# ELECTROSTATIC AND ELECTROMAGNETIC FIELDS ACTUATORS FOR MEMS AD/DA CONVERTERS

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**Abstract**

MEMS Analog -to-digital and digital-to- analog converters are proposed using electrostatic field and electromagnetic fields actuators. For the former, parallel deformable plates supported by springs are used with bias applied voltage which determines the amount of static displacement needed for equilibrium condition. For the latter, coil winding(s) are embedded in a rotating plate, which is exposed to a constant field of a permanent magnet, causing the plate to deflect according to the currents in the windings. In the analog-to-digital arrangement, different spring displacements are tapped off either the spring in case of electrostatic or the moving plate in case of electromagnetic actuators, corresponding to the binary decoded currents. At these off tapping points, logic high signal levels are applied at these locations so that when a certain analog voltage is applied on the moving plate of the capacitor, the spring is displaced to one of these locations, enabling different binary voltages to all switches up to that level. Similar result occurs when an analog voltage is applied on the winding. The digital binary voltages are fed to a priority encoder to obtain the digital value. In digital-to-analog arrangement, the input binary voltage is decoded to different spring locations which correspond to resistances making up a potentiometer circuit for the output analog voltage. Similarly; for the electromagnetic actuator, a number of different length coil windings are embedded within the moving plate, causing different deflections corresponding to one bit of the binary input.

**Keywords:** ADC, DAC, MEMS, Electrostatic, Electromagnetic

## 1. INTRODUCTION

### 1.1 Electrostatic field actuator

The parallel plate capacitor with one movable plate supported by a mechanical spring is depicted in Figure (1), where the top plate is supported by a spring with the force constant being $K_m$. At rest the applied voltage, displacement and the mechanical restoring forces are zero. Gravity does not play an important role in the static analysis of micro devices because the mass of plates is generally very small and the gravitational force would not cause appreciable static displacement.

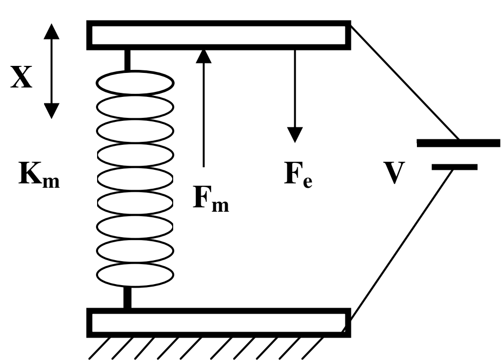

**FIGURE 1:** A Coupled Electromechanical Model

When a voltage is applied an electrostatic force $F_e$ will be developed with a magnitude of

$$\mathbf{F_e = \varepsilon\, A\, V^2/[2\, d^2]}$$

with the movable plate is at its starting position. This force tends to decrease the gap which gives rise to displacement and the mechanical restoring force. Under static equilibrium the mechanical restoring force has an equal magnitude but opposite direction as the electrostatic force. The magnitude of the electrostatic force is itself a function of the displacement. It's to be noted too that this electrostatic force affects the spring constant as well, due to the spring being softer due to this force. The spatial gradient of the electric force is defined as an electrical spring constant

$$\mathbf{K_e = \Delta F_e/\Delta d = CV^2/d^2}$$

As seen the magnitude of electric spring constant changes with position d and the biasing voltage V. This is ignored for small displacements. Thus the effective spring constant is mechanical spring constant minus the electrical spring constant.

To derive the equilibrium displacement of a spring supported electrode plate under a bias voltage V, consider the resulting equilibrium displacement being x in the direction of increasing gap. With displacement x, the gap between the two electrode plates is d+x and thus the electrostatic force at equilibrium is

$$\mathbf{F_e = \varepsilon\, A\, V^2/[2\, (X_o+X)^2] = C(X)\, V^2/[2\, (X_o+X)]}$$

Whereas the mechanical force is $\mathbf{F_m = -K_m\, X}$

By equating these two forces, and rearranging terms,

$$\mathbf{X = F_m/K_m = F_e/K_m = C(x)\, V^2/[2\, (X_o+X)\, K_m]}$$

The displacement at equilibrium can be calculated from the above quadratic equation as shown. This can be visualized graphically as shown in Figure (2). The horizontal axis represents space between the two plates, and the vertical axis is the mechanical or electrical force irrespective to their directions. The movable plate is displaced $X_o$ from the rigid fixed plate at origin. Two curves representing both mechanical and electrical forces are plotted with electrode positions according to their quoted equations and their intersecting points correspond to the solutions of the above quadratic equation. It can be noted that more than one intersecting points exist, but only one is achieved in reality. The solution that is closest to the rest position is realized first and is generally the realistic solution.

