Intuitionistic Fuzzy W- Closed Sets and Intuitionistic Fuzzy W -Continuity

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Abstract

The aim of this paper is to introduce and study the concepts of intuitionistic fuzzy w-closed sets, intuitionistic fuzzy w-continuity and inttuitionistic fuzzy w-open & intuitionistic fuzzy w-closed mappings in intuitionistic fuzzy topological spaces.

Key words: Intuitionistic fuzzy w-closed sets, Intuitionistic fuzzy w-open sets, Intuitionistic fuzzy w-connectedness, Intuitionistic fuzzy w-compactness, intuitionistic fuzzy w-continuous mappings.

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1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [23] in 1965 and fuzzy topology by Chang [4] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [7], fuzzy connectedness [21], fuzzy separation axioms [3], fuzzy continuity [8], fuzzy g-closed sets [15] and fuzzy g-continuity [16] have been generalized for intuitionistic fuzzy topological spaces. In the present paper we introduce the concepts of intuitionistic fuzzy w-closed sets; intuitionistic fuzzy w-open sets, intuitionistic fuzzy w-connectedness, intuitionistic fuzzy w-compactness and intuitionistic fuzzy w-continuity obtain some of their characterization and properties.

2. PRELIMINARIES

Let X be a nonempty fixed set. An intuitionistic fuzzy set A[1] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. The intutionistic fuzzy sets $\tilde{\mathbf{0}} = \{< x, 0, 1 > : x \in X\}$ and $\tilde{\mathbf{1}} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{ <x, \gamma_A(x), \mu_A(x) >: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) > : x \in X, (i \in \wedge)\}$ of X be the intuitionistic fuzzy set $\cap A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup A_i = \{<x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) >: x \in X\}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [5] on X if the intuitionistic fuzzy sets $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [5].

Lemma 2.1 [5]**:** Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \Im)$. Then:

(a). $\rceil(A_qB) \Leftrightarrow A \subseteq B^c$.
(b). A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A.
(c). A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.
(d). $cl(A^c) = (int(A))^c$.
(e). $int(A^c) = (cl(A))^c$.
(f). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$.
(g). $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$.
(h). $cl(A \cup B) = cl(A) \cup cl(B)$.
(i). $int(A \cap B) = int(A) \cap int(B)$

Definition 2.1 [6]**:** Let X is a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha+\beta \le 1$ then:

(a) $c(\alpha,\beta) = < x, c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of non membership of $c(\alpha,\beta)$.
(b) $c(\beta) = < x, 0, 1-c_{1-\beta} >$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2[7] **:** A family $\{ G_i : i \in \wedge\}$ of intuitionistic fuzzy sets in X is called an intuitionistic fuzzy open cover of X if $\cup\{ G_i : i \in \wedge\} = \tilde{\mathbf{1}}$ and a finite subfamily of an intuitionistic fuzzy open cover $\{ G_i: i \in \wedge\}$of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of $\{ G_i: i \in \wedge\}$.

Definition 2.3[7]**:** An intuitionistic fuzzy topological space $(X,\Im)$ is called fuzzy compact if every intuitionistic fuzzy open cover of X has a finite sub cover.

Definition 2.4[8]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) if there exists a intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) U such that $U \subseteq A \subseteq cl(A)$ (resp.$int(U) \subseteq A \subseteq U$)

Definition 2.5 [21]: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy closed .

Definition 2.6[15]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy g-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.

(b) Intuitionistic fuzzy g-open if its complement A^c is intuitionistic fuzzy g-closed.

Remark 2.1[15]: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true.

Definition 2.7[18]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy sg-closed if $scl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.

(b) Intuitionistic fuzzy sg -open if its complement A^c is intuitionistic fuzzy sg-closed.

Remark 2.2[18]: Every intuitionistic fuzzy semi-closed (resp. Intuitionistic fuzzy semi-open) set is intuitionistic fuzzy sg-closed (intuitionistic fuzzy sg-open) but its converse may not be true.

Definition 2.8[12]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy gs-closed if $scl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.

(b) Intuitionistic fuzzy gs -open if its complement A^c is intuitionistic fuzzy gs-closed.

Remark 2.3[12]: Every intuitionistic fuzzy sg-closed (resp. Intuitionistic fuzzy sg-open) set is intuitionistic fuzzy gs-closed (intuitionistic fuzzy gs-open) but its converse may not be true.

Definition 2.9: [5] Let X and Y are two nonempty sets and $f: X \to Y$ is a function. :

(a) If $B = \{<y, \mu_B(y), \gamma_B(y)> : y \in Y\}$is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = <x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)>: x \in X\}.$$

(b) If $A = \{<x, \lambda_A(x), \nu_A(x)> : x \in X\}$is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{<y, f(\lambda_A)(y), f(\nu_A)(y)>: y \in Y\}$$

Where $f(\nu_A) = 1 - f(1- \nu_A)$.

Definition 2.10[8]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let $f: X\to Y$ be a function. Then f is said to be

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

(b) Intuitionistic fuzzy semi continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy semi open set in X.

(c) Intuitionistic fuzzy closed if the image of each intuitionisic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.

(d) Intuitionistic fuzzy open if the image of each intuitionisic fuzzy open set in X is an intuitionistic fuzzy open set in Y.

Definition 2.6[12, 16,17 19]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let $f: X \to Y$ be a function. Then *f* is said to be

(a) Intuitionistic fuzzy g-continuous [16] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.
(b) Intuitionistic fuzzy gc-irresolute[17]if the pre image of every intuitionistic fuzzy g-closed in Y is intutionistic fuzzy g-closed in X
(c) Intuitionistic fuzzy sg-continuous [19] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg –closed in X.
(d) Intutionistic fuzzy gs-continuous [12] if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gs –closed in X.

