Comparative Evaluation of Ordinary Least Square Regression and Principal Component Regression Models for Reliability-Based Optimization of Concrete Mix Proportions

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Abstract

This paper explores Reliability Based Design Optimization (RBDO) technique for finding optimal concrete mixture compositions that are less sensitive to uncertainties involved in concrete mix design process. The optimization problem is formulated to determine optimal concrete mix parameters, namely, water content (w), fine aggregate content (fa), coarse aggregate content (ca) and cement content (c). This is achieved by minimizing the cost of concrete for a given compressive strength and target reliability. The compressive strength is considered for 28 days and 56 days curing periods. Compressive strength models are developed using Ordinary Least Square Regression (OLSR) and Principal Component Regression (PCR) techniques. SPSS 12.0 and MATLAB 5.3 are used to develop these models. An attempt has also been made to demonstrate the effect of prediction models on optimal concrete mix parameters. The RBDO problems are solved using Sequential Optimization and Reliability Assessment (SORA) method which is implemented using Altair Hyperstudy 10.0. Optimal mixes for a wide range of target compressive strengths and different reliability levels are reported. It is seen that optimization results based on PCR models are more reliable than the results obtained using OLSR models.

Keywords: Reliability, Concrete, Optimization, Principal Component Regression.

1. INTRODUCTION

Concrete stands as the foremost choice for construction worldwide, owing to its versatility, longevity, and cost-effectiveness. Concrete mixture design involves meticulously selecting the type and amount of constituents to design a concrete mixture that satisfies precise design criteria for a given purpose (DeRousseau et al., 2018). Hence, it becomes crucial to strike a balance between competing design demands to maximize the efficient utilization of natural resources.

Optimization plays a key role in determining the optimum concrete mixture composition that meets a given set of requirements. Yeh (1999, 2003, 2007, and 2009) attained optimal concrete mix proportions meeting specific design requirements at minimum cost. Tensile strength and ductility for a given compressive strength of fibre reinforced concrete mixes were simultaneously optimized using compromise programming technique by Karihaloo and Kornbak (2001). A design method for optimal high performance concrete mixure using Genetic Algorithms (GA) was presented by Lim et al. (2004). Őzbay et al. (2006) exercised Taguchi method and GA to determine optimal concrete mix proportions maximizing compressive strength. Jayaram et al. (2009) proposed elitist GA models for high volume fly ash concrete mix optimization. Lee et al. (2009) implemented a methodology based on GA, Artificial Neural Network (ANN), and convex hull to obtain concrete mix proportions with minimum cost under a given compressive strength requirement. Parichatprecha and Nimityongskul (2009) developed minimum cost models for high performance concrete using GA. A two-step approach to optimize High Strength Concrete (HSC) parameters was followed by Baykasoğlu et al. (2009). In first step, regression analysis, neural networks and Gene Expression Programming (GEP) were exploited to predict HSC parameters.

Afterwards, multi-objective optimization model was solved using GA. Ozbay et al. (2010) exercised genetic programming and genetic algorithm to achieve HSC mixes having minimum cost while satisfying requirements of workability and strength. Şimşek and Uygunoğlu (2016) applied TOPSIS based Taguchi optimization method to determine optimal mixture proportions of polymer blended concrete. A support vector regression and multi-objective firefly algorithm based optimization model was proposed by Huang et al. (2020) for mixture design of steel fibre reinforced concrete. Sharifi et al. (2020) used improvised Taguchi optimization method to find optimum mix design of high strength self-consolidating concrete (HSSCC). Kondapally et al.(2023) compared the concrete mix designs obtained manually with optimal designs obtained using GA. Chen et al.(2023) combined GA, ANN and Scipy library for hybrid intelligent modeling and multi-objective optimization of concrete mix design parameters. Oveido et al. (2024) presented optimized concrete mixtures using machine learning (ML) models to predict compressive strength, and genetic algorithms to find optimal the mixture cost under quality constraints. Li et al. (2024) employed the fusion of response surface method (RSM) and non-dominated sorting genetic algorithm-II (NSGA-II) for multi-objective optimization of hydraulic asphalt concrete mix ratios. However, the formulation of a structural optimization problem used in the above studies ignores the scattering of the various design parameters is termed as Deterministic Design Optimization (DDO).

It is important to note that concrete mix design is always influenced by its surroundings. Variations in material quality, methods of curing, placing of concrete, techniques of mixing and transportation, testing procedures, etc. are the sources of randomization. Because of this, the concrete's actual compressive strength in a structure will never match the specimen produced in controlled laboratory conditions. The gap between the expected and obtained performances is even greater when the mix design is deterministically optimized, because DDO usually yields optimal designs that are pushed to the boundaries of design constraints, ignoring any possibility for uncertainty in manufacturing process, modeling, and design variables. Typically, in DDO, the uncertainties are taken into consideration by adding safety factors, as explained by the design codes of practice IS 456 (2000). In actuality, these safety factors are unable to ensure steady reliability levels for specific design environments because they are standardized for average design conditions. Thus, for robust and economical designs, the Reliability Based Design Optimization (RBDO) becomes highly effective. The RBDO procedure adjusts safety margins while considering for each variable's uncertainty effect during the optimization process. In this manner, as compared to deterministic design, where the safety factors are predetermined and then subjected to optimization, the safety factors are optimally set within the system in RBDO. Consequently, reliability-based concrete mix proportion optimization is significantly more efficient in practice than deterministic optimization (Chateauneuf, 2008).

Solving a RBDO problem is computationally very expensive. Several tools are developed by researchers to handle RBDO problems (Thanedar and Kodiyalam, 1992; Enevoldsen and Sorensen, 1993; Wang and Grandhi, 1995; Luo and Grandhi, 1995; Chen et al., 1997; Royset et al., 2001; Aggarwal, 2004; Du and Chen, 2004; Zou and Mahadevan, 2006). The Sequential Optimization and Reliability Assessment (SORA) approach was designed by Du and Chen (2004). It is a single loop strategy which integrates reliability assessment with deterministic optimization in a series of cycles for efficient probabilistic design. In this study, the optimal proportions of the concrete mixture are achieved by the application of the SORA method.

