rate

2. INTRODUCTION

Desalination is a water treatment process that removes salt from sea water or brackish water. It is the only option in arid regions, since the rainfall is marginal. This can be achieved by a major process known as Multi-stage Flash distillation Process which is very expensive and involves sophisticatedsystems. Since, desalination plants are designed to fulfill the requirement of water supply for a larger sector in arid regions, they are normally kept in continuous production modeexcept for emergency/forced/planned outages. It is therefore, essential to maintain the efficiency ofthese desalination plantsusing good maintenance practices to avoid big losses.

Many researchers have analyzed systems and obtained various reliability indicesthat are useful for effective equipment/plant maintenance.G. Taneja&V. Naveen [1] studied models with patience time and chances of non-availability of expert repairman,B. Parashar&G. Taneja [2] evaluated the reliability and profit of a PLC hot standby system based on master slave concept and two types of repair facilities;Rizwan et. al. [3], [4] &[5] have analyzed aPLC system, desalination plant system and a CC plant system. Recently, Padmavathi et al. [6] explored a possibility of analyzing desalination plant with online repair and emergency shutdowns situation. In all these papers various measures of system effectiveness are obtained under different failure possibilities. The novelty of the work lies in the application of modeling methodology for reliability analysis of systems as real case studies under different failure possibilities. Interesting variation on the reliability results could be obtained for a situation of a desalination plant when the annual maintenance of the plant is planned during winter season and the plant is shut down for one month; and the priority is given to repair over maintenance.

Thus, the present paper offers a probabilistic analysis of adesalination plantwhere the annual maintenance of the plant is carried out during winter season and the plant is shut down for one month for this purpose; and the priority is given to repair over maintenance on failure of a unit. The desalination plant under discussion operates round the clock for water purification and ensures the continuous production of water for domestic usage. The plant consists of seven evaporators and at any given time; six out of seven evaporators are operative whereas one is always under maintenance and works as standby. Any major failure/annual maintenance brings the evaporator to a complete halt and the water production stops until the fault restored. Seven years maintenance data has been extracted from the operations and maintenance report of a desalination plant in Oman. A robust model embedding the real failure situations, as categorized in the data with priority of repair over maintenance, has been developed (Fig. 1). The real values of various failure rates and probabilities are being used in this analysis for achieving the reliability indicators.

Using the data, the following values are estimated:

Estimated rate of failure of any component of the unit (λ) = 0.00002714 per hour

Estimated rate of the unit moving from winter to summer (β_1) =0.0002315 per hour
Estimated rate of the unit moving from summer to winter (β_2) = 0.0002315 per hour

Estimated rate of Maintenance (γ) = 0.0014881

Estimated rate of shutting down (γ_1) = 0.0001142 per hour

Estimated rate of recovery after shut down during winter (γ_2) = 0.0069444 per hour

Estimated value of repair rate (α) = 0.001577 per hour

The system is analyzed probabilistically by using semi-Markov processes and regenerative point techniques. Various measures of system effectiveness such as mean time to system shut down, system availability, busy period analysis of repairman, busy period analysis for repair, expected busy period during shut down and the expected number of repairs are estimated numerically.

3. MODEL DESCRIPTION AND ASSUMPTIONS

- The desalination plant has seven evaporators out of which six are operative and one is under maintenance.
- If a unit is failed in one season, it gets repaired in that season only.
- Maintenance of no unit is done if the repair of some other unit is going on.
- Not more than two units fail at a time.
- During the maintenance of one unit, more than one of the other units cannot get failed.
- All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.

- After each repair the unit works as good as new.
- The unit is brought into operation as soon as possible.

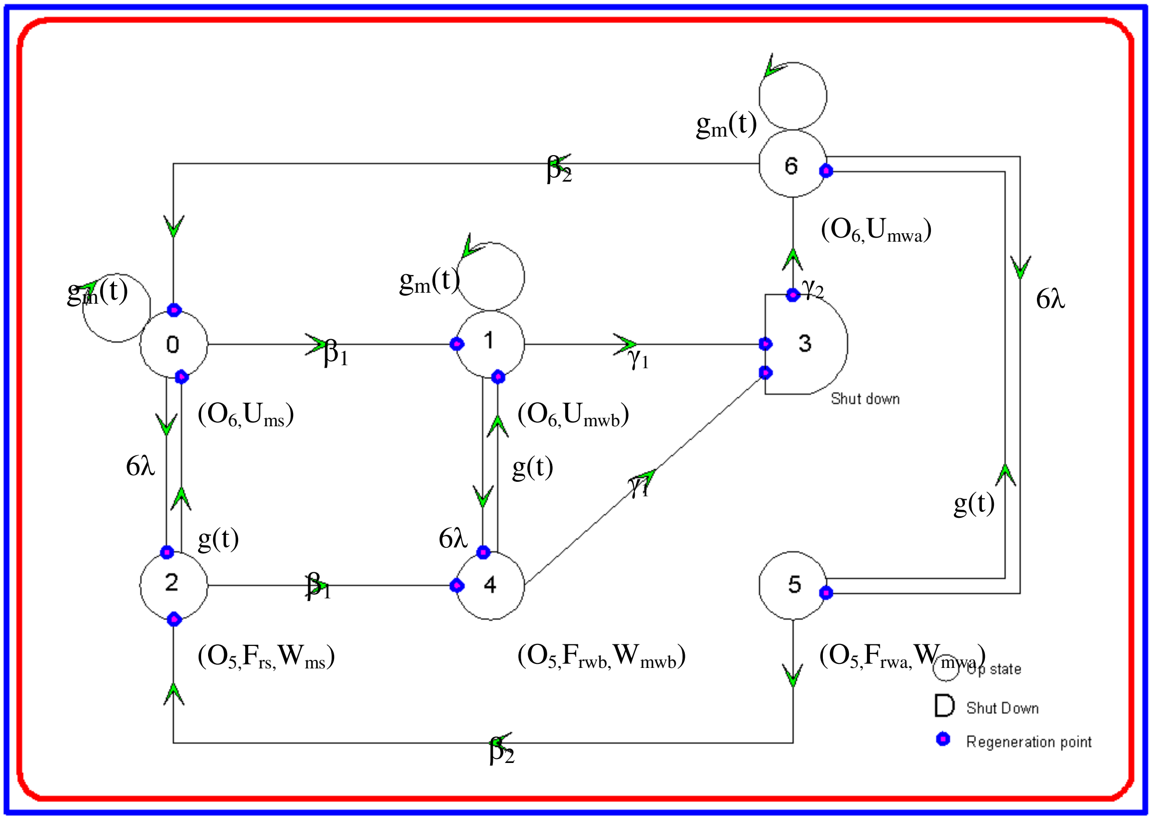

FIGURE 1: State Transition Diagram.

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram showing the possible states of transition of the plant is shown in figure 1. The epochs of entry into states 0, 1, 2, 3, 4, 5 and 6 are regeneration points and hence these states are regenerative states. The transition probabilities are as under:

$$dQ_{00} = \gamma e^{-(\gamma+6\lambda+\beta_1)t}dt,\ dQ_{01} = \beta_1 e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,\ dQ_{02} = 6\lambda e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{11} = \gamma e^{-(\gamma+6\lambda+\gamma_1)t}dt,\ dQ_{13} = \gamma_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,\ dQ_{14} = 6\lambda e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{20} = g(t)dt = e^{-(\alpha+\beta_1)t}dt\ \ dQ_{24} = \beta_1 e^{-\beta_1 t}\overline{G}(t) = \beta_1 e^{-(\alpha+\beta_1)t}dt,\ dQ_{36} = \gamma_2 e^{-\gamma_2 t}dt,$$
$$dQ_{41} = \alpha e^{-(\alpha+\gamma_1)t}dt,\ dQ_{43} = \gamma_1 e^{-(\alpha+\gamma_1)t}dt,\ dQ_{52} = \beta_2 e^{-(\alpha+\beta_2)t}dt,\ dQ_{56} = \alpha e^{-(\alpha+\beta_2)t}dt$$
$$dQ_{60} = \beta_2 e^{-(\gamma+6\lambda+\beta_2)t}dt,\ dQ_{65} = 6\lambda e^{-(\gamma+6\lambda+\beta_2)t}dt,\ dQ_{66} = \gamma e^{-(\gamma+6\lambda+\beta_2)t}dt \qquad (1\text{-}16)$$

Therefore, the non-zero elements p_{ij}can be obtained as $p_{ij} = \lim_{s\to 0}\int_0^\infty q_{ij}(t)dt$ andare given below:

$$p_{00} + p_{01} + p_{02} = 1,\ p_{11} + p_{13} + p_{14} = 1,\ p_{20} + p_{24} = 1,\ p_{36} = 1$$
$$p_{41} + p_{43} = 1,\ p_{52} + p_{56} = 1,\ p_{60} + p_{65} + p_{66} = 1 \qquad (17\text{-}23)$$

The mean sojourn time (μ_i) in the regenerative state '*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state '*i*', then:

$$\mu_i = E(T) = \Pr[T > t]$$

$$\mu_0 = \int_0^\infty e^{-\gamma t} e^{-6\lambda t} e^{-\beta_1 t} dt = \frac{1}{\gamma + 6\lambda + \beta_1}; \ \mu_1 = \int_0^\infty e^{-\gamma t} e^{-6\lambda t} e^{-\gamma_1 t} dt = \frac{1}{\gamma + 6\lambda + \gamma_1};$$

$$\mu_2 = \frac{1}{\alpha + \beta_1}, \ \mu_3 = \frac{1}{\gamma_2}, \ \mu_4 = \frac{1}{\alpha + \gamma_1}, \ \mu_5 = \frac{1}{\alpha + \beta_2}, \ \mu_6 = \frac{1}{6\lambda + \gamma + \beta_2} \qquad (24\text{–}30)$$