Remark 2.4[12, 16, 19]:

(a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [16].
(b) Every intuitionistic fuzzy semi continuous mapping is intuitionistic fuzzy sg-continuous, but the converse may not be true [19].
(c) Every intuitionistic fuzzy sg- continuous mapping is intuitionistic fuzzy gs-continuous, but the converse may not be true [12].
(d) Every intuitionistic fuzzy g- continuous mapping is intuitionistic fuzzy gs-continuous, but the converse may not be true [12].

3. INTUITIONISTIC FUZZY W-CLOSED SET

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called an intuitionistic fuzzy w-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.

Remark 3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy w-closed but its converse may not be true.

Example 3.1: Let X = {a, b} and $\Im = \{\tilde{0}, \tilde{1}, U\}$ be an intuitionistic fuzzy topology on X, where U= {< a,0.5,0.5>,< b, 0.4, 0.6 > }.Then the intuitionistic fuzzy set A = {<a,0.5,0.5>,<b,0.5,0.5>} is intuitionistic fuzzy w -closed but it is not intuitionistic fuzzy closed.

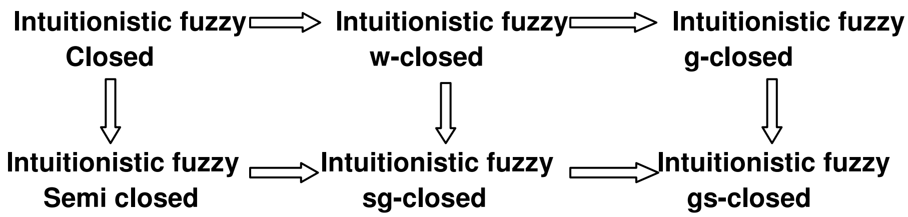
Remark 3.2: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed but its converse may not be true.

Example 3.2: Let X = {a, b} and $\Im = \{\tilde{0}, \tilde{1}, U\}$ be an intuitionistic fuzzy topology on X, where U= {< a,0.7,0.3>,< b, 0.6, 0.4 >}.Then the intuitionistic fuzzy set A = {<a,0.6,0.4>,<b,0.7,0.3>} is intuitionistic fuzzy g -closed but it is not intuitionistic fuzzy w-closed.

Remark 3.3: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy sg-closed but its converse may not be true.

Example 3.3: Let X = {a, b} and $\Im = \{\tilde{0}, \tilde{1}, U\}$ be an intuitionistic fuzzy topology on X, where U= {< a,0.5,0.5>,< b, 0.4, 0.6 >}.Then the intuitionistic fuzzy set A ={<a,0.5,0.5>,<b,0.3,0.7>} is intuitionistic fuzzy sg -closed but it is not intuitionistic fuzzy w-closed.

Remark 3.4: Remarks 2.1, 2.2, 2.3, 3.1, 3.2, 3.3 reveals the following diagram of implication.



Theorem 3.1: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy w-closed if and only if $\rceil(AqF) \Rightarrow \rceil(cl\ (A)qF)$ for every intuitionistic fuzzy semi closed set F of X.

Proof: **Necessity**: Let F be an intuitionistic fuzzy semi closed set of X and $\rceil\ (AqF)$. Then by Lemma 2.1(a), $A \subseteq F^c$ and F^c intuitionistic fuzzy semi open in X. Therefore $cl(A) \subseteq F^c$ by Def 3.1 because A is intuitionistic fuzzy w-closed. Hence by lemma 2.1(a), $\rceil\ (cl\ (A)qF)$.

Sufficiency: Let O be an intuitionistic fuzzy semi open set of X such that $A \subseteq O$ i.e. $A \subseteq ((O)^c)^c$ Then by Lemma 2.1(a), $\rceil(A_qO^c)$ and O^c is an intuitionistic fuzzy semi closed set in X. Hence by hypothesis $\rceil\ (cl\ (A)_qO^c)$. Therefore by Lemma 2.1(a), $cl\ (A) \subseteq ((O)^c)^c$ i .e. $cl\ (A) \subseteq O$ Hence A is intuitionistic fuzzy w-closed in X.

Theorem 3.2: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space $(X,\Im)$ and $c(\alpha,\beta)$ be an intuitionistic fuzzy point of X such that $c(\alpha,\beta)_q cl\ (A)$ then $cl(c(\alpha,\beta))qA$.

Proof: If $\rceil cl(c(\alpha,\beta))_qA$ then by Lemma 2.1(a),$cl(c(\alpha,\beta) \subseteq A^c$ which implies that $A \subseteq (cl(c(\alpha,\beta)))^c$ and so $cl(A) \subseteq (cl(c(\alpha,\beta)))^c \subseteq (c(\alpha,\beta))^c$**,** because A is intuitionistic fuzzy w-closed in X. Hence by Lemma 2.1(a), $\rceil\ (c(\alpha,\beta)_q\ (cl\ (A)))$, a contradiction.

Theorem 3.3: Let A and B are two intuitionistic fuzzy w-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$, then $A \cup B$ is intuitionistic fuzzy w-closed.

Proof: Let O be an intuitionistic fuzzy semi open set in X, such that $A\cup B \subseteq O$. Then $A \subseteq O$ and $B \subseteq O$. So, $cl\ (A) \subseteq O$ and $cl\ (B) \subseteq O$. Therefore $cl\ (A) \cup cl\ (B) = cl\ (A\cup B) \subseteq O$. Hence $A\cup B$ is intuitionistic fuzzy w-closed.