The development of prediction models for concrete mix parameters that are sufficiently accurate is crucial to the optimization process since the accuracy of the results of the optimization rely on the quality of these models. Concrete's cost is a linear function of its constituents, but as concrete's compressive strength is only known through discrete outcomes, it may not be a linear function of its constituents. Therefore, the form of the prediction model for compressive strength is unknown. However, several researchers use a second order polynomial for compressive strength model (Melchers, 2002). In the present study, quadratic models are developed for the compressive strength of concrete at different curing ages.

Ordinary Least Square Regression (OLSR) technique and its modifications have been used by many researchers for modeling compressive strength of concrete (Namyong et al., 2004; Wu et al., 2010; Riad et al., 2011; You et al., 2012). One of the major challenges faced by OLSR technique is multicollinearity, if the data used for building regression models is not generated by a statistically designed experiment. Multicollinearity occurs in case of near-constant linear functions of two or more predictors. OLSR technique under the influence of multicollinearity affects the three important features of the regression model, namely, magnitude, sign and standard errors or variance of the regression coefficients (Ryan, 1996).Thus, the model may give erroneous results if used for optimization.

In this study, Principal Component Regression (PCR) technique has been used to handle multicollinearity. It produces stable and significant estimates of regression coefficients. PCR technique deals with multicollinearity problem by eliminating those dimensions of sample space that are causing multicollinearity. This is conceptually similar to dropping an independent variable from the model when there is insufficient dispersion in that variable to contribute meaningful information on response variable (Rawling et al., 1998).

The objective of the present study is to develop, validate and use statistical prediction models (OLSR and PCR) with full-quadratic terms for compressive strength at 28 and 56 days; formulate RBDO problems using those models and solve them with the SORA method to obtain cost-optimal concrete mixes that satisfy reliability targets of 0.90, 0.95 and 0.99 across a range of target compressive strengths.

2. DATA USED FOR STUDY

Kumar (2002) performed experiments under controlled laboratory conditions to generate the compressive strength data examined in this study. Water-cementitious material ratio, cementitious content, aggregate zones, water content, percentage replacement of cement by fly ash, workability and curing ages were the seven parameters considered in the mentioned work. The range of values for these parameters is summarized in Table 1. The coarse aggregates, as described in Table 1, are classified into three zones: A, B and C. The principal characteristics of these aggregates zones are set out in Table 2. The physical properties of fine aggregates used in this study are summed up in Table 3 and that of coarse aggregates: CA-I, CA-II and CA-III are shown in Table 4. For each concrete mix, forty five 150 mm cubes were cast and tested at 28 days, 56 days and 91 days of curing time. The compressive strength data for 28 days and 56 days of treatment and without fly ash has been considered in this study.

Water cementitious material ratio	0.42-0.55
Cementitious content	350-475@25 kg/m^3
Water content	180-230@10 kg/m^3
Percentage replacement of cement by fly ash	0 and 15%
Workability	Medium and High
Aggregate zones	A, B, C
Curing ages	28,56,91 days

TABLE 1: Variation in parameters.

Zone	Percentage passing 20 mm sieve and retained on 10 mm sieve (CA –I)	Percentage passing 10 mm sieve and retained on 4.75 mm sieve (CA –II)	Percentage passing 4.75 mm sieve and retained on 2.36 mm sieve (CA –III)	Fineness Modulus
A	67	33	-	6.67
B	50	50	-	6.50
C	-	50	50	6.50

TABLE 2: Principal characteristics zones of coarse aggregates.

S. No.	Property	Observed values
1.	Unit mass (compact)	1,680 kg/m^3
2.	Unit mass (loose)	1,590 kg/m^3
3.	Specific gravity (oven-dry basis)	2.54
4.	Percentage voids (compact)	33.7 percent
5.	Percentage voids (loose)	37.4 percent
6.	Percentage absorption	0.5 percent
7.	Fineness modulus	2.09

TABLE 3: Physical properties of fine aggregates.

S.No.	Property	Observed values		
		CA - I	CA - II	CA - III
1.	Unit mass (compact)	1,580 kg/m^3	1,480 kg/m^3	2,150 kg/m^3
2.	Unit mass (loose)	1,380 kg/m^3	1,350 kg/m^3	1,980 kg/m^3
3.	Specific gravity (a) Saturated surface dry (b) Oven-dry	 2.61 2.68	 2.63 2.68	 2.58 2.60
4.	Percentage voids (compact)	41.2 percent	43.7 percent	17.3 percent
5.	Percentage voids (loose)	48.6 percent	48.7 percent	23.85 percent
6.	Percentage absorption	1.8 percent	1.18 percent	1.20 percent

TABLE 4: Physical properties of coarse aggregates.

3. METHODOLOGY

The study develops and applies statistical and reliability-based optimization techniques for concrete mix design. Data used in the study has been described in section 2.

Methodology adopted in this study is as follows:

a) Develop linear regression models for the cost of concrete.
b) Develop full quadratic prediction models for compressive strength of concrete for 28 days and 56 days curing period using OLSR and PCR techniques.
c) Formulation of RBDO problem with the objective of minimizing the cost per cubic meter of concrete subject to probabilistic constraints on target compressive strength.
d) Perform RBDO separately using OLSR and PCR prediction models and compare optimal mixes, costs for a given reliability level.
e) Comparison of RBDO results with DDO results in terms of margin of compressive strength.

