Averaging Method for PWM DC-DC Converters Operating in Discontinuous Conduction Mode With Feedback

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Abstract

In this paper, a one-cycle-average (OCA) discrete-time model for PWM dc-dc converter based onclosed-loop control method for discontinuous conduction mode (DCM) is presented. It leads toexact discrete-time mathematical representation of the OCA values of the output signal even atlow frequency. It also provides the exact discrete-time mathematical representation of theaverage values of other internal signals with little increase in simulation time. A comparison of thismodel to other existing models is presented through a numerical example of boost converter.Detailed simulation results confirm the better accuracy and speed of the proposed model.

Keywords: Switched Systems, Pulse-width Modulation (PWM), Power Converter, Discrete TimeModeling, one-cycle-averaging, Sampled Data Model.

1. INTRODUCTION

PWM converters are widely used for operating switch controlled systems. These systems areusually operated in two modes of operation, namely: continuous and discontinuous conductionmodes [5]. The discontinuous conduction mode (DCM) of operation typically occurs in dc/dcconverters at light load.The boundary between the continuous conduction mode (CCM) andDCM depends on the ripple current in the inductor or the ripple voltage in the capacitor. Forlowpower applications, many designers prefer to operate in the DCM in order to avoid the reverserecovery problem of the diode. DCM operation has also been considered a possible solution tothe right-half plane (RHP) zero problem encountered in buck-boost and boost derived topologies. In single-phase ac/dc converters with active power factor correction (PFC), the input inductorcurrent becomes discontinuous in the vicinity of the voltage zero crossing; some PFC circuits areeven purposely designed to operate in DCM over the entire line cycle in order to simplify thecontrol. Proper analytical models for DCM operation of PWM converters are therefore essentialfor the analysis and design of converters in a variety of applications (see [11] and referencestherein). These modes of operation are also very much useful for efficiently extracting maximumpower from the photovoltaic panel (PV) which is another main application [12]. These powerconverters are connected between the PV and load or bus. Due to the variety of applications ofPWM converters operating in DCM, there is a need for an accurate model for the analysis anddesign of such converters. Many efforts have been taken in this view for past three decades [8,11].

Most power electronic design procedures rely on averaging techniques. Averaging techniquesprovide the analytical foundation for most power electronic design procedures of the system level. These techniques are widely used in the analysis and control design of PWM power electronicsystem. In fact classical averaging theory is not applicable when there are state discontinuities.This is significant because all feedback controller converters are state discontinuous systems.

