A Thresholding Method to Estimate Quantities of Each Class

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Abstract

Thresholding method is a general tool for classification of a population. Various thresholding methods have been proposed by many researchers. However, there are some cases in which existing methods are not appropriate for a population analysis. For example, this is the case when the objective of analysis is to select a threshold to estimate the total number of data (pixels) of each classified population. In particular, If there is a significant difference between the total numbers and/or variances of two populations, error possibilities in classification differ excessively from each other. Consequently, estimated quantities of each classified population could be very different from the actual one.

In this report, a new method which could be applied to select a threshold to estimate quantities of classes more precisely in the above mentioned case is proposed. Then verification of features and ranges of application of the proposed method by sample data analysis is presented.

Keywords: Thresholding, Classification, Quantity of a class, Counting accuracy, Synthetic aperture radar.

1. INTRODUCTION

Thresholding method is a general tool for classification of a population. This method is one of the picture binarization techniques. Various thresholding methods have been proposed by many researchers. These thresholding methods were listed and evaluated by Dr. Sahoo and Dr. Wong in the field of image processing [1]. These methods ware categorized as point dependent techniques[2][3][4][5][6][7], region dependent techniques[8][9][10][11][12] and multi-thresholding[13][14][15]. In the field of classical image processing “Uniformity index” and “Shape index” are used to evaluate an analysis and a threshold. These indexes can evaluate for visibility and legibility of a character and an object in a classified image. By contrast, “overall accuracy” is used to evaluate in field of classification of many applications. Especially, some methods proposed by Dr. Otsu [16][17], Dr. Kittler et al. [18][19] and Dr. Kurita et al. [20][21] have a simple algorithm and are easy for data handling and can get high accuracy results. Hence, these techniques have been applied for a lot of classification analysis. These are effective methods for increasing the overall accuracy of analysis. However, there are some cases in which these methods are not appropriate for a population analysis.

For example, this is the case when the objective of analysis is to select a threshold to estimate the total number of data (pixels) of each classified population. In particular, If there is significant difference between the total numbers and/or variances of two populations, error possibilities in classification differ excessively from each other. Consequently, estimated total number of each classified population could be very different from the actual one.

For the field of remote sensing, there are many cases which the area of a target is estimated using a satellite data [22][23][24][25]. The purpose of these analyses is just to estimate the total number of data (pixels) of each classified population.

In this report, authors propose a new method which could be applied to specify a threshold to estimate the quantity of data more precisely in the above mentioned case. And this method is applied for estimation of planting area of rice paddy using a synthetic aperture radar (hereinafter referred to as SAR) data. Then some advantageous effects of the proposed method are shown.

2. METHOD FOR THRESHOLD SELECTION

Lets us consider the most appropriate threshold for classifying pixels from an image represented in gray scale. Of course, the definition of the most appropriate threshold depends on its purpose. In this paper, we propose a threshold to estimate the quantities, total amount of data, of each classified population most precisely.

In the past, purpose of most thresholding methods was to decrease the number of misclassifications. This analysis is to improve the overall accuracy of a classification. However, the most appropriate threshold to estimate the quantities of each classified population differs from a threshold to make the overall accuracy highest. For example, even if the overall accuracy is high, it could happen that an estimated quantity differs from true value grossly when there is an excessive difference between the parameters including total amount of pixels and deviations of each class. On the contrary, even if there may be many misclassifications, estimated quantities of each class could be very similar to true values when the numbers of misclassifications of each class are very similar to each other. Therefore the most appropriate threshold to estimate quantities of each class is a threshold which can minimize the difference between the numbers of misclassifications of each class.

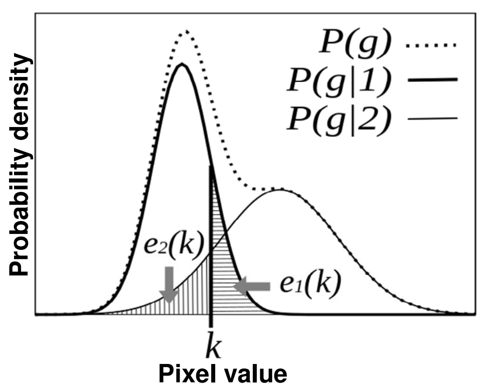


FIGURE 1: Probability density functions and misclassifications of each class

Let the pixels of given image be represented in gray levels g = {1,2,…,L}. The histogram of the gray levels in this image is denoted by h(g). Then the probability density function of gray levels is given by p(g)=h(g)/N, where N is the total number of pixels in the image. Now supposing that the p(g) is a mixed population compounded of class 1 (i=1) and class 2 (i=2), distributions of each class are denoted by p(g|i) and a prior probabilities by P_i. The probability density function of gray levels is also given by