4. PREDICTION MODELS FOR CONCRETE MIX PARAMETERS

4.1 Design Variables

The variables used for prediction are water content (w), fine aggregate content (fa), coarse aggregate content (ca) and cement content (c). There are three response variables, namely, cost of concrete ($cost$), 28 days compressive strength ($st28$) and 56 days compressive strength ($st56$). The basic descriptive statistics of the variables used in this study are depicted in Table 5. The predictor variable contents are measured in kg/m^3, compressive strength of concrete is measured in MPa and cost of concrete is measured in Indian rupee ($Rs.$). Coefficient of determination (r^2) is used to assess the overall prediction accuracy of the developed models. Regression analysis is carried out using SPSS 12.0 and MATLAB 5.3.

Variable	Minimum (kg/m^3)	Maximum (kg/m^3)	Mean (kg/m^3)	Standard deviation (kg/m^3)
w	180.00	230.00	202.44	12.69
fa	416.93	642.18	535.64	57.29
ca	798.48	1252.05	1064.85	133.42
c	350.00	475.00	424.49	37.32
$st28$	31.66	54.49	45.80	5.42
$st56$	37.18	58.65	51.11	5.03

TABLE 5: Descriptive statistics.

4.2 Overview of Principal Component Regression (PCR) Technique

PCR is a method designed to address multicollinearity issues and enhance the reliability of regression coefficient estimates. When multicollinearity is pronounced, it leads to at least one eigenvalue of the predictors' correlation matrix nearing zero. This signifies that a subset of the sample space has minimal impact on explaining the dispersion within the data. PCR tackles this by transforming the initial predictors into a fresh ensemble of orthogonal or uncorrelated variables known as Principal Components (PCs). These PCs are derived through linear combinations of the original variables after they have been centered and scaled. The principal component matrix contains exactly the same information as the original centered and scaled data set. This transformation arranges the new orthogonal variables based on their significance in elucidating the variance within the sample space. PCR serves as a technique for reducing dimensionality, where principal components are gradually discarded until the remaining components account for a predetermined proportion of the total variance. Various stopping criteria have been proposed in the literature for this elimination process. The Kaiser-Gutman rule, commonly employed, suggests retaining principal components associated with eigenvalues greater than 1. Guiot et al. (1982) introduced another selection criterion, keeping principal components corresponding to eigenvalue products exceeding 1. Alternatively, the scree test involves plotting the number of principal components against the eigenvalues of the predictor variables' correlation matrix. As the number of components increases, eigenvalues decline, resulting in a curve with an "elbow." An intuitive approach is to discard components following the elbow point.

Jollife (2002) argued that rejecting PCs with small eigenvalues might lead to the elimination of some of the PCs having high correlation with the dependent variable. Therefore, it is essential to simultaneously achieve two objectives: deleting PCs with small variances while retaining those that serve as effective predictors of the dependent variables. After selection of PCs, regression model is constructed using the OLSR technique, enabling estimation of the response variable. Subsequently, the set of regression coefficients obtained is transformed back into a new set of coefficients corresponding to the original correlated set of variables.

4.3 Model for Cost of Concrete

In this study, the water content is assumed to have no associated cost, while the cost of concrete is represented as a linear function of fine aggregate content, coarse aggregate content, and cement content. Initially, the dataset from Kumar (2002) is scrutinized to ascertain the presence of multicollinearity among the predictor variables. The analysis reveals eigenvalues of 1.383, 1.013, and 0.604 for the correlation matrix of the predictors, none of which approach zero. These findings reveal that multicollinearity among the predictor variables is not sufficiently strong to adversely impact the linear ordinary least squares regression models for the cost of concrete.

Parameter	$cost$
fa	0.629
ca	0.333
c	4.892
$Intercept$	236.461
r^2	0.997

TABLE 6: OLSR models for cost of concrete.

Accordingly, linear OLSR models are constructed to predict the cost of concrete. The regression coefficients for the concrete cost variable are presented in Table 6. Notably, the coefficient of determination (r^2) approaches unity for the cost model, indicating a near-perfect fit of the model to the data.

4.4 Models for Compressive Strength of Concrete

Full quadratic prediction models for concrete compressive strength for curing periods of 28 days and 56 days are developed using both OLSR and PCR techniques.

OLSR coefficients corresponding to various parameters for compressive strength at various curing periods are documented in Table 7. It can be noted from Table 7 that value of r^2 is greater than or equal to 0.950 in both cases indicating that OLSR models fit the given data very well.

Parameter	$st28$	$st56$
w	-3.409483	-8.394017
fa	0.577980	1.376687
ca	-0.100047	-0.226823
c	1.053982	2.019124
w^2	0.006410	0.062986
fa^2	-0.001368	0.001193
ca^2	0.000039	0.000053
c^2	-0.001014	0.000462
$w*fa$	0.004913	-0.018626
$w*ca$	-0.002528	0.000440
$w*c$	0.001411	-0.018352
$fa*ca$	0.000569	0.000012
$fa*c$	-0.001726	0.002542
$ca*c$	0.000565	0.000067
$Intercept$	57.911390	216.915630
r^2	0.973490	0.949940

TABLE 7: OLSR models for compressive strength of concrete.

Table 8 displays the eigenvalues of the correlation matrix of predictor variables along with the percentage of variance explained by each PC. Analysis of the table reveals that the eigenvalues of the 5th to 14th PCs are nearly zero, indicating a significant presence of multicollinearity.

Furthermore, the first three PCs collectively account for 99.487% of the variance in the sample space, while the dispersion in the remaining eleven PC dimensions is limited.

Component	Eigen value	% of Variance explained
1	6.897134	49.265407
2	4.802748	34.305240
3	2.228281	15.916181
4	0.056556	0.403989
5	0.007233	0.051671
6	0.003756	0.026828
7	0.002365	0.016893
8	0.000968	0.006917
9	0.000571	0.004080
10	0.000327	0.002335
11	0.000052	0.000372
12	0.000011	0.000075
13	0.000001	0.000010
14	0.000000	0.000001

TABLE 8: Eigen Analysis.