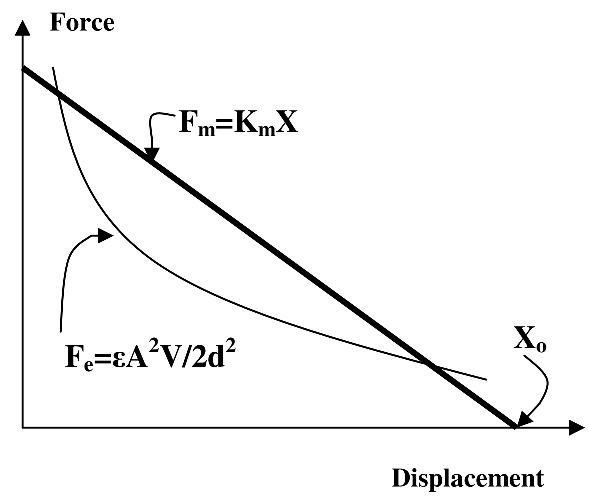


**FIGURE 2:** Electrical and Mechanical Forces as Functions of Spring Displacement

The graphic solution can be used to track the equilibrium position as the bias voltage is increased. As the voltage increases, the family of curves corresponding to electrostatic force shifts upwards, shifting the x coordinates of the interception points further away from the rest position.

### 1.2 Electromagnetic field actuator

A moving coil electromagnetic capable of one-axis rotation is proposed as depicted in Figure (3). A plate is supported by torsional hinge structure of embedded conducting wires, constituting multi windings positioned at different locations. The conducting wires of these windings are therefore of different lengths. Two permanent magnets are placed on the side of the plate, such that the magnetic field lines are parallel to the plane and orthogonal to the torsional hinges.

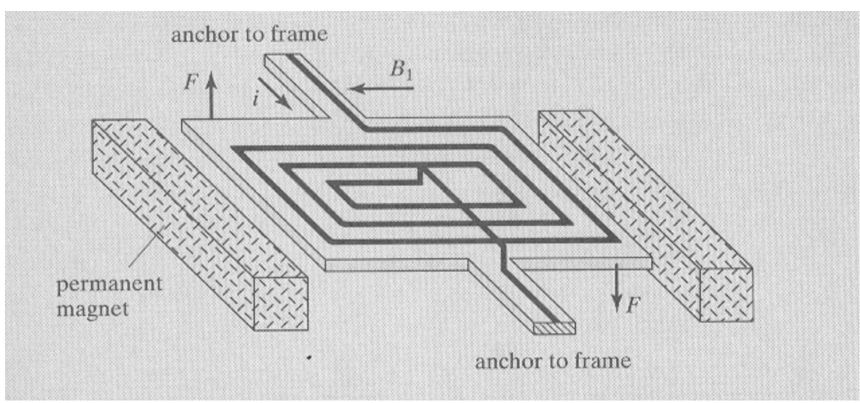


**FIGURE 3:** Electromagnetic field actuator

When current passes through the coils, Lorentz forces will develop and cause rotational torque on the plate. The direction of the torque depends on the direction of input currents. The magnitude of torque acting on this actuator is:

$$\mathbf{T=iBl_1l_2N}$$

where i is the winding current, B is the magnetic field density, $l_1$ & $l_2$ are the length and width of the coil and N is the number of turns per winding.

Since torque is linearly proportional with both winding current and winding turn length & width, different geometrical dimensions will give different resonant frequencies and thus different rotational angles for the same input current.

## 2. PULL-IN VOLTAGE

At a particular bias voltage the two curves intercept at one point tangentially as shown in Figure (4). At this interception point the electrostatic and mechanical force balance each other. At this point, the magnitude of electric and mechanical spring constants is equal. This is given by the gradient of the electrostatic force curve at the interception point, making the effective spring constant equal to zero, I.e. extremely soft. The bias voltage that invokes this condition is called the pull-in voltage $V_p$.

