

## Position Control of Robot Manipulator: Design a Novel SISO Adaptive Sliding Mode Fuzzy PD Fuzzy Sliding Mode Control

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### Abstract

This research focuses on design Single Input Single Output (SISO) adaptive sliding mode fuzzy PD fuzzy sliding mode algorithm with estimates the equivalent part derived in the Lyapunov sense. The stability of the closed-loop system is proved mathematically based on the Lyapunov method. Proposed method introduces a SISO fuzzy system to compensate for the model uncertainties of the system and eliminate the chattering by linear boundary layer method. This algorithm is used a SISO fuzzy system to alleviate chattering and to estimate the control gain in the control law and presented a scheme to online tune of sliding function. To attenuate the chattering phenomenon this method developed a linear boundary layer and the parameter of the sliding function is online tuned by adaptation laws. This algorithm will be analyzed and evaluated on robotic manipulators and design adaption laws of adaptive algorithms after that writing Lyapunov function candidates and prove the asymptotic convergence of the closed-loop system using Lyapunov stability theorem mathematically. Compare and evaluate proposed method and sliding mode algorithms under disturbance. In regards to the former, we will be looking at the availability of online tuning methodology and the number of fuzzy if-then rules inherent to the fuzzy system being used and the corresponding computational load. Our analysis of the results will be limited to tracking accuracy and chattering.

**Keywords:** Sliding Mode Algorithm, Adaptive Sliding Mode Fuzzy PD Fuzzy Sliding Mode Algorithm, Estimator, Lyapunov Method, Model Uncertainties, Linear boundary Layer Method, Chattering Phenomenon.

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## 1. INTRODUCTION

Dynamic of robotic manipulators have strong nonlinear and time variant characteristic [1, 6]. Conventional linear control technologies are not quite gratifying to control robotic manipulators [1-6]. Nonlinear control technologies can arrangement with highly nonlinear equations in dynamic parameters. Conventional nonlinear control strategies cannot provide good robustness for controlling robotic manipulators. The control system designer is often unsure of the exact value of the manipulator parameters that describe the dynamic behavior of the manipulator. Sliding mode control methods can manage uncertainties in the dynamic parameters of the robotic manipulator. Sliding mode controllers are robust controllers for controlling uncertain plant. Classical sliding mode control is robust to control model uncertainties and external disturbances. A sliding mode method with a switching control law guarantees the stability of the certain and/or uncertain system, but the addition of the switching control law introduces chattering into the system. One way to reduce or eliminate chattering is to insert a boundary layer method [1-15] inside of a boundary layer around the sliding surface. Chattering phenomenon can cause some problems such as saturation and heats the mechanical parts of robot manipulators or drivers. To reduce or eliminate the chattering, various papers have been reported by many researchers which classified into two most important methods: boundary layer saturation method and estimated uncertainties method [9-20]. In boundary layer saturation method, the basic idea is the discontinuous method replacement by saturation (linear) method with small neighborhood of the switching surface. This replacement caused to increase the error performance against with the considerable chattering reduction. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[21]. Slotine has presented sliding mode with boundary layer to improve the industry application [22]. R. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the chattering and control the result performance[23]. Moreover, C. C. Weng and W. S. Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method[24]. As mentioned [24]sliding mode fuzzy controller (SMFC) is fuzzy controller based on sliding mode technique to simple implement, most exceptional stability and robustness. Conversely above method has the following advantages; reducing the number of fuzzy rule base and increasing robustness and stability, the main disadvantage of SMFC is need to define the sliding surface slope coefficient very carefully. To eliminate the above problems control researchers have applied artificial intelligence method (e.g., fuzzy logic) in nonlinear robust controller (e.g., sliding mode controller) besides this technique is very useful in order to implement easily. Estimated uncertainty method used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic part which it is in classical sliding mode controller. If estimator has an acceptable performance to compensate the uncertainties, the chattering is reduced. Research on estimated uncertainty to reduce the chattering is significantly growing as their applications such as industrial automation and robot manipulator. For instance, the applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [25-28]. Wu et al. [30] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Their design had three main parts i.e. equivalent, discontinuous and fuzzy estimator tuning part which has reduced the chattering very well. Elmali et al. [27]and Li and Xu [29]have addressed sliding mode control with perturbation estimation method (SMCPE) to reduce the classical sliding mode chattering. This method was tested for the tracking control of the first two links of a SCARA type HITACHI robot. In this technique, digital controller is used to increase the system's response quality. Conversely this method has the following advantages; increasing the controller's response speed and reducing dependence on dynamic system model by on-line control, the main disadvantage are chattering phenomenon and need to improve the performance.

Classical sliding mode control method has difficulty in handling unstructured model uncertainties. One can overcome this problem by combining a sliding mode controller and artificial intelligence (e.g. fuzzy logic).

Zadeh [31] introduced fuzzy sets in 1965. After 40 years, fuzzy systems have been widely used in different fields, especially on control problems. Fuzzy systems transfer expert knowledge to mathematical models. Fuzzy systems used fuzzy logic to estimate dynamics of proposed systems. Fuzzy controllers including fuzzy if-then rules are used to control proposed systems. Conventional control methods use mathematical

models to controls systems [31-40]. Fuzzy control methods replace the mathematical models with fuzzy if then-rules and fuzzy membership function to controls systems. Both fuzzy and conventional control methods are designed to meet system requirements of stability and convergence. When mathematical models are unknown or partially unknown, fuzzy control models can use fuzzy systems to estimate the unknown models. This is called the model-free approach [31-40]. Conventional control models can use adaptive control methods to achieve the model-free approach. When system dynamics become more complex, nonlinear systems are difficult to handle by conventional control methods. From the universal approximation theorem, fuzzy systems can approximate arbitrary nonlinear systems. In practical problems, systems can be controlled perfectly by expert. Experts provide linguistic description about systems. Conventional control methods cannot design controllers combined with linguistic information. When linguistic information is important for designing controllers, we need to design fuzzy controllers for our systems. Fuzzy control methods are easy to understand for designers. The design process of fuzzy controllers can be simplified with simple mathematical models. Research on applied fuzzy logic methodology in sliding mode controller (FSMC) to reduce or eliminate the high frequency oscillation (chattering), to compensate the unknown system dynamics and also to adjust the linear sliding surface slope in pure sliding mode controller considerably improves the robot manipulator control process [41-47]. H.Temeltas [46] has proposed fuzzy adaption techniques for SMC to achieve robust tracking of nonlinear systems and solves the chattering problem. Conversely system's performance is better than sliding mode controller; it is depended on nonlinear dynamic equation. C. L. Hwang *et al.* [47] have proposed a Tagaki-Sugeno (TS) fuzzy model based sliding mode control based on  $N$  fuzzy based linear state-space to estimate the uncertainties. A multi-input multi-output FSMC reduces the chattering phenomenon and reconstructs the approximate the unknown system has been presented for a robot manipulator [42].