The inflation of variances in the coefficients of OLSR model for all independent variables is observed due to near-singularities. According to the Kaiser-Gutman rule, the first three PCs, which collectively explain more than 95% of the variance in the sample space, should be selected. Guiot's method advocates for the use of the first four PCs, a suggestion supported by the scree plot depicted in Figure 1. Consequently, the first four PCs are chosen for the development of the concrete compressive strength model.

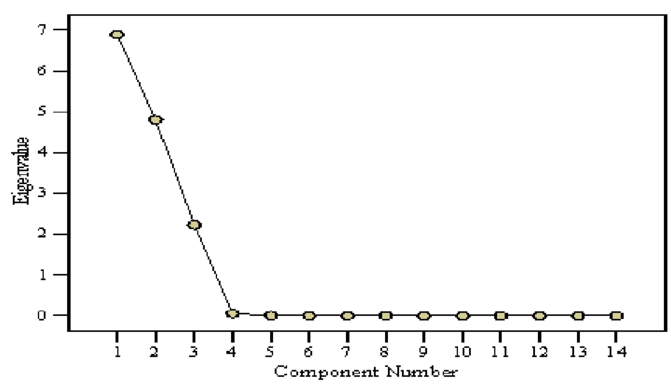

FIGURE 1: Scree plot.

Moreover, it is ensured that the excluded PCs do not possess any predictive significance for concrete mix parameters. To verify this, the compressive strength at various ages is regressed against each dropped PC, yielding r^2 values ranging from 0.002 to 0.051. These low r^2 values confirm that the excluded PCs do not impact the predictive capacity of the models. Subsequently, least square regression is employed to formulate regression models for estimating compressive strength using the selected PCs as predictors. Eventually, the regression equations with selected PCs as parameters are transformed back to determine the regression coefficients of the original variables. The outcomes of PCR are summarized in Table 9. The r^2 values for PCR models are lower compared to their Ordinary Least Squares Regression (OLSR) counterparts for all response variables. This reduction in r^2 amounts to 2.37% and 5.33% for the 28-day and 56-day compressive strength models, respectively. Such a decline was anticipated, as the stability of regression coefficients is enhanced at the expense of the model's predictive ability in Principal Component Regression technique.

Parameter	$st28$	$st56$
w	-0.124246	-0.101777
fa	0.000585	-0.000530
ca	-0.002207	-0.001770
c	0.059438	0.052720
w^2	-0.000349	-0.000286
fa^2	-0.000001	-0.000002
ca^2	-0.000002	-0.000002
c^2	0.000069	0.000061
$w*fa$	-0.000045	-0.000039
$w*ca$	-0.000027	-0.000022
$w*c$	0.000045	0.000045
$fa*ca$	0.000001	-0.000001
$fa*c$	0.000035	0.000030
$ca*c$	0.000028	0.000025
$Intercept$	37.984695	41.254346
r^2	0.950410	0.899350

TABLE 9: PCR models for compressive strength of concrete.

5. RELIABILITY BASED DESIGN OPTIMIZATION OF CONCRETE MIX PARAMETERS

5.1 Description of the Problem

RBDO problems involve the consideration of three distinct types of variables: deterministic design variables, random design variables, and random design parameters. These variables are incorporated into the objective function of the RBDO problem, comprising both deterministic and random design variables. The formulation of the RBDO problem is structured as follows:

$$\left.\begin{array}{ll} & \underset{d,\mu_x}{Minimize}\, f(\boldsymbol{d},\boldsymbol{X},\boldsymbol{P}) \\ Subject\ to: & Prob\{g_i(\boldsymbol{d},\boldsymbol{X},\boldsymbol{P}) \le 0\} \ge R_i \\ & h_k(\boldsymbol{d},\boldsymbol{X},\boldsymbol{P}) \le 0 \\ & \boldsymbol{d}_l \le \boldsymbol{d} \le \boldsymbol{d}_u \\ & \boldsymbol{X}_l \le \boldsymbol{X} \le \boldsymbol{X}_u \end{array}\right\} \quad (1)$$

where f is the objective function, $\boldsymbol{d}$ is the vector of deterministic design variables, $\boldsymbol{X}$ is the vector of random design variables, $\boldsymbol{P}$ is the vector of random design parameters. $\boldsymbol{d}$ and $\boldsymbol{\mu_x}$ are the design variables where $\boldsymbol{\mu_x}$ is the mean of the random design variables **X**. $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{P})$ $(i - 1, 2, \dots, m)$ are reliability constraint functions, $R_i(i - 1, 2, \dots, m)$ are desired probabilities of constraints satisfaction. Here, $h_k(\boldsymbol{d},\boldsymbol{X},\boldsymbol{P})(k - 1, 2, \dots, n)$ are deterministic constraint functions. Last two constraints in (1) are the boundary constraints.

Traditionally, the RBDO problem has been addressed using double loop methods as outlined by Enevoldsen and Sorensen (1993), Wang and Grandhi (1995), and Luo and Grandhi (1995). In this approach, the loop of reliability analysis is embedded within the loop of optimization. However, this method proves to be slow and inefficient as multiple reliability analyses are required to evaluate each probabilistic constraint in every iteration of optimization. To address this inefficiency, various approaches have been proposed in the literature, including those by

Thanedar and Kodiyalam (1992), Chen et al. (1997), Royset *et al*. (2001), Aggarwal (2004), Du and Chen (2004), and Zou and Mahadevan (2006). One notable approach is the Sequential Optimization and Reliability Assessment (SORA) method developed by Du and Chen (2004), which aims to streamline the RBDO process by decoupling the double loop into a deterministic optimization problem followed by an inverse reliability assessment problem.

The SORA method operates by shifting the boundaries of violated constraints (those with low reliability) towards the feasible direction based on reliability information obtained in previous cycles. This involves employing a percentile formulation of probabilistic constraints, utilizing an efficient and robust inverse Most Probable Point (MPP) search algorithm, and executing sequential cycles of optimization and reliability assessment. These measures collectively render SORA a less computationally intensive method for solving RBDO problems.