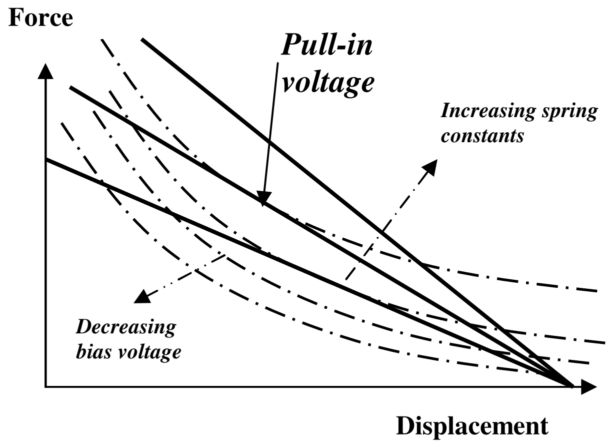


**FIGURE 4:** Balance of Electrical and Mechanical Forces at Different Bias Voltages And Different Spring Constants

This pull-in voltage can be calculated as

$$\mathbf{V^2= -2k_mX(X+X_O)^2 /[\varepsilon A] = -2k_mX(X+X_O)/C}$$

The value of x is negative when the spacing between the electrodes decreases.
If the bias voltage is increased further beyond $V_p$ the two curves will not intercept and no equilibrium solution exists. In reality the electrostatic force continues to grow while the mechanical force is unable to catch up and match it. The two plates are thus pulled against each other rapidly until they contact, at which the mechanical force will finally balance the electric one. This is termed as the pull in or snap in condition. Now substituting the pull in voltage in the electric force constant equation $\mathbf{K_e= CV^2/d^2}$ yields

$$\mathbf{K_e= CV^2/[X+X_o]^2 = -2K_mX/[X+X_o]}$$

The only solution in which $\mathbf{K_e=K_m}$ is when $\mathbf{x= -X_o/3}$.
This states that the relative displacement of the plates from its rest position is one third of the original spacing at the critical pull-in voltage irrespective of the actual mechanical force constant or actual pull-in voltage value. Substituting this displacement in the pull-in voltage equation yields

$$V_p^2 = 4X_o^2 K_m/9C$$

or $$V_p = 2X_o/3 \ [K_m/1.5 C_o]^{1/2}$$

## 3. ELECTROSTATIC ADC

The electrostatic field force within two plated capacitor is used to move the spring contacts at 8 different locations according to the pull-in voltages found from Figure (5), each one is a multiple of the previous contacts ones. This constitutes the binary digital values. Once connected, these contacts apply a zero voltage on a PMOS switch, thus continuing the circuit to the next contact and finally to the reference high voltage. This is depicted in Figure (5), which also shows the use of 7-3 priority encoder converting the contacts tapped voltages to binary digital voltage. Table (I) lists the truth table of the encoder.

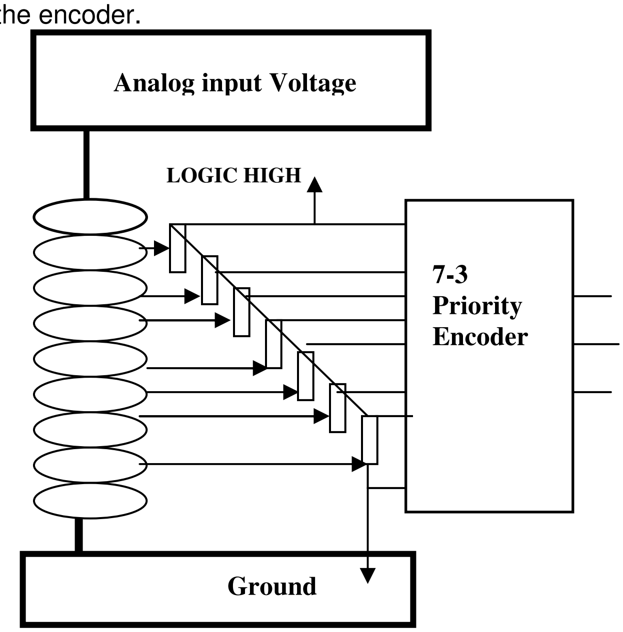


**FIGURE 5:** ADC using 7 PMOS switches at spring taps and an 7-3 priority encoder

## 4. ELECTROSTATIC DAC

In a similar manner, the two plated capacitor is used with a decoder and a spring operated potentiometer to implement a DAC. In this case a 3-8 decoder is used to energize one output at a time. This output is spring position switch which enables a current source to flow in the spring resistance thus dropping an output voltage according to I X R value, with the help of NMOS switches. This is depicted in Figure (6).