Adaptive control uses a learning method to self-learn the parameters of systems. For system whose dynamics are varying, adaptive control can learn the parameters of system dynamics. In traditional adaptive control, we need some information about our system such as the structure of system or the order of the system. In adaptive fuzzy control we can deal with uncertain systems. Due to the linguistic characteristic, adaptive fuzzy controllers behave like operators: adaptively controlling the system under various conditions. Adaptive fuzzy control provides a good tool for making use of expert knowledge to adjust systems. This is important for a complex unknown system with changing dynamics. Investigation on applied sliding mode methodology in fuzzy logic controller (SMFC) to reduce the fuzzy rules and refine the stability of close loop system in fuzzy logic controller has grown specially in recent years as the robot manipulator control [23, 48-50]. Lhee *et al.* [48] have presented a fuzzy logic controller based on sliding mode controller to more formalize and boundary layer thickness. Emami *et al.* [51] have proposed a fuzzy logic approximate inside the boundary layer. H.K.Lee *et al.* [52] have presented self tuning SMFC to reduce the fuzzy rules, increase the stability and to adjust control parameters control automatically. We divide adaptive fuzzy control into two categories: direct adaptive fuzzy control and indirect adaptive fuzzy control. A direct adaptive fuzzy controller adjusts the parameters of the control input. An indirect adaptive fuzzy controller adjusts the parameters of the control system based on the estimated dynamics of the plant. This research is used fuzzy indirect method to estimate the nonlinear equivalent part in order to used sliding mode fuzzy algorithm to tune and adjust the sliding function (direct adaptive).

In this research we will highlight the SISO adaptive sliding mode fuzzy PD fuzzy sliding mode algorithm with estimates the equivalent part derived in the Lyapunov sense. This algorithm will be analyzed and evaluated on robotic manipulators. Section 2, serves as an introduction to the classical sliding mode control algorithm and its application to a two degree of-freedom robot manipulator, describe the objectives and problem statements. Part 3, introduces and describes the methodology algorithms and proves Lyapunov stability. Section 4 presents the simulation results of this algorithm applied to a 2 degree-of-freedom robot manipulator and the final section is describe the conclusion.

## 2. OBJECTIVES, PROBLEM STATEMENTS AND SLIDING MODE ALGORITHM

When system works with various parameters and hard nonlinearities design linear controller technique is very useful in order to be implemented easily but it has some limitations such as working near the system operating point[2-20]. Sliding mode controller is used in wide range areas such as in robotics, in control process, in aerospace applications and in power converters because it has an acceptable control performance and solve some main challenging topics in control such as resistivity to the external

disturbance. Even though, this controller is used in wide range areas but, pure sliding mode controller has the following disadvantages: chattering problem; which caused the high frequency oscillation in the controllers output and equivalent dynamic formulation; calculate the equivalent control formulation is difficult because it depends on the dynamic equation [20]. Conversely pure FLC works in many areas, it cannot guarantee the basic requirement of stability and acceptable performance[30-40]. Although both SMC and FLC have been applied successfully in many applications but they also have some limitations. The linear boundary layer method is used to reduce or eliminate the chattering and fuzzy estimator is used instead of dynamic equivalent equation to implement easily and avoid mathematical model base controller. To reduce the effect of uncertainty in proposed method, self tuning sliding mode fuzzy method is applied in fuzzy sliding mode controller in robot manipulator in order to solve above limitation.

The dynamic equation of an n-link robot manipulator is define as [53-62]

$$M(q)\ddot{q} + c(q, \dot{q}) + G(q) = \tau \tag{1}$$

Where  $q \in R^n$  is the vector of joint position,  $M(q) \in R^{n \times n}$  is the inertial matrix,  $C(q, \dot{q}) \in R^n$  is the matrix of Coriolis and centrifugal forces,  $G(q) \in R^n$  is the gravity vector and  $\tau \in R^n$  is the vector of joint torques. This work focuses on two-degree-of-freedom robot manipulator (Figure 1)

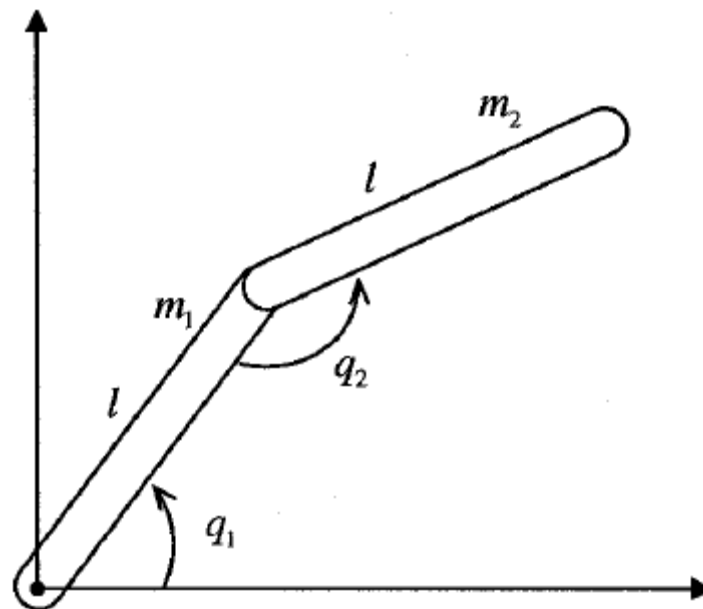


FIGURE 1: Two-link robotic manipulator

The dynamics of this robotic manipulator is given by [1, 6, 9-14]

$$\tau = M(q)\ddot{q} + B(q)[\dot{q} \dot{q}] + C(q)[\dot{q}]^2 + G(q) \tag{2}$$

Where

$$M(q) = \begin{bmatrix} m_1 l^2 + 2m_2 l^2 + 2m_2 l^2 \cos q_2 & m_2 l^2 + m_2 l^2 \cos q_2 \\ m_2 l^2 + m_2 l^2 \cos q_2 & m_2 l^2 \end{bmatrix} \tag{3}$$

$$C(q, \dot{q}) = \begin{bmatrix} -2m_2 l^2 \dot{q}_1 \dot{q}_2 \sin q_2 - m_2 l^2 \dot{q}_2^2 \sin q_2 \\ m_2 l^2 \dot{q}_1^2 \sin q_2 \end{bmatrix} \tag{4}$$

Our target is to track the desired trajectories  $q_d$  of the robotic manipulators (2) by using a sliding mode controller. We extract  $\ddot{q}$  from  $C(q, \dot{q})$  in (2) and rewrite (2) as

$$\tau = M(q)\ddot{q} + c(q, \dot{q})\dot{q} \tag{5}$$

Where

$$C(q, \dot{q}) = \begin{bmatrix} -m_2 l^2 \dot{q}_2 \sin q_2 - m_2 l^2 \dot{q}_1 \sin q_2 - m_2 l^2 \dot{q}_2 \sin q_2 \\ m_2 l^2 \dot{q}_1 \sin q_2 & 0 \end{bmatrix} \tag{6}$$

We define the tracking error as

$$e = q - q_d \tag{7}$$

Where  $q = [q_1, q_2]^T$ ,  $q_d = [q_{1d}, q_{2d}]^T$ . The sliding surface is expressed as

$$s = \dot{e} + \lambda e \tag{8}$$

Where  $\lambda = \text{diag}[\lambda_1, \lambda_2]$ ,  $\lambda_1$  and  $\lambda_2$  are chosen as the bandwidth of the robot controller.

