

# Accuracy Assessment of GPS\_RTK Grid to Ground Solutions

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## Abstract

Global Navigation Satellite Systems (GNSS) become the main observation technique of the surveying work. Almost, it takes place the traditional methods because of its ability to do the surveying work economically, effectively and rapidly. The Real Time Kinematic (RTK) is a basic technique for topographic and engineering surveys. The common issue with RTK measurements in civil projects is the difference in the surfaces used for design and that used for measuring. While projects are designed on ground, the RTK measurements are done on flat (Grid) surface. This cause a problem for engineering drawings and project implementation. For compatibility between design and measurements, a conversion of distances from grid to ground or vice versa should be performed. In this paper, three approaches to solve the linear distortion problem are investigated. These approaches include applying scale factor, using control points and designing Low Distortion Projection (LDP) surface. To achieve the research goal, the coordinates of 13 check points were computed and adjusted after total station measurements of the traverse that connect these points with one control point. The traverse stretched for about 1400 m in the east west direction which affected by projection. The coordinates of the check points were obtained again through RTK\_GPS measurements considering the control point as base station. The three solution approaches were utilized, and the results were analyzed. The Root Mean Square Error (RMSE) of relative position of points were 2.3, 2.8 and 3.2 cm when using control points, scale factor and LDP respectively. The RMSE of the absolute positions were 3.8, 13.3 and 13.4 cm.

Keywords: RTK, Distance, Control points, LDP, Scale Factor.

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## 1. INTRODUCTION

Real Time Kinematic (RTK) positioning is a GPS observation technique in which the corrected coordinates of a rover receiver point are determined with the help of a reference point of known coordinates occupied by a base receiver [1]. It uses the same static GPS principles, but the corrections are sent from the base to the rover receiver through a communication protocol to enable determination of rover corrected coordinates at the time of measuring [2]. The ability of RTK technique to obtain coordinates instantaneously and accurately make it widely used in surveying and engineering applications [3] and [4]. The cost reduction and ease of use of RTK techniques enlarge its customers in the surveying field [5] and [6]. Lowering field operation crew and steps, decreasing the number of traverse and control points make the RTK technique the best compromise between usability and accuracy [7].

Many researchers study the accuracy of RTK positioning technique comparing to total station and traditional static GPS. Reference [8] stated that RTK technique is more practical, fast and can produce accurate topographic maps. Reference [9] achieved 1 cm accuracy in horizontal coordinates while reference [10] achieved 2 cm accuracy in vertical coordinates. The same results were stated by [11]. Reference [12] stated that errors within 9 mm in horizontal and 15 mm in vertical has been gotten when RTK accuracy tested. Reference [13] concluded that they achieved a 2.5 cm accuracy. Reference [14] investigated the time required for accurate RTK observations and they concluded that 1 minute is enough for short base lines and 2 minutes or

more is needed for long baselines. The greatest difference from the mean was observed to be 5 cm in horizontal positioning and 20 cm in vertical positioning. The previous researchers didn't explain how they compare the RTK and Total Station (TS) measurements while they are performed on different surfaces. Another drawback of previous studies is most of results derived from measurements performed in small sites and short ranges.

The research objective is investigating the accuracy of three approaches to transform RTK measurements from grid to ground in a wide study area of about 296000 m<sup>2</sup> and the distance between base and rover extend to about 1400 m.

## 2. DATA ACQUISITION

This research is concerned to check the accuracy of approaches used to convert RTK observations from grid to ground. The coordinates of 14 points connected through closed loop traverse established in the King Abdulaziz University (KAU) campus as shown in figure (1) were determined using total station and RTK measurements. The points were chosen in sky free locations where GPS receiver can be setup. Point S177 is a control point established by Jeddah Municipality, used as reference point for total station measurements and as base station for RTK observations. The instrument used in traversing was Topcon total station GPT-7501 with distance accuracy of 2 mm + 2 ppm and angle accuracy of 1 sec. The traverse misclosure error was computed and adjusted consequently the adjusted coordinates of the check points were determined. The traverse precision was 1:34000 which meet the standards of high accurate surveying works. For RTK observations, two Topcon GR3 geodetic receivers were used. The observations were performed with epoch interval of 5 sec., mask angle of 15° and observation duration of 3 minutes at each point.