$$p(g)=\frac{h(g)}{N}=\sum_{i=1}^{2}P_i p(g \mid i)\ . \tag{1}$$

These parameters are shown in Figure 1. Where Broken line is the distribution of input data, Solid lines are distributions of each class estimated by optimization and Striped areas, $e_1(k)$ and $e_2(k)$ denote misclassifications should be considered at first. Supposing that the pixels are categorized into two classes by threshold at level k, pixels not more than threshold k can be classified into class 1. On the other hand, pixels which are more than threshold k can be classified into class 2. In this case, the total number of class 1 and class 2 are given by

$$\begin{aligned} n_1(k)&=N\sum_{g=1}^{k}p(g)=N\sum_{g=1}^{k}\sum_{i=1}^{2}P_i p(g \mid i)\\ n_2(k)&=N\sum_{g=k+1}^{L}p(g)=N\sum_{g=k+1}^{L}\sum_{i=1}^{2}P_i p(g \mid i)\end{aligned}\ . \tag{2}$$

The number of pixels belonging to class 1 which are misclassified into class 2 is denoted by $e_1(k)$. And the number of pixels belonging to class 2 which are misclassified into class 1 is denoted by $e_2(k)$. These are given by

$$\begin{aligned} e_1(k)&=N\sum_{g=k+1}^{L}P_1 p(g \mid 1)\\ e_2(k)&=N\sum_{g=1}^{k}P_2 p(g \mid 2)\end{aligned}\ . \tag{3}$$

Then, an assumption that $e_1(k)$ and $e_2(k)$ are approximated when the threshold k is equal to the value τ is introduced. In other words, the value τ can minimize the criterion function

$$\varepsilon(k)=\left(e_1(k)-e_2(k)\right). \tag{4}$$

In this case, the total number of the pixels classified into class 1 can be evaluated as

$$\begin{aligned} n_1(\tau)&=N\sum_{g=1}^{\tau}\left(P_1 p(g \mid 1)+P_2 p(g \mid 2)\right)\\ n_1(\tau)&\approx N\sum_{g=1}^{\tau}P_1 p(g \mid 1)+N\sum_{g=\tau+1}^{L}P_1 p(g \mid 1)\\ n_1(\tau)&\approx N\sum_{g=1}^{L}P_1 p(g \mid 1)\\ n_1(\tau)&\approx NP_1\end{aligned}\ . \tag{5}$$

Thus, the total number of pixels classified into class 1 is very close to the true value. Similarly the total number of the pixels classified into class 2 can be evaluated as

$$n_2(\tau)\approx NP_2\ . \tag{6}$$

Also the total number of pixels classified into class 2 is very close to the true value.
As a result, using the threshold τ which can minimize the difference between the numbers of misclassification pixels of class 1 and class 2, the quantity of classified pixels can be approximated to the true value. In order to compute the threshold τ which can minimize the criterion function $\varepsilon(k) = |e_1(k)- e_2(k)|$, following procedures are needed.

- To formulate a histogram and a probability density function from an input image.
- To optimize a histogram to a mixed population.
- To calculate the threshold τ which can minimize the criterion function from the mixed population.

In order to optimize a histogram and the threshold τ an optimization technique (e.g., the steepest descent method, the downhill simplex [26], the simulated annealing [27]) has to be used.

The result of classification is evaluated using a confusion matrix first. The confusion matrix is shown in Table 1, where, n is the number of evaluated pixels, the first subscript denotes a class number of the classified image, the second subscript denotes a class number of the correct classification result and a symbol "+" denotes summation. In particular, n_{11} and n_{22} denote the number of pixels which were classified correctly. n_{12} and n_{21} denote the number of pixels which were classified as false. n_{i+} denote the number of pixels which were classified into class i. And N_i denotes the correct number of pixels belonging to class 1.