The unconditional mean time taken by the system to transit to any of the regenerative state '*j*' when time is counted from the epoch of entry into state '*i*' is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}{}^{*\prime}(0)$$

$$m_{00} + m_{01} + m_{02} = \mu_0, \ m_{11} + m_{13} + m_{14} = \mu_1$$

$$m_{20} + m_{24} = \mu_2, \ m_{36} = \mu_3, \ m_{41} + m_{43} = \mu_4, \ m_{52} + m_{56} = \mu_5, \ m_{60} + m_{65} + m_{66} = \mu_6 \qquad (31\text{–}37)$$

5. THE MATHEMATICAL ANALYSIS

5.1 Mean Time to System Shut Down

The mean time to system shut down can be found by considering the failed states as absorbing states. Let $\Phi_i(t)$ be the c.d.f. of the first passage time from regenerative state ' i ' to a failed state ' j ' Applying simple probabilistic arguments, the following recursive relations for $\phi_i(t)$ are obtained:

$$ø_0(t) = Q_{00}(t) Ⓢ ø_0(t) + Q_{01}(t) Ⓢ ø_1(t) + Q_{02}(t) Ⓢ ø_2(t)$$

$$ø_1(t) = Q_{11}(t) Ⓢ ø_1(t) + Q_{13}(t) + Q_{14}(t) Ⓢ ø_4(t)$$

$$ø_2(t) = Q_{20}(t) Ⓢ ø_0(t) + Q_{24}(t) Ⓢ ø_4(t)$$

$$ø_4(t) = Q_{41}(t) Ⓢ ø_1(t) + Q_{43}(t) \qquad (38\text{–}41)$$

Now the mean time to shut down when the unit started at the beginning of state 0, is given by

$$\lim_{s \to 0} \frac{1 - \varphi_0^{**}(s)}{s} = \frac{N(s)}{D(s)} \qquad (42)$$

Where,

$$N(s) = Q_{01}^{**}(s) Q_{13}^{**}(s) + Q_{01}^{**}(s) Q_{14}^{**}(s) Q_{43}^{**}(s) + Q_{02}^{**}(s) Q_{24}^{**}(s) Q_{43}^{**}(s) - Q_{02}^{**}(s) Q_{24}^{**}(s) Q_{43}^{**}(s) Q_{11}^{**}(s) + Q_{02}^{**}(s) Q_{24}^{**}(s) Q_{41}^{**}(s) Q_{13}^{**}(s)$$

$$D(s) = 1 - Q_{00}^{**}(s) - Q_{11}^{**}(s) + Q_{00}^{**}(s) Q_{11}^{**}(s) - Q_{02}^{**}(s) Q_{20}^{**}(s) + Q_{02}^{**}(s) Q_{11}^{**}(s) Q_{20}^{**}(s) - Q_{14}^{**}(s) Q_{41}^{**}(s) + Q_{00}^{**}(s) Q_{14}^{**}(s) Q_{41}^{**}(s) + Q_{02}^{**}(s) Q_{20}^{**}(s) Q_{14}^{**}(s) Q_{41}^{**}(s)$$

5.2 Availability Analysis of the Unit of the Plant

For repairable systems, an essential significant measure is availability. Using the probabilistic arguments and defining $A_i(t)$ as the probability of unit entering into upstate at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained:

$A_0(t) = M_0(t) + q_{00}(t) © A_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$

$A_1(t) = M_1(t) + q_{11}(t) © A_1(t) + q_{13}(t) © A_3(t) + q_{14}(t) © A_4(t)$

$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{24}(t) © A_4(t)$

$A_3(t) = q_{36}(t) © A_6(t)$

$A_4(t) = M_4(t) + q_{41}(t) © A_1(t) + q_{43}(t) © A_3(t)$

$A_5(t) = M_5(t) + q_{52}(t) © A_2(t) + q_{56}(t) © A_6(t)$

$A_6(t) = M_6(t) + q_{60}(t) © A_0(t) + q_{65}(t) © A_5(t) + q_{66}(t) © A_6(t)$ (43–49)

Where $M_0(t) = e^{-(6\lambda+\beta_1+\gamma)t}$, $M_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$,
$M_2(t) = e^{-(\alpha+\beta_1)t}$, $M_4(t) = e^{-(\alpha+\gamma_1)t}$, $M_5(t) = e^{-(\alpha+\beta_2)t}$, $M_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$.

On taking Laplace Transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability is given by,

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} \qquad (50)$$

Where,

$N_2(0) = \mu_0 + p_{01}\mu_1 - p_{11}\mu_0 + p_{02}\mu_2 - p_{02}p_{11}\mu_2 + p_{01}p_{14}\mu_4 + p_{02}p_{24}\mu_4 - p_{02}p_{24}\mu_4 p_{11} + p_{01}p_{13}p_{36}\mu_6 + p_{43}p_{36}\mu_6(p_{01}p_{14} + p_{02}p_{24} - p_{11}p_{02}p_{24}) + p_{36}p_{65}\mu_5(p_{01}p_{13} + p_{01}p_{14}p_{43} + p_{02}p_{24}p_{43} - p_{11}p_{02}p_{24}p_{43}) + p_{01}p_{13}p_{36}p_{65}p_{52}(\mu_2 + p_{41}p_{24}) + p_{43}p_{36}p_{65}p_{52}(p_{01}p_{14}\mu_2 - p_{24}\mu_0 - p_{01}\mu_1 p_{24} + p_{11}p_{24}\mu_0) + p_{56}p_{65}(-\mu_0 - p_{01}\mu_1 + p_{11}\mu_0) + p_{02}\mu_2 p_{56}p_{65}(-1 + p_{11}) + p_{56}p_{65}\mu_4(p_{01}p_{14} - p_{02}p_{24} + p_{02}p_{24}p_{11}) + p_{14}p_{41}(-\mu_0 - p_{02}\mu_2) + p_{02}p_{24}p_{41}(\mu_1 + p_{13}p_{36}\mu_6) + p_{24}p_{41}p_{36}p_{65}(p_{02}p_{13}\mu_5 - p_{01}p_{13}p_{52}) + p_{14}p_{41}p_{56}p_{65}(\mu_0 + p_{02}\mu_2) - p_{02}p_{24}p_{41}p_{56}p_{65}\mu_1 - p_{66}\mu_0 - p_{01}p_{66}\mu_1 + p_{11}p_{66}\mu_0 + p_{02}p_{66}\mu_2(1+p_{11}) + p_{66}\mu_4(p_{01}p_{14} - p_{02}p_{24} + p_{02}p_{24}p_{11}) + p_{41}p_{66}(p_{14}\mu_0 + p_{02}p_{14}\mu_2 - p_{02}p_{24}\mu_1)$

$D_2'(0) = p_{00}\mu_0 + p_{11}\mu_1 - p_{11}p_{00}(\mu_0 + \mu_1) + p_{02}p_{20}(\mu_0 + \mu_2) - p_{11}p_{02}p_{20}(\mu_0 + \mu_1 + \mu_2) + p_{01}p_{13}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_3 + \mu_6) + p_{01}p_{14}p_{43}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_3 + \mu_4 + \mu_6) + p_{02}p_{24}p_{43}p_{36}p_{60}(\mu_0 + \mu_2 + \mu_3 + \mu_4 + \mu_6) - p_{11}p_{02}p_{24}p_{43}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_6) + p_{01}p_{13}p_{52}p_{20}p_{36}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_5 + \mu_6) + p_{01}p_{14}p_{43}p_{36}p_{65}p_{52}p_{20}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{00}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_0 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{11}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{00}p_{11}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{56}p_{65}(\mu_5 + \mu_6) - p_{00}p_{56}p_{65}(\mu_0 + \mu_5 + \mu_6) - p_{11}p_{56}p_{65}(\mu_1 + \mu_5 + \mu_6) + p_{00}p_{11}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_5 + \mu_6) - p_{02}p_{20}p_{56}p_{65}(\mu_0 + \mu_2 + \mu_5 + \mu_6) + p_{11}p_{02}p_{20}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_5 + \mu_6) + p_{14}p_{41}(\mu_1 + \mu_4) - p_{00}p_{14}p_{41}(\mu_0 + \mu_1 + \mu_4) - p_{02}p_{20}p_{14}p_{41}(\mu_0 + \mu_1 + \mu_2 + \mu_4) + p_{02}p_{24}p_{41}p_{13}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_6) + p_{24}p_{41}p_{13}p_{36}p_{52}p_{65}(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{00}p_{24}p_{41}p_{13}p_{36}p_{52}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{14}p_{41}p_{56}p_{65}(\mu_1 + \mu_4 + \mu_5 + \mu_6) + p_{00}p_{14}p_{41}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_4 + \mu_5 + \mu_6) + p_{02}p_{20}p_{14}p_{41}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_4 + \mu_5 + \mu_6) + p_{66}\mu_6 - p_{00}p_{66}(\mu_0 + \mu_6) - p_{11}p_{66}(\mu_1 + \mu_6) + p_{00}p_{11}p_{66}(\mu_0 + \mu_1 + \mu_6) - p_{02}p_{20}p_{66}(\mu_0 + \mu_2 + \mu_6) + p_{11}p_{66}p_{02}p_{20}(\mu_0 + \mu_1 + \mu_2 + \mu_6) - p_{14}p_{41}p_{66}(\mu_1 + \mu_4 + \mu_6) + p_{00}p_{14}p_{41}p_{66}(\mu_0 + \mu_1 + \mu_4 + \mu_6) + p_{02}p_{20}p_{14}p_{41}p_{66}(\mu_0 + \mu_1 + \mu_2 + \mu_4 + \mu_6)$ (51 – 52)

5.4 Busy Period Analysis of Repairman

In steady sate, the total fraction of time $B_0^M(t)$ for which the unit is under repair is given by the following recursive relations:

$B_0^M(t) = W_0(t) + q_{00}(t) © B_0^M(t) + q_{01}(t) © B_1^M(t) + q_{02}(t) © B_2^M(t)$