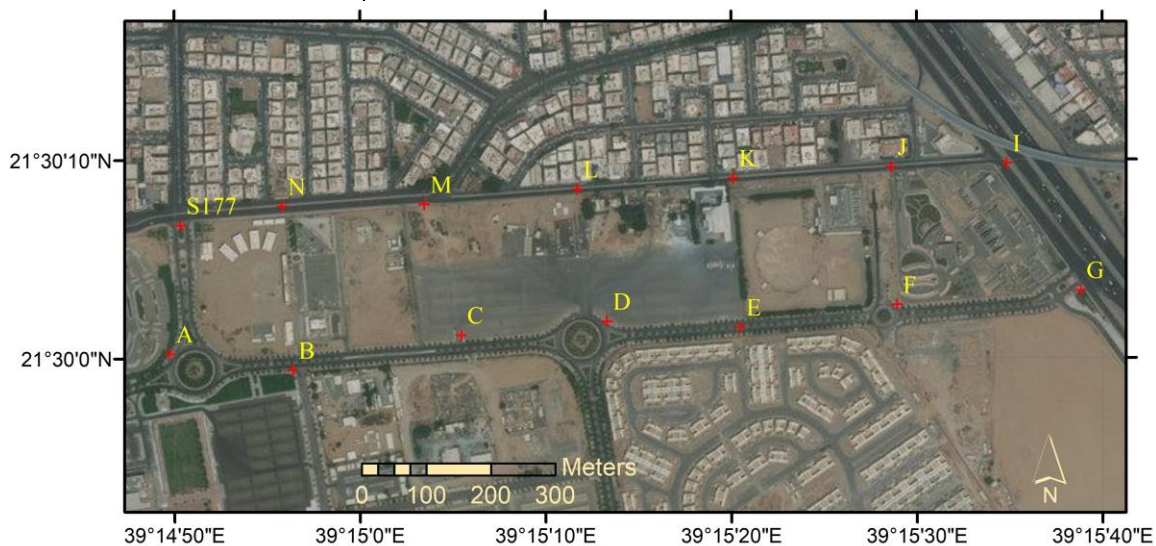


FIGURE 1: The Accuracy Assessment Check Points.

## 3. GRID TO GROUND APPROACHES

The difference in distance between projected (Grid) coordinates obtained by RTK measurements and the true horizontal distance measured at Ground surface could cause problems for various engineering plans and surveys. This problem is known as linear distortion. There are some methods to minimize the effect of this problem because it cannot be eliminated. The accuracy of three approaches deal with linear distortion will be study in this research. These approaches are: scale factor, control points and the Low Distortion Projection (LDP).

### 3.1 Scale Factor

The GPS observations are done on a flat surface or Grid while engineering projects are designed and implemented on ground surface. The distances between points are not the same on those surfaces as shown in figure 2. Transform distances pass through two steps, one to transform from grid to ellipsoid and the second from ellipsoid to ground. Each step requires its own scale factor.

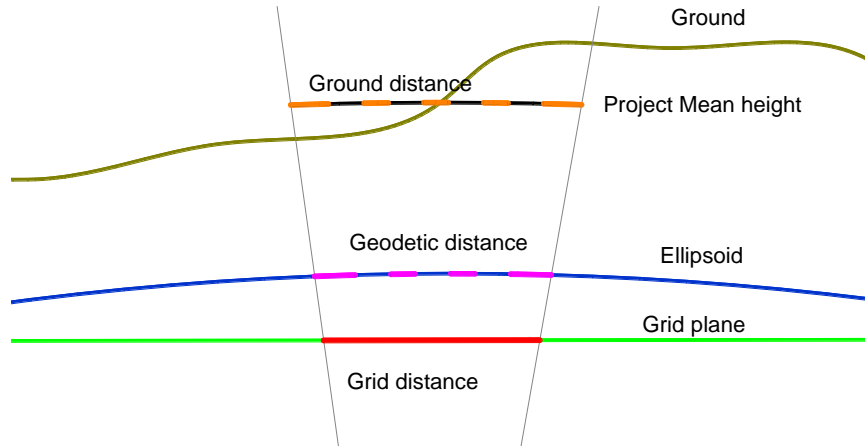


FIGURE 2: Distances and Surfaces

The scale value that used to convert distances from flat surface (Grid) to curved surface (ellipsoid) is known as scale factor (SF). The scale factor at each point can be determined utilizing equation (1).