Remark 3.2: The intersection of two intuitionistic fuzzy w-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$ may not be intuitionistic fuzzy w-closed. For,

Example 3.2: Let X = {a, b, c} and U, A and B be the intuitionistic fuzzy sets of X defined as follows:

U = {<a, 1, 0>, <b, 0, 1 >, < c, 0, 1>}

A = {<a, 1, 0 >, < b, 1, 0 >, < c, 0, 1>}

B = {<a, 1, 0 >, < b, 0, 1>, < c, 1, 0>}

Let $\mathfrak{I}$ = {$\tilde{\mathbf{0}}$, $\tilde{\mathbf{1}}$, U} be intuitionistic fuzzy topology on X. Then A and B are intuitionistic fuzzy w-closed in (X,$\mathfrak{I}$) but A $\cap$ B is not intuitionistic fuzzy w-closed.

Theorem 3.4: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space (X,$\mathfrak{I}$) and A $\subseteq$ B $\subseteq$ cl (A). Then B is intuitionistic fuzzy w-closed in X.

Proof: Let O be an intuitionistic fuzzy semi open set such that B $\subseteq$ O. Then A $\subseteq$ O and since A is intuitionistic fuzzy w-closed, cl (A) $\subseteq$ O. Now B $\subseteq$ cl (A) $\Rightarrow$ cl (B) $\subseteq$ cl (A) $\subseteq$ O. Consequently B is intuitionistic fuzzy w-closed**.**

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\mathfrak{I}$) is called intuitionistic fuzzy w-open if and only if its complement A^c is intuitionistic fuzzy w-closed.

Remark 3.5 Every intuitionistic fuzzy open set is intuitionistic fuzzy w-open. But the converse may not be true. For

Example 3.4: Let X = {a, b} and $\mathfrak{I}$ = {$\tilde{\mathbf{0}}$, $\tilde{\mathbf{1}}$, U} be an intuitionistic fuzzy topology on X, where U= {<a, 0.5, 0.5>, <b, 0.4, 0.6>}. Then intuitionistic fuzzy set B defined by B={ <a,0.5,0.5>, <b,0.5,0.5>}is an intuitionistic fuzzy w-open in intuitionistic fuzzy topological space (X, $\mathfrak{I}$) but it is not intuitionistic fuzzy open in (X, $\mathfrak{I}$).

Remark 3.6: Every intuitionistic fuzzy w-open set is intuitionistic fuzzy g-open but its converse may not be true.

Example 3.5: Let X = {a, b} and $\mathfrak{I}$ = {$\tilde{\mathbf{0}}$, $\tilde{\mathbf{1}}$, U} be an intuitionistic fuzzy topology on X, where U= {<a,0.5,0.5>,<b,0.4,0.6>}.Then the intuitionistic fuzzy set A={<a,0.4,0.6>,<b,0.3,0.7>} is intuitionistic fuzzy g-open in (X, $\mathfrak{I}$) but it is not intuitionistic fuzzy w-open in (X, $\mathfrak{I}$).

Theorem 3.5: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\mathfrak{I}$) is intuitionistic fuzzy w-open if F$\subseteq$ int (A) whenever F is intuitionistic fuzzy semi closed and F $\subseteq$ A.

Proof: Follows from definition 3.1 and Lemma 2.1

Remark 3.4: The union of two intuitionistic fuzzy w-open sets in an intuitionistic fuzzy topological space (X,$\mathfrak{I}$) may not be intuitionistic fuzzy w-open. For the intuitionistic fuzzy set C ={ <a,0.4,0.6> ,<b,0.7,0.3> } and D = {< a,0.2,0.8> ,<b,0.5,0.5>} in the intuitionistic fuzzy topological space (X,$\mathfrak{I}$) in Example 3.2 are intuitionistic fuzzy w-open but their union is not intuitionistic fuzzy w-open.

Theorem 3.6: Let A be an intuitionistic fuzzy w-open set of an intuitionistic fuzzy topological space (X,$\mathfrak{I}$) and int (A) $\subseteq$ B $\subseteq$ A. Then B is intuitionistic fuzzy w-open.

Proof: Suppose A is an intuitionistic fuzzy w-open in X and int(A) $\subseteq$ B $\subseteq$A. $\Rightarrow A^c \subseteq B^c \subseteq (\text{int}(A))^c \Rightarrow A^c \subseteq B^c \subseteq \text{cl}(A^c)$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy w-closed it follows from theorem 3.4 that B^c is intuitionistic fuzzy w-closed .Hence B is intuitionistic fuzzy w-open.

Definition 3.3: An intuitionistic fuzzy topological space (X, $\mathfrak{I}$) is called intuitionistic fuzzy semi normal if for every pair of two intuitionistic fuzzy semi closed sets F_1 and F_2 such that $\rceil(F_{1q}F_2)$, there exists two intuitionistic fuzzy semi open sets U_1 and U_2 in X such that $F_1 \subseteq U_1$, $F_2 \subseteq U_2$ and $\rceil(U_{1q}U_2)$.

Theorem 3.7: If F is intuitionistic fuzzy semi closed and A is intuitionistic fuzzy w--closed set of an intuitionistic fuzzy semi normal space $(X,\Im)$ and $\rceil(A_qF)$. Then there exists intuitionistic fuzzy semi open sets U and V in X such that cl (A) $\subset$U, F$\subset$V and $\rceil(U_qV)$.