TABLE 1: Confusion matrix for evaluation

		True Data		
		Class1	Class2	Σ
Classifier	Class1	n_{11}	n_{12}	n_{1+}
	Class2	n_{21}	n_{22}	n_{2+}
	Σ	N_1	N_2	N

After evaluation by the confusion matrix, typically the result of classification is evaluated using the overall accuracy O, the producer's accuracy P and the user's accuracy U [28]. These accuracies are given by

$$O=\frac{\sum_{i=1}^{2} n_{ii}}{N}, \qquad P_i=\frac{n_{ii}}{N_i}, \qquad U_i=\frac{n_{ii}}{n_{i+}}. \tag{7}$$

These accuracies are useful to evaluate a result of classification. However those may not be suitable for evaluation of a threshold value. For example, this is the case when the numbers of each class differ widely. In this case, an accuracy of majority class becomes dominant in the overall accuracy. And an accuracy of minority class is neglected. In the result, the greater the number of estimated pixels which are classified to the majority class, the greater the overall accuracy is increased. So the overall accuracy is not suitable to evaluate a threshold value. Furthermore, the producer's accuracy is improved when a threshold changes in a direction which classifies pixels more into a target class. The user's accuracy is also improved when a threshold changes in a direction which classifies pixels less into a target class. Thus, these accuracies are not suitable to evaluate a threshold because it is possible that those are improved by an unfair threshold.

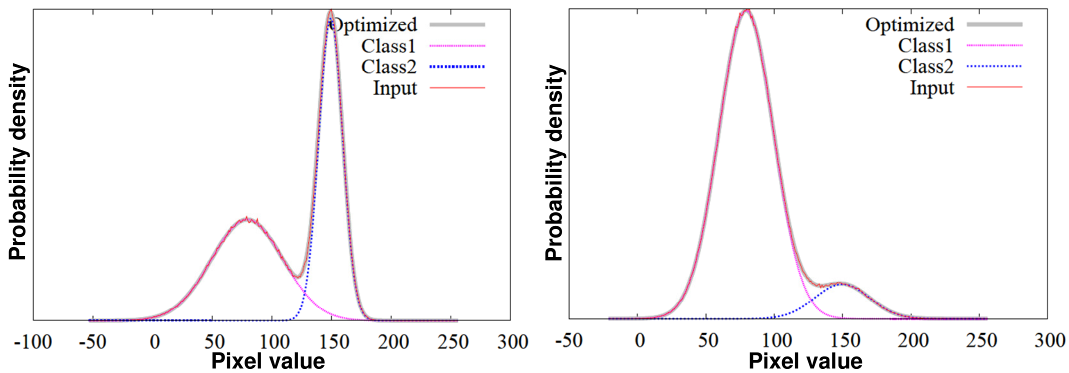
Accordingly, we use a new accuracy to evaluate a threshold to estimate the quantities of class populations. We call the new accuracy a "counting accuracy" in this paper and define it as

$$C_i=\frac{n_{i+}}{N_i}. \tag{8}$$

The counting accuracy is a ratio between a quantity classified into class i and a correct quantity of class i. It directly compares a quantity classified into class i and a correct quantity of class i. So the counting accuracy can evaluate the result of classification directly and can evaluate also the threshold indirectly, when our purpose of classification is to estimate the quantities of classified populations.

3. FEATURE

Theory of the proposed method was presented in the second chapter. However, can the expected result be taken by the proposed method? A validation of the effects and the works using a data set is needed. To confirm the feature of the suggested method, comparison analysis between the results of the proposed method and existing methods are conducted. At first, ten sample input images are prepared. Every sample images have two classes and consist of 1,000,000 pixels. Each class goes along gaussian distribution. Two examples of the probability density functions are shown in Figure 2. Then number of pixels N_i, average μ_i and standard deviation σ_i for each class are shown in Table 2, where i denotes class number. Each image has a different number of pixels and a different standard deviation for each class. The results in case that there are various distributions of each class can be confirmed by the analysis of using these sample images.