$$\text{scale factor} = K_0 + 1.23 (E - 500000)^2 \times 10^{-14} \quad (1)$$

Where:

$K_0$  = Grid scale factor at central meridian (0.999600)

E = Easting coordinate of the point

As scale factor has a distinct value at each end of the distance, a compromised or effective scale factor ( $SF_{eff}$ ) is calculated using equation (2) [15]. The ellipsoidal distance can be determined by dividing grid distance by the effective scale factor.

$$SF_{eff} = \frac{SF_a + 4SF_{ab} + SF_b}{6} \quad (2)$$

Where:

$SF_a$  : is the scale factor for one of the points,

$SF_b$  : is the scale factor for the other point

$SF_{ab}$  : is grid scale factor for the point midway between the two points.

To convert distance from ellipsoid surface to the ground surface it is divided by the Elevation Factor (EF). The Elevation Factor (EF) is the change in distances between these two surfaces because of their distant from the center of the earth. Equation (3) can be used to determine the elevation Factor.

$$EF = R / (R + h_{avr}) \quad (3)$$

Where:

R = Mean earth radius

$h_{avr}$  = Average ellipsoidal height

To transform grid distance directly to ground distance a Combination Factor (CF) is used. The Combination Factor is calculated using equation (4).

$$CF = SF_{eff} \cdot EF \tag{4}$$

### 3.2 Ground Reference Points

For this approach, two or more control points with both grid and ground coordinates are needed. These control points are used to determine the transformation parameters explained in Figure (3) and Equation (5) [16]. The ground coordinates could be calculated from RTK measurements using these transformation parameters. Point (S177) and point (G) are used as control points.

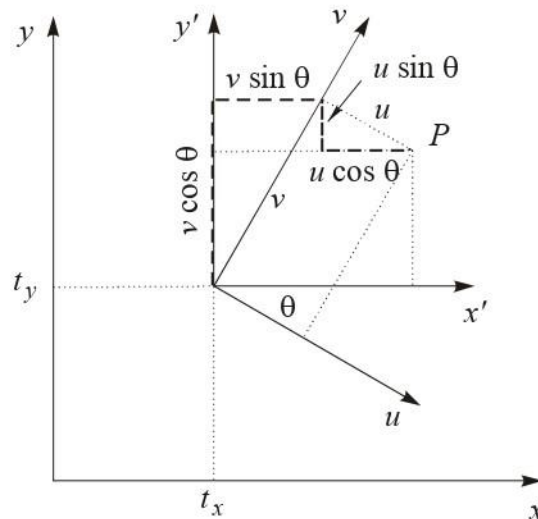


FIGURE 3: Rotation and Translation of Axes.

The transformation equations could be derived using Figure 3. The  $x'$ ,  $y'$  coordinates obtained by rotating and scaling  $u$ ,  $v$  coordinates and  $x$ ,  $y$  coordinates obtained by adding translations  $t_x$  and  $t_y$ .

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = s \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{5}$$

### 3.3 Low Distortion Projection (LDP)

Low Distortion Projection (LDP) is a custom designed projection to enable the direct conversion of GPS observations from grid to ground. The purpose of LDP is minimizing linear distortion and ensure compatibility with surveying datasets [17]. Reference [18] summarized the six steps of designing LDP.

According to [17] and [19] a Low Distortion Projection (LDP) was designed with the following parameters:

- WGS84 was selected as the reference datum

- Create a projection surface that is tangent with the average ellipsoidal height of the project area.
- The ellipsoidal distance is scaled up using inverted elevation factor( $k_t$ ) equation 6.

$$k_t = 1/EF \quad (6)$$

- Lowering the Projection surface slightly to increase the extents of the usable zone and compute a reduction factor  $k_r$  using equation 7.

$$k_r = \cos\{\sin^{-1}(w/2R)\} \quad (7)$$

Where

$k_r$  = scale reduction factor

w = project width

R = ellipsoidal radius (6378137 for WGS84)

- The secant scale factor  $k_s$  is computed using equation 8.

$$k_s = k_t \cdot k_r \quad (8)$$

- The distortion  $\delta$  can be computed using equation 9 and should not exceeds the limits.

$$\delta = k_r - 1 \quad (9)$$

- Select the project midpoint as origin latitude and central meridian.
- Select a distinct False Northings and Eastings to avoid confusion with other standard coordinate systems for the area.