Proof: Since A is intuitionistic fuzzy w-closed set and $\rceil(A_qF)$, by Theorem (3.1), $\rceil(\text{cl }(A)_qF)$ and **(X,**$\Im$) is intuitionistic fuzzy semi normal. Therefore by Definition 3.3 there exists intuitionistic fuzzy semi open sets U and V in X such that cl (A) $\subset$ U, F $\subset$ V and $\rceil(U_qV)$.

Theorem 3.8: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space $(X,\Im)$ and f: $(X,\Im) \to (Y,\Im^*)$ is an intuitionistic fuzzy irresolute and intuitionistic fuzzy closed mapping then f (A) is an intuitionistic w-closed set in Y.

Proof: Let A be an intuitionistic fuzzy w-closed set in X and f: $(X,\Im) \to (Y,\Im^*)$ is an intuitionistic fuzzy continuous and intuitionistic fuzzy closed mapping. Let f(A) $\subseteq$ G where G is intuitionistic fuzzy semi open in Y then A $\subseteq$ $f^{-1}(G)$ and $f^{-1}(G)$ is intutionistic fuzzy semi open in X because f is intuitionistic fuzzy irresolute .Now A be an intuitionistic fuzzy w-closed set in X , by definition 3.1 cl(A) $\subseteq$ $f^{-1}(G)$. Thus f(cl(A)) $\subseteq$ G and f(cl(A)) is an intuitionistic fuzzy closed set in Y(since cl(A) is intuitionistic fuzzy closed in X and f is intuitionistic fuzzy closed mapping). It follows that cl (f (A) $\subseteq$ cl (f (cl (A))) = f (cl (A)) $\subseteq$ G. Hence cl (f (A)) $\subseteq$ G whenever f (A) $\subseteq$ G and G is intuitionistic fuzzy semi open in Y. Hence f (A) is intuitionistic fuzzy w-closed set in Y.

Theorem 3.9: Let$(X,\Im)$ be an intuitionistic fuzzy topological space and IFSO(X) (resp.IFC(X)) be the family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy closed) sets of X. Then IFSO(X) = IFC(X) if and only if every intuitionisic fuzzy set of X is intuitionistic fuzzy w -closed.

Proof :Necessity : Suppose that IFSO(X) = IFC(X) and let A is any intuitionistic fuzzy set of X such that A$\subseteq$ U $\in$ IFSO(X) i.e. U is intuitionistic fuzzy semi open. Then cl (A) $\subseteq$cl (U) = U because U$\in$IFSO(X) =IFC(X). Hence cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open. Hence A is w- closed set.

Sufficiency: Suppose that every intuitionistic fuzzy set of X is intuitionistic fuzzy w- closed. Let U $\in$ IFSO(X) then since U $\subseteq$ U and U is intuitionistic fuzzy w- closed, cl (U) $\subseteq$ U then U $\in$ IFC(X).Thus IFSO(X) $\subseteq$ IFC(X). If T $\in$ IFC(X) then T^c $\in$ IFO(X) $\subseteq$IFSO $\subseteq$ IFC(X) hence T$\in$ IFO(X) $\subseteq$ IFSO(X). Consequently IFC(X) $\subseteq$ IFSO(X) and IFSO(X) =IFC(X).

4: INTUITIONISTIC FUZZY W-CONNECTEDNESS AND INTUITIONISTIC FUZZY W-COMPACTNESS

Definition 4.1: An intuitionistic fuzzy topological space (X $\Im$)is called intuitionistic fuzzy w – connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy w- open and intuitionistic fuzzy w- closed .

Theorem 4.1: Every intuitionistic fuzzy w-connected space is intuitionistic fuzzy connected.

Proof: Let $(X, \Im)$ be an intuitionisic fuzzy w –connected space and suppose that $(X, \Im)$ is not intuitionistic fuzzy connected .Then there exists a proper intuitionistic fuzzy set A(A≠ $\tilde{\mathbf{0}}$, A≠ $\tilde{\mathbf{1}}$) such that A is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic w-open ((resp. intuitionistic fuzzy w-closed), X is not intuitionistic fuzzy w-connected, a contradiction.

Remark 4.1: Converse of theorem 4.1 may not be true for ,

Example 4.1: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$, $\tilde{\mathbf{1}}$, U} be an intuitionistic fuzzy topology on X, where U = {< a,0.5,0.5>,< b, 0.4, 0.6 > }.Then intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy connected but not intuitionistic fuzzy w-connected because there exists a proper intuitionistic fuzzy set A={<a,0.5,0.5>,<b,0.5,0.5>} which is both intuitionistic fuzzy w -closed and intuitionistic w-open in X.

Theorem 4.2: An intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy w-connected if and only if there exists no non zero intuitionistic fuzzy w-open sets A and B in X such that A=B^c.

Proof: Necessity: Suppose that A and B are intuitionistic fuzzy w-open sets such that A≠ $\tilde{\mathbf{0}}$≠ B and A = B^c. Since A=B^c, B is an intuitionistic fuzzy w-open set which implies that B^c = A is intuitionistic fuzzy w-closed set and B≠ $\tilde{\mathbf{0}}$ this implies that B^c≠ $\tilde{\mathbf{1}}$ i.e. A≠ $\tilde{\mathbf{1}}$ Hence there exists a proper intuitionistic fuzzy set A(A≠ $\tilde{\mathbf{0}}$**,** A≠ $\tilde{\mathbf{1}}$) such that A is both intuitionistic fuzzy w- open and intuitionistic fuzzy w-closed. But this is contradiction to the fact that X is intuitionistic fuzzy w- connected.