We need to choose  $\tau$  to satisfy the sufficient condition (9). We define the reference state as

$$\frac{1}{2} \frac{d}{dt} s^2(x, t) = S \cdot \dot{s} = [f - \hat{f} - K \text{sgn}(s)] \cdot S = (f - \hat{f}) \cdot S - K|S| \tag{9}$$

$$\dot{q}_e = \dot{q} - s = \dot{q}_d - \lambda e \tag{10}$$

Now we pick the control input  $\tau$  as

$$\tau = M^{\hat{}} \ddot{q}_r + C_1^{\hat{}} \dot{q}_r - A s - K \text{sgn}(s) \tag{11}$$

Where  $M^{\hat{}}$  and  $C_1^{\hat{}}$  are the estimations of  $M(q)$  and  $C_1(q, \dot{q})$ ;  $A = \text{diag}[a_1, a_2]$  and  $K = \text{diag}[k_1, k_2]$  are diagonal positive definite matrices. From (7) and (11), we can get

$$M \dot{s} + (C_1 + A) s = \Delta f - K \text{sgn}(s) \tag{12}$$

Where  $\Delta f = \Delta M \ddot{q}_r + \Delta C_1 \dot{q}_r$ ,  $\Delta M = M^{\hat{}} - M$  and  $\Delta C_1 = C_1^{\hat{}} - C_1$ . We assume that the bound  $|\Delta f_i|_{\text{bound}}$  of  $\Delta f_i$  ( $i = 1, 2$ ) is known. We choose  $K$  as

$$K_i \geq |\Delta f_i|_{\text{bound}} \tag{13}$$

We pick the Lyapunov function candidate to be

$$V = \frac{1}{2} s^T M s \tag{14}$$

Since  $M$  is positive symmetric definite,  $V > 0$  for  $s \neq 0$ . Take the derivative of  $M$  with respect to time in (6) and we get

$$\dot{M} = \begin{bmatrix} -2 m_2 l^2 \dot{q}_2 \sin q_2 & -m_2 l^2 \dot{q}_2 \sin q_2 \\ -m_2 l^2 \dot{q}_2 \sin q_2 & 0 \end{bmatrix} \tag{15}$$

From (11) and (15) we get

$$\dot{M} - 2C_1 = \begin{bmatrix} 0 & 2m_2 l^2 \dot{q}_1 \sin q_2 + m_2 l^2 \dot{q}_2 \sin q_2 \\ -2m_2 l^2 \dot{q}_1 \sin q_2 - m_2 l^2 \dot{q}_2 \sin q_2 & 0 \end{bmatrix} \tag{16}$$

Which is a skew-systemmetric matrix satisfying

$$s^T (\dot{M} - 2C_1) s = 0 \tag{17}$$

Then  $\dot{V}$  becomes

$$\begin{aligned} \dot{V} &= s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s \\ &= s^T (M \dot{s} + C_1 s) \\ &= s^T [-A s + \Delta f - K \text{sgn}(s)] \\ &= \sum_{i=1}^2 (s_i [\Delta f_i - K_i \text{sgn}(s_i)]) - s^T A s \end{aligned} \tag{18}$$

For  $K_i \geq |\Delta f_i|$ , we always get  $s_i [\Delta f_i - K_i \text{sgn}(s_i)] \leq 0$ . We can describe  $\dot{V}$  as

$$\dot{V} = \sum_{i=1}^2 (s_i [\Delta f_i - K_i \text{sgn}(s_i)]) - s^T A s \leq -s^T A s < 0 \quad (s \neq 0) \tag{19}$$

To attenuate chattering problem, we introduce a saturation function in the control law instead of the sign function in (9). The control law becomes

$$\tau = M^s \ddot{q}_r + C_1^s \dot{q}_r - As - Ksat(s/\Phi) \tag{20}$$

In this classical sliding mode control method, the model of the robotic manipulator is partly unknown. To attenuate chattering, we use the saturation function described in (20). Our control law changes to

$$\tau = M^s \ddot{q}_r + C_1^s \dot{q}_r - As - Ksat(s) \tag{21}$$

The main goal is to design a position controller for robot manipulator with acceptable performances (e.g., trajectory performance, torque performance, disturbance rejection, steady state error and RMS error). Robot manipulator has nonlinear dynamic and uncertain parameters consequently; following objectives have been pursuit in the mentioned study.

- To develop a chattering in a position pure sliding mode controller against uncertainties.
- To design and implement a position fuzzy estimator sliding mode controller in order to solve the equivalent problems in the pure sliding mode control.
- To develop a position sliding mode fuzzy adaptive fuzzy sliding mode controller in order to solve the disturbance rejection.

Figure 2 is shown the classical sliding mode methodology with linear saturation function to eliminate the chattering.

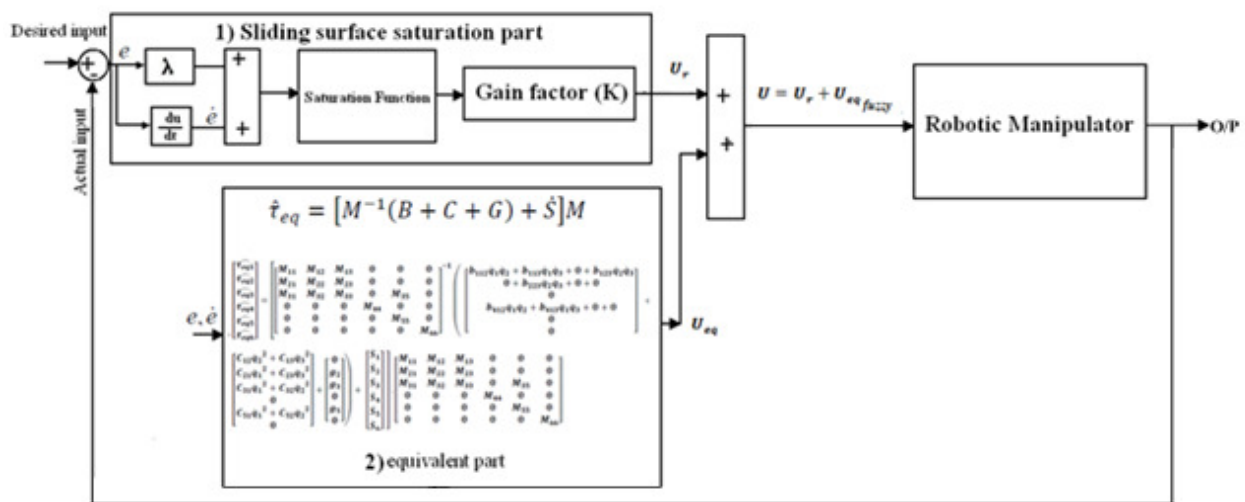


FIGURE 2: Classical sliding mode controller: applied to two-link robotic manipulator

### 3. METHODOLOGY: DESIGN A NOVEL SISO ADAPTIVE SLIDING MODE FUZZY PD FUZZY ESTIMATE SLIDING MODE CONTROL

**First part** is focuses on design chattering free sliding mode methodology using linear saturation algorithm. A time-varying sliding surface  $s(x, t)$  is given by the following equation:

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} = 0 \tag{22}$$

where  $\lambda$  is the constant and it is positive. The derivation of  $\hat{S}$ , namely,  $\hat{S}$  can be calculated as the following formulation [5-16, 41-62]:

$$\hat{S} = (\tilde{x} - \tilde{x}_d) + \lambda(\tilde{x} - \tilde{x}_d) \tag{23}$$

The control law for a multi degrees of freedom robot manipulator is written as:

$$U = U_{eq} + U_r \tag{24}$$

Where, the model-based component  $U_{eq}$  is the nominal dynamics of systems and it can be calculate as follows:

$$U_{eq} = [M^{-1}(B + C + G) + \dot{S}]M \tag{25}$$

Where  $M(q)$  is an inertia matrix which it is symmetric and positive,  $V(q, \dot{q}) = B + C$  is the vector of nonlinearity term and  $G(q)$  is the vector of gravity force and  $U_r$  with minimum chattering based on [9-16] is computed as;

$$U_r = K \cdot (\mu u + b) \left( \frac{S}{\phi} \right) \tag{26}$$

Where  $\phi_{sa} = \mu u + b = \text{saturation function}$  is a dead zone (saturation) function and, u and b are unlimited coefficient, by replace the formulation (5) in (3) the control output can be written as;

$$U = U_{eq} + K \cdot (\mu u + b) \left( \frac{S}{\phi} \right) = \begin{cases} U_{eq} + K \cdot \text{sgn}(S) & , |S| \geq \phi \\ U_{eq} + K \cdot \frac{S}{\phi} & , |S| < \phi \end{cases} \tag{27}$$

Where the function of  $\text{sgn}(S)$  defined as;

$$\text{sgn}(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \tag{28}$$

**Second part** is focuses on design fuzzy estimator to estimate nonlinear equivalent part. However the application area for fuzzy control is really wide, the basic form for all command types of controllers consists of;

- Input fuzzification (binary-to-fuzzy[B/F]conversion)
- Fuzzy rule base (knowledge base)
- Inference engine
- Output defuzzification (fuzzy-to-binary[F/B]conversion) [30-40].