The designed LDP parameters were as follows

Linear Unit:	Meter
Geodetic Datum:	WGS84
Vertical Datum:	WGS84
Projection:	UTM
Origin Latitude ( $\phi_0$ ):	N 21°30'
Central Meridian ( $\lambda_0$ ):	E 39°15'
Scale Factor:	1.000005115
False Easting:	350,000.000
False Northing:	650,000.000

#### 4. RESULTS

The three solution approaches for converting distances from grid to ground have been applied on the RTK observations. The differences between RTK converted distances and the corresponding distances from total station observations were calculated. The average, maximum, standard deviation ( $\sigma$ ) and Root mean square error (RMSE) for the residuals were calculated and shown in Table (1) and represented graphically in Figure (4). From Table (1) it's clear that the linear distortion increases with increasing the distance to the base station. Applying scale factor to transform the grid distances to project distances reduces the RMSE from 30.2 cm to 2.8 cm. Using control points to transform RTK measurements to ground coordinates lowers the RMSE to 2.3 cm while applying LDP approach reduces the RMSE to 3.2 cm. These results coincide with that of [20] and [21].

Point	Distance	RTK_Measurements		RTK_scaled distances		Using two R.P.		Applying LDP	
		L	$\delta L$ (m)	L	$\delta L$ (m)	L	$\delta L$ (m)	L	$\delta L$ (m)
S177	0.000	0.000	0.000	0.000	0.000	0	0.000	0.000	0.000
N	159.971	159.919	0.052	159.982	-0.011	159.98	-0.010	159.983	-0.012
A	199.845	199.779	0.066	199.857	-0.012	199.86	-0.010	199.858	-0.013
B	283.400	283.292	0.108	283.403	-0.003	283.4	-0.001	283.404	-0.004
M	379.163	379.043	0.120	379.192	-0.029	379.19	-0.026	379.194	-0.031
C	467.107	466.966	0.141	467.149	-0.042	467.15	-0.038	467.152	-0.044
L	617.782	617.573	0.208	617.815	-0.034	617.81	-0.029	617.819	-0.037
D	677.404	677.181	0.223	677.446	-0.042	677.44	-0.036	677.450	-0.045
K	859.590	859.283	0.307	859.620	-0.030	859.61	-0.023	859.624	-0.034
E	881.571	881.266	0.304	881.612	-0.041	881.6	-0.034	881.616	-0.045
J	1105.676	1105.273	0.402	1105.706	-0.031	1105.7	-0.022	1105.712	-0.036
F	1117.341	1116.928	0.413	1117.365	-0.024	1117.4	-0.016	1117.371	-0.030
I	1284.029	1283.550	0.479	1284.052	-0.023	1284	-0.014	1284.059	-0.030
G	1398.728	1398.192	0.537	1398.739	-0.011	1398.7	0.000	1398.746	-0.018
Average (cm)			25.8		-2.6		-2.1		-2.9
Maximum (cm)			53.7		-4.2		-3.8		-4.5
$\sigma$ (cm)			16.2		1.3		1.3		1.4
RMSE (cm)			30.2		2.8		2.3		3.2

TABLE 1: Statistics of Relative Position To The Base Station.

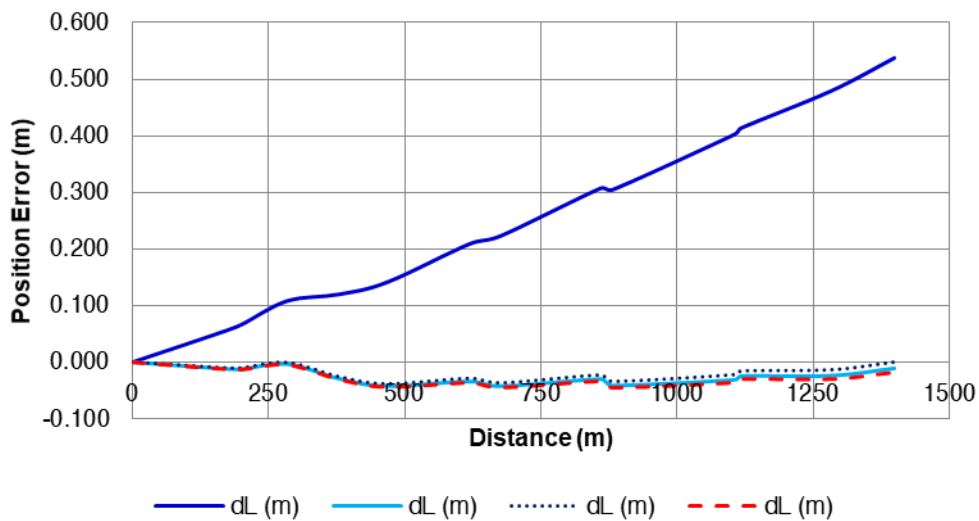


FIGURE 4: Errors in direct and converted RTK distances to the base station.