5.2 Formulation of RBDO Models for Concrete Mix Parameters

The RBDO problem for the study carried out in this paper is formulated as:

$$\left.\begin{array}{ll} & \underset{\mu_w,\mu_{fa},\mu_{ca},\mu_c}{Minimize}\ cost\ (fa, ca, c) \\ Subject\ to: & Prob(g(w, fa, ca, c) \geq f_c) \geq R \\ & 0.42 \leq {}^{W}/_{C} \leq\ 0.55 \\ & w_l \leq w \leq w_u \\ & fa_l \leq fa \leq fa_u \\ & ca_l \leq ca \leq ca_u \\ & c_l \leq c \leq c_u \end{array}\right\} \qquad (2)$$

where $\mu_w, \mu_{fa}, \mu_{ca}, \mu_c$ are, respectively, the mean values of water content, fine aggregate content, coarse aggregate content and cement content. In the present study, all the four design variables are considered as random design variables. There are no deterministic variables and random parameters. $g(w, fa, ca, c)$ is the compressive strength function for a given curing age and f_c is target value for compressive strength for that particular curing age and R is the target reliability level. Water-cement content ratio ${}^{W}/_{C}$ is kept between 0.42 and 0.55. This constraint is taken as deterministic constraint. w_l, fa_l, ca_l, c_l, respectively, are lower bounds for water, fine aggregate, coarse aggregate and cement content. w_u, fa_u, ca_u, c_u, respectively are upper bounds for water, fine aggregate, coarse aggregate and cement content. The lower and upper bounds for the design variables are taken from Table 5.

5.3 RBDO Results and Discussion

The RBDO models developed are solved using the SORA method, which is implemented through Altair Hyperstudy 10.0. Both OLSR and PCR models are utilized in RBDO models to illustrate the impact of prediction models on RBDO outcomes. The study also examines the influence of reliability levels on optimization results. To this end, the RBDO problem formulated in the previous section is tackled for three target reliability levels: 0.90, 0.95, and 0.99. Optimal proportions for concrete mixtures are determined across a wide range of target compressive strength values, and these optimal mix proportions are detailed in Tables 10-11.

The process of identifying optimal concrete mix proportions involves setting the minimum target compressive strength at 27MPa and incrementing it in 3 MPa intervals. Results are reported up to the maximum target compressive strength for which the SORA optimizer converged at a given reliability level. Additionally, the compressive strength predicted by the respective regression model at the reliably optimal design is provided for each case. Furthermore, the safety margin employed by the RBDO process to ensure the target reliability is computed for each optimal mix. The range of safety margins considered by the RBDO process across various reliability levels and curing periods is presented in Table 12.

From Table 10, it is observed that PCR models perform better than OLSR models for RBDO problems solved for the 28 days compressive strength. The salient observations leading to the aforementioned conclusion are as follows:

- The RBDO model based on OLSR model converged for a very narrow range of target compressive strength (27 MPa-33 MPa) for R = 0.90. Convergence is obtained only for one value of target compressive strength, *i.e.*, 27 MPa for R = 0.95 and no optimal solution is obtained for R = 0.99. Whereas, for the RBDO problem based on PCR model, optimal mixture compositions are obtained for target 28 days compressive strength lying between 27 MPa and 46 MPa with reliability level of 0.90. Maximum attained compressive strengths for reliability levels of 0.95 and 0.99 are 43 MPa and 37 MPa, respectively.
- The comparison of optimal costs depicts that optimal costs are much higher in case of OLSR based models.
  Compressive strength predicted by OLSR model at optimal design in each case is much above the upper bound for 28 days compressive strength given in Table 5. But, compressive strength predicted by PCR model lies between the bounds given in Table 5 in every case.
- Safety margin assumed by RBDO process is very high in case of OLSR based models for a given target strength and reliability, whereas, moderate safety margins are taken up by PCR based RBDO model for each reliability level (Table 12).

The salient observations relating to the RBDO problem for 56 days compressive strength (Table 11(a)-(b)) follow a similar trend and are presented below:

- OLSR model based RBDO problem converged for exceptionally wide range of target 56 days compressive strength indicating the non-reliability of the model used. Optimal solutions are obtained for target $st56$ lying between 27 MPa and 70 MPa for R = 0.90.
- Maximum attained target compressive strengths for reliability level of 0.95 and 0.99 are 66 MPa and 54 MPa, respectively. Thus, the results obtained using OLSR models give an idea that as if there is no relationship between 28 days compressive strength and 56 days compressive strength. However, RBDO problem based on PCR model converged for a moderate range of target $st56$ lying between 27 MPa and 47 MPa with R = 0.90. Maximum attained compressive strengths for reliability levels of 0.95 and 0.99 are 45 MPa and 40 MPa, respectively.
- Predicted values of $st56$ by OLSR model go as high as 176.11 MPa. The predicted values obtained are much beyond the range of experimentally generated data used for analysis. However, Predicted $st56$ by PCR model lie between the bounds given for $st56$ in Table 5.
- In almost every corresponding case, optimal costs for OLSR based models are less than the optimal costs for PCR based models.
- Same optimal mixture compositions are obtained for both OLSR and PCR based RBDO problem with target $st56$ of 27 MPa and R = 0.90 and 0.95. But, there is a sharp difference between predicted $st56$ by the two regression models. Predicted $st56$ by OLSR model is 57.69 MPa while that by PCR model is 39.53 MPa.
- It is expected that as reliability level increases for a particular value of target compressive strength, optimal cost should rise. But, it can be seen from this table that optimal costs for target $st56$ of 48 MPa, 51 MPa, 54 MPa, 63 MPa and 66 MPa do not consistently increase with the increase in reliability level in OLSR model based RBDO solution.
- The safety margins taken by OLSR based RBDO process are very high in comparison to the safety margins taken by PCR based RBDO process (Table 12).