The basic structure of a fuzzy controller is shown in Figure 3.

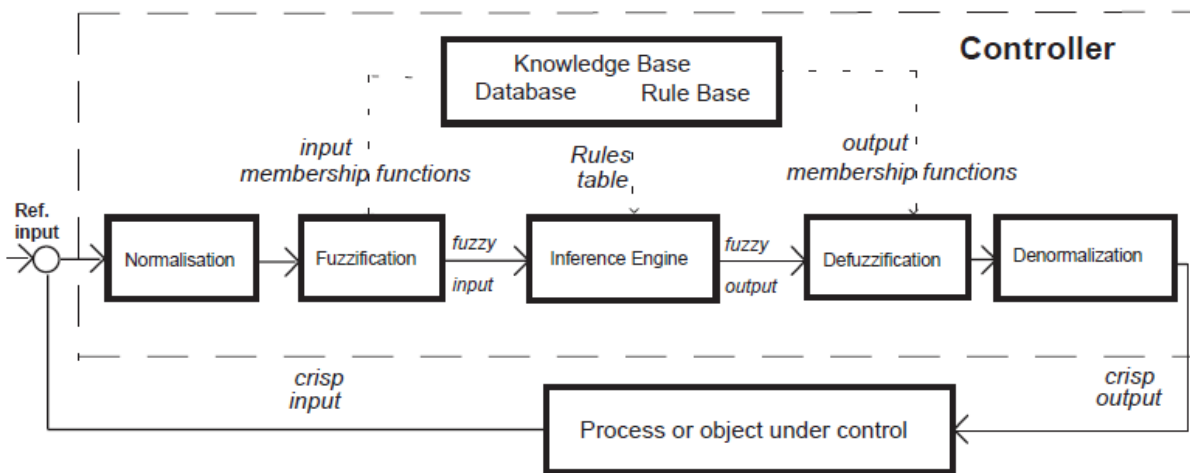


FIGURE 3: Block diagram of a fuzzy controller with details.

The fuzzy system can be defined as below [38-40]

$$f(x) = U_{fuzzy} = \sum_{i=1}^M \theta^i \zeta^i(x) = \psi(S) \tag{29}$$

where  $\theta = (\theta^1, \theta^2, \theta^3, \dots, \theta^M)^T, \zeta(x) = (\zeta^1(x), \zeta^2(x), \zeta^3(x), \dots, \zeta^M(x))^T$

$$\zeta^1(x) = \frac{\sum_i \mu_i(x) x_i}{\sum_i \mu_i(x)} \tag{30}$$

where  $\theta = (\theta^1, \theta^2, \theta^3, \dots, \theta^M)$  is adjustable parameter in (8) and  $\mu_{A_j^l}$  is membership function. error base fuzzy controller can be defined as

$$U_{fuzzy} = \psi(S) \tag{31}$$

In this work the fuzzy controller has one input which names; sliding function. Fuzzy controller with one input is difficult to implementation, because it needs large number of rules, to cover equivalent part estimation [16-25]. Proposed method is used to a SISO fuzzy system which can approximate the residual coupling effect and alleviate the chattering. The robotic manipulator used in this algorithm is defined as below: the tracking error and the sliding surface are defined as:

$$e = q - q_d \tag{32}$$

$$s = \dot{e} + \lambda_e \tag{33}$$

We introduce the reference state as

$$\dot{q}_r = \dot{q} - s = \dot{q}_d - \lambda_e \tag{34}$$

$$\ddot{q}_r = \ddot{q} - \dot{s} = \ddot{q}_d - \lambda \dot{e} \tag{35}$$

The control input is given by

$$\tau = M^* \ddot{q}_r + C_1^* \dot{q}_r - As - K \tag{36}$$

Where  $A = \text{diag}[a_1, \dots, a_m]$  and  $a_1, \dots, a_m$  are positive constants;  $K = [k_1, \dots, k_m]^T$  and  $K_j$  is defined as the fuzzy gain estimated by fuzzy systems.

The fuzzy if-then rules for the  $j$ th joint of the robotic manipulator are defined as

$$R^{(l)}: \text{if } s_j \text{ is } A_j^l \text{ , then } y \text{ is } B_j^l \tag{37}$$

Where  $j = 1, \dots, m$  and  $l = 1, \dots, M$ .

We define  $K_j$  by

$$K_j = \frac{\sum_{l=1}^M \theta_j^l [\mu_{A_j^l}(s_j)]}{\sum_{l=1}^M [\mu_{A_j^l}(s_j)]} = \theta_j^T s_j(s_j) \tag{38}$$

Where

$$\varepsilon_j(s_j) = [\varepsilon_j^1(s_j), \varepsilon_j^2(s_j), \dots, \varepsilon_j^M(s_j)]^T, \tag{39}$$

$$\varepsilon_j^l(s_j) = \frac{\sum_{i=1}^M \mu_{A_j^i}(s_j)}{\sum_{i=1}^M [\mu_{A_j^i}(s_j)]} \tag{40}$$

The membership function  $\mu_{A_j^l}(s_j)$  is a Gaussian membership function defined in bellows:

$$\mu_{A_j^l}(s_j) = \exp \left[ - \left( \frac{s_j - \alpha_j^l}{\delta_j^l} \right)^2 \right] \quad (j = 1, \dots, m). \tag{41}$$

The Lyapunov function candidate is given by

$$V = \frac{1}{2} s^T M s + \frac{1}{2} \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \phi_j \tag{42}$$

Where  $\phi_j = \theta_j^* - \theta_j$ . The derivative of  $V$  is

$$\dot{V} = s^T M \dot{s} + \frac{1}{2} \dot{s}^T M s + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \tag{43}$$



Since  $\dot{M} - 2C_1$  is a skew-symmetric matrix, we can get  $s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s = s^T (M \dot{s} + C_1 s)$ . From (2) and (36), we get

$$\tau = M(q)\ddot{q} + c(q, \dot{q})\dot{q} + G(q) = M^{\hat{}} \ddot{q}_r + C_1^{\hat{}} \dot{q}_r + G^{\hat{}} - A s - K \tag{44}$$

Since  $\dot{q}_r = \dot{q} - s$  and  $\ddot{q}_r = \ddot{q} - \dot{s}$  in (43) and (42), we get

$$M \dot{s} + (C_1 + A) s = \Delta F - K \tag{45}$$

Where  $\Delta F = \Delta M \ddot{q}_r + \Delta C_1 \dot{q}_r + \Delta G$ ,  $\Delta M = M^{\hat{}} - M$ ,  $\Delta C_1 = C_1^{\hat{}} - C_1$  and  $G = G^{\hat{}} - G$ , then  $\dot{V}$  becomes

$$\begin{aligned} \dot{V} &= s^T (M \dot{s} + C_1 s) + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j^1 \\ &= -s^T (-A s + \Delta f - K) + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j^1 \\ &= \sum_{j=1}^m [s_j (\Delta f_j - K_j)] - s^T A s + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j^1 \\ &= \sum_{j=1}^m [s_j (\Delta f_j - \theta_j^T \varepsilon_j(s_j))] - s^T A s + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j^1 \\ &= \sum_{j=1}^m [s_j (\Delta f_j - (\theta_j^*)^T \varepsilon_j(s_j) + \phi_j^T \varepsilon_j(s_j))] - s^T A s + \sum_{j=1}^m \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j^1 \\ &= \sum_{j=1}^m [s_j (\Delta f_j - (\theta_j^*)^T \varepsilon_j(s_j))] - s^T A s + \sum_{j=1}^m \left( \frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} s_j \varepsilon_j(s_j) + \dot{\phi}_j^1] \right) \end{aligned}$$