Sufficiency: Let (X,$\Im$) is an intuitionistic fuzzy topological space and A is both intuitionistic fuzzy w-open set and intuitionistic fuzzy w-closed set in X such that $\tilde{\mathbf{0}}$ **≠** A ≠ $\tilde{1}$**.** Now take B = A^c .In this case B is an intuitionistic fuzzy w-open set and A≠ $\tilde{\mathbf{1}}$**.**This implies that B =A^c ≠ $\tilde{0}$ which is a contradiction. Hence there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy w- open and intuitionistic fuzzy w- closed. Therefore intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy w-connected

Definition 4.2: Let (**X**,$\Im$) be an intuitionistic fuzzy topological space and Abe an intuitionistic fuzzy set X. Then w-interior and w-closure of A are defined as follows.

wcl (A) = $\cap$ {K: K is an intuitionistic fuzzy w-closed set in X and A$\subseteq$ K}

wint (A) = $\cup$ {G: G is an intuitionistic fuzzy w-open set in X and G$\subseteq$ A}

Theorem 4.3: An intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy w-connected if and only if there exists no non zero intuitionistic fuzzy w-open sets A and B in X such that B= A^c , B ={wcl(A))^c , A= $(wcl(B))^c$.

Proof: Necessity : Assume that there exists intuitionistic fuzzy sets A and B such that A≠ $\tilde{\mathbf{0}}$ **≠** B in X such that B=A^c ,B =$(wcl(A))^c$,A=$(wcl(B))^c$. Since $(wcl\ (A))^c$ and $(wcl\ (B))^c$ are intuitionistic fuzzy w-open sets in X, which is a contradiction.

Sufficiency: Let A is both an intuitionistic fuzzy w-open set and intuitionistic fuzzy w-closed set such that $\tilde{\mathbf{0}}$ **≠** A ≠ $\tilde{1}$**.** Taking B= A^c, we obtain a contradiction.

Definition 4.3: An intuitionistic fuzzy topological space (X,$\Im$) is said to be intuitionistic fuzzy w- $T_{1/2}$ if every intuitionistic fuzzy w-closed set in X is intuitionistic fuzzy closed in X.

Theorem 4.4: Let (X,$\Im$) be an intuitionistic fuzzy w- $T_{1/2}$ space, then the following conditions are equivalent:

(a) X is intuitionistic fuzzy w-connected.

(b) X is intuitionistic fuzzy connected.

Proof: (a) ⇒(b) follows from Theorem 4.1

(b) ⇒(a): Assume that X is intuitionistic fuzzy w- $T_{1/2}$ and intuitionistic fuzzy w-connected space. If possible, let X be not intuitionistic fuzzy w-connected, then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy w-open and w-closed. Since X is intuitionistic fuzzy w-$T_{1/2}$, A is intuitionistic fuzzy open and intuitionistic fuzzy closed which implies that X is not intuitionistic fuzzy connected, a contradiction.

Definition 4.4 : A collection $\{ A_i : i\in \Lambda\}$ of intuitionistic fuzzy w- open sets in intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy w- open cover of intuitionistic fuzzy set B of X if $B \subseteq \cup\{ A_i : i\in \Lambda\}$

Definition 4.5: An intuitionistic fuzzy topological space $(X,\Im)$ is said to be intuitionistic fuzzy w-compact if every intuitionistic fuzzy w- open cover of X has a finite sub cover.

Definition 4.6 : An intuitionistic fuzzy set B of intuitionistic fuzzy topological space $(X,\Im)$ is said to be intuitionistic fuzzy w- compact relative to X, if for every collection $\{ A_i : i\in \Lambda\}$ of intuitionistic fuzzy w- open subset of X such that $B \subseteq \cup\{ A_i : i\in \Lambda\}$ there exists finite subset Λ_o of Λ such that $B \subseteq \cup\{ A_i : i\in \Lambda_o\}$
.

Definition 4.7: A crisp subset B of intuitionistic fuzzy topological space $(X,\Im)$ is said to be intuitionistic fuzzy w- compact if B is intuitionistic fuzzy w- compact as intuitionistic fuzzy subspace of X .

Theorem 4.5: A intuitionistic fuzzy w-closed crisp subset of intuitionistic fuzzy w- compact space is intuitionistic fuzzy w- compact relative to X.

Proof: Let A be an intuitionistic fuzzy w- closed crisp subset of intuitionistic fuzzy w- compact space($X,\Im$). Then A^c is intuitionistic fuzzy w- open in X. Let M be a cover of A by intuitionistic fuzzy w- open sets in X. Then the family $\{M, A^c\}$ is intuitionistic fuzzy w- open cover of X. Since X is intuitionistic fuzzy w-compact, it has a finite sub cover say $\{G_1, G_2, G_3 \ldots\ldots, Gn\}$. If this sub cover contains A^c, we discard it. Otherwise leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy w – open sub cover of A. Therefore A is intuitionistic fuzzy w – compact relative to X.

5: INTUTIONISTIC FUZZY W- CONTINUOUS MAPPINGS

Definition 5.1:A mapping $f : (X,\Im). \rightarrow(Y, \sigma)$ is intuitionistic fuzzy w- continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy w-closed set in X.

Theorem 5.1: A mapping $f : (X,\Im). \rightarrow(Y,\sigma)$ is intuitionistic fuzzy w- continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy w- open in X.