In deterministic design procedures, a safety margin is established prior to the optimization process. According to *IS 10262* (2009), for a specified target compressive strength f_c MPa, the concrete mix should be proportioned to achieve an average strength of at least $(f_c + 1.65s)$ MPa, ensuring that no more than 5% of the results fall below f_c MPa. Here, s represents the assumed standard deviation of the compressive strength data. The standard deviations assumed for different grades of concrete are outlined in Table 13 of *IS 10262* (2009).

Target Reliability R	Target compressive strength (MPa)	Optimization results based on OLSR models							Optimization results based on PCR models						
		$cost$ $(Rs.)$	Predicted $st28$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c	$cost$ $(Rs.)$	Predicted $st28$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c
0.90	27	2988.45	66.99	180.00	523.02	1252.05	410.07	0.439	2476.80	36.66	190.00	416.93	798.48	350.00	0.543
	30	3038.73	69.82	180.00	516.45	1252.05	421.20	0.427	2476.80	39.44	180.00	416.93	798.48	350.00	0.514
	33	3081.72	71.95	180.00	524.22	1252.05	428.98	0.420	2544.39	41.57	180.00	416.93	798.48	363.82	0.495
	36	-	-	-	-	-	-	-	2641.66	44.67	180.00	416.93	798.48	383.70	0.469
	39	-	-	-	-	-	-	-	2737.28	47.77	180.00	416.93	798.48	403.24	0.446
	42	-	-	-	-	-	-	-	2831.31	50.88	180.00	416.93	798.48	422.47	0.426
	45	-	-	-	-	-	-	-	3089.30	54.41	191.92	558.96	798.48	456.94	0.420
	46	-	-	-	-	-	-	-	3196.16	55.63	199.25	593.02	798.48	474.41	0.420
0.95	27	3134.68	63.18	230.00	490.65	798.48	475.00	0.484	2476.80	38.94	181.84	416.93	798.48	350.00	0.520
	30	-	-	-	-	-	-	-	2476.80	39.44	180.00	416.93	798.48	350.00	0.514
	33	-	-	-	-	-	-	-	2623.21	44.08	180.00	416.93	798.48	379.93	0.474
	36	-	-	-	-	-	-	-	2720.01	47.21	180.00	416.93	798.48	399.71	0.450
	39	-	-	-	-	-	-	-	2815.19	50.34	180.00	416.93	798.48	419.17	0.429
	42	-	-	-	-	-	-	-	3046.75	53.89	191.93	491.09	798.48	456.97	0.420
	43	-	-	-	-	-	-	-	3161.78	55.21	198.49	552.40	798.48	472.60	0.420
0.99	27	-	-	-	-	-	-	-	2574.29	42.52	180.00	416.93	798.48	369.93	0.487
	30	-	-	-	-	-	-	-	2673.53	45.70	180.00	416.93	798.48	390.21	0.461
	33	-	-	-	-	-	-	-	2771.08	48.88	180.00	416.93	798.48	410.15	0.439
	36	-	-	-	-	-	-	-	2887.27	52.14	182.13	419.04	798.48	433.63	0.420
	37	-	-	-	-	-	-	-	3025.48	53.61	192.77	441.66	798.48	458.98	0.420

TABLE 10: Reliability based design optimization results for 28 days compressive strength

Target Reliability R	Target Compressive strength (MPa)	Optimization results based on OLSR models							Optimization results based on PCR models						
		$cost$ $(Rs.)$	Predicted $st56$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c	$cost$ $(Rs.)$	Predicted $st56$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c
0.90	27	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2476.80	39.53	190.00	416.93	798.48	350.00	0.543
	30	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2476.80	39.53	190.00	416.93	798.48	350.00	0.543
	33	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2476.80	41.19	182.67	416.93	798.48	350.00	0.522
	36	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2537.73	43.49	180.00	416.93	798.48	362.46	0.497
	39	2477.48	62.49	192.58	416.93	798.48	350.14	0.550	2646.64	46.58	180.00	416.93	798.48	384.72	0.468
	42	2506.69	69.01	195.86	416.93	798.48	356.11	0.550	2753.52	49.67	180.00	416.93	798.48	406.56	0.443
	45	2535.51	76.10	199.10	416.93	798.48	362.00	0.550	2858.45	52.76	180.00	416.93	798.48	428.01	0.421
	47	2746.43	108.31	180.00	642.17	798.48	376.16	0.479	3059.33	55.06	197.01	416.93	798.48	469.08	0.420
	48	2757.55	110.02	180.00	642.17	798.48	378.43	0.476	-	-	-	-	-	-	-
	51	2791.01	115.18	180.00	642.17	798.48	385.27	0.550	-	-	-	-	-	-	-
	54	2824.16	120.33	180.00	642.17	798.48	392.04	0.550	-	-	-	-	-	-	-
	57	2639.54	107.13	210.80	416.93	798.48	383.27	0.550	-	-	-	-	-	-	-
	60	2661.07	114.62	213.22	416.93	798.48	387.67	0.550	-	-	-	-	-	-	-
	63	2922.14	135.82	180.00	642.17	798.48	412.07	0.550	-	-	-	-	-	-	-
	66	2954.39	141.00	180.00	642.17	798.48	418.66	0.550	-	-	-	-	-	-	-
	69	2986.36	146.17	180.00	642.17	798.48	425.20	0.423	-	-	-	-	-	-	-
	70	2996.96	147.89	180.00	642.17	798.48	427.37	0.421	-	-	-	-	-	-	-
0.95	27	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2476.80	39.53	190.00	416.93	798.48	350.00	0.543
	30	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2476.80	39.62	189.58	416.93	798.48	350.00	0.542
	33	2476.80	57.69	190.00	416.93	798.48	350.00	0.543	2504.72	42.56	180.00	416.93	798.48	355.71	0.506
	36	2491.91	65.62	194.20	416.93	798.48	353.09	0.550	2615.12	45.68	180.00	416.93	798.48	378.27	0.476
	39	2521.48	72.56	197.52	416.93	798.48	359.13	0.550	2723.43	48.79	180.00	416.93	798.48	400.41	0.450
	42	2551.33	80.27	200.88	416.93	798.48	365.23	0.550	2829.75	51.91	180.00	416.93	798.48	422.15	0.426
	45	2581.42	88.74	204.26	416.93	798.48	371.38	0.550	3086.83	55.37	199.37	416.93	798.48	474.70	0.420
	48	2877.59	128.73	180.00	642.17	798.48	402.97	0.447	-	-	-	-	-	-	-
	51	2916.56	134.93	180.00	642.17	798.48	410.93	0.550	-	-	-	-	-	-	-
	54	2669.84	117.78	214.20	416.93	798.48	389.46	0.550	-	-	-	-	-	-	-
	57	2696.43	127.71	217.19	416.93	798.48	394.90	0.550	-	-	-	-	-	-	-
	60	2720.64	137.24	219.91	416.93	798.48	399.84	0.550	-	-	-	-	-	-	-
	63	2742.57	146.27	222.38	416.93	798.48	404.33	0.550	-	-	-	-	-	-	-
	66	2762.55	154.83	224.63	416.93	798.48	408.41	0.550	-	-	-	-	-	-	-