We choose the adaptation law  $\dot{\theta}_j = \gamma_{sj} s_j \varepsilon_j(s_j)$ . Since  $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} s_j \varepsilon_j(s_j)$ ,  $\dot{V}$  becomes

$$\dot{V} = \sum_{j=1}^m [s_j (\Delta f_j - (\theta_j^*)^T \varepsilon_j(s_j))] - s^T A s \tag{46}$$

We define the minimum approximation error as

$$\omega_j = \Delta f_j - (\theta_j^*)^T \varepsilon_j(s_j) \tag{47}$$

Then  $\dot{V}$  change to

$$\begin{aligned} \dot{V} &= \sum_{j=1}^m s_j \omega_j - s^T A s \\ &\leq \sum_{j=1}^m |s_j| |\omega_j| - s^T A s \\ &= \sum_{j=1}^m (|s_j| |\omega_j| - a_j s_j^2) \\ &= \sum_{j=1}^m (|s_j| (|\omega_j| - a_j |s_j|)) \end{aligned} \tag{48}$$

According to Universal Approximation theorem in sliding mode algorithm, the minimum approximation error  $\omega_j$  is as small as possible. We can simply pick  $a_j$  to make  $a_j |s_j| > |\omega_j|$  ( $s_j \neq 0$ ). Then we get  $\dot{V} < 0$  for  $s \neq 0$ .

The fuzzy division can be reached the best state when  $s \cdot \dot{s} < 0$  and the error is minimum by the following formulation

$$\theta^* = \arg \min [Sup_{x \in U} | \sum_{i=1}^M \theta^T \zeta(x) - U_{equ} |] \tag{49}$$

Where  $\theta^*$  is the minimum error,  $sup_{x \in U} | \sum_{i=1}^M \theta^T \zeta(x) - \tau_{equ} |$  is the minimum approximation error.

suppose  $K_j$  is defined as follows

$$K_j = \frac{\sum_{i=1}^M \theta_j^i [\mu_A(S_j)]}{\sum_{i=1}^M [\mu_A(S_j)]} = \theta_j^T \zeta_j(S_j) \tag{50}$$

Where  $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \dots, \zeta_j^M(S_j)]^T$

$$\zeta_j^1(s_j) = \frac{\mu_{(A_j)^1}(s_j)}{\sum_i \mu_{(A_j)^1}(s_j)} \tag{51}$$

where the  $\gamma_{sj}$  is the positive constant.

According to the nonlinear dynamic equivalent formulation of robot manipulator the nonlinear equivalent part is estimated by (8)

$$[M^{-1}(B + C + G) + \dot{S}]M = \sum_{l=1}^M \theta^l \zeta(x) - \lambda S - K \tag{52}$$

Based on (3) the formulation of proposed fuzzy sliding mode controller can be written as;

$$U = U_{eqfuzzy} + U_r \tag{53}$$

Where  $U_{eqfuzzy} = [M^{-1}(B + C + G) + \dot{S}]M + \sum_{l=1}^M \theta^l \zeta(x) + K$

Figure 4 is shown the proposed fuzzy sliding mode controller.

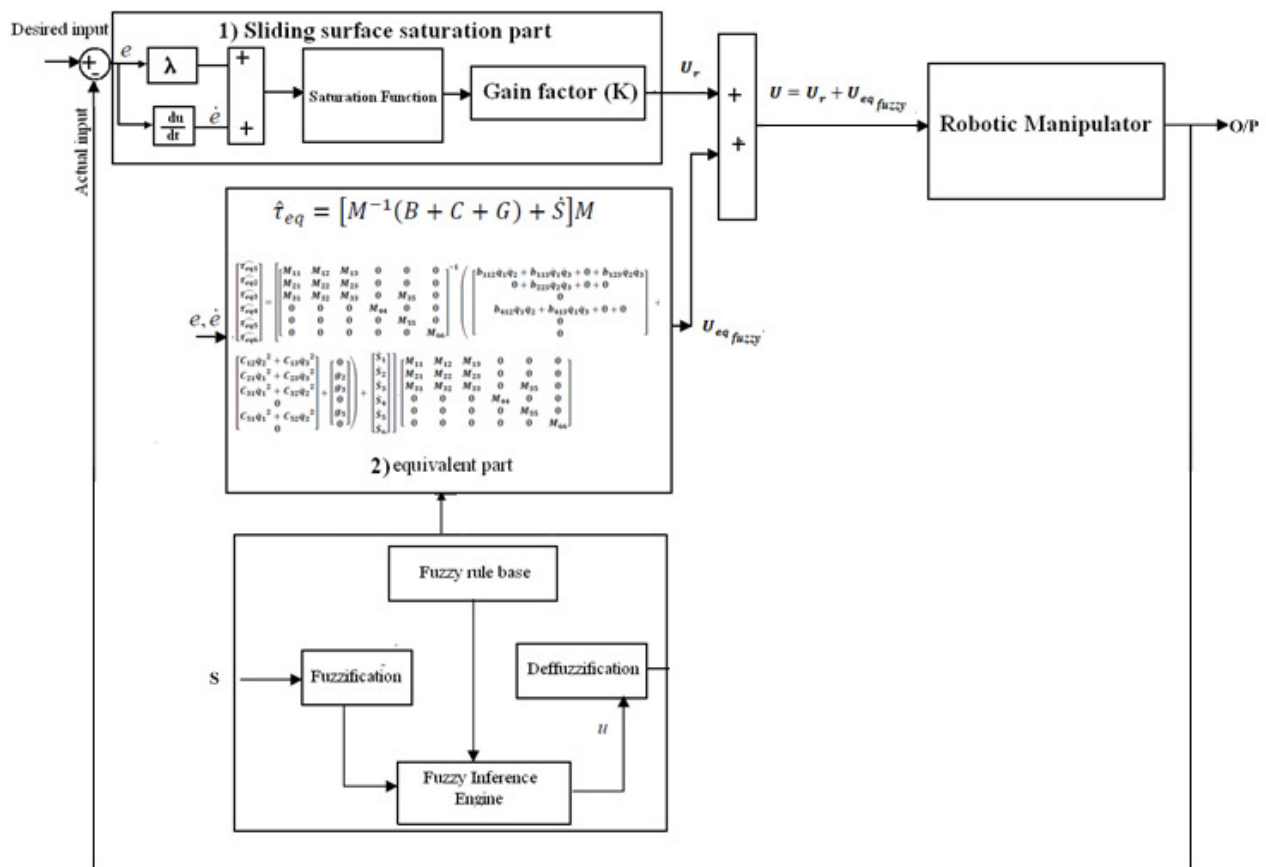


FIGURE 4: Proposed fuzzy estimator sliding mode algorithm: applied to robot manipulator