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Remark5.1 Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy w-continuous, but converse may not be true. For,

Example 5.1 Let X = {a, b}, Y ={x, y } and intuitionistic fuzzy sets U and V are defined as follows :

U= {< a, 0.5, 0.5>, < b, 0.4, 0.6>}

V= {<x, 0.5, 0.5>, <y, 0.5, 0.5>}

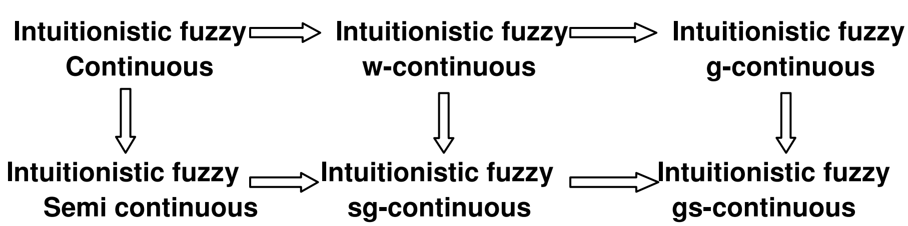
Let $\Im$ = {$\tilde{0}$**,** $\tilde{1}$, **U}** and σ = {$\tilde{0}$**,** $\tilde{1}$, V**}** be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$). →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy w- continuous but not intuitionistic fuzzy continuous.

Remark5.2 Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy g-continuous, but converse may not be true. For,

Example 5.2: Let X = {a, b}, Y ={x, y} and intuitionistic fuzzy sets U and V are defined as follows:
U= {< a, 0.7, 0.3>, < b, 0.6, 0.4>}
V= {<x, 0.6, 0.4>, <y, 0.7, 0.3>}
Let $\Im$ = { $\tilde{0}$**,** $\tilde{1}$ **, U }** and σ ={ $\tilde{0}$**,** $\tilde{1}$ **, V }** be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$). →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy g- continuous but not intuitionistic fuzzy w- continuous.

Remark5.3 Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy sg-continuous, but converse may not be true. For,
Example 5.1 Let X = {a, b}, Y ={x, y} and intuitionistic fuzzy sets U and V are defined as follows:
U= {< a, 0.5, 0.5>, < b, 0.4, 0.6>}
V= {<x, 0.5, 0.5>, <y, 0.3, 0.7>}
Let $\Im$ = { $\tilde{0}$**,** $\tilde{1}$ **, U }** and σ ={ $\tilde{0}$**,** $\tilde{1}$ **, V }** be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping *f*: (X,$\Im$). →(Y, σ) defined by *f* (a) = x and *f* (b) = y is intuitionistic fuzzy sg- continuous but not intuitionistic fuzzy w- continuous.

Remark 5.4: Remarks 2.4, ,5.1, 5.2, 5.3 reveals the following diagram of implication:



Theorem 5.2: If *f*: (X,$\Im$). →(Y, σ) is intuitionistic fuzzy w- continuous then for each intuitionistic fuzzy point c(α,β) of X and each intuitionistic fuzzy open set V of Y such that *f*(c(α,β)) ⊆ V there exists a intuitionistic fuzzy w- open set U of X such that c(α,β) ⊆ U and *f*(U) ⊆ V.
Proof : Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that *f*(c(α,β)) ⊆ V. Put U = f^{-1}(V). Then by hypothesis U is intuitionistic fuzzy w- open set of X such that c(α,β) ⊆ U and f (U) = *f* (f^{-1}(V)) ⊆ V.

Theorem 5.3: Let *f:* (X,$\Im$). →(Y,σ) is intuitionistic fuzzy w- continuous then for each intuitionistic fuzzy point c(α,β) of X and each intuitionistic fuzzy open set V of Y such that *f*(c(α,β))qV, there exists a intuitionistic fuzzy w- open set U of X such that c(α,β)qU and f(U) ⊆ V.
Proof: Let c(α,β) be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that *f*(c(α,β))q V. Put U = f^{-1}(V). Then by hypothesis U is intuitionistic fuzzy w- open set of X such that c(α,β)q U and *f*(U)= *f*(f^{-1}(V)) ⊆ V.

Theorem 5.4: If $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous, then $f(wcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.

Proof: Let A be an intuitionistic fuzzy set of X. Then cl(f(A)) is an intuitionistic fuzzy closed set of Y. Since f is intuitionistic fuzzy w –continuous, $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy w-closed in X. Clearly $A \subseteq f^{-1}$(cl ((A)). Therefore $wcl(A) \subseteq wcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Hence $f(wcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.

.

Theorem 5.5: A mapping f from an intuitionistic fuzzy w-$T_{1/2}$ space $(X,\Im)$ to a intuitionistic fuzzy topological space (Y, σ) is intuitionistic fuzzy semi continuous if and only if it is intuitionistic fuzzy w – continuous.

Proof: Obvious

Remark 5.5: The composition of two intuitionistic fuzzy w – continuous mapping may not be Intuitionistic fuzzy w – continuous. For

Example 5-5: Let X = {a, b}, Y= {x, y} and Z= {p, q} and intuitionstic fuzzy sets U,V and W defined as follows :

U = {< a, 0.5, 0.5>, < b, 0.4, 0.6>}

V = {<x, 0.5, 0.5>, <y, 0.3, 0.7>}

W = {< p, 0.6, 0.4>, < q, 0.4, 0.6>}

Let $\Im = \{ \tilde{0}, \tilde{1}, U \}$, $\sigma = \{ \tilde{0}, \tilde{1}, V \}$ and $\mu = \{ \tilde{0}, \tilde{1}, W \}$ be intuitionistic fuzzy topologies on X , Y and Z respectively. Let the mapping $f: (X,\Im). \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ and $g : (Y,\sigma) \to (Z,\mu)$ defined by $g(x) = p$ and $g(y) = q$. Then the mappings f and g are intuitionistic fuzzy w-continuous but the mapping $gof: (X,\Im) \to (Z, \mu)$ is not intuitionistic fuzzy w-continuous**.**

Theorem 5.6: If $f: (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous and $g :(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy continuous. Then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy w-continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy w – closed in X. Hence gof is intuitionistic fuzzy w –continuous.