$B_1^M(t) = W_1(t) + q_{11}(t) © B_1^M(t) + q_{13}(t) © B_3^M(t) + q_{14}(t) © B_4^M(t)$,

$B_2^M(t) = q_{20}(t)$ © $B_0^M(t) + q_{24}(t)$ ©$B_4^M(t)$,

$B_3^M(t) = q_{36}(t)$ © $B_6^M(t)$,

$B_4^M(t) = q_{41}(t)$ © $B_1^M(t) + q_{43}(t)$ © $B_3^M(t)$,
$B_5^M(t) = q_{52}(t)$ © $B_2^M(t) + q_{56}(t)$ © $B_6^M(t)$,

$B_6^M(t) = W_6(t)+q_{60}(t)$ © $B_0^M(t) + q_{65}(t)$ ©$B_5^M(t) + q_{66}(t)$ ©$B_6^M(t)$

Where $W_0(t)=e^{-(6\lambda+\beta_1+\gamma)t}$, $W_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$, $W_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$ (53 - 60)

Taking Laplace Transforms of the above equations and solving them for $B_0^{M*}(s)$, the following is obtained:

$$B_0^M = \lim_{s\to 0} sB_0^{M*}(s) = \frac{N_3(0)}{D_2'(0)} \tag{61}$$

Where,
$N_3(0) = \mu_0 + p_{01}\mu_1 - p_{11}\mu_0 - p_{14}p_{41}\mu_0 + p_{01}p_{13}p_{36}\mu_6 + p_{02}\,p_{24}\,p_{41}\mu_1 + p_{02}\,p_{24}\,p_{41}\,p_{13}p_{36}\mu_6 + p_{01}\,p_{14}\,p_{43}p_{36}\mu_6 + p_{02}\,p_{24}\,p_{43}p_{36}\mu_6 - p_{11}\,p_{02}\,p_{24}\,p_{43}p_{36}\mu_6 - p_{13}p_{36}\,p_{41}\,p_{24}\,p_{65}\,p_{52}\mu_0 - p_{24}\,p_{43}\,p_{36}p_{65}\,p_{52}\mu_0 - p_{01}\,p_{24}\,p_{43}\,p_{36}p_{65}\,p_{52}\mu_1 + p_{11}\,p_{24}\,p_{43}\,p_{36}p_{65}\,p_{52}\mu_0 - p_{56}p_{65}\mu_0 - p_{01}\,p_{56}p_{65}\mu_1 + p_{11}\,p_{56}p_{65}\mu_0 + p_{14}\,p_{4156}p_{65}\mu_0 - p_{02}\,p_{24}\,p_{41}\,p_{56}p_{65}\mu_1 - p_{66}\mu_0 - p_{01}\,p_{66}\mu_1 + p_{11}\,p_{66}\mu_0 + p_{14}\,p_{41}\,p_{66}\mu_0 - p_{01}\,p_{14}\,p_{41}\,p_{66}\mu_1$ (62)
And $D_2'(0)$ as already mentioned in equation (52).

Proceeding in the same way, the other reliability measures could also be obtained:

- Expected busy period for repair $[B_0^{R*}(s)]$:

$$B_0^S = \lim_{s\to 0} sB_0^{S*}(s) = \frac{N_5(0)}{D_2'(0)} \tag{63}$$

Where,

$N_5(0) = p_{01}\,p_{13}\mu_3 + p_{01}\,p_{14}\mu_3 - p_{02}\,p_{24}\mu_3 - p_{11}\,p_{02}\,p_{24}\mu_3 + p_{02}\,p_{24}\,p_{41}\,p_{13}\mu_3 - p_{01}\,p_{13}\,p_{56}\,p_{65}\mu_3 - p_{01}\,p_{14}\,p_{56}\,p_{65}\mu_3 + p_{11}\,p_{02}\,p_{24}\,p_{56}\,p_{65}\mu_3 - p_{02}\,p_{24}\,p_{56}\,p_{65}\mu_3 - p_{02}\,p_{24}\,p_{41}\,p_{13}\,p_{56}\,p_{65}\mu_3 - p_{01}\,p_{13}\,p_{66}\mu_3 - p_{01}\,p_{14}\,p_{66}\mu_3 - p_{02}\,p_{24}\,p_{66}\mu_3 + p_{11}\,p_{02}\,p_{24}\,p_{66}\mu_3 - p_{02}\,p_{24}\,p_{13}p_{41}p_{66}\mu_3$ (64)

- Expected busy period during shutdown $[B_0^{S*}(s)]$:

$$B_0^S = \lim_{s\to 0} sB_0^{S*}(s) = \frac{N_5(0)}{D_2'(0)} \tag{65}$$

Where,

$N_5(0) = p_{01}\,p_{13}\mu_3 + p_{01}\,p_{14}\mu_3 - p_{02}\,p_{24}\mu_3 - p_{11}\,p_{02}\,p_{24}\mu_3 + p_{02}\,p_{24}\,p_{41}\,p_{13}\mu_3 - p_{01}\,p_{13}\,p_{56}\,p_{65}\mu_3 - p_{01}\,p_{14}\,p_{56}\,p_{65}\mu_3 + p_{11}\,p_{02}\,p_{24}\,p_{56}\,p_{65}\mu_3 - p_{02}\,p_{24}\,p_{56}\,p_{65}\mu_3 - p_{02}\,p_{24}\,p_{41}\,p_{13}\,p_{56}\,p_{65}\mu_3 - p_{01}\,p_{13}\,p_{66}\mu_3 - p_{01}\,p_{14}\,p_{66}\mu_3 - p_{02}\,p_{24}\,p_{66}\mu_3 + p_{11}\,p_{02}\,p_{24}\,p_{66}\mu_3 - p_{02}\,p_{24}\,p_{13}p_{41}p_{66}\mu_3$ (66)

- Expected number of repairs $[R_0^*(s)]$:

$$R_0 = \lim_{s\to 0} sR_0^*(s) = \frac{N_6(0)}{D_2'(0)} \tag{67}$$

Where,

$N_6(0) = p_{02}p_{20} - p_{11}\,p_{02}p_{20} + p_{01}p_{14}\,p_{41} + p_{02}p_{24}\,p_{41} - p_{11}p_{02}p_{24}\,p_{41} - p_{02}p_{20}p_{14}p_{41} + p_{01}p_{20}p_{13}p_{36}p_{65}p_{52} + p_{01}p_{24}p_{41}\,p_{13}p_{36}p_{65}p_{52} + p_{01}\,p_{14}\,p_{43}p_{36}p_{65}p_{52}\,p_{20} - p_{02}p_{20}p_{56}p_{65} + p_{11}$

$p_{02}p_{20}p_{56}p_{65} - p_{01}\, p_{14}p_{41}p_{56}p_{65} - p_{02}p_{24}\, p_{41}\, p_{56}p_{65} + p_{11}p_{02}p_{24}\, p_{41}p_{56}p_{65} + p_{02}p_{14}\, p_{41}\, p_{56}p_{65} + p_{01}\, p_{13}p_{36}\, p_{56}p_{65} + p_{02}p_{24}\, p_{41}\, p_{13}p_{36}\, p_{56}p_{65} + p_{01}\, p_{14}p_{43}p_{36}\, p_{56}p_{65} + p_{02}\, p_{24}p_{43}p_{36}\, p_{56}p_{65} - p_{02}\, p_{24}p_{43}p_{36}\, p_{56}p_{65}\, p_{11} - p_{02}p_{20}\, p_{66} + p_{11}\, p_{02}p_{20}\, p_{66} - p_{01}p_{14}\, p_{41}p_{66} - p_{02}p_{24}\, p_{41}p_{66} + p_{11}\, p_{02}p_{24}\, p_{66} + p_{02}p_{20}\, p_{41}\, p_{14}\, p_{66}$
(68)

and $D_2'(0)$ as already mentioned in equation (52).

6. PARTICULAR CASE

For the particular case, it is assumed that the failure and repair rates are exponentially distributed and therefore the following have been assumed:

$g(t) = \alpha e^{-\alpha t}$, $g_m(t) = \gamma e^{-\gamma t}$, $g_{sr}(t) = \gamma_2 e^{-\gamma_2 t}$

Using (1-16) and (24-30), the following are obtained:

$$p_{00} = g^*_m(6\lambda+\beta_1) = \frac{\gamma}{\gamma+6\lambda+\beta_1};\quad p_{01} = \frac{\beta_1}{6\lambda+\beta_1}[1-g^*_m(6\lambda+\beta_1)] = \frac{\beta_1}{\gamma+6\lambda+\beta_1};$$

$$p_{02} = \frac{6\lambda}{6\lambda+\beta_1}[1-g^*_m(6\lambda+\beta_1)] = \frac{6\lambda}{\gamma+6\lambda+\beta_1}$$

$$p_{11} = g^*_m(6\lambda+\gamma_1) = \frac{\gamma}{\gamma+6\lambda+\gamma_1};\quad p_{13} = \frac{\gamma_1}{6\lambda+\gamma_1}[1-g^*_m(6\lambda+\gamma_1)] = \frac{\gamma_1}{\gamma+6\lambda+\gamma_1}$$

$$p_{14} = \frac{6\lambda}{6\lambda+\gamma_1}[1-g^*_m(6\lambda+\gamma_1)] = \frac{6\lambda}{\gamma+6\lambda+\gamma_1}$$

$$p_{20} = g^*(\beta_1) = \frac{\alpha}{\alpha+\beta_1};\quad p_{24} = 1-g^*(\beta_1) = \frac{\beta_1}{\alpha+\beta_1};\ p_{36} = 1$$

$$p_{41} = g^*(\gamma_1) = \frac{\alpha}{\alpha+\gamma_1};\quad p_{43} = 1-g^*(\gamma_1) = \frac{\gamma_1}{\alpha+\gamma_1};$$

$$p_{52} = 1-g^*(\beta_2) = \frac{\beta_2}{\alpha+\beta_2};\quad p_{56} = g^*(\beta_2) = \frac{\alpha}{\alpha+\beta_2};$$

$$p_{60} = \frac{\beta_2}{6\lambda+\beta_2}[1-g^*_m(6\lambda+\beta_2)] = \frac{\beta_2}{\gamma+6\lambda+\beta_2}$$

$$p_{65} = \frac{6\lambda}{6\lambda+\beta_2}[1-g^*_m(6\lambda+\beta_2)] = \frac{6\lambda}{\gamma+6\lambda+\beta_2}\quad p_{66} = g^*_m(6\lambda+\beta_2) = \frac{\gamma}{\gamma+6\lambda+\beta_2};$$

$$\mu_0 = \int_0^\infty e^{-\gamma t}e^{-6\lambda t}e^{-\beta_1 t}dt = \frac{1}{\gamma+6\lambda+\beta_1};\quad \mu_1 = \int_0^\infty e^{-\gamma t}e^{-6\lambda t}e^{-\gamma_1 t}dt = \frac{1}{\gamma+6\lambda+\gamma_1};$$

$$\mu_2 = \frac{1}{\alpha+\beta_1},\ \mu_3 = \frac{1}{\gamma_2},\ \mu_4 = \frac{1}{\alpha+\gamma_1},\ \mu_5 = \frac{1}{\alpha+\beta_2},\ \mu_6 = \frac{1}{6\lambda+\gamma+\beta_2}$$