TABLE 11(a): Reliability based design optimization results for 56 days compressive strength

Target Reliabiliy R	Target Compressive strength (MPa)	Optimization results based on OLSR models							Optimization results based on PCR models						
		$cost$ $(Rs.)$	Predicted $st56$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c	$cost$ $(Rs.)$	Predicted $st56$ (MPa)	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c
0.99	27	2488.18	64.80	193.78	416.93	798.48	352.33	0.550	2476.80	41.29	182.19	416.93	798.48	350.00	0.521
	30	2516.30	71.30	196.94	416.93	798.48	358.07	0.550	2539.79	43.54	180.00	416.93	798.48	362.88	0.496
	33	2545.03	78.58	200.17	416.93	798.48	363.95	0.550	2651.06	46.70	180.00	416.93	798.48	385.62	0.467
	36	2574.51	86.73	203.48	416.93	798.48	369.97	0.550	2760.24	49.87	180.00	416.93	798.48	407.94	0.441
	39	2604.92	95.86	206.90	416.93	798.48	376.19	0.550	2882.42	53.08	181.82	416.93	798.48	432.91	0.420
	40	2615.30	99.14	208.07	416.93	798.48	378.31	0.550	3000.26	54.39	191.94	416.93	798.48	457.00	0.420
	42	2636.47	106.10	210.45	416.93	798.48	382.64	0.550	-	-	-	-	-	-	-
	45	2669.43	117.62	214.16	416.93	798.48	389.38	0.550	-	-	-	-	-	-	-
	48	2704.07	130.66	218.05	416.93	798.48	396.46	0.550	-	-	-	-	-	-	-
	51	2740.51	145.40	222.15	416.93	798.48	403.90	0.550	-	-	-	-	-	-	-
	54	2777.93	161.63	226.35	416.93	798.48	411.55	0.550	-	-	-	-	-	-	-

TABLE 11 (b): Reliability based design optimization results for 56 days compressive strength

Target Reliability R	Curing Period	Safety margin obtained for OLSR based optimization results		Safety margin obtained for PCR based optimization results	
		Minimum (MPa)	Maximum (MPa)	Minimum (MPa)	Maximum (MPa)
0.90	28 days	38.95	39.99	8.57	9.66
	56 days	21.69	77.89	7.49	12.53
0.95	28 days	36.18	36.18	9.44	11.94
	56 days	24.69	88.83	9.56	12.53
0.99	28 days	-	-	15.52	16.14
	56 days	37.80	107.63	13.54	14.29

TABLE 12: Range of safety margins

Upon examination of Table 13, it is evident that for the cases under study, the value of s ranges between 4.0 and 5.0. Consequently, the safety margin should fall within the range of 6.60 MPa to 8.25 MPa. However, Table 12 illustrates that the safety margins adopted by the PCR based RBDO process are notably higher than these aforementioned safety margins.

It is also noted from both the Tables 10-11 that the parameters that vary predominantly in optimal solutions as target compressive strength and reliability level changes, are water-cement content ratio and cement content. Graphs are plotted to investigate the relationship between target compressive strength and the above mentioned parameters for different reliability levels (Figs. 2-3).

Grade of concrete	Assumed standard deviation (MPa)
M10 M15	3.5
M20 M25	4.0
M30 M35 M40 M45 M50 M55	5.0

TABLE 13: Assumed standard deviation.

The effect of reliability level and target compressive strength on each parameter is discussed below:

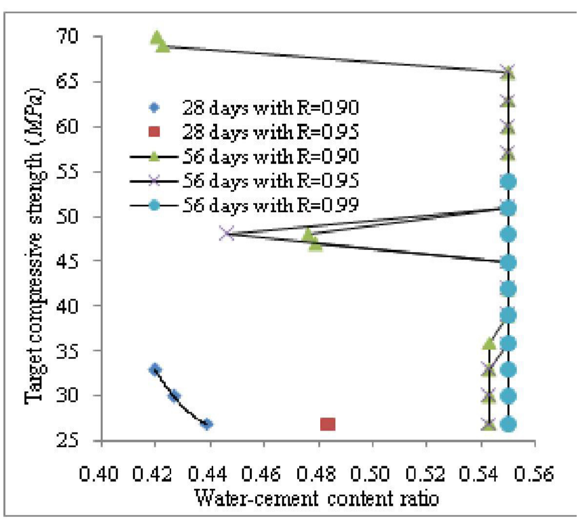

FIGURE 2(a): Variation of OLSR based optimal water-cement content ratio with target compressive strength for different reliability levels.