(a) for sample input image No.4 (b) for sample input image No.8

FIGURE 2: Example of the probability density function for each sample input image

TABLE 2: Then number of pixels N_i, average μ_i and standard deviation σ_i for each class

Sample input image	N_1	μ_1	σ_1	N_2	μ_2	σ_2
No.1	500,000	80	10	500,000	150	10
No.2	500,000	80	10	500,000	150	30
No.3	500,000	80	20	500,000	150	20
No.4	500,000	80	30	500,000	150	10
No.5	500,000	80	30	500,000	150	30
No.6	900,000	80	10	100,000	150	10
No.7	900,000	80	10	100,000	150	30
No.8	900,000	80	20	100,000	150	20
No.9	900,000	80	30	100,000	150	10
No.10	900,000	80	30	100,000	150	30

Selections of three thresholds using three methods are conducted respectively. One method is the proposed method while the other two methods are the Otsu's thresholding (hereinafter referred to as the Otsu thresholding) and the minimum error thresholding by Kittler and Illingworth's (hereinafter referred to as the Kitter thresholding). In case of Otsu thresholding, a threshold which makes $\eta=\sigma_B/\sigma_W$ maximum is selected by the downhill simplex method [26]. Where σ_B is intra-class variance and σ_W is inter-class variance. On the other hand, in case of the Kitller thresholding, a threshold which makes an evaluation function J minimum is selected by the downhill simplex method. In the case of the proposed method, we optimized some parameters of a mixed gaussian distribution to match a probability density function of input image using the multivariate downhill simplex method. Then, we got a threshold which makes the criterion function

$\varepsilon(k) = |e_1(k) - e_2(k)|$ minimum from some parameters of the mixed gaussian distribution using the downhill simplex method [26].

In the result, when numbers of pixels and standard deviations of two classes were equal (for sample image No.1, No.3 and No.5), each of the three thresholding methods selected same thresholds.

For each sample input image, we classified a pixel less than or equal to a threshold into class 1. In reverse, we classified a pixel bigger than a threshold into class 2. Then some classified images were created.

To evaluate the result of classifications, we compared the classified images with the correct classification results which we made first. Then, we made a confusion matrix which is shown in Table 1 from each result of evaluations. All elements of the confusion matrices for all evaluations are shown in Table 3. At the end, we calculated a overall accuracy O, a producer's accuracy P, a user's accuracy U and a counting accuracy C from the elements of the confusion matrices for each classifications. The results are shown in Table 4.

For sample input image No.1 and No.6 of which two classes were separated clearly, any accuracy of all methods was just about 100%. For sample input images of which the number of pixels and the standard deviations of the two classes were equal, the accuracies of each method did not differ clearly. However, for sample image No.2 and No.4 of which the standard deviations between the two classes were different, the most accurate overall accuracies were obtained by the Kittler thresholding. And the most exact counting accuracies were obtained by the proposed method. Furthermore, the Otsu method and the Kittler method make a big difference between a user's accuracy and a producer's accuracy. By contrast, the proposed method made a user's accuracy and a producer's accuracy approximate each other. Also for sample input images from No.6 to No.10, the most accurate counting accuracies were obtained by the proposed method being compared with other methods. Sample image No. 10 is convex shape. That means it is not biphasic distribution. However the proposed method can classify sample image No.10 with high accuracy. And the proposed method made the least difference between a user's accuracy and a producer's accuracy.

TABLE 3: Evaluated elements of confusion matrices

Sample image	Method	n_{11}	n_{12}	N_{1+}	N_1	n_{21}	n_{22}	n_{2+}	N_2
No.1	Otsu	499,886	102	499,988	500,000	114	499,898	500,012	500,000
	Kittler	499,888	106	499,994	500,000	112	499,894	500,006	500,000
	proposed	499,887	102	499,989	500,000	113	499,898	500,011	500,000
No.2	Otsu	414,754	6	414,760	500,000	85,246	499,994	585,240	500,000
	Kittler	466,625	3,144	469,769	500,000	33,375	496,856	530,231	500,000
	proposed	479,767	19,959	499,726	500,000	20,233	480,041	500,274	500,000
No.3	Otsu	479,842	19,984	499,826	500,000	20,158	480,016	500,174	500,000
	Kittler	479,904	20,051	499,955	500,000	20,096	479,949	500,045	500,000
	proposed	479,904	20,051	499,955	500,000	20,096	479,949	500,045	500,000
No.4	Otsu	499,995	84,938	584,933	500,000	5	415,062	415,067	500,000
	Kittler	496,668	32,943	529,611	500,000	3,332	467,057	470,389	500,000
	proposed	479,879	20,076	499,955	500,000	20,121	479,924	500,045	500,000
No.5	Otsu	439,178	60,812	499,990	500,000	60,822	439,188	500,010	500,000
	Kittler	438,272	59,885	498,157	500,000	61,728	440,115	501,843	500,000
	proposed	436,012	57,771	493,783	500,000	63,988	442,229	506,217	500,000
No.6	Otsu	99,974	238	100,212	100,000	26	899,762	899,788	900,000
	Kittler	99,922	64	99,986	100,000	78	899,936	900,014	900,000
	proposed	99,924	73	99,997	100,000	76	899,927	900,003	900,000
No.7	Otsu	84,797	36	84,833	100,000	15,203	899,964	915,167	900,000
	Kittler	90,152	718	90,870	100,000	9,848	899,282	909,130	900,000