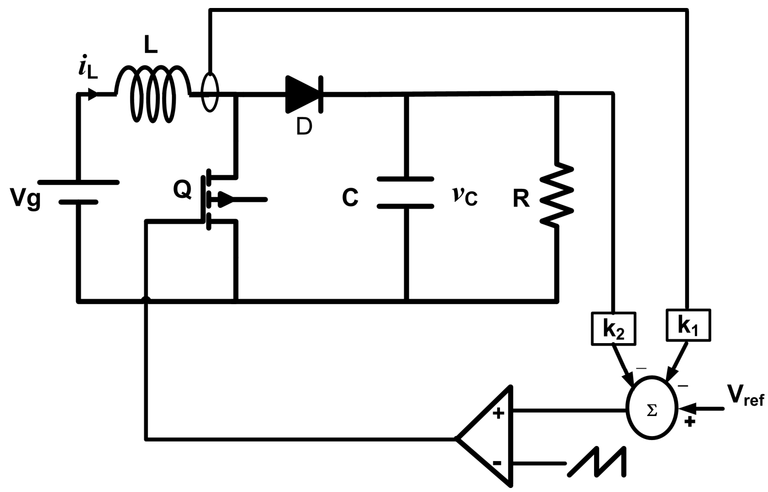

FIGURE 1: Closed loop Boost converter

Theperiodic solution of PWM converters is averaged to equilibrium. Although the periodic solutionat high switching operations has small amplitude (ripple), and averaging seems justified, theinherent dynamics for a periodic solution and at equilibrium are completely different. This issueis generally neglected in most power electronics literature. It is known that the averaged modelsare approximate by design [4, 9]. Moreover, it has been found that the directly obtained averagedmodels are inaccurate for converters operated in discontinuous conduction mode (DCM). Thisleads to the need of more efforts to obtain more accurate averaging methods.Averaging methods are sometimes used to produce approximate continuous-time models forPWM systems by neglecting the switching period of the switches and the sampling period ofthe controller [1, 2]. In averaging process the ripple in the current or in the voltage is also notconsidered. To overcome the above disadvantage, the sampled-data modeling techniques areadapted. This provides the most accurate result, which replicates the actual behavior of PWMsystems and is also suitable for digital control process. Sampled-data models allow us to focuson cycle-to-cycle behavior, ignoring intra cycle ripples. This makes them effective in generalsimulation, analysis and design. These models predict the values of signals at the beginningof each switching period, which most of the times represent peaks or valleys of the signals ratherthan average values. To better understand the average behavior of the system, a discrete-timemodel for the OCA signals was presented in [1]. Averaged discrete-time models for continuousconduction mode (CCM) and discontinuous conduction mode (DCM) without PWM feedbackcontrol are available. They are shown to be more accurate than continuous-time models andfaster [1, 2].

In this paper, a sampled-data model for PWM converters operating in DCM with feedback isformulated. This gives the exact discrete-time mathematical representation of the values of theoutput and internal signals with feedback loop at low frequency. A discrete-time model to providethe one-cycle-average (OCA) signals of PWM converters operating in DCM with feedback is proposed.This model provides the exact discrete-time mathematical representation of the averagedvalues of the output signal. It also provides the average values of other internal signals with littleincrease in simulation time. The main motivation for the new model is based on the fact that, inmany power electronic applications, it is the average values of the voltage and current rather thantheir instantaneous values that are of greatest interest. Numerical simulations show the accuracyof the propose model.

2. EXISTING AVERAGE MODELS

Different averaging methods for PWM converters are used for comparison, analysis and design.The mathematical models of the boost converter with PWM feedback control shown in Figure 1are presented in this section and will be used in Numerical example.

2.1 Switched Model

The DCM PWM converter can be described by

$$\dot{x}(t) = \begin{cases} A_1 x(t) + B_1 u(t), \; t \in \tau_1 \\ A_2 x(t) + B_2 u(t), \; t \in \tau_2 \\ A_3 x(t) + B_3 u(t), \; t \in \tau_3 \end{cases} \quad (1)$$

$$y(t) = \begin{cases} C_1 x(t), \; t \in \tau_1 \\ C_2 x(t), \; t \in \tau_2 \\ C_3 x(t), \; t \in \tau_3 \end{cases} \quad (2)$$

Where $u \in R^m$ is the input vector, $x \in R^n$ is the state vector, and $y \in R^p$ is the output vector. Thesystem switches between three topologies (A_1, B_1, C_1), (A_2, B_2, C_2), and (A_3, B_3, C_3), withswitching intervals determined by

$$\begin{aligned} \tau_1 &:= kT \le t < kT + d_k^1 T \\ \tau_2 &:= kT + d_k^1 T \le t < kT + (d_k^1 + d_k^2)T \\ \tau_3 &:= kT + (d_k^1 + d_k^2)T \le t < kT + T \end{aligned}$$

Where T is the switch period, $(d_k^1+d_k^2) \in [0, 1]$ are the switch duty ratios, and k is the discrete-time index. All auxiliary inputs will be assumed to be piecewise constants, i.e. $u(t) = u_k$ for all $t \in [kT, (k+1)T]$. This assumption is not necessary and is made for convenience only; more general cases would only require more complex notations. This is the exact switching model which will be used as the base model for comparison of different methods.

The state space matrices A_1,A_2,A_3,B_1,B_2,B_3,C_1,C_2, and $C_3$of boost converter shown in Figure1 are defined as

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}; \; B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}; \; C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}; \; B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}; \; C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}; \; B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \; C_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The control scheme given in is applied, where the modulation signal is $m(t)=V_{ref}- k_1\, i(t) - k_2\, v(t)$ with $V_{ref}=0.13$, $k_1 =0.174$, and $k_2 = -0.0435$ as in [1, 7].