Using the above equations and the values estimated from the data, the following are obtained:

μ_0 = 531.22543; μ_1 = 566.5273; μ_2 = 552.94443; μ_3 = 144.00092; μ_4 = 591.29612;

μ_5 = 552.9444; μ_6= 531.22543.

p_{00} =0.790516564, p_{01} = 0.122978687, p_{02}= 0.086504749;

p_{11}=0.843049277, p_{13} = 0.092253306, p_{14} = 0.064697418;

p_{20}= 0.871993365,p_{24}=0.128006635;

$p_{36=}1$;

p_{41}= 0.932473983,p_{43}=0.06752602;

p_{52}=0.128006635, p_{56}=0.871993365;

p_{60}=0.122978687,p_{65}=0.086504749, p_{66}=0.790516564;

Using the summarized data and the expressions of section 5, various measures of system effectiveness are estimated:

Mean time for the unit to shut down = 424 days

Availability of the unit (A_0) = 0.991790084

Expected busy period for repairman (B_0^M) = 0.902028086

Expected busy period for repair (B_0^R) = 0.008209916

Expected busy period during shut down (B_0^S) = 0.089761998

Expected number of repairs (R_0) =0.000141555

As a future direction, it would be interesting to study the variations on these results of the plant when repair or maintenance is done on first come first served basis.

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Performance and Profit Evaluations of a Stochastic Model on Centrifuge System Working in Thermal Power Plant Considering Neglected Faults

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Abstract

The paper formulates a stochastic model for a single unit centrifuge system on the basis of the real data collected from the Thermal Power Plant, Panipat (Haryana). Various faults observed in the system are classified as minor, major and neglected faults wherein the occurrence of a minor fault leads to degradation whereas occurrence of a major fault leads to failure of the system. Neglected faults are taken as those faults that are neglected /delayed for repair during operation of the system until the system goes to complete failure such as vibration, abnormal sound, etc. However these faults may lead to failure of the system. There is assumed to be single repair team that on complete failure of the system, first inspects whether the fault is repairable or non repairable and accordingly carries out repairs/replacements. Various measures of system performance are obtained using Markov processes and regenerative point technique. Using these measures profit of the system is evaluated. The conclusions regarding the reliability and profit of the system are drawn on the basis of the graphical studies.

Keywords: Centrifuge System, Neglected Faults, Mean Time to System Failure, Expected Uptime, Profit, Markov Process and Regenerative Point Technique.

1. INTRODUCTION

The centrifuge system or simply centrifuge is related to continuously operating machines with inertial discharge of deposit. These are used for extracting solid deposits and suspensions of liquid media, and separation of medium and highly concentrated suspensions. For instance it is used in Thermal Power Plants for oil purification, milk plants, laboratories for blood fractionation, and liquor industries for wine clarification. As in many practical situations centrifuge systems are used and act as the main systems or sub-systems and therefore play a very significant and crucial role in determining the reliability and cost of the whole system.

In the design, manufacture and operation of centrifuge system evaluation of their reliability is recommended in order to provide accident-free operation [1]. Various authors in the field of reliability modeling including [2-7] analyzed several one and two-unit systems considering various aspects such as different types of failure, maintenances, repairs/replacements policies, inspections, operational stages etc. In the literature of reliability modeling not much work has been reported to analyze the centrifuge systems in terms of their performance and cost.

However [8] carried out reliability and cost analyses of a centrifuge system considering minor and major faults wherein a minor fault leads to down state while a major fault leads to complete failure of the system. In fact while collecting data on faults/ failures and repairs of a centrifuge system working in Thermal Power Plant, Panipat (Haryana), it was also observed that some faults such as vibration, abnormal sound, etc are neglected/ delayed for repair during the operation of the system until system fails. These faults even sometimes lead to complete failure of the system. The aspect of neglected faults in the system was not taken up in [8]. The values of various rates and probabilities estimated from the data collected for the centrifuge system are as under:

Estimated value of rate of occurrence of major faults	=	0.0019
Estimated value of rate of occurrence of minor faults	=	0.0022
Estimated value of rate of occurrence of neglected faults	=	0.0018
Probability that a fault is non repairable major faults	=	0.3672
Probability that a fault is repairable major faults	=	0.6328
Estimated repair rate on occurrence of minor faults	=	0.3846
Estimated repair rate on occurrence of repairable major faults	=	0.3097
Estimated replacement rate on occurrence of non repairable major faults	=	0.3177

Keeping this in view, the present paper formulates a stochastic model for a single unit centrifuge system considering minor, major and neglected faults wherein a minor fault degrades the system whereas a major fault leads to complete failure of the system. The neglected fault is taken as the fault that may be neglected for repair during the operation of the system until system goes to complete failure. During the complete failure the repair team first inspect whether the fault is repairable or non repairable and accordingly carry out repair or replacement of the faulty components. Various measures of system performance such as mean time to system failure, expected up time and expected down time, expected number of repairs/replacements are obtained using Markov processes and regenerative point technique. Using these measures profit of the system is computed. Various conclusions regarding the reliability and profit of the system are drawn on the basis of graphical analysis for a particular case.

2. ASSUMPTIONS

1. Faults are self- announcing.
2. The repair team reaches the system in negligible time.
3. The system is as good as new after each repair/replacement.
4. The neglected faults may occur when system is either operative or degraded**.**
5. Switching is perfect and instantaneous.
6. The failure time distributions are exponential while other time distributions are general.

3. NOTATIONS

λ_1 / λ_2 / λ_3	Rate of occurrence of a major/minor/neglected faults
a/b	Probability that a fault is non repairable/repairable, b = 1- a
i(t)/I(t)	p.d.f./c.d.f. of time to inspection of the unit
$g_1(t)$/ $G_1(t)$,	p.d.f./c.d.f. of time to repair the unit at down state
$g_2(t)$/$G_2(t)$	p.d.f./c.d.f. of time to repair the unit at failed state
h(t)/H(t)	p.d.f./c.d.f. of time to replacement of the unit
k(t)/K(t)	p.d.f./c.d.f. of time to delay in repair of the neglected fault

O	Operative state
O_n/ O_r	Operative state under neglected fault/repair
F_i/ F_r / F_{rp}	Failed unit under inspection/ repair/ replacement

4. THE MODEL

A diagram showing the various states of transition of the system is shown in Figure 1. The epochs of entry in to state 0, 1, 2, 3, 4, 5 and 6 are regenerative point and thus all the states are regenerative states.