	proposed	93,617	6,303	99,920	100,000	6,383	893,697	900,080	900,000
No.8	Otsu	99,059	117,060	216,119	100,000	941	782,940	783,881	900,000
	Kittler	80,660	3,773	84,433	100,000	19,340	896,227	915,567	900,000
	proposed	89,054	10,398	99,452	100,000	10,946	889,602	900,548	900,000
No.9	Otsu	100,000	302,704	402,704	100,000	0	597,296	597,296	900,000
	Kittler	99,977	109,844	209,821	100,000	23	790,156	790,179	900,000
	proposed	81,186	18,659	99,845	100,000	18,814	881,341	900,155	900,000
No.10	Otsu	97,213	297,742	394,955	100,000	2,787	602,258	605,045	900,000
	Kittler	87,928	109,509	197,437	100,000	12,072	790,491	802,563	900,000
	proposed	71,065	34,317	105,382	100,000	28,935	865,683	894,618	900,000

TABLE 4: Result of evaluation for each sample images (Overall accuracy, Producer's accuracy, User's accuracy and Counting accuracy for each class)

Sample image	Method	O	P_1	U_1	C_1	P_2	U_2	C_2
No.1	Otsu	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Kittler	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	proposed	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
No.2	Otsu	91.5%	83.0%	100.0%	83.0%	100.0%	85.4%	117.1%
	Kittler	96.4%	93.3%	99.3%	94.0%	99.4%	93.7%	106.1%
	proposed	96.0%	96.0%	96.0%	100.0%	96.0%	96.0%	100.1%
No.3	Otsu	96.0%	96.0%	96.0%	100.0%	96.0%	96.0%	100.0%
	Kittler	96.0%	96.0%	96.0%	100.0%	96.0%	96.0%	100.0%
	proposed	96.0%	96.0%	96.0%	100.0%	96.0%	96.0%	100.0%
No.4	Otsu	91.5%	100.0%	85.5%	117.0%	83.0%	100.0%	83.0%
	Kittler	96.4%	99.3%	93.8%	105.9%	93.4%	99.3%	94.1%
	proposed	96.0%	96.0%	96.0%	100.0%	96.0%	96.0%	100.0%
No.5	Otsu	87.8%	87.8%	87.8%	100.0%	87.8%	87.8%	100.0%
	Kittler	87.8%	87.7%	88.0%	99.6%	88.0%	87.7%	100.4%
	proposed	87.8%	87.2%	88.3%	98.8%	88.5%	87.4%	101.2%
No.6	Otsu	100.0%	100.0%	99.8%	100.2%	100.0%	100.0%	100.0%
	Kittler	100.0%	99.9%	99.9%	100.0%	100.0%	100.0%	100.0%
	proposed	100.0%	99.9%	99.9%	100.0%	100.0%	100.0%	100.0%
No.7	Otsu	98.5%	84.8%	100.0%	84.8%	100.0%	98.3%	101.7%
	Kittler	98.9%	90.2%	99.2%	90.9%	99.9%	98.9%	101.0%
	proposed	98.7%	93.6%	93.7%	99.9%	99.3%	99.3%	100.0%
No.8	Otsu	88.2%	99.1%	45.8%	216.1%	87.0%	99.9%	87.1%
	Kittler	97.7%	80.7%	95.5%	84.4%	99.6%	97.9%	101.7%
	proposed	97.9%	89.1%	89.5%	99.5%	98.8%	98.8%	100.1%
No.9	Otsu	69.7%	100.0%	24.8%	402.7%	66.4%	100.0%	66.4%
	Kittler	89.0%	100.0%	47.7%	209.8%	87.8%	100.0%	87.8%
	proposed	96.3%	81.2%	81.3%	99.9%	97.9%	97.9%	100.0%
No.10	Otsu	70.0%	97.2%	24.6%	395.0%	66.9%	99.5%	67.2%
	Kittler	87.8%	87.9%	44.5%	197.4%	87.8%	98.5%	89.2%
	proposed	93.7%	71.1%	67.4%	105.4%	96.2%	96.8%	99.4%