2.2 DCM State-space Average Model (SSA)

The SSA mathematical model of the boost converter is given as [11]

$$\dot{x}_1(t) = \frac{2x_1}{dT}\left(1 - \frac{x_2}{u}\right) + \frac{dx_2}{L} \quad (3)$$

$$\dot{x}_2(t) = \frac{x_1}{C} - \frac{d^2Tu}{2LC} - \frac{x_2}{RC} \quad (4)$$

2.3 Conventional Discrete-Time Model (CDTM)

The conventional discrete-time mode is given by

$$x_{k+1} = A(d_k^1, d_k^2)x_k + B(d_k^1, d_k^2)u_k \quad (5)$$

Where the input nonlinearities $A\ (d^1, d^2)$and $B\ (d^1, d^2)$ are given by

$$A(d^1, d^2) := \Phi_3\Phi_2\Phi_1$$
$$B(d^1, d^2) := \Phi_3(\Phi_2\Gamma_1 + \Gamma_2) + \Gamma_3$$

The arguments d^1T, d^2T, and $(1\text{-}d^1\text{-}d^2)T$ for ($\Phi_1$andΓ_1), ($\Phi_2$andΓ_2) and ($\Phi_3$andΓ_3) respectively are omitted from the above equations for notation simplicity. Where

$$\Phi_i(t) := e^{A_i t}$$
$$\Gamma_i(t) := \int_0^t e^{A_i \tau} b_i d\tau$$

3. PROPOSED MODEL

This section introduces the new average discrete-time model for PWM converter operating inDCM with feedback loop. Description of the original system and derivation of the proposed modelare discussed here. The one-cycle average (OCA) representation of the output signal [1] is givenby

$$y^*(t) := \frac{1}{T}\int_{t-T}^{t} y(\tau)d\tau \quad (6)$$

The signal, $y^*(t)$ is used to develop a new discrete-time model for PWM converters operating inDCM. This model provides the basis for discrete-time simulation of the averaged value of anystate in the DCM PWM system, even during transient non-periodic operating conditions.

3.1 OCA Discrete-Time Model

It is desired to compute, without approximation, the evolution of all system variables at thesampling instants, $t = kT$ assuming three different topologies for the system. Since the state andoutput equations (1) - (2) are piecewise-linear with respect to time t, the desired discrete-time model can be obtained symbolically. Using the notation, $x_k := x(kT)$ and $y_k^* := y^*(kT)$, the result is the OCA large signal model

$$x_{k+1} = A(d_k^1,\ d_k^2)x_k + B(d_k^1,\ d_k^2)u_k \quad (7)$$
$$y_{k+1}^* = C(d_k^1,\ d_k^2)x_k + D(d_k^1,\ d_k^2)u_k \quad (8)$$

Where the input nonlinearities $A(d^1, d^2)$, $B(d^1, d^2)$, $C(d^1, d^2)$and $D(d^1, d^2)$ are given by

$$A(d^1, d^2) :=\ \Phi_3\Phi_2\Phi_1$$
$$B(d^1, d^2) := \Phi_3(\Phi_2\Gamma_1 + \Gamma_2) + \Gamma_3$$
$$C(d^1, d^2) := C_1\Phi_1^* + C_2\Phi_2^*\Phi_1 + C_3\quad \Phi_3^*\Phi_2\Phi_1$$
$$D(d^1, d^2) := C_1\Gamma_1^* + C_2(\Phi_2^*\Gamma_1 + \Gamma_2^*) + C_3(\Phi_3^*(\Phi_2\Gamma_1 + \Gamma_2) + \Gamma_3^*)$$

The arguments d^1T, d^2T, and $(1 - d^1 - d^2)T$ for (Φ_1, Φ_1^*, Γ_1, and Γ_1^*), (Φ_2, Φ_2^*, Γ_2, and Γ_2^*) and (Φ_3, Φ_3^*, Γ_3, and Γ_3^*) respectively are omitted from the above equations for notation simplicity. Where

$$\Phi_i(\mathrm{t}) := e^{A_i t}$$

$$\Gamma_i(\mathrm{t}) := \int_0^t e^{A_i \tau} b_i d\tau$$

$$\Phi_i^*(\mathrm{t}) := \frac{1}{T}\int_0^t \Phi_i(\tau) d\tau$$

$$\Gamma_i^*(\mathrm{t}) := \frac{1}{T}\int_0^t \Gamma_i(\tau) d\tau$$

Note that the averaging operation adds "sensor" dynamics to the system;as a consequence, thelarge-signal model equations (7) and (8) is not in standard state-space form. By defining theaugmented state vector $x^* \in R^{n+p}$such that

$$x_{k+1}^* := \begin{bmatrix} x_{k+1} \\ C(d_k^1, d_k^2)x_k + D(d_k^1, d_k^2)u_k \end{bmatrix}$$