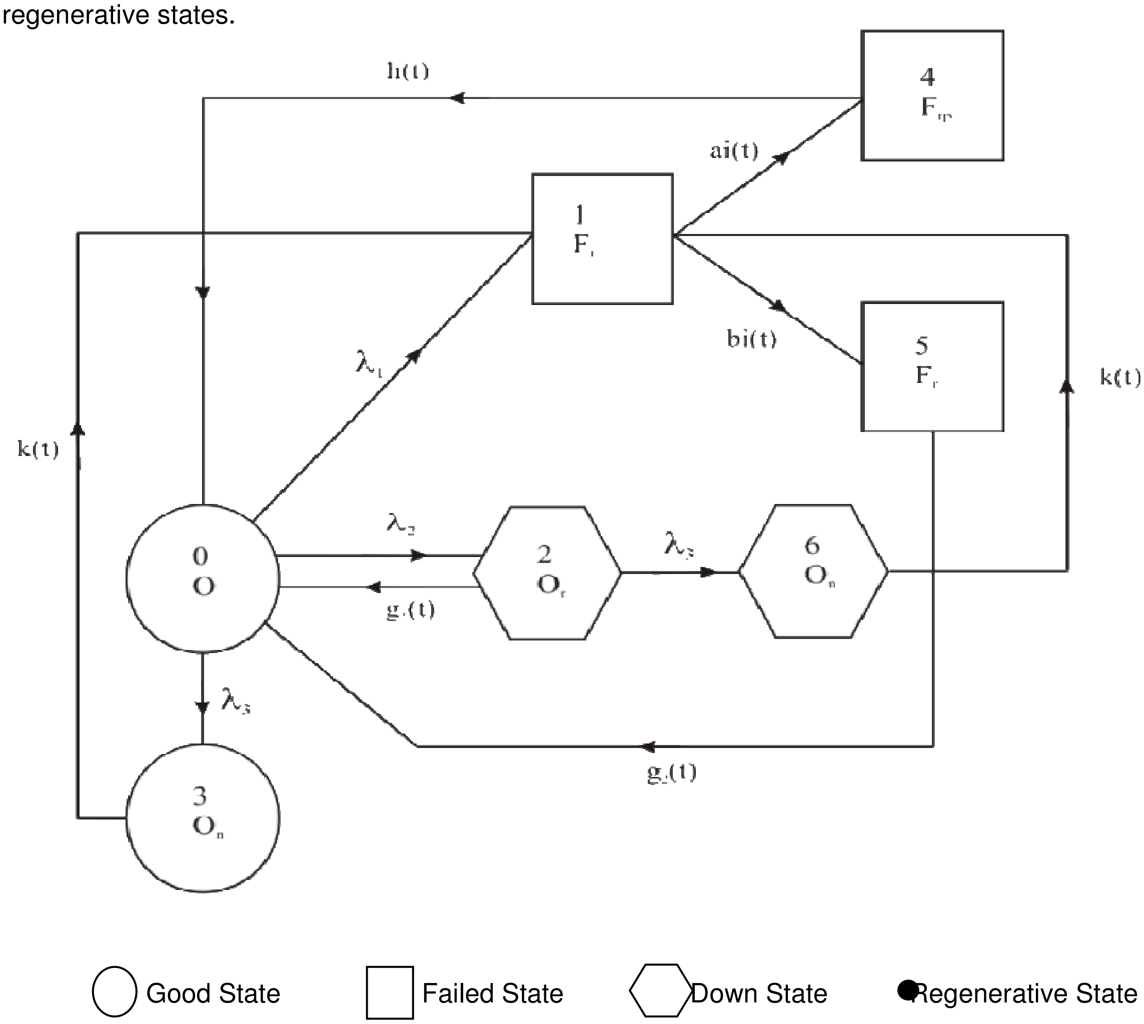

FIGURE 1: State Transition Diagram.

5. Transition Probabilities and Mean Sojourn Time

The transition probabilities are

$$dQ_{01}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2+\lambda_3)t}dt \qquad dQ_{02}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2+\lambda_3)t}dt \qquad dQ_{03}(t) = \lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3)t}dt$$

$$dQ_{14}(t)=ai(t)dt \qquad dQ_{15}(t) = bi(t)dt \qquad dQ_{20}(t) = g_1(t)e^{-\lambda_3 t}dt$$

$$dQ_{26}(t) = \lambda_3 e^{-\lambda_3 t}\overline{G}_1(t)dt \qquad dQ_{31}(t) = k(t)dt = dQ_{61}(t) \qquad dQ_{40}(t) = h(t)dt$$

$$dQ_{50}(t) = g_2(t)dt$$

The non-zero elements p_{ij} are:

$$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \qquad p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \qquad p_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{14} = ai^*(0) \qquad p_{15} = bi^*(0) \qquad p_{20} = g_1^*(\lambda_3)$$

$$p_{26} = 1 - g_1^*(\lambda_3) \qquad p_{31} = k^*(0) = p_{61} \qquad p_{40} = h^*(0)$$

$$p_{50} = g_2^*(0)$$

By these transition probabilities, it can be verified that:

$p_{01}+p_{02}+p_{03}=1$, $p_{14}+p_{15}=1$, $p_{20}+p_{26}=1$, $p_{31} = p_{40}= p_{50}=p_{61}=1$

The mean sojourn time (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state I, then

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} \qquad \mu_1 = -i^{*/}(0) \qquad \mu_2 = \frac{1 - g_1^*(\lambda_3)}{\lambda_3}$$

$$\mu_3 = -k^{*/}(0) = \mu_6 \qquad \mu_4 = -h^{*/}(0) \qquad \mu_5 = -g_2^{*/}(0)$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as:

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -Q_{ij}^{*'}(s)$$

Thus,

$m_{01} + m_{02} + m_{03} = \mu_0$ $m_{14} + m_{15} = \mu_1$ $m_{20} + m_{26} = \mu_2$

$m_{31} = \mu_3$ $m_{40} = \mu_4$ $m_{50} = \mu_5$

$m_{61} = \mu_6$

6. OTHER MEASURES OF SYSTEM PERFORMANCE

Using probabilistic arguments for regenerative processes, various recursive relations are obtained and are solved to derive important measures of the system performance that are as given below:

Mean time to system failure (T_0) **= N/D**

Expected up time of the system (A_0) **= N_1/ D_1**

Expected down time of the system (A_{01}) **= N_2/ D_1**

Busy period of repair man (Inspection time only)(B_i) **= N_3/ D_1**

Busy period of repair man (Repair time only)(B_r) **= N_4/ D_1**

Busy period of repair man (Replacement time only) (B_{rp}) **= N_5/ D_1**

where

$N = \mu_0 + p_{02}\mu_2 + p_{03}\mu_3 + p_{02}p_{26}\mu_6$

$D = 1 - p_{02}p_{20}$

$N_1 = \mu_0 + p_{03}\mu_3$

$N_2 = p_{02}\mu_2 + p_{02}\, p_{26}\mu_6$

$N_3 = (p_{01}+p_{02}p_{26}p_{61}+p_{03}p_{31})\ \mu_1$

$N_4 = p_{02}\mu_2+(p_{01}+p_{02}p_{26}p_{61}+p_{03}p_{31})\ p_{15}\ \mu_5$

$N_5 = (p_{01}+p_{02}p_{26}p_{61}+p_{03}p_{31})\ p_{14}\ \mu_4$

$D_1 = \mu_0 + p_{02}\mu_2 + p_{03}\mu_3 + p_{02}\, p_{26}\mu_6 + (\mu_1 + p_{14}\mu_4 + p_{15}\mu_5)(p_{01}+p_{02}p_{26}p_{61}+ p_{03}p_{31})$

7. PROFIT ANALYSIS

The expected profit incurred of the system is

$P = C_0\, A_0 - C_1\, A_{01} - C_2\, B_i - C_3\, B_r - C_4 B_{rp} - C$

where

C_0 = revenue per unit uptime of the system
C_1 = revenue per unit downtime of the system
C_2 = cost per unit inspection of the failed unit
C_3 = cost per unit repair of the failed unit
C_4 = cost per unit replacement of the failed unit
C = cost of installation of the unit

8. GRAPHICAL INTERPRETATION AND CONCLUSIONS

For graphical analysis the following particular cases are considered:

$$g_1(t) = \beta_1 e^{-\beta_1(t)} \qquad g_2(t) = \beta_2 e^{-\beta_2(t)} \qquad i(t) = \alpha e^{-\alpha(t)} \qquad k(t) = \delta e^{-\delta(t)}$$
$$h(t) = \gamma e^{-\gamma(t)}$$

Various graphs are drawn for the MTSF, the expected uptime (A_0) and expected profit (P) of the system for the different values of the rate of occurrence of faults (λ_1, λ_2, λ_3), repairs (β_1, β_2), replacement (□), inspection (α) and delay (δ) on the basis of these plotted graphs.

Figure 2 gives the graphs between MTSF (T_0) and the rate of occurrence of neglected faults (λ_3), for different values of rate of occurrence of major faults (λ_1). The graph reveals that the MTSF deceases with increase in the values of rates of occurrence of major and neglected faults.

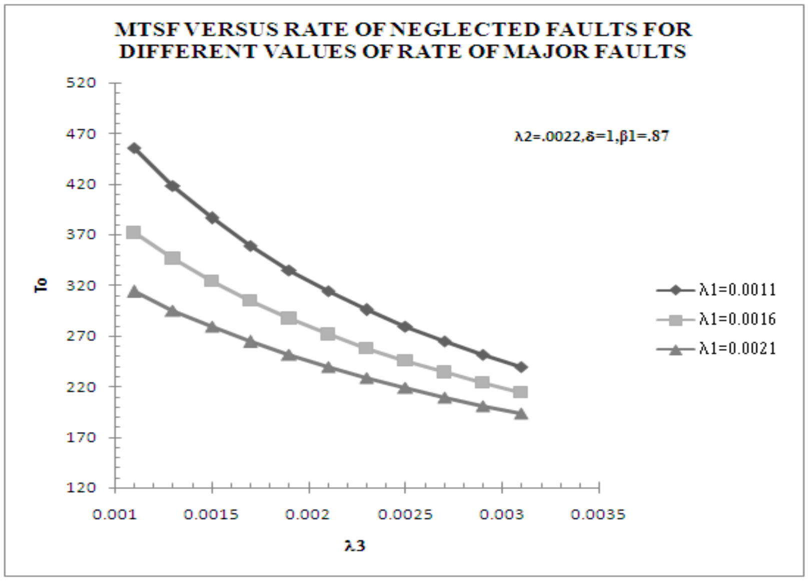

FIGURE 2

Figure 3 gives the graphs between MTSF (T_0) and the rate of occurrence of neglected faults (λ_3) for different values of rate of delay in repair of neglected faults (δ). The graph reveals that the MTSF deceases with increase in the values of rates of occurrence of neglected faults and delay in repair of neglected faults.

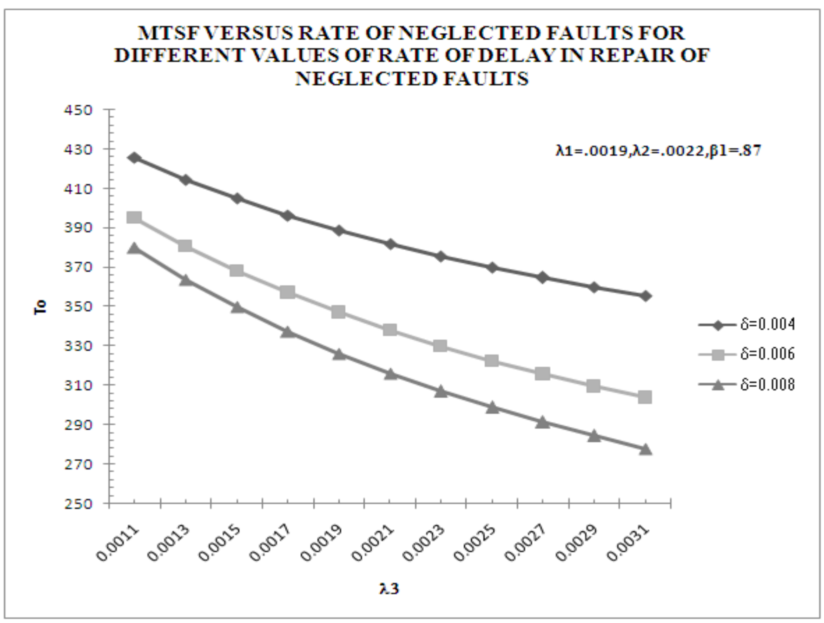

FIGURE 3

Figure 4 gives the graphs of expected uptime (A_0) of the system and rate of occurrence of minor faults (λ_2) for different values of rates of occurrence of major faults (λ_1). The graphs reveal that the expected uptime of the system deceases with increase in the values of failure rates.