It should be noted that the Otsu method and the Kittler method make a big difference between a user's accuracy and a producer's accuracy, because these methods make a big difference between the number of classified pixels n_{i+} and the number of correct classification pixel N_i, too. On the other hand, the proposed method decreases the difference between a user's accuracy and a producer's accuracy, because the method makes the number of classified pixels n_{i+} and the number of correct classification pixel N_i approximate each other. Accordingly, the proposed method can select a threshold to improve the counting accuracy C. In other words, the proposed method can estimate the number of pixels of a class with more precision than other methods.

4. RANGE OF APPLICATION

The proposed method selects a threshold by statistical algorithm. Thus, a sufficient number of input data are required in order to stabilize a result. Verifying analysis of a sufficient quantity of input data to stabilize a threshold and a classification under the assumption of estimating a planting area of paddy using a SAR data (RADARSAT-2, Ultra-Fine, 2009/6/6) from a satellite is conducted.

At first, inspection of a distribution of backscattering coefficients of SAR data for each field is implemented. Then it becomes clear that the distribution is similar to a mixed gaussian distribution（refer to Figure 3）. Some input data sets which have 10 to 10,000,000 sample data and the same distribution are prepared. At the Time, a class of the sample data belongs was defined in order to evaluate accuracies after classifications.

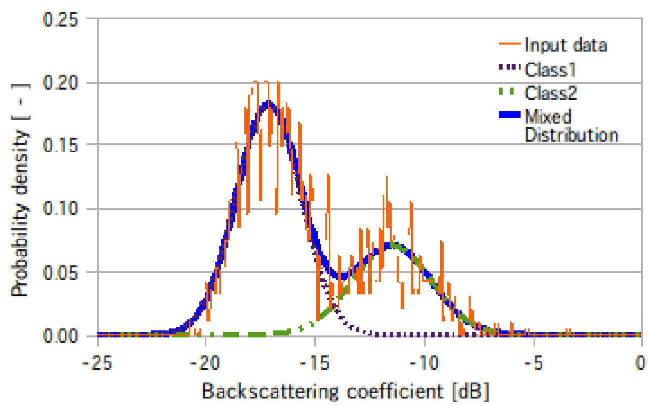


FIGURE 3: A result of optimization of a mixed distribution which used RADARSAT-2 data

As a result of validation of the stable number of input samples, stable thresholds were not obtained when we used less than 1000 samples with this distribution data. In contrast, stable thresholds were obtained using more than 1000 samples by any threshold method (refer to Figure 4).

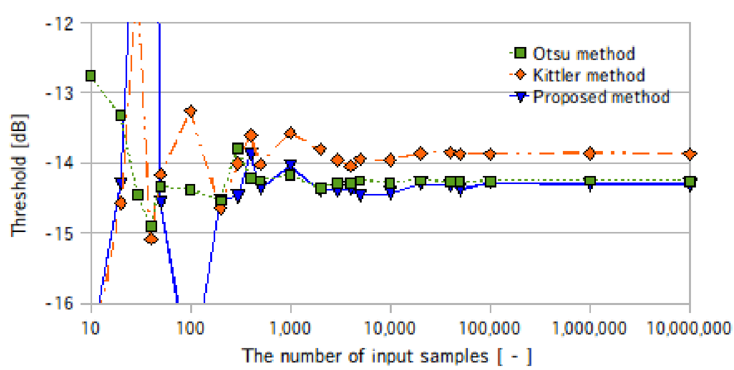


FIGURE 4: Variation in a threshold when the number of input samples was changed.