An equivalent (but standard form) representation of the OCA large-signal model is given by:

$$x_{k+1}^* = A^*(d_k^1, d_k^2)x + B^*(d_k^1, d_k^2)u_k$$
$$y_k^* = C^* x_k^*$$

Where

$$A^*(d_k^1, d_k^2) := \begin{bmatrix} A(d^1, d^2) & 0_{n\times p} \\ C(d^1, d^2) & 0_{p\times p} \end{bmatrix}$$
$$B^*(d_k^1, d_k^2) := \begin{bmatrix} B(d^1, d^2) \\ D(d^1, d^2) \end{bmatrix}$$
$$C^*(d_k^1, d_k^2) := \begin{bmatrix} 0_{p\times n} & I_{n\times p} \end{bmatrix}$$

Note that not only the OCA values of output signal will be available but also the values of thesignals (without averaging) at the beginning of every switching period as well.

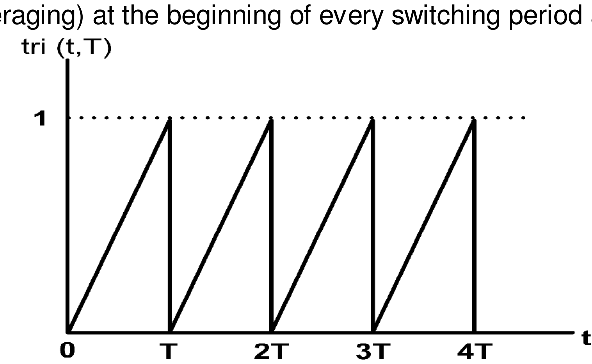

FIGURE 2: PWM sawtooth function

3.2 Feedback Computation

The modulation signal for feedback control is $m(t)=V_{ref}-k_1\ i(t) - k_2\ v(t) = V_{ref}- K\ x(t)$ and the duty ratio at each switching period is $d_k=t^*/T$. The time instant t^* at which the modulation signal crosses the sawtooth is computed by solving the nonlinear equation

$$tri(t^*, T) = V_{ref} - Kx(kT + t^*) \\ = V_{ref} - K\{\Phi_1(t^*)x_k + \Gamma_1(t^*)u_k\} \quad (9)$$

at each time instant *k*, where the sawtooth function is shown in Fig. 2 and mathematicallyrepresented by *tri(t, T)= (t/T)-floor(t/T)*. For reasonably high switching frequency, the value of$x(kT+t^*)$ can be approximated by neglecting the higher order terms in the Taylor expansion of the nonlinear functions $\Phi_1$andΓ_1. That is

$$\Phi_1(t^*) = I + A_1t^* + \frac{A_1^2}{2!}t^{*^2} + \cdots \approx I + A_1t^*$$

$$\Gamma_1(t^*) = (It^* + A_1\frac{t^{*^2}}{2!} + \cdots)B_1 \approx It^*B_1$$

And hence, a good approximation of (9) becomes

$$tri(t^*, T) \approx V_{ref} - K\{(I + A_1t^*)x_k + B_1t^*u_k\}$$

Noting that $tri(t^*,T)$ equals to t^*/T for $t^* \in [kT, (k+1)T]$, we get

$$\frac{t^*}{T} \approx V_{ref} - K\{(I + A_1t^*)x_k + B_1t^*u_k\}$$

Or

$$d_k = \frac{t^*}{T} \approx \frac{V_{ref} - Kx_k}{KT(A_1x_k + B_1u_k) + 1}$$

Which provides a closed from solution for d_k. The duty ratio d_k can be computed without approximation by solving the nonlinear equation (9) for t^*.