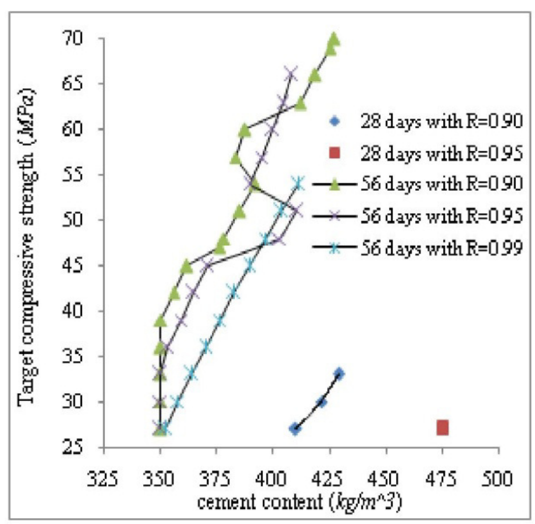

FIGURE 2(b): Variation of OLSR based optimal cement content with target compressive strength for different reliability levels.

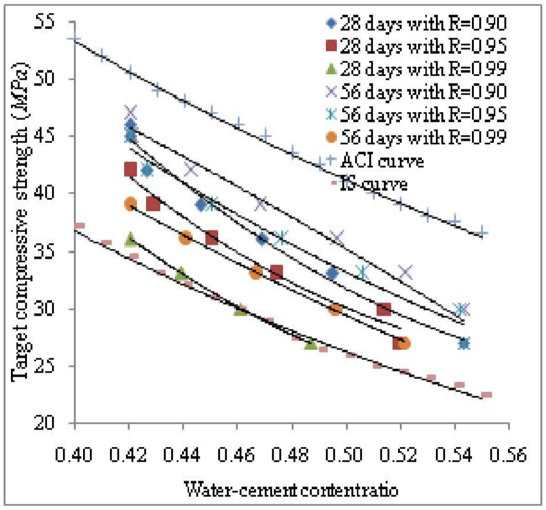

FIGURE 3(a): Variation of PCR based optimal water-cement content ratio with target compressive strength for different reliability levels.

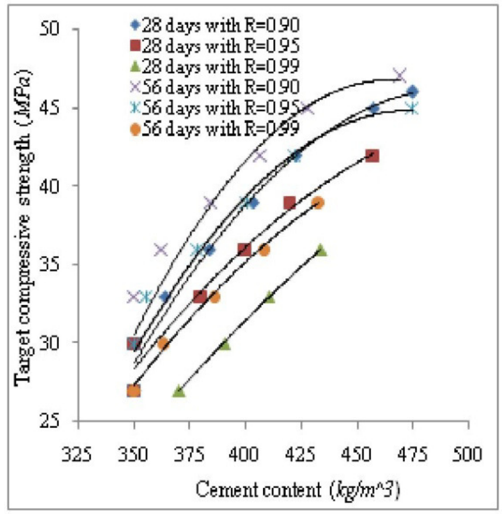

FIGURE 3(b): Variation of PCR based optimal cement content with target compressive strength for different reliability levels.

- *Water-cement ratio* (Figs. 2(a)- 3(a))

It can be observed from Fig. 2(a) that for OLSR based optimal results, w/c ratio decreases as 28 days compressive strength increases when R = 0.90. But, for 56 days curing period, w/c ratio is equal to its upper bound, i.e., 0.55 in most of the cases and for each reliability level. As per established facts, for increase in compressive strength, w/c ratio must decrease, and this trend is not found to be followed for results obtained using OLSR models. From a similar analysis carried out using PCR models, it is observed from Fig. 3(a) that as target compressive strength and reliability level increases, w/c ratio decreases in PCR based RBDO results. w/c ratio decreases quadratically as compressive strength increases for each reliability level. Value of coefficient of determination r^2 is greater than 0.970 for each curve. It is seen that all the curves lie below the ACI curve and above the IS curve except for 28 days compressive strength curve with R = 0.99. The part of this curve for low compressive strength values lies below the IS curve.

- *Cement content (Figs. 2(b)-3(b))*

The results of variation of cement content with target compressive strength are shown in Figs. 2(b)- 3(b). It can be observed from Fig. 2(b) that for OLSR model based optimization results for 56 days curing period, graph lines cross each other. However, for target compressive strength up to 45 MPa, higher the reliability level, higher is the required cement content for a given target compressive strength. But, there are portions in the graph showing that lesser cement content is required to satisfy higher reliability level for a given target 56 days compressive strength.

The graph shown in Fig. 3(b) is based on PCR based RBDO results. For both curing periods, cement content increases as reliability level increases for a given target compressive strength. Also, Cement content increases in quadratic manner with target compressive strength for each reliability level. Coefficient of determination r^2 is greater than 0.950 for each quadratic curve except for 56 days curve for R = 0.90. Value of r^2 is 0.933 for this curve.

6. CONCLUSION

This paper presents optimal designs of concrete mix having minimum cost and satisfying specific performance with given target reliability. RBDO models are formulated using prediction models based on OLSR and PCR techniques. RBDO is implemented using SORA method. It is seen that RBDO results are extensively affected by the compressive strength modeling techniques. Following conclusions are drawn:

- The PCR based RBDO models avoid extrapolation of predicted compressive strength, but in most of the cases OLSR models extrapolated to give unexpected values of predicted compressive strength when used for RBDO.
- The PCR based models yield mix proportions that are both cost-effective and within realistic bounds of material usage, preventing overdesign that often occurs in deterministic or poorly conditioned regression models.
- The results indicate that cement content and water–cement ratio are key control parameters. Adjusting these in line with reliability targets can help practitioners fine-tune mix designs for desired performance levels.

The outcomes provide strong evidence supporting the integration of reliability-based design optimization into concrete mix proportioning standards. This approach can move current practice beyond deterministic safety factors to quantifiable and verifiable reliability targets. In conclusion, the study establishes that PCR-based RBDO offers a practical, data-driven path for achieving consistent, verifiable, and cost-effective reliability in concrete mix design, paving the way for its application in modern performance-based design frameworks.

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