Applying grid to ground solution approaches reduce the linear distortion error to approximately 2 to 3 cm for all check points. While the error in relative position is the same all over the project area, the error in absolute position depends upon the point location to the base station. Table (2) shows the error in coordinates of check points. From Figure (1) and Table (2) it is clear that for points close to the south of base station (e.g. points A and B), the errors in northing coordinates are less than the errors in easting coordinates conversely for points close to the east of base

station (e.g. points I and G), the errors in northing are bigger than that in easting coordinates. The RMSE in easting coordinates is approximately 4.5 cm when using scale factor and LDP and it is 2.7 cm when using control points to convert coordinates from grid to ground. The RMSE in northing coordinates is 12.5 cm when using scale factor and LDP and it is 1.4 cm when using control points to the conversion process.

Point	Distance	RTK_Measurements		RTK_scaled distances		Using two R.P.		Applying LDP	
		$\delta E$ (m)	$\delta N$ (m)	$\delta E$ (m)	$\delta N$ (m)	$\delta E$ (m)	$\delta N$ (m)	$\delta E$ (m)	$\delta N$ (m)
S177	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
N	159.971	0.059	-0.032	-0.003	-0.044	-0.006	-0.021	-0.003	-0.044
A	199.845	-0.065	-0.061	-0.058	0.017	-0.030	0.013	-0.058	0.018
B	283.400	0.008	-0.130	-0.060	-0.043	-0.026	-0.019	-0.061	-0.041
M	379.163	0.125	-0.060	-0.022	-0.073	-0.024	-0.019	-0.024	-0.073
C	467.107	0.100	-0.131	-0.070	-0.065	-0.042	-0.004	-0.073	-0.064
L	617.782	0.216	-0.072	-0.025	-0.095	-0.028	-0.005	-0.028	-0.095
D	677.404	0.192	-0.167	-0.067	-0.109	-0.041	-0.015	-0.071	-0.108
K	859.590	0.317	-0.100	-0.019	-0.129	-0.022	-0.005	-0.023	-0.130
E	881.571	0.274	-0.196	-0.066	-0.135	-0.037	-0.011	-0.071	-0.134
J	1105.676	0.415	-0.140	-0.016	-0.176	-0.021	-0.016	-0.022	-0.176
F	1117.341	0.390	-0.230	-0.045	-0.183	-0.018	-0.023	-0.050	-0.182
I	1284.029	0.491	-0.136	-0.010	-0.174	-0.014	0.011	-0.017	-0.174
G	1398.728	0.521	-0.239	-0.025	-0.200	0.000	0.000	-0.032	-0.200
Average (cm)		23.4	-13.0	-3.7	-10.8	-2.6	-1.0	-4.1	-10.8
Maximum (cm)		52.1	-23.9	-7.0	-20.0	-4.2	-2.3	-7.3	-20.0
$\sigma$ (cm)		18.6	6.5	2.4	6.6	1.3	1.2	2.4	6.6
RMSE (cm)		29.5	14.5	4.4	12.5	2.7	1.4	4.7	12.5

TABLE 2: Error In Absolute Position.

The error in position at each point was calculated and shown in Table (3) and drawn in figure (5). Applying both of scale factor and LDP approaches generate RMSE of 13.4 cm while using control points gave 3.8 cm RMSE. When applying scale factor and LDP approaches the error in absolute position of points is directly proportional to the distance to the base station. The error in absolute position is directly proportional to the distance to the control points when using them to convert RTK measurements to ground coordinates.