4. NUMERICAL EXAMPLE

Since all of the aforementioned averaged models have been controlled with the same controllerdesign, a comparative study is carried out to investigate the accuracy and speed of the proposedmodel as compared to the existing averaged model. It should be noted that no approximation ismade in deriving the new discrete-time model, and all simulations were performed using Matlab.The results of all models are computed using built-in Matlab nonlinear equation solver. The state variables are $x_1= i_L$and $x_2= v_C$.

4.1 Ideal Condition

Ideal boost converter with PWM feedback control shown in Figure 1 is simulated using existingaveraged models and proposed model. The simulated voltage and current waveforms are shownin Figures 3 - 4. The steady-state average values predicted by the proposed model are moreaccurate than the ones obtained by the SSA models for the parameters R = 45 Ω, L = 100 μH,C = 4.4 μF, V_g= 5 V, and T_S= 100 μs.

The steady-state average values of the output voltage are v_C= 8.1125 V for SSA model andv_C= 8.3174 V for the proposed model. It should be noted that the accuracy of the SSA methoddecreases as the switching frequency decreases, while the proposed model does not depend onthe switching frequency as discussed below.

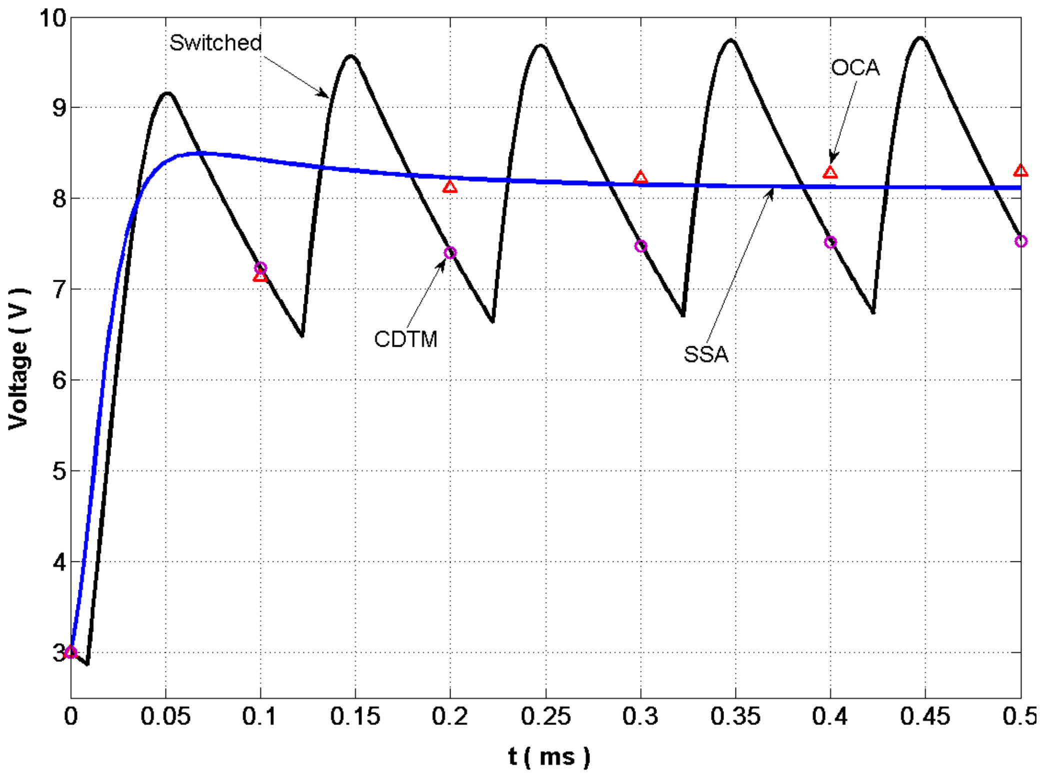

FIGURE 3: Simulation comparison of voltage waveform

4.2 Effect of Switching Frequency

To demonstrate the effect of switching frequency, consider operating the converter in a Higherfrequency, for example TS = 80µs i.e. f_S= 12.5 KHz. Figures 5 - 6 shows the effect ofincreasing the switching frequency on the simulation results of the output waveform. The steadystateaverage values of the output voltage are v_C= 8.055 V for SSA model and v_C= 8.475V for the proposed model. The accuracy of SSA averaged model decreases and move awayfrom the actual average as the switching frequency increases. The proposed model captures thecycle-to-cycle behavior of the system with no approximation regardless of changes in switchingfrequency.