Theorem 5.7 : If $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous and $g : (Y,\sigma) . \to (Z,\mu)$ is intuitionistic fuzzy g-continuous and (Y,σ) is intuitionistic fuzzy $(T_{1/2})$ then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy w-continuous.

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionstic fuzzy g-closed in Y. Since Y is $(T_{1/2})$, then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y. Hence $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy w – closed in X. Hence gof is intuitionistic fuzzy w – continuous.

Theorem 5.8: If $f: (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy gc-irresolute and $g :(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy w-continuous. Then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy g-continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionstic fuzzy w-closed in Y, because g is intuitionistic fuzzy w-continuous. Since every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed set, therefore $g^{-1}(A)$ is intuitionstic fuzzy g-closed in Y .Then $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g-closed in X ,because f is intuitionistic fuzzy gc- irresolute. Hence $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy g-continuous**.**

Theorem 5.9: An intuitionistic fuzzy w – continuous image of a intuitionistic fuzzy w-compact space is intuitionistic fuzzy compact.

Proof: Let. $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous map from a intuitionistic fuzzy w-compact space $(X,\Im)$ onto a intuitionistic fuzzy topological space (Y, σ). Let $\{Ai: i\in \Lambda\}$ be an intuitionistic fuzzy open cover of Y then $\{f^{-1}(Ai) : i\in \Lambda\}$ is a intuitionistic fuzzy w –open cover of X. Since X is intuitionistic fuzzy w- compact it has finite intuitionistic fuzzy sub cover say $\{ f^{-1}(A_1) , f^{-1}(A_2) ,\text{----} f^{-1}(An) \}$. Since f is onto $\{A_1, A_2, \text{------------} A_n\}$ is an intuitionistic fuzzy open cover of Y and so (Y, σ) is intuitionistic fuzzy compact.

Theorem 5.10: If $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-continuous surjection and X is intuitionistic fuzzy w-connected then Y is intuitionistic fuzzy connected.

Proof: Suppose Y is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy set of X, which is both intuitionistic fuzzy w- open and intuitionistic fuzzy w – closed , because f is intuitionistic fuzzy w– continuous surjection. Hence X is not intuitionistic fuzzy w –connected, which is a contradiction.

6. INTUITIONISTIC FUZZY W-OPEN MAPPINGS

Definition 6.1: A mapping $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-open if the image of every intuitionistic fuzzy open set of X is intuitionistic fuzzy w-open set in Y.

Remark 6.1 : Every intuitionistic fuzzy open map is intuitionistic fuzzy w-open but converse may not be true. For,

Example 6.1: Let X = {a, b} , Y = {x , y} and the intuitionistic fuzzy set U and V are defined as follows :

U = {< a, 0.5. 0.5 > , < b ,0.4,0.6>}
V = {< x, 0.5, 0.5 >, <y, 0.3, 0.7 >}

Then $\Im = \{\tilde{0} ,U, \tilde{1} \}$ and $\sigma = \{ \tilde{0} ,V, \tilde{1} \}$ be intuitionistic fuzzy topologies on X and Y respectively . Then the mapping $f : (X,\Im). \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) =y$ is intuitionistic fuzzy w-open but it is not intuitionistic fuzzy open.

Theorem 6.1: A mapping $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-open if and only if for every intuitionisic fuzzy set U of X $f(\text{int}(U)) \subseteq \text{wint}(f(U))$.

Proof: **Necessity** Let f be an intuitionistic fuzzy w-open mapping and U is an intuitionistic fuzzy open set in X. Now $\text{int}(U) \subseteq U$ which implies that $f (\text{int}(U) \subseteq f(U)$. Since f is an intuitionistic fuzzy w-open mapping, $f(\text{Int}(U)$ is intuitionistic fuzzy w-open set in Y such that $f(\text{Int}(U) \subseteq f(U)$ therefore $f(\text{Int}(U) \subseteq \text{wint } f(U)$.

Sufficiency: For the converse suppose that U is an intuitionistic fuzzy open set of X. Then $f (U) = f (\text{Int } (U) \subseteq \text{wint } f(U)$. But $\text{wint } (f (U)) \subseteq f (U)$. Consequently $f (U) = \text{wint } (U)$ which implies that $f(U)$ is an intuitionistic fuzzy w-open set of Y and hence f is an intuitionistic fuzzy w-open .

Theorem 6.2: If $f : (X,\Im). \to (Y, \sigma)$ is an intuitionistic fuzzy w-open map then $\text{int } (f^{-1}(G) \subseteq f^{-1}(\text{wint } (G)$ for every intuitionistic fuzzy set G of Y.

Proof: Let G is an intuitionistic fuzzy set of Y. Then $\text{int } f^{-1}(G)$ is an intuitionistic fuzzy open set in X. Since f is intuitionistic fuzzy w-open $f(\text{int } f^{-1}(G))$ is intuitionistic fuzzy w-open in Y and hence $f(\text{Int } f^{-1}(G)) \subseteq \text{wint}(f(f^{-1}(G)) \subseteq \text{wint}(G)$. Thus $\text{int } f^{-1}(G) \subseteq f^{-1}(\text{wint } (G)$.

