A Review of Some Local Feature Detection Algorithms

John Zhang *zhang@iup.edu*
Department of Mathematics
Indiana University of Pennsylvania
Indiana, PA 15705, USA

Tao Sun *tsun@gmail.com*
Department of Mathematics
Indiana University of Pennsylvania
Indiana, PA 15705, USA

Abstract

This paper reviews and summarizes the results of some local feature detection algorithms. Due to the large number of publications, this review just covers the publications prior to 2010. Methods are classified by grayscale and color applications. The grayscale methods are further classified into gradient based methods, orientation/spatial analysis based methods, model/template fitting or matching methods, methods based on fuzzy logic, and statistical learning. The evaluation of the performance of the different algorithms is also discussed. This is not a comprehensive review but aims to cover most of the important developments. It can be used as the first step in literature review for new researchers in edge detection.

Keywords: Edge Detection, Edge Detection Algorithms, Edges in Color Images, Gaussian Filter, Multi-scale Transformation, Morphological Gradient Operators, Fuzzy Logic, Statistical Learning, Algorithm Evaluation.

1. INTRODUCTION

Changes or discontinuities in an image's amplitude attributes, such as luminance or tristimulus value, are fundamentally important because they often provide an indication of the physical characteristics of objects within the image. In computer vision, *edge detection* is a process which attempts to capture the significant properties of objects in the image. These properties include *changes, variations and discontinuities* in the photometrical, geometrical and physical characteristics of objects. Edge detection is an important stage in low-level image processing. As Marr (1976) pointed out, the purpose of early visual processing is to construct a raw *primal sketch*, a primitive but rich description of the image. One part of this raw primal sketch is the description of the intensity changes, obtained using a primitive language of edge-segments, bars, blobs and terminations. Edge detection is widely used as a low-level process, which extracts important information for high-level processing, such as object recognition, stereo matching, motion tracking, 3-D object construction, and many others.

Intuitively, edges can be seen as the boundaries which separate the areas of images that have different intensities. Although there is no canonical definition of *edge* in computer vision, the definition presented by Ziou (1999) is generally accepted. Ziou stated that an edge can be seen as a local change, or variation in the image's intensity function. In essence, an edge is a local phenomenon in a digital image. As a result, most of the current edge detection methods are based on comparing the pixels within a small neighborhood and using the differences between these pixels to determine whether or not an edge occurs within that neighborhood. An often mentioned edge model in literature is the step edge, corresponding to a discontinuity in image intensity. Step edges are usually generated by the borders of objects in the image. There are edge types other than the step edge, and some of these include ramp edges, roof edges, and staircase edges.

In 2-D digital image there are other important local features besides edges. Some of these include lines, corners, junctions, line terminations, and points where multiple edges meet. Edges, as they have been discussed above, can be classified as one type of these local features in 2-D. These classifications are due to the fact that corners and junctions carry robust information about the image and traditional edge detection methods can't handle these local features. Actually, the development of specific local feature detectors has gained more attention recently and has become an approach independent of traditional edge detection. Thus, we distinguish the traditional edge and the local feature in our review hereafter.

The area or neighborhood, in which image variations or changes occur, known as the scale of the variation or change, leads to the development of different techniques to characterize them. For example, boundary segmentation has been developed to identify global variations and texture segmentation has been developed to identify the changes between textured regions. Generally, edge detection is used to identify the *local* variations or changes in an image.

1.1 Edges in Grayscale Images

Intensity variations can be classified based on their 1-D and 2-D features. The commonly observed variations are discontinuities, classified as step edges, local extrema, classified as line edges, and other 2-D features formed by at least two edges, which are typically classified as junctions or corners.

Based on the 1-D signal features, edges can be modeled as *step, ramp, roof, or staircase*. Important characteristics of these edges include *height (amplitude)*, *slope angle,* and *horizontal coordinate of the slope,* also known as *slope midpoint*.

Based on 2-D features, edges can be part of any of the following: *points* (classified into categories such as interest point, key point, dominant point, salient point,) *lines, junctions,* or *corners*.

- **Point:** Pratt (P489) pointed out that a spot, which we will refer to as a point, can only be defined in two dimensions, and consists of a plateau of a high amplitude against a lower amplitude background, or vice versa. One of the most commonly used point types is the *interest point*. Schmid et al. (2000) simply define an "interest point" as any point in the image at which the signal changes two-dimensionally. It could be a conventional corner (Y type, T type, X type), a single black dot against a white background, or any location with significant 2-D texture. Tuytelaars et al. (2007) pointed out that the definition can actually be application dependent. They stated that the term *interest point* should be used if only the location information of the point (usually sub-pixel accuracy) is used for future processes. For example, camera calibration or 3-D reconstruction. If the immediate neighborhood information, which is used to localize the interest point, is also used for future processes, the term *region* is preferred.
- **Line:** Deschenes and Ziou (Tech report 259) defined lines as curvilinear image events in which the intensity surface form a roof, a valley or a ridge with a narrow width. These edges can result from mutual illumination, from the placement of thin objects against a background, or from roads and rivers in remotely sensed images.
- **Junction/Corner:** the intersection of several edges (steps) or lines constitutes a junction or corner. Typical corners include T type, Y type, X type and L type corners. Some researchers, Bergevin et al. (2004) for example, also use characterized and labeled junction points to define junctions.

The purpose of the classification of edges based on their 1-D feature is that it enables researchers to use the proper mathematical model/techniques to effectively identify and localize these variations by designing optimal edge detectors (or filters.) In the 1990's, researchers designed different optimal filters for different types of edges. Most of the edge detection algorithms, such as Canny's, were intended for only one edge type. Another term "characterization" was also used for classification of edges and it was based on the behavior of the edge with respect to different edge detectors and at different scales. These developments were crucial for edge detection algorithms based on multiscale techniques. It is also important for hybrid edge detection, which is intended to either combine different edge detection algorithms or choose a particular one in order to accomplish the edge detection. Extensive work has been done in this area regarding to certain edge types (e.g. step, ramp) and certain edge detectors (e.g. Gaussian, especially Laplacian of Gaussian). Chidiac and Ziou (1999) proposed an edge classification algorithm based on the classification of 1-D edges and their behavior with respect to the Laplacian of Gaussian.

There are two main reasons to further classify the edges in 2-D images. First, junctions and corners carry robust information in 2-D images. A number of psychophysical experiments have shown the importance of junctions in shape recovery and object recognition. For instance, some studies showed that human visual system tends to focus on image regions containing junctions. A brief review on this can be found at Bergevin et al. (2004). Second, traditional edge detectors, such as Canny's and LoG (Laplacian of Gaussian), perform poorly at junctions, where intensity behavior is more complex and is not well-modeled by simple step edge models. For example, trihedral junctions (e.g. Y corner, T corner) are frequently missed by Canny's algorithm. It is also known that the LoG breaks down at corners. Thus, the detection of junctions calls for different techniques other than traditional edge detectors. Surveys on these techniques can be found at Deschenes and Ziou (Tech report 259), Schmid et al. (2000), and Bergevin et al. (2004.)

1.2 Edges in Color Images

Color is perceived by humans as combination of tristimuli R (red), G (green), and B (blue). Color space is used to represent the colors. The commonly used color spaces include RGB, Nrgb (normalized RGB), HSI, and CIE spaces (XYZ, L*u*v).

- The RGB space is the most commonly used model for television systems and digital camera pictures. Since the RGB components are highly correlated, and the measurement of a color in RGB space does not represent color differences in a uniform scale, it is impossible to evaluate the similarity of two colors from their distance in RGB space.
- The Nrgb space can be obtained from RGB space through a nonlinear transformation. The individual color components are independent of the brightness of the image. However, it is very noisy at low intensities due to the nonlinear transformation.
- The HSI space is more intuitive to human vision and can also be transformed from the RGB space through nonlinear transformation. It has non-removable singularity and is numerically unstable at low saturation due to the nonlinear transformation.
- CIE spaces (XYZ, L*u*v) can control color and intensity information independently. Direct color comparison can be performed based on the geometric separation (e.g. Euclidean distance) within CIE space. It has the same singularity problem as HSI system.

In color images, the information about edges is much richer than the monochrome case. Hue, saturation, shading, shadow, transparency, and highlight play important roles in how humans perceive edges from a color image. For example, edges between two objects with the same brightness can be detected in color image. Compared to the gray-level image, which can be considered as a two dimensional