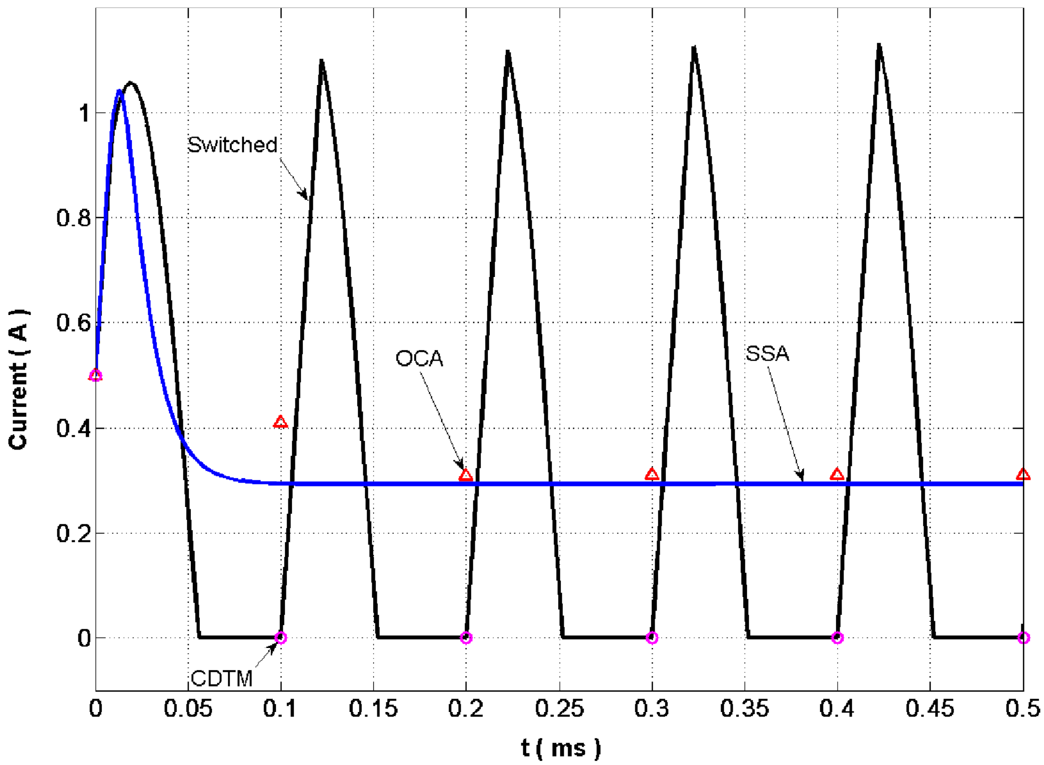

FIGURE 4: Simulation comparison of current waveform

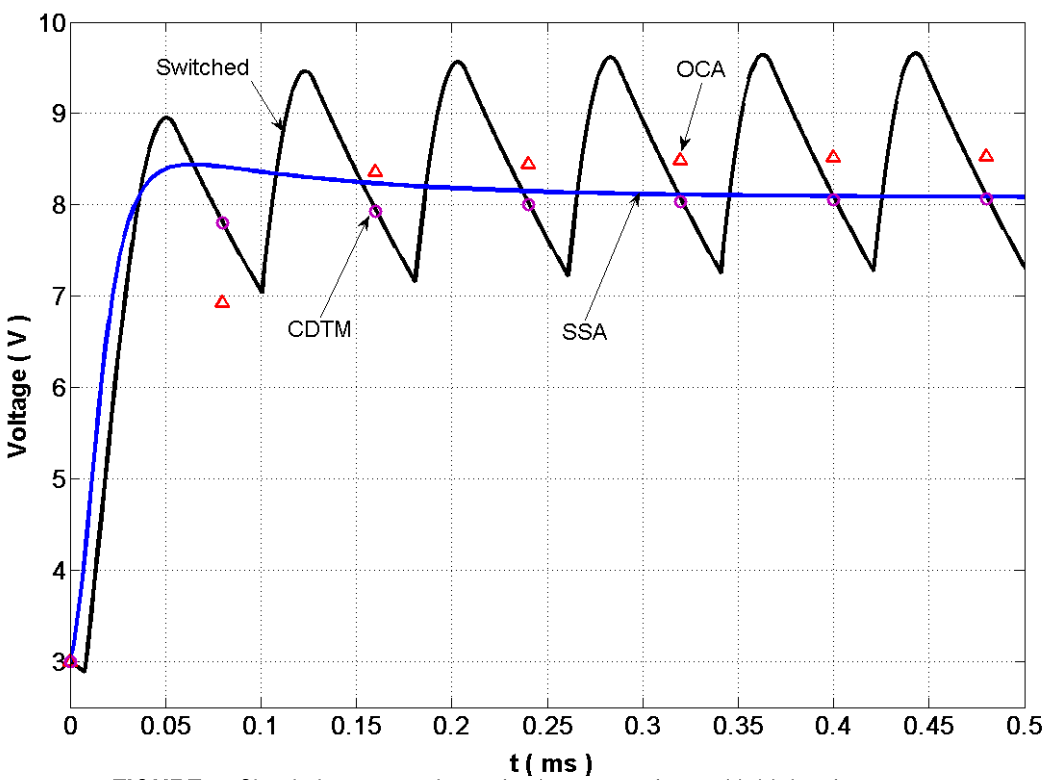

FIGURE 5: Simulation comparison of voltage waveform with higher frequency

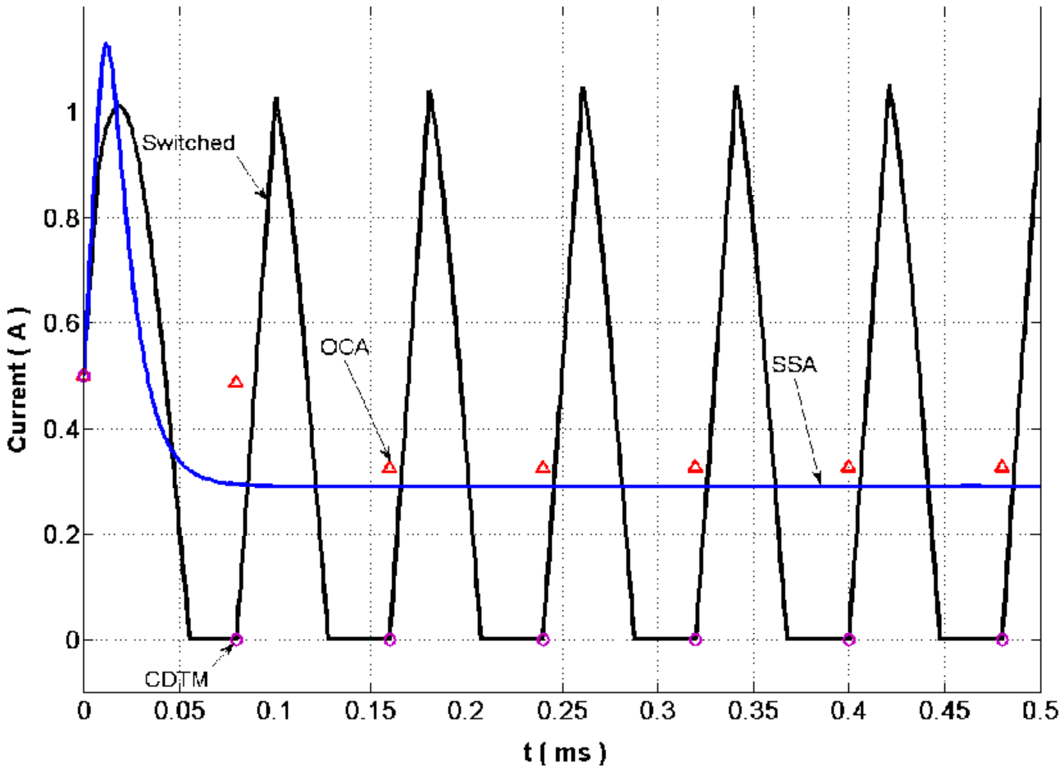

FIGURE 6: Simulation comparison of current waveform with higher frequency

4.3 Effect of Change in load

To study the effect of load resistance on the simulation results, a step change on the loadresistance R from 45 Ωto 55 Ωat time instant, t = 0.6 ms has been simulated and the resultsare shown in Figure 7. The steady-state average values of the output voltage are v_C= 8.1125V and v_C= 9.1125 V for SSA model and v_C= 8.3174 V and v_C= 9.625 V for the proposedmodel