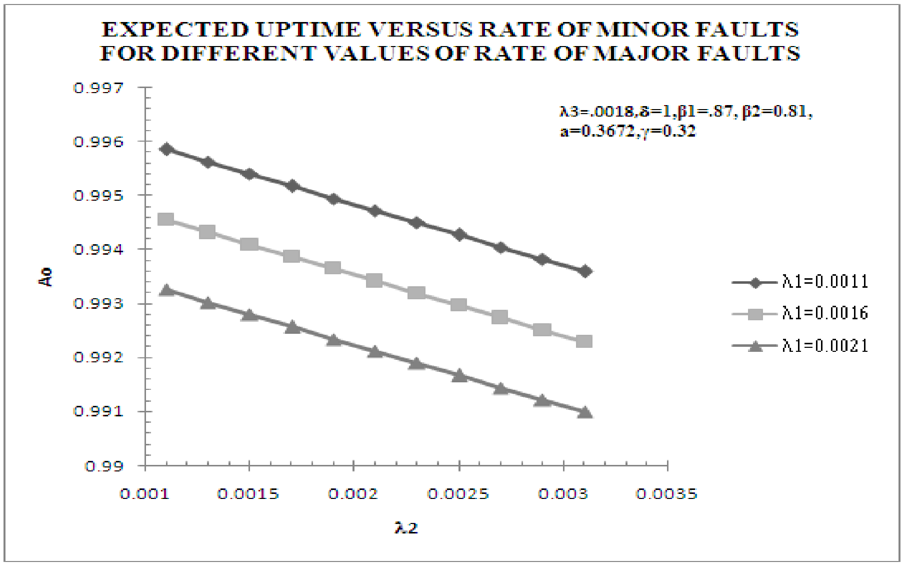

FIGURE 4

The curves in the figure 5 show the behavior of the profit with respect to rate of occurrence of minor faults (λ_2) of the system for the different values of rate of occurrence major faults (λ_1). It is evident from the graph that profit decreases with the increase in the rate due to occurrence of minor faults and major faults respectively when other parameters remain fixed. From the figure 5 it may also be observed that for $\lambda_1 = 0.0001$, the profit is > or = or < 0 according as λ_2 is < or = or > 0.0851. Hence the system is profitable to the company whenever $\lambda_2 \leq 0.0851$. Similarly, for $\lambda_1 = 0.0081$ and $\lambda_1 = 0.0161$ respectively the profit is > or = or < 0 according as λ_2 is < or = or > 0.0762 and 0.0671 respectively. Thus, in these cases, the system is profitable to the company whenever $\lambda_2 \leq 0.0762$ and 0.0671 respectively

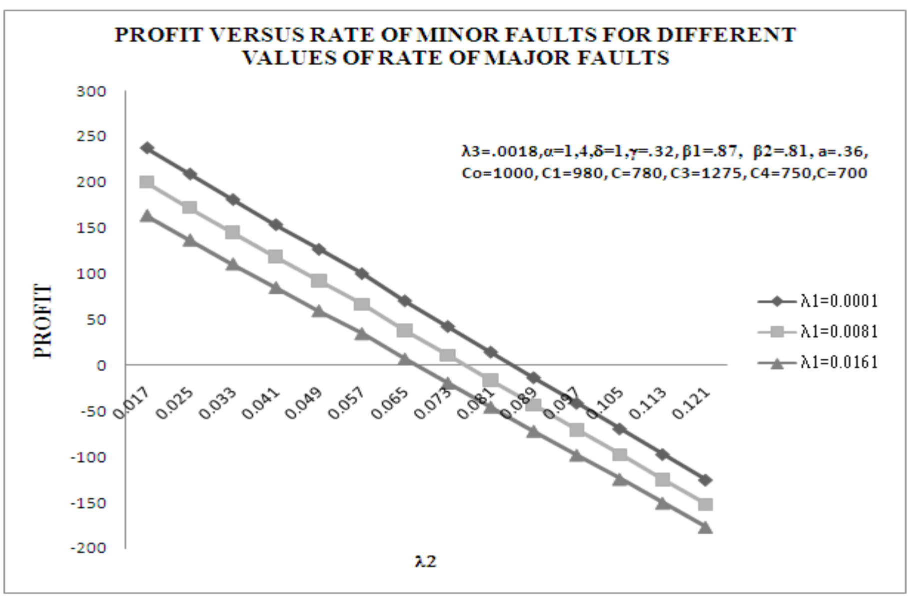

FIGURE 5

The curves in the figure 6 show the behavior of the profit with respect to rate of occurrence of minor faults (λ_2) of the system for the different values of rate of delay in repair of neglected faults (δ). It is evident from the graph that profit decreases with the increase in the rate due to occurrence of minor faults and delay in repair of neglected faults respectively when other parameters remain fixed. From the figure 6 it may also be observed that for δ =0.003, the profit is > or = or < 0 according as λ_2 is < or = or > 0.1038. Hence the system is profitable to the company whenever $\lambda_2 \leq 0.1038$. Similarly, for δ = 0.005 and δ = 0.007 respectively the profit is > or = or < 0 according as λ_2 is < or = or > 0.098 and 0.095 respectively. Thus, in these cases, the system is profitable to the company whenever $\lambda_2 \leq$ 0.098 and 0.095 respectively.

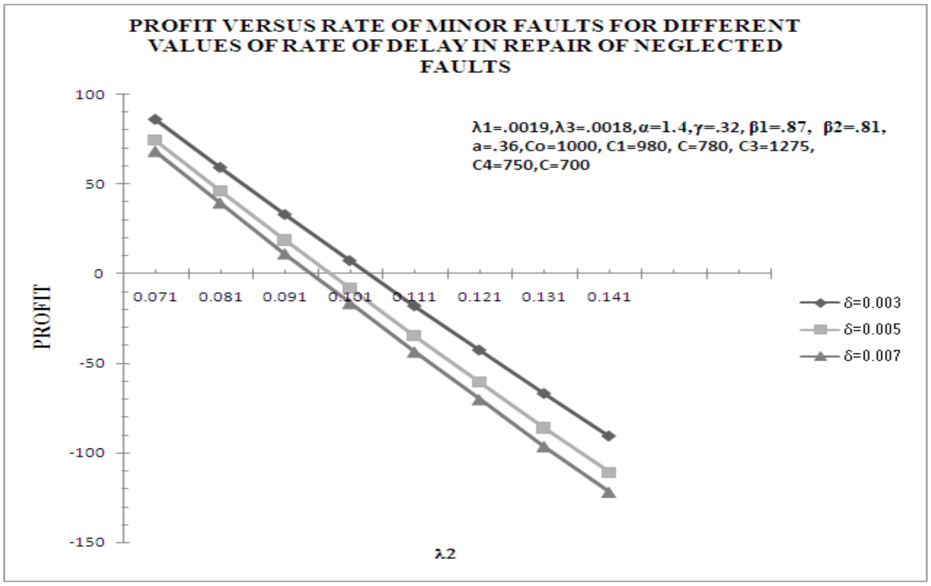

FIGURE 6

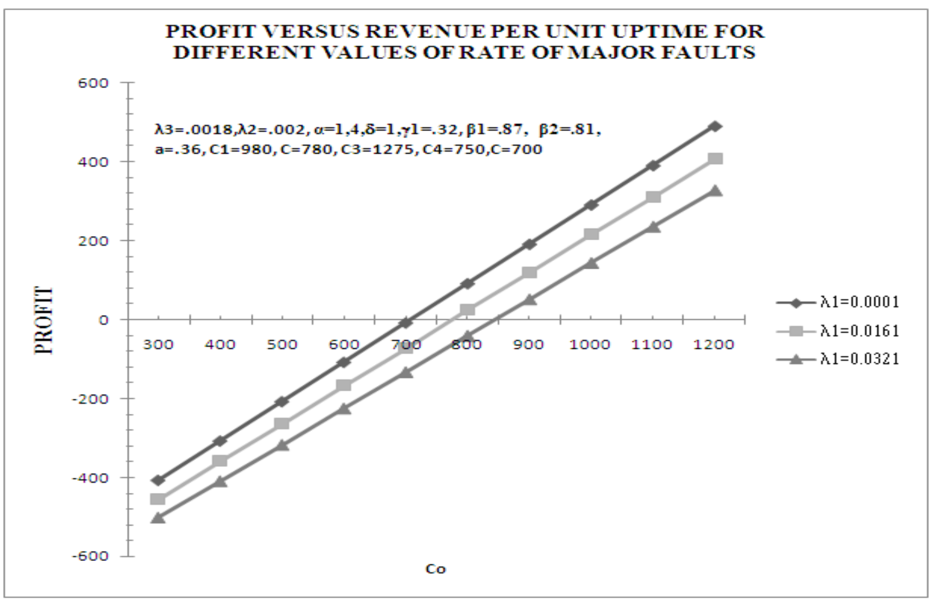

FIGURE 7

The curves in the figure 7 show the behavior of the profit with respect to the revenue per unit up time (C_0) of the system for the different values of rate of occurrence of major faults (λ_1). It is evident from the graph that profit increases with the increase in revenue up time of the system for fixed value of the rate of occurrence of major faults. From the figure 7 it may also be observed that for λ_1 = 0.0001, the profit is > or = or < 0 according as C_0 is > or = or < 707.88. Hence the system is profitable to the company whenever $C_0 \geq$ Rs. 707.88. Similarly, for λ_1 = 0.0161 and λ_1 = 0.0321 respectively the profit is > or = or < 0 according as C_0 is > or = or < Rs.775.98 and Rs.844.08 respectively. Thus, in these cases, the system is profitable to the company whenever $C_0 \geq$ Rs.775.98 and Rs.844.08 respectively.