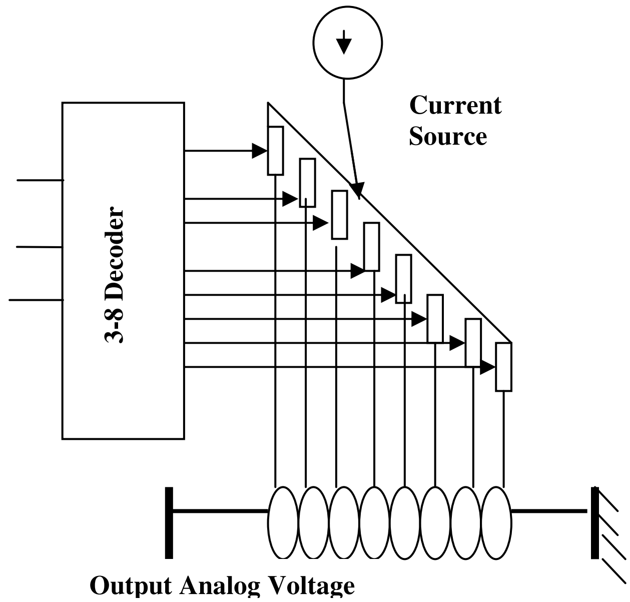


**FIGURE 6:** DAC using 8 NMOS switches at spring tap

| Input | | | | | | | Output | | |
|---|---|---|---|---|---|---|---|---|---|
| $I_6$ | $I_5$ | $I_4$ | $I_3$ | $I_2$ | $I_1$ | $I_O$ | $O_2$ | $O_1$ | $O_O$ |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **0** | **0** | **0** | **0** | **1** | **0** | **0** | **1** |
| **0** | **0** | **0** | **0** | **0** | **1** | **1** | **0** | **1** | **0** |
| **0** | **0** | **0** | **0** | **1** | **1** | **1** | **0** | **1** | **1** |
| **0** | **0** | **0** | **1** | **1** | **1** | **1** | **1** | **0** | **0** |
| **0** | **0** | **1** | **1** | **1** | **1** | **1** | **1** | **0** | **1** |
| **0** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **0** |
| **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** |

**Table 1:** 7-to-3 Priority Encoder

### 5. ELECTROMAGNETIC AD/DA CONVERTERS

In the case of electromagnetic field AD/DA converters the same arrangements of tapping positions are used here. Their positions and the coil geometrical dimensions, thought, need to be calculated for proper functioning.

In the case of ADC, only one coil of a certain dimension is used for the input analog voltage. The torque or deflection and hence tapping positions, are all linearly proportional to the input current, since both, the magnetic field and coil lengths are constant. Therefore tapping or contact positions are distributed linearly on the perimeter of the plate deflection path. It must be noted that this path should not extend beyond $90^o$ deflection. As the deflection plate moves, logic high signals are inputted to the priority encoder, resulting; a binary digital output to be generated.

In the case of DAC, a number of coils, depending on the input bits, are embedded within the actuating plate. The dimensions [length X width] of these coils are proportional to their bits order. Since square windings are used, thus the torque is linearly proportional with the square of windings lengths, i.e. $\mathbf{T= kl^2}$, where k is a constant. Figure (7) depicts the relation between torque and winding lengths and positions of the embedded winding coils.

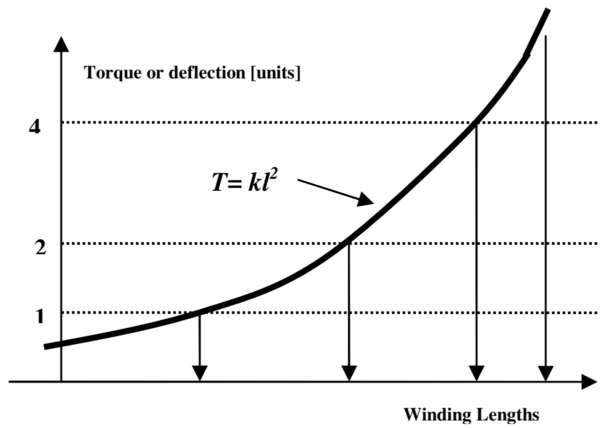


**FIGURE 7:** Electromagnetic DAC

## 6. CONCLUSION

It has been demonstrated that MEMS analog-to-digital and digital-to-analog converters can be formed using electrostatic and electromagnetic fields actuators. In the former, two plated capacitors with a damping spring arrangement is, whereas in the latter, coils embedded in a deflecting plate exposed to constant field, are employed.

MOS switches and contact positions of the converters are calculated according to the formulae governing the two fields. Whereas a certain input/output bits number is used, this arrangement can be expanded to a larger number.

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