respectively. It can be observed that the average values produce by SSA model deviatemore from the exact waveforms as the load resistance increased. On the other hand the proposedmodel provides the same accuracy of waveforms regardless of the change in load resistance.

Table 1 summarizes the normalized simulation times for ideal boost converter with feedback fordifferent simulation methods.

Method	Normalized Simulation Time
Switched	42
SSA	1
CDTM	8.4
DCM OCA	8.81

TABLE 1: Simulation time for boost converter in DCM with feedback

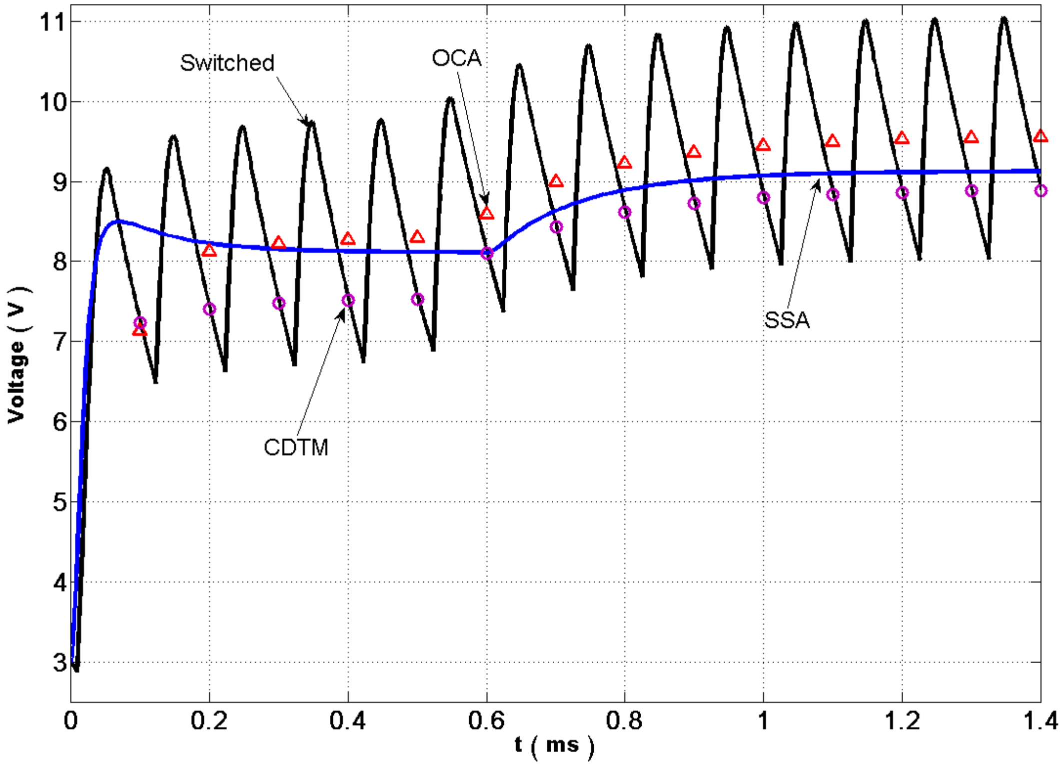

FIGURE 7: Simulation comparison of voltage waveform with variable load resistance

5. CONCLUSION

This paper proposed a new model which provides a discrete-time response of the one-cycle-average(OCA) value of the output signal for PWM dc-dc converters operating in the DCM withfeedback. It is compared to existing models through a numerical example of boost converter. Asa result of variations in the circuit parameters such as switching frequency and load resistance,a significant deviation in the average values of the converter's signals is predicted by the existingaveraging method. On the other hand, the proposed model is fast and can accurately simulatethe average behavior of the output voltage even though there is a large variation in the circuitparameters without any approximation in the design.

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