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Logistic Loglogistic With Long Term Survivors For Split Population Model

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Abstract

The split population model postulates a mixed population with two types of individuals, the susceptibles and long-term survivors. The susceptibles are at the risk of developing the event under consideration, and the event would be observed with certainty if complete follow-up were possible. However, the long-term survivors will never experience the event. We known that populations are immune in the Stanford Heart Transplant data. This paper focus on the long term survivors probability vary from individual to individual using logistic model for loglogistic survival distribution. In addition, a maximum likelihood method to estimate parameters in the split population model using the Newton-Raphson iterative method.

Keywords: Split Population Model, Logistic Loglogistic Model, Split Loglogistic Model.

1. INTRODUCTION

Split population models are also known as mixture model. The data used in this paper is Stanford Heart Transplant data. Survival times of potential heart transplant recipients from their date of acceptance into the Stanford Heart Transplant program [3]. This set consists of the survival times, in days, uncensored and censored for the 103 patients and with 3 covariates are considered Ages of patients in years, Surgery and Transplant, failure for these individuals is death. Covariate methods have been examined quite extensively in the context of parametric survival models for which the distribution of the survival times depends on the vector of covariates associated with each individual. See [6] for approaches which accommodate censoring and covariates in the ordinary exponential model for survival.

Currently, such mixture models with immunes and covariates are in use in many areas such as medicine and criminology. See for examples [4][5][7]. In our formulation, the covariates are incorporated into a split loglogistic model by allowing the proportion of ultimate failures and the rate of failure to depend on the covariates and the unknown parameter vectors via logistic model. Within this setup, we provide simple sufficient conditions for the existence, consistency, and asymptotic normality of a maximum likelihood estimator for the parameters involved. As an application of this theory, the likelihood ratio test for a difference in immune proportions is shown to have an asymptotic chi-square distribution. These results allow immediate practical applications on the covariates and also provide some insight into the assumptions on the covariates and the censoring mechanism that are likely to be needed in practice. Our models and analysis are described in section 5.

2. PREVIOUS METHODOLOGY AND DISCUSSION

[2] was the first to publish in a discussion paper of the Royal Statistical Society. He used the method of maximum likelihood to estimate the proportion of cured breast cancer patients in a population represented by a data set of 121 women from an English hospital. The follow-up time for each woman varied up to a maximum of 14 years. [2] approach was to assume a lognormal

distribution as the survival distribution of the susceptibles, although, curiously he noted that an exponential distribution is in fact a better fit to the particular set of data analyzed, and to treat deaths from causes other than the cancer under consideration as a censoring mechanism.

[8] used a model consisting of a mixture of the exponential distribution and a degenerate distribution, to allow for a cured proportion, they fitted this model to a large data set consisting of 2682 patients from the Mayo clinic who suffered from cancer of the stomach. The follow-up time on some of their patients was as much as 15 years. They noted that a correct interpretation of the existence of patients cured of the disease should be that the death rates for individual with long follow-up drop to the baseline death rate of the population.

[9] applied a Weibull mixture model with allowance for immunes to a prospective study on breast cancer. Information on various factors was collected at a certain time for approximately 5000 women, of whom 48 subsequently developed breast cancer. [9] wished to estimate that proportion and to investigate how it may be influenced by risk factors, as well as to investigate how risk factors might affect the time to development of the cancer, if this occurred.

Similarly above, we will allow the covariates associated with individuals, relate the loglogistic with long term survivors, and relate to the probability of being immune in logistic model.

3. SPLIT MODELS

In this section, we will consider 'split population models' (or simply 'split models') in which the probability of eventual death is an additional parameter to be estimated, and may be less than one. Split models in the biometrics literature, i.e., part of the population is cured and will never experience the event, and have both a long history [2] and widespread applications and extensions in recent years [4]. The intuition behind these models is that, while standard duration models require a proper distribution for the density which makes up the hazard (i.e., one which integrates to one; in other words, that all subjects in the study will eventually fail), split population models allow for a subpopulation which never experiences the event of interest. This is typically accomplished through a mixture of a standard hazard density and a point mass at zero [6]. That is, split population models estimate an additional parameter (or parameters) for the probability of eventual failure, which can be less than one for some portion of the data. In contrast, standard event history models assume that eventually all observations will fail, a strong and often unrealistic assumption.

In standard survival analysis, data come in the form of failure times that are possibly censored, along with covariate information on each individual. It is also assumed that if complete follow-up were possible for all individual, each would eventually experience the event. Sometimes however, the failure time data come from a population where a substantial proportion of the individuals does not experience the event at the end of the observation period. In some situations, there is reason to believe that some of these survivors are actually "cured" or "long–term survivors" the sense that even after an extended follow-up, no further events are observed on these individuals. Long-term survivors are those who are not subject to the event of interest. For example, in a medical study involving patients with a fatal disease, the patients would be expected to die of the disease sooner or later, and all deaths could be observed if the patients had been followed long enough. However, when considering endpoints other than death, the assumption may not be sustainable if long-term survivor are present in population. In contrast, the remaining individuals are at the risk of developing the event and therefore, they are called *susceptibles*.

Using the notation of [7], we can express a split model as follows. Suppose that $F_R(t)$ is the usual cumulative distribution function for death only, and ω is the probability of being subject to reconviction, which is also usually known as the eventual death rate. The probability of being immune is (1-ω), which is sometimes described as the rate of termination. This second group of immune individuals will never reoffend. Therefore their survival times are infinite (with probability one) and so their associated cumulative distribution function is identically zero, for all finite $t > 0$. If we now define $F_S(t) = \omega\ F_R(t)$, as the new cumulative distribution function of failure for the split-population, then this an improper distribution, in the sense that, for $0 < \omega < 1$, $F_S(\infty) = \omega < 1$.

Let Y_i be an indicative variable, such that

$$Y_i = \begin{cases} 0; & \text{ith individual will never fail} \\ 1; & \text{ith individual will eventually fail} \end{cases}$$

and follows the discrete probability distribution

$$\Pr[Y_i = 1] = \omega$$

and

$$\Pr[Y_i = 0] = (1-\omega).$$

For any individual belonging to the group of death, we define the density function of eventual failure as $F_R(t)$ with corresponding survival function $S_R(t)$, while for individual belonging to the other (immune) group, the density function of failure is identically zero and the survival function is identically one, for all finite time *t*.

Suppose the conditional probability density function for those who will eventually fail (death) is

$$f(t \mid Y = 1) = f_R(t) = F_R^{'}(t)$$

wherever $F_R(t)$ is differentiable. The unconditional probability density function of the failure time is given by

$$f_s(t) = f(t \mid Y = 0)\Pr[Y = 0] + f(t \mid Y = 1)\Pr[Y = 1]$$

$$= 0\ (1-\omega) + f_R(t)\ \omega = \omega\ f_R(t).$$

Similarly, the survival function for the recidivist group is defined as

$$S_R(t) = \Pr[T > t \mid Y = 1] = \int_t^{\infty} f(u \mid Y = 1)du$$

$$= \int_t^{\infty} f_R(u)du = 1 - F_R(t).$$

The unconditional survival time is then defined for the split population as

$$S_S(t) = \Pr[T > t] = \int_t^{\infty} \{f(u \mid Y = 0)\Pr[Y = 0] + f(u \mid Y = 1)\Pr[Y = 1]\}du$$

$$= (1-\omega) + \omega\ S_R(t)$$

which corresponds to the probability of being a long-term survivor plus the probability of being a recidivist who reoffends at some time beyond *t*.
In this case,

$$F_S(t) = \omega F_R(t)$$

is again an improper distribution function for $\omega < 1$.

4. THE LIKELIHOOD FUNCTION

The likelihood function can then be written as

$$L(\omega,\theta)=\prod_{i=1}^{n}[\omega f_R(t_i)]^{\delta_i}[(1-\omega)+\omega S_R(t_i)]^{1-\delta_i}$$

and the log-likelihood function becomes

$$l(\omega,\theta)=\ln L(\omega,\theta)=\sum_{i=1}^{n}\{\delta_i[\ln\omega+\ln f_R(t_i)]+(1-\delta_i)\ln[(1-\omega)+\omega S_R(t_i)]\}$$

where δ_i is an indicator of the censoring status of observation t_i, and θ is vector of all unknown parameters for $f_R(t)$ and $S_R(t)$. The existence of these two types of release, one type that simply does not reoffend and another that eventually fails according to some distribution, leads to what may be described as simple split-model. When we modify both $f_R(t)$ and $S_R(t)$ to include covariate effects, $f_R(t\mid z)$ and $S_R(t\mid z)$ respectively, then these will be referred to as *split models*.

We fit split models to our data using the same three distributions as were considered in the section (exponential, Weibull and loglogistic). The likelihood values achieved were -511.21, -495.60 and -489.17, respectively. The loglogistic model fits the estimation better than other two distributions, while the exponential model better than weibull model. The value of the 'splitting parameter' ω implied by our models were 0.81, 0.84 and 0.78 for the exponential, Weibull and loglogistic distributions, respectively.