**Third part** is focuses on design sliding mode fuzzy adaptive algorithm for fuzzy estimator to estimate nonlinear equivalent part. Adaptive control uses a learning method to self-learn the parameters of systems. For system whose dynamics are varying, adaptive control can learn the parameters of system dynamics. In traditional adaptive control, we need some information about our system such as the structure of system or the order of the system. In adaptive fuzzy control we can deal with uncertain systems. Due to the linguistic characteristic, adaptive fuzzy controllers behave like operators: adaptively controlling the system under various conditions. Adaptive fuzzy control provides a good tool for making use of expert knowledge

to adjust systems. This is important for a complex unknown system with changing dynamics. The adaptive fuzzy systems is defined by

$$f(x) = \sum_{i=1}^M \theta^i \varepsilon^i(x) = \theta^T \varepsilon(x) \tag{54}$$

Where  $\theta = (\theta^1, \dots, \theta^M)^T$ ,  $\varepsilon(x) = (\varepsilon^1(x), \dots, \varepsilon^M(x))^T$ , and  $\varepsilon^i(x) = \prod_{l=1}^n \mu_{A_l^i}(x_l) / \sum_{i=1}^M (\prod_{l=1}^n \mu_{A_l^i}(x_l))$  define in the previous part.  $\theta^1, \dots, \theta^M$  are adjustable parameters in (50).  $\mu_{A_1^1}(x_1), \dots, \mu_{A_n^M}(x_n)$  are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

$$f(x) = \frac{\sum_{i=1}^M \theta^i \left[ \prod_{l=1}^n \exp\left(-\left(\frac{x_l - \alpha_l^i}{\delta_l^i}\right)^2\right) \right]}{\sum_{i=1}^M \left[ \prod_{l=1}^n \exp\left(-\left(\frac{x_l - \alpha_l^i}{\delta_l^i}\right)^2\right) \right]} \tag{55}$$

Where  $\theta^i$ ,  $\alpha_l^i$  and  $\delta_l^i$  are all adjustable parameters.

From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust  $\theta^i$  in (50). We define  $f^*(x|\theta)$  as the approximator of the real function  $f(x)$ .

$$f^*(x|\theta) = \theta^T \varepsilon(x) \tag{56}$$

We define  $\theta^*$  as the values for the minimum error:

$$\theta^* = \arg \min_{\theta \in \Omega} \left[ \sup_{x \in U} |f^*(x|\theta) - g(x)| \right] \tag{57}$$

Where  $\Omega$  is a constraint set for  $\theta$ . For specific  $x$ ,  $\sup_{x \in U} |f^*(x|\theta^*) - f(x)|$  is the minimum approximation error we can get.

The fuzzy system can be defined as below

$$f(x) = \tau_{fuzzy} = \sum_{l=1}^M \theta^l \zeta(x) = \psi(\theta, \xi) \tag{58}$$

where  $\theta = (\theta^1, \theta^2, \theta^3, \dots, \theta^M)^T$ ,  $\zeta(x) = (\zeta^1(x), \zeta^2(x), \zeta^3(x), \dots, \zeta^M(x))^T$

$$\zeta^1(x) = \frac{\sum_l \mu(x_l) x_l}{\sum_l \mu(x_l)} \tag{59}$$

where  $\theta = (\theta^1, \theta^2, \theta^3, \dots, \theta^M)$  is adjustable parameter in (58) and  $\mu(x_l)$  is membership function.

error base fuzzy controller can be defined as

$$\tau_{fuzzy} = \psi(\theta, \xi) \tag{60}$$

According to the formulation sliding function

$$if \ S = 0 \ then \ -\dot{e} = \lambda e \tag{61}$$

the fuzzy division can be reached the best state when  $S \cdot \dot{S} < 0$  and the error is minimum by the following formulation

$$\theta^* = \arg \min [ \sup_{x \in U} | \sum_{l=1}^M \theta^l \zeta(x) - \tau_{equ} | ] \tag{62}$$

Where  $\theta^*$  is the minimum error,  $\sup_{x \in U} | \sum_{l=1}^M \theta^l \zeta(x) - \tau_{equ} |$  is the minimum approximation error. The adaptive controller is used to find the minimum errors of  $\theta - \theta^*$ .

suppose  $K_j$  is defined as follows

$$K_j = \frac{\sum_{i=1}^M \theta_j^i [\mu_A(S_j)]}{\sum_{i=1}^M [\mu_A(S_j)]} = \theta_j^T \zeta_j(S_j) \tag{63}$$

Where  $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \dots, \zeta_j^M(S_j)]^T$

$$\zeta_j^1(S_j) = \frac{\mu_{(A)}^1(S_j)}{\sum_i \mu_{(A)}^i(S_j)} \tag{64}$$

the adaption law is defined as

$$\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j) \tag{65}$$

where the  $\gamma_{sj}$  is the positive constant.

According to the formulation (63) and (64) in addition from (60) and (58)

$$M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) = \sum_{i=1}^M \theta^T \zeta(x) - \lambda S - K \tag{66}$$

The dynamic equation of robot manipulator can be written based on the sliding surface as;

$$M\dot{S} = -VS + M\dot{S} + VS + G - \tau \tag{67}$$

It is supposed that

$$S^T(M - 2V)S = 0 \tag{68}$$

it can be shown that

$$M\dot{S} + (V + \lambda)S = \Delta f - K \tag{69}$$

where  $\Delta f = [M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q)] - \sum_{i=1}^M \theta^T \zeta(x)$

as a result  $\dot{V}$  is became

$$\begin{aligned} \dot{V} &= \frac{1}{2} S^T \dot{M} S - S^T V S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= S^T (-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - K_j)] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - ((\theta_j^*)^T \zeta_j(S_j))] - S^T \lambda S] + \sum (\frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} \zeta_j(S_j) S_j + \dot{\phi}_j]) \end{aligned}$$

where  $\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j)$  is adaption law,  $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} S_j \zeta_j(S_j)$ ,

consequently  $\dot{V}$  can be considered by

$$\dot{V} = \sum_{j=1}^m [S_j \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j))] - S^T \lambda S \tag{70}$$

the minimum error can be defined by

$$e_{mj} = \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j)) \tag{71}$$

$\dot{V}$  is intended as follows

$$\begin{aligned}
 \dot{V} &= \sum_{j=1}^m [S_j \dot{e}_{mj}] - S^T \lambda S \\
 &\leq \sum_{j=1}^m |S_j| |e_{mj}| - S^T \lambda S \\
 &= \sum_{j=1}^m |S_j| |e_{mj}| - \lambda_j S_j^2 \\
 &= \sum_{j=1}^m |S_j| (|e_{mj}| - \lambda_j S_j)
 \end{aligned} \tag{72}$$

For continuous function  $g(x)$ , and suppose  $\varepsilon > 0$  it is defined the fuzzy logic system in form of (56) such that

$$\text{Sup}_{x \in U} |f(x) - g(x)| < \varepsilon \tag{73}$$

the minimum approximation error ( $e_{mj}$ ) is very small.

$$\text{if } \lambda_j = \alpha \text{ that } \alpha |S_j| > e_{mj} (S_j \neq 0) \text{ then } \dot{V} < 0 \text{ for } (S_j \neq 0) \tag{74}$$

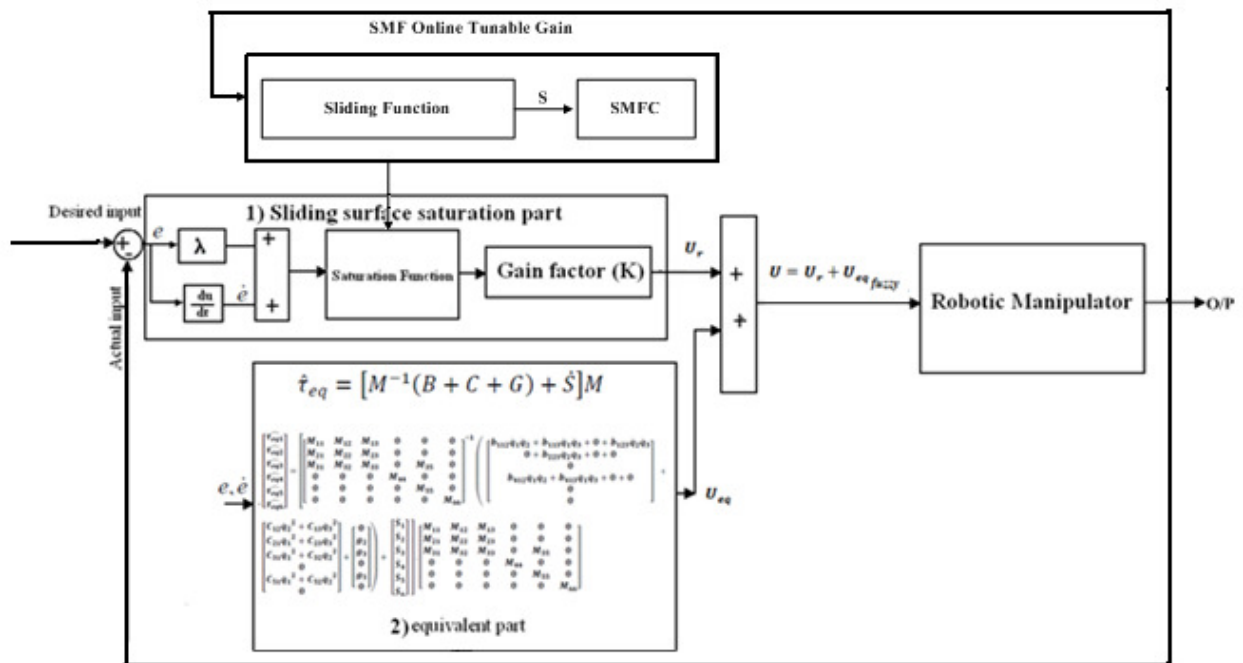


FIGURE 5: Sliding mode fuzzy adaptive proposed fuzzy estimator sliding mode algorithm: applied to robot manipulator

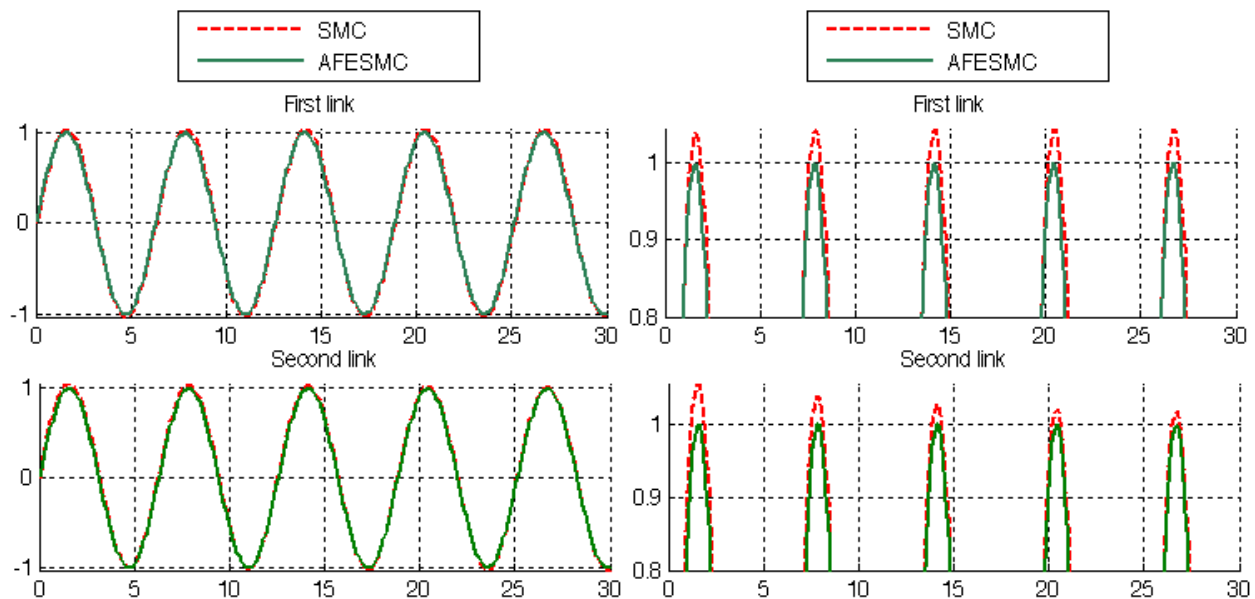
#### 4. SIMULATION RESULTS

PD sliding mode controller (PD-SMC) and SISO proposed adaptive sliding mode fuzzy algorithm Fuzzy Estimate Sliding Mode Controller (AFESMC) were tested to sinus response trajectory. This simulation applied to two degrees of freedom robot arm therefore the first and second joints are moved from home to final position without and with external disturbance. The simulation was implemented in Matlab/Simulink environment. Trajectory performance, torque performance, disturbance rejection, steady state error and

RMS error are compared in these controllers. It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude which the sample time is equal to 0.1. This type of noise is used to external disturbance in continuous and hybrid systems.

### Tracking performances

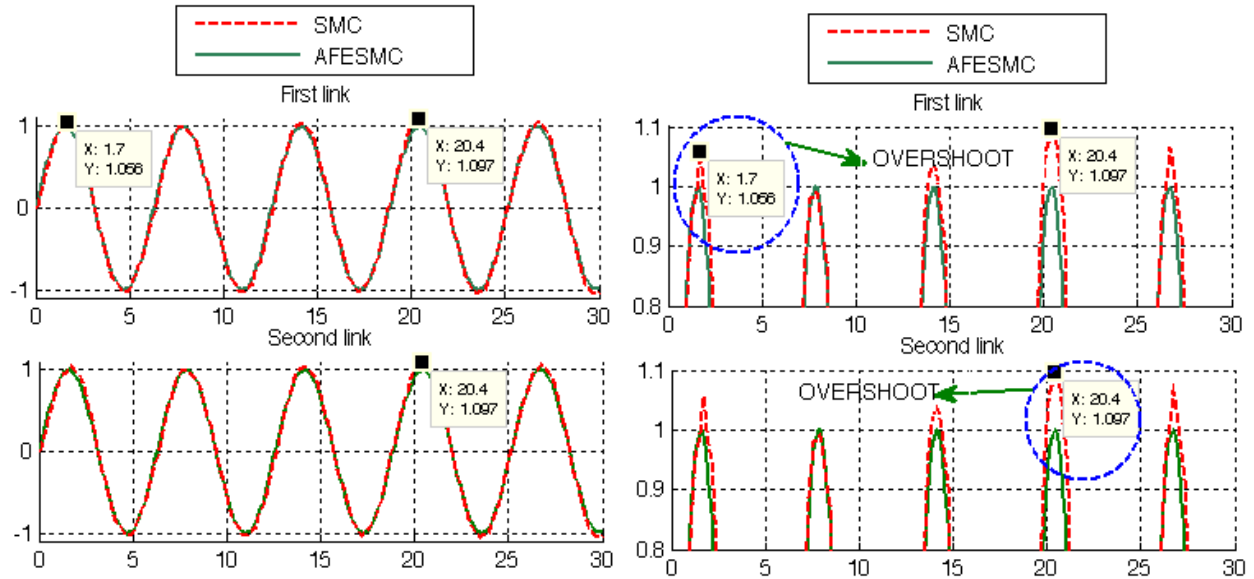
Figure 6 is shown tracking performance for first and second link in SMC, and AFESMC without disturbance for sinus trajectories. By comparing sinus response trajectory without disturbance in SMC and AFESMC it is found that the SMC's overshoot (8%) is higher than AFESMC (0%), although all of them have about the same rise time.



**FIGURE 6:** AFESMC and SMC trajectory: applied to robot manipulator.

### Disturbance Rejection

Figure 7 has shown the power disturbance elimination in SMC and AFESMC. The main target in these controllers is disturbance rejection as well as reduces the chattering. A band limited white noise with predefined of 40% the power of input signal is applied to above controllers. It found fairly fluctuations in SMC trajectory responses.

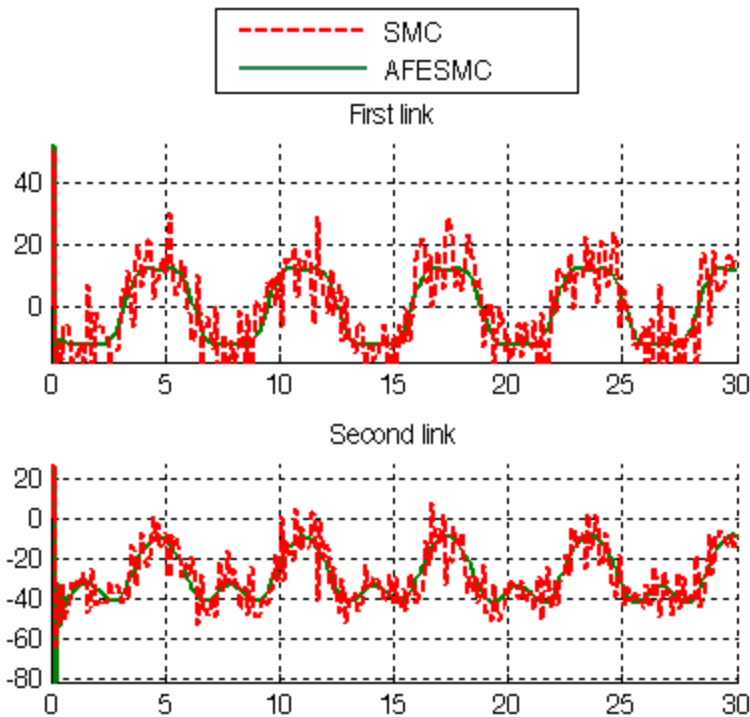


**FIGURE 7:** AFESMC and SMC trajectory with external disturbance: applied to robot manipulator

Among above graph relating to trajectory following with external disturbance, SMC has fairly fluctuations. By comparing some control parameters such as overshoot and rise time it found that the SMC's overshoot (10%) is higher than AFESMC (0%).

### Torque Performance

Figure 8 has shown the torque performance in presence of unstructured uncertainties in SMC and AFESMC. The main target in these controllers is chattering free in proposed method in presence of external disturbance.



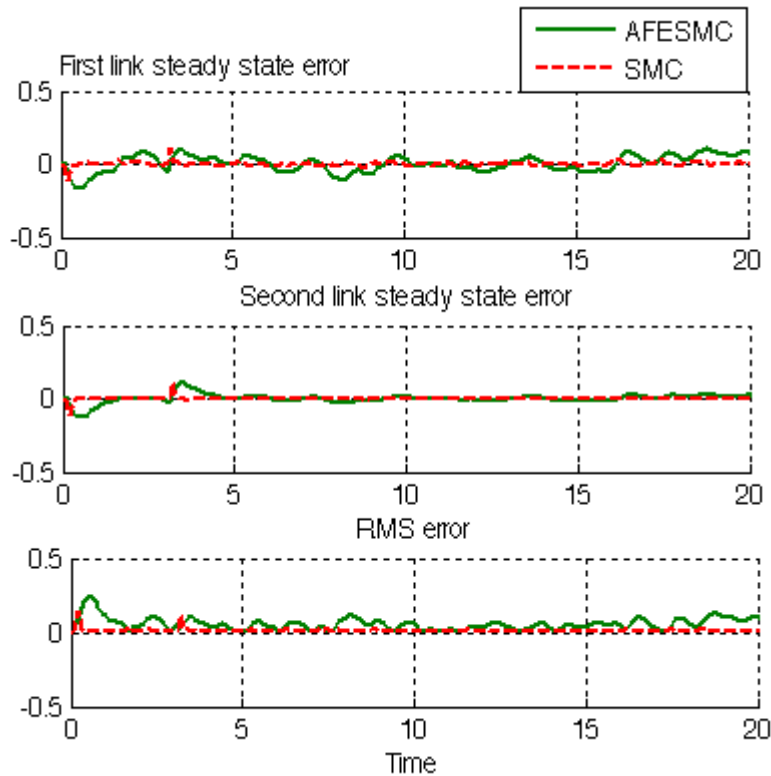
**FIGURE 8:** AFESMC and SMC torque performance with external disturbance: applied to robot manipulator

**Error Calculation:** Figure 9 and Table 1 are shown error performance in SMC and AFESMC in presence of external disturbance. SMC has oscillation in tracking which causes chattering phenomenon. As it is obvious in Table 2 FSMC is a SMC which estimate the equivalent part so FSMC have acceptable performance with regard to SMC in presence of certain and uncertainty and AFESMC also is fuzzy estimate sliding mode controller which online tuning by sliding mode fuzzy algorithm. Figure 9 is shown steady state and RMS error in SMC and AFESMC in presence of external disturbance.

**TABLE 1:** RMS Error Rate of Presented controllers

<i>RMS Error Rate</i>	<b>SMC</b>	<b>FSMC</b>	<b>AFESMC</b>
<b>Without Noise</b>	<b>1e-3</b>	<b>0.9e-3</b>	<b>0.6e-6</b>
<b>With Noise</b>	<b>0.012</b>	<b>0.0012</b>	<b>0.65e-6</b>





**FIGURE 9:** AFESMC and SMC error performance with external disturbance: applied to robot manipulator

In these methods if integration absolute error (IAE) is defined by (75), table 2 is shown comparison between these two methods.

$$IAE = \int_0^{\infty} |e(t)| dt \tag{75}$$

**TABLE 2:** Calculate IAE

Method	Traditional SMC	Fuzzy Estimator SMC	AEFSMC
IAE	490.1	406	208

### 5. CONCLUSIONS

In this work, a SISO sliding mode fuzzy adaptive fuzzy estimate sliding mode controller is design, analysis and applied to robot manipulator. This method focuses on design AFSMC algorithm with the adaptation laws derived in the Lyapunov sense. The stability of the closed-loop system is proved mathematically based on the Lyapunov method. The first objectives in proposed method is remove the chattering which linear boundary layer method is used to solve this challenge. The second target in this work is compensate the model uncertainty by SISO fuzzy inference system, in the case of the m-link robotic manipulator, if we define  $k_1$  membership functions for each input variable, the number of fuzzy rules applied for each joint is  $R_1$  which will result in a low computational load. In finally part sliding mode fuzzy algorithm with minimum rule base is used to online tuning and adjusted the sliding function and eliminate the chattering with minimum computational load. In this case the performance is improved by using the advantages of sliding mode algorithm, artificial intelligence compensate method and adaptive algorithm while the disadvantages removed by added each method to previous method. Fuzzy logic method by adding to the sliding mode controller has covered negative points in fuzzy and sliding algorithms.

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