Theorem 6.3: A mapping $f : (X,\Im). \to (Y,\sigma)$ is intuitionistic fuzzy w-open if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy closed set U of X containing $f^{-1}(S)$ there is a intuitionistic fuzzy w-closed V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy w- open map. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy closed set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is intuitionistic fuzzy w- closed set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy open set of X. Then $f^{-1}((f(F))^c \subseteq F^c$ and F^c is intuitionistic fuzzy closed set in X. By hypothesis there is an intuitionistic fuzzy w-closed set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ .Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f (F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy w-open set of Y. Hence f (F) is intuitionistic fuzzy w-open in Y and thus f is intuitionistic fuzzy w-open map.

Theorem 6.4: A mapping $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-open if and only if $f^{-1}(wcl (B) \subseteq cl f^{-1}(B)$ for every intuitionistic fuzzy set B of Y.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy w- open map. For any intuitionistic fuzzy set B of Y $f^{-1}(B) \subseteq cl(f^{-1}(B))$ Therefore by theorem 6.3 there exists an intuitionistic fuzzy w-closed set F in Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq cl(f^{-1}(B))$. Therefore we obtain that $f^{-1}(wcl(B)) \subseteq f^{-1}(F) \subseteq cl f^{-1}((B))$.

Sufficiency: For the converse suppose that B is an intuitionistic fuzzy set of Y. and F is an intuitionistic fuzzy closed set of X containing $f^{-1}(B)$. Put V= cl (B) , then we have $B \subseteq V$ and Vis w-closed and $f^{-1}(V) \subseteq cl (f^{-1}(B)) \subseteq F$. Then by theorem 6.3 f is intuitionistic fuzzy w-open.

Theorem 6.5: If $f: (X,\Im). \to (Y, \sigma)$ and $g :(Y, \sigma) \to (Z, \mu)$ be two intuitionistic fuzzy map and $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy w-open**.** If $g :(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy w-irresolute then $f: (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-open map.

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space$(X,\Im)$. Then $(go f)$ (H) is intuitionistic fuzzy w-open set of Z because gof is intuitionistic fuzzy w- open map. Now since $g :(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy w-irresolute and (gof) (H) is intuitionistic fuzzy w-open set of Z therefore $g^{-1} (gof (H)) = f(H)$ is intuitionistic fuzzy w-open set in intuitionistic fuzzy topological space Y. Hence f is intuitionistic fuzzy w-open map.

7. INTUITIONISTIC FUZZY W-CLOSED MAPPINGS

Definition 7.1: A mapping $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy w-closed if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy w-closed set in Y.

Remark 7.1 Every intuitionistic fuzzy closed map is intuitionistic fuzzy w-closed but converse may not be true. For,

Example 7.1: Let X = {a, b}, Y = {x, y}

Then the mapping $f : (X,\Im). \to (Y, \sigma)$ defined in Example 6.1 is intuitionistic fuzzy w- closed but it is not intuitionistic fuzzy closed.

Theorem 7.1: A mapping $f : (X,\Im). \to (Y,\sigma)$ is intuitionistic fuzzy w-closed if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy open set U of X containing $f^{-1}(S)$ there is a intuitionistic fuzzy w-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy w- closed map. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy open set of X such that $f^{-1}(S) \subseteq U$. Then $V = Y - f^{-1}(U^c)$ is intuitionistic fuzzy w- open set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy closed set of X. Then $(f(F))^c$ is an intuitionistic fuzzy set of Y and F^c is intuitionistic fuzzy open set in X such that $f^{-1}((f(F))^c) \subseteq F^c$. By hypothesis there is an intuitionistic fuzzy w-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ .Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy w-closed set of Y. Hence f (F) is intuitionistic fuzzy w-closed in Y and thus f is intuitionistic fuzzy w-closed map.

Theorem 7.2: If f: $(X,\Im)$. $\rightarrow(Y, \sigma)$ is intuitionistic fuzzy semi continuous and intuitionistic fuzzy w-closed map and A is an intuitionistic fuzzy w-closed set of X ,then f(A) intuitionistic fuzzy w-closed.

Proof: Let $f(A) \subseteq O$ where O is an intuitionistic fuzzy semi open set of Y. Since f is intuitionistic fuzzy semi continuous therefore $f^{-1}(O)$ is an intuitionistic fuzzy semi open set of X such that $A \subseteq f^{-1}(O)$. Since A is intuitionistic fuzzy w-closed of X which implies that $cl(A) \subseteq (f^{-1}(O))$ and hence $f(cl(A) \subseteq O$ which implies that $cl(f(cl(A)) \subseteq O$ therefore $cl(f((A)) \subseteq O$ whenever $f(A) \subseteq O$ where O is an intuitionistic fuzzy semi open set of Y. Hence f(A) is an intuitionistic fuzzy w-closed set of Y.

Corollary 7.1: If f: $(X,\Im)$. $\rightarrow(Y, \sigma)$ is intuitionistic fuzzy w-continuous and intuitionistic fuzzy closed map and A is an intuitionistic fuzzy w-closed set of X ,then f(A) intuitionistic fuzzy w-closed.

Theorem 7.3: If f: $(X,\Im)$. $\rightarrow(Y, \sigma)$ is intuitionistic fuzzy closed and g :$(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy w-closed. Then $g \circ f : (X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-closed**.**

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space$(X,\Im)$. Then f(H) is intuitionistic fuzzy closed set of (Y, σ) because f is inuituionistic fuzzy closed map. Now$(g \circ f)(H) = g(f(H))$ is intuitionistic fuzzy w-closed set in intuitionistic fuzzy topological space Z because g is intuitionistic fuzzy w-closed map. Thus $g \circ f : (X,\Im) \rightarrow (Z,\mu)$ is intuitionistic fuzzy w-closed.

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