5. MODEL WITH EXPLANATORY VARIABLES

We now consider models with explanatory variables. This is obviously necessary if we are to make predictions for individuals, or even if we are to make potentially accurate predictions for groups which differ systematically from our original sample. Futhermore, in many applications in economics or criminology the coefficients of the explanatory variables may be of obvious interest. We begin by fitting a parametric model based on the loglogistic distribution. The model in its most general form is a split model in which the probability of eventual death follows a logistic model, while the distribution of the time until death is loglogistic, with its scale parameter depending on explanatory variables. The estimate are based on the usual MLE method.

To be more explicit, we follow the notation of section 3. For individual *i*, there is an unobservable variable Y_i which indicates whether or not individual *i* will eventually return to prison. The probability of eventual failure for individual *i* will be denoted ω_i so that $P(Y_i=1)=\omega_i$. Let Z_i be a (row) vector of individual characteristics (explanatory variables), and let α be the corresponding vector of parameters. Then we assume a logistic model for eventual death:

$$\omega_i=\frac{\exp(\alpha^T z_i)}{\left[1+\exp(\alpha^T z_i)\right]}.$$

Next, we assume that the distribution of time until death is loglogistic , with scale parameter λ and shape parameter κ .

The likelihood function for this model is

$$l(\omega_{\mathrm{i}},\theta)=\ln L(\omega_{\mathrm{i}},\theta)$$
$$=\sum_{i=1}^{n}\{\delta_i[\ln\omega_{\mathrm{i}}+\ln f_R(t_i)]+(1-\delta_i)\ln[(1-\omega_{\mathrm{i}})+\omega_{\mathrm{i}}\,S_R(t_i)]\}.$$

We can now define special cases of this general model. First, the model in which $\omega_{\mathrm{i}}=0,$ but in which the scale parameter depends on individual characteristics, will be called *Loglogistic model* (with explanatory variables) , it is not a split model. Second, the model in which ω_{i} is replaced by a single parameter ω will be referred to as the *split Loglogistic model* (with explanatory variables). In this model the probability of eventual death is a constant, though not necessarily equal to one, while the scale parameter of the distribution of time until death varies over individuals or depend on individual characteristic Z_{i}, so that $\lambda_i=\exp(\beta^T z_i)$.

The likelihood function for this model is

$$l(\omega_{\mathrm{i}},\beta,\kappa)=$$
$$\sum_{i=1}^{n}\left\{\begin{array}{l}\delta_i\left[\ln\omega_{\mathrm{i}}+\beta^T z_i+\ln\kappa+(\kappa-1)\ln(t_i\exp(\beta^T z_i))-2\ln\left(1+\left(\exp(\beta^T z_i)\kappa t_i\right)^{\kappa}\right)\right]+\\(1-\delta_i)\ln\left[\dfrac{(1-\omega_{\mathrm{i}})\left[1+\left(\exp(\beta^T z_i)\kappa t\right)^{\kappa}\right]^2+\omega_{\mathrm{i}}}{\left[1+\left(\exp(\beta^T z_i)\kappa t\right)^{\kappa}\right]^2}\right]\end{array}\right\}.$$

Third, the model in which λ_{i} is replaced by a single parameter λ will be called the *logistic Loglogistic model*. In this model the probability of eventual death varies over individual , while the distribution of time until death (for the eventual death) does not depend on individual characteristics. The likelihood function for this model is

$$l(\alpha,\lambda,\kappa)=$$
$$=\sum_{i=1}^{n}\left\{\begin{array}{l}\delta_i\left[\ln\left(\dfrac{\exp(\alpha^T z_i)}{1+\exp(\alpha^T z_i)}\right)+\ln\lambda+\ln\kappa+(\kappa-1)\ln(\lambda t_i)-2\ln\left(1+(\lambda\kappa t)^{\kappa}\right)\right]+\\(1-\delta_i)\ln\left(\dfrac{[1+(\lambda\kappa t)^{\kappa}]^2+\exp(\alpha^T z_i)}{\left[1+\exp(\alpha^T z_i)\right]\left[1+(\lambda\kappa t)^{\kappa}\right]^2}\right)\end{array}\right\}.$$

Finally, the general model as presented above will be called the *logistic / individual Loglogistic model*. In this model both the probability of eventual death and the distribution of time until death vary over individuals, the likelihood function for this model is

$$l(\alpha,\beta,\kappa)=$$

$$=\sum_{i=1}^{n}\left\{\begin{array}{l}\delta_i\left[\begin{array}{l}\ln\left(\dfrac{\exp(\alpha^T z_i)}{1+\exp(\alpha^T z_i)}\right)+\beta^T z_i+\ln\kappa+(\kappa-1)\ln\left(t_i\exp(\beta^T z_i)\right)-\\ 2\ln\left(1+\left(\kappa t_i\exp(\beta^T z_i)\right)^{\kappa}\right)\end{array}\right]+\\ (1-\delta_i)\ln\left(\dfrac{[1+\left(\kappa t_i\exp(\beta^T z_i)\right)^{\kappa}]^2+\exp(\alpha^T z_i)}{\left[1+\exp(\alpha^T z_i)\right]\left[1+\left(\kappa t_i\exp(\beta^T z_i)\right)^{\kappa}\right]^2}\right)\end{array}\right\}.$$

In the tables 1 gives the results for the split loglogistic model and the logistic loglogistic model. The split loglogistic model dominates the logistic loglogistic models. For example, the likelihood value of -18.2007 for the split loglogistic model is noticably higher than the values for the logistic loglogistic models with likelihood value of -19.7716. We now turn to the logistic /individual loglogistic model, in which both the probability of eventual death and the distribution of time until death vary according to individual characteristics. These parameter estimates are given in table 2. They are somewhat more complicated to discuss than the results from our other models, in part because there are simply more parameters, and some of them turn out to be statistically insignificant.

In table 2, we can see that two covariates have significant on the probability of immune, Age and Transplant with (p - value 0.0081 and 0.031, respectively) but different on the loglogistic regression, Age is fail significant with p -value of 0.1932, while Transplant to be significant with p - value of 0.0002. Surgery just fail to be significant on the probability of immune with p - value of 0.9249 but significant on the loglogistic regression with p -value of 0.077.

Furthermore, these results are reasonably similar to the results we obtained using a logistic/individual exponential model [1]. There are similars on the probability of immune that Age and Transplant are significant with (p - value 0.0081 and 0.031, respectively) for logistic /individual loglogistic model and with (p - value 0.0359 and 0.000, respectively) for logistic /individual exponential model, while Surgery did not have significant on both the loglogistic and exponential model with (p -value 0.9249 and 0.0662, respectively). Next, we analyzing statistically significant on the distribution of time until death using both the logistic/individual loglogistic and exponential model. Age did not have significant with p -value of 0.1932 for loglogistic model but significant with p - value of 0.0184 for the exponential model, Surgery is significant with p -value of 0.0077 for loglogistic model but just fail significant with p -value of 0.8793 for the exponential model and finally Transplant is significant with p -value 0.0002 for loglogistic model but marginally significant for exponential model with p -value 0.0655.

Variable	Split loglogistic		Logistic loglogistic	
	Coefficient	p - value	Coefficient	p - value
intercept	9.683928	0.0000	-0.207918	0.8972
Age	-2.240139	0.0000	0.097019	0.0039
Surgery	-8.762389	0.2153	-0.983625	0.187
Transplant	-6.673942	0.1429	-3.074790	0.0468
	κ = 0.094981		λ = 0.021881	
	ω = 0.808882 ln L = -18.2007		κ = 0.566241 ln L= -19.7716	

TABLE 1: Split Loglogistic Model and Logistic Loglogistic Model.

Variable	E q u a t ion for Pr(never fail)		E q u a t ion for duration, given eventual failure (Loglogistic regression)	
	Coefficient	p - value	Coefficient	p - value
intercept	-0.529806	0.6852	-4.387064	0.0012
Age	0.087290	0.0081	0.037826	0.1932
Surgery	0.130583	0.9249	-2.250424	0.0077
Transplant	-2.254443	0.031	-2.064279	0.0002
κ = 0.767896 ln L = -468.533				

TABLE 2: Logistic / Individual Loglogistic Model.

6. CONCLUSION

In this section, we will summaries the result above about significantly covariates in the data for those models which we presented in section 4, and we shown in table 3. As we can see in table 3, There are similars on both the split loglogistic and logistic/ individual loglogistic model that Age is significant with (p -value 0.9379 and 0.1555, respectively) and Surgery just fail to be significant with (p -value 0.2153 and 0.9249, respectively), while the different that Transplant did not have significant with p -value of 0.1429 for split loglogistic but to be significant with p -value of 0.031 for logistic/ individual loglogistic model.

* not relevant

Variable	p **-value**		
	Split Loglogistic	***Logistic* Loglogistic**	**Logistic/ individual Loglogistic model.**
β_0 *(intercept)*	0.0000	*	0.6852
β_1 *(Age)*	0.0000	*	0.0081
β_2 *(Surgery)*	0.2153	*	0.9249
β_3 *(Transplant)*	0.1429	*	0.031
α_0 *(intercept)*	*	0.8972	0.0012
α_1 *(Age)*	*	0.0039	0.1932
α_2 *(Surgery)*	*	0.1870	0.0077
α_3 *(Transplant)*	*	0.0468	0.0002
λ	*	0.0044	-
κ	0.0000	0.0000	0.0000
ω *(Population split)*	0.0000	-	-

TABLE 3: Significantly Covariates for the Stanford Heart Transplant Data.

Now, we can see that Age and Surgery have different significant effects on both the logistic loglogistic and logistic/ individual loglogistic model. Age is found to be the significant with a p-value of 0.0039 for logistic loglogistic but not on the logistic/ individual loglogistic model where p-value of 0.1932. Surgery just fail significant for logistic loglogistic with p-value of 0.1870 but significant on the logistic/ individual loglogistic model with p-value of 0.0077, and finally Transplant have similar significant on both the logistic loglogistic and logistic/ individual loglogistic model with (p -value 0.0468 and 0.0002, respectively).

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