		RTK_Measurements	RTK_Scaled distances	Using two R.P.	Applying LDP
Point	Distance	e(m)	e(m)	e(m)	e(m)
S177	0.000	0.000	0.000	0.000	0.000
N	159.971	0.067	0.044	0.023	0.044
A	199.845	0.089	0.061	0.034	0.061
B	283.400	0.131	0.074	0.033	0.074
M	379.163	0.139	0.077	0.035	0.077
C	467.107	0.165	0.096	0.048	0.097
L	617.782	0.228	0.098	0.037	0.099
D	677.404	0.254	0.128	0.051	0.129
K	859.590	0.332	0.131	0.034	0.132
E	881.571	0.337	0.150	0.049	0.151
J	1105.676	0.438	0.177	0.040	0.178
F	1117.341	0.453	0.188	0.039	0.189
I	1284.029	0.509	0.174	0.033	0.175
G	1398.728	0.573	0.202	0.018	0.202
Average (cm)		28.6	12.293	3.8	12.4
Maximum (cm)		57.3	20.181	5.1	20.2
$\sigma$ (cm)		16.8	5.231	0.9	5.2
RMSE (cm)		32.8	13.280	3.8	13.4

TABLE 3: Displacement In Absolute Position.

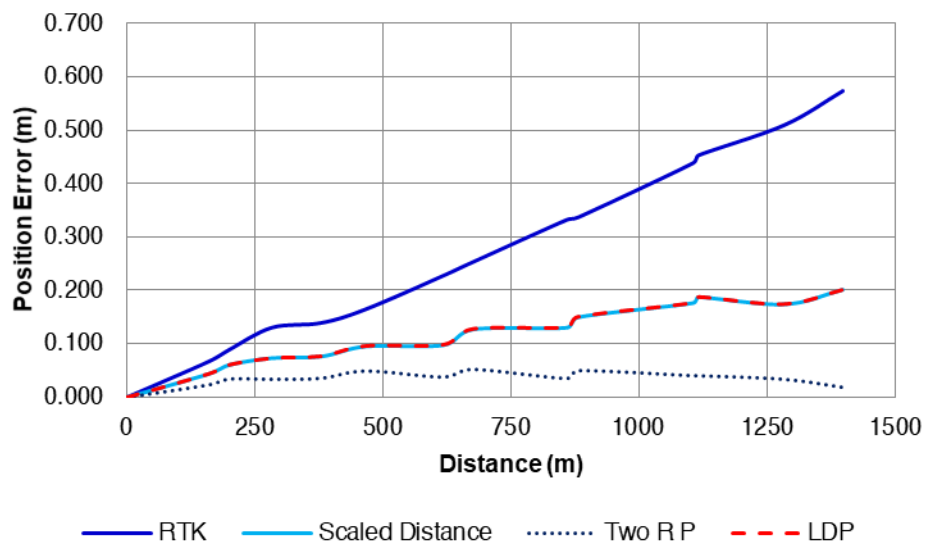


FIGURE 5: Displacement In Absolute Position.

## 5. DISCUSSION AND CONCLUSIONS

The results confirm that the accuracy of converting RTK measurements from grid to ground is almost the same for the three studied approaches when dealing with distances or relative position of points related to the base station. The maximum RMSE was 3.2 cm for a distance up to 1400 m which make the precision at least equal to 1:40000. In case of absolute positioning, which is more important, the RMSE increased and consequently the precision decreased. This big error in the absolute position may be due to the shift in azimuth of line connects each point to the base



station which lead to displacement in point coordinates. This is clear for scale factor and LDP, in which the RMSE reached 13 cm and the precision is 1:10000. Using control points improves the precision to be 1:30000 and the RMSE decreased to 3.8 cm. Referring to Table (3), to keep the error in absolute position within 10 cm when applying scale factor or LDP, the project width should be within 600 m. An absolute error within 5 cm could be achieved when using control points with distance within 500 m apart. From the above discussion and analyzed results, one can conclude:

1. The three studied approaches, scale factor, control points and LDP gave almost the same results in converting grid distances to ground one.
2. linear distortion in RTK observations increases with increase the distance to the base station.
3. The precision of the studied approaches in converting distances from grid to ground (relative positioning) is 1:40000.
4. When applying scale factor and LDP approaches the error in absolute position of points is directly proportional to the distance to the base station.
5. The error in absolute position is directly proportional to the distance to the control points when using them to convert RTK measurements to ground coordinates.
6. The least precision of the studied approaches in converting coordinates from grid to ground (absolute positioning) is 1:10000.
7. Using control points is the most appropriate solution for both relative and absolute position.
8. A 500 m distance between control points assure errors under 5 cm.
9. Scale factor and LDP solutions gave almost the same results.

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