Furthermore, as a result of evaluating accuracies of classifications, accuracies of each method were reversed at each data set when we used less than 10,000 samples for evaluation. In other hand, stable accuracies were obtained when we used more than 10,000 samples. This means that 10,000 samples are required at least (refer to Figure 5 and Figure 6).

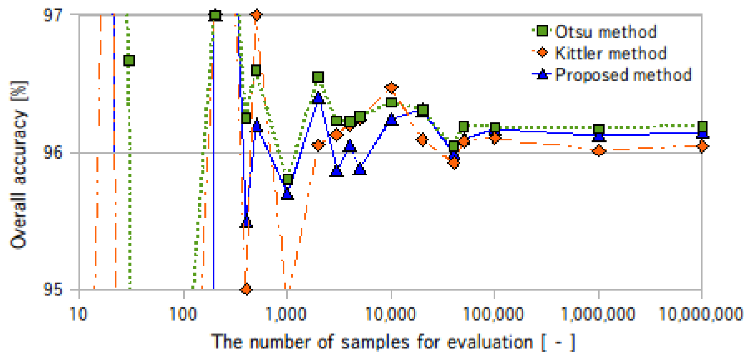


FIGURE 5: Variation in a overall accuracy when the number of input samples was changed.

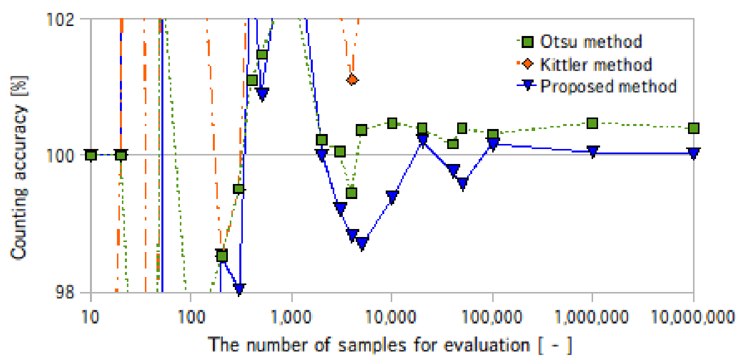


FIGURE 6: Variation in a counting accuracy when the number of input samples was changed.

Accordingly, distributions with at least 1,000 samples are required in order to obtain a stable threshold. And at least 10,000 samples are required in order to compare the accuracies of each thresholding method.

5. CONCLUSION

In conclusion, the method we proposed can select a threshold which equalizes two amounts of incorrect classifications of each class. This method has a unique objective which is to select a threshold to estimate the total number of data (pixels) of each classified population. Furthermore by comparing the proposed method with the existing methods, we showed the proposed method has the following advantages.

• A higher counting accuracy can be obtained than with the existing methods.
•The method can equalize a user's accuracy and a producer's accuracy more than the other methods.
• The proposed method is valid even if there are biases of amounts and deviations in each class.
• The proposed method is valid even if input data do not have a biphasic distribution.

The distribution characteristics of input data must be known when the proposed method is utilized. In addition, the application of proposed method has the following characteristics:

• The proposed method can be used not only for an image data but also for a numerical data.
• At least 1,000 samples are required in order to obtain a stable threshold.
• At least 10,000 samples are required in order to compare with the accuracies of some thresholding methods.

For actual applications, sometimes the number of input data and true data for evaluation are not sufficiency. In this case, it is difficult to apply the proposed method into the analysis. Furthermore, it is also difficult to use an input data having a unknown distribution as input class. This is because an assuming or a knowing a distributed shape is needed for the proposed method.

It should be noted that the result is an example for estimating a planting area of paddy using some specific data. We showed some characteristics and ranges of application of the proposed method. The method has some advantages for thresholding and classification. However, the most specified characteristic of the proposed method is its objective. A purpose of the existing methods is to decrease errors of a classification. On the other hand, the purpose of the proposed method is to estimate quantities, data volumes, of classes through a classification analysis. Of course, classical statistic method could estimate quantity of a class. But it cannot classify sample data. The proposed method can coordinate a result of a classification and a result of quantity estimation. The proposed method is definitely useful when both classification and quantity estimation of a class are required.

Almost all data have some noise. So, validations of resistance to some noises will be conducted in the future. After an adaptive limits are known, solutions which can be applied the proposed method will be findable. Application of the proposed method to various field analyses would be a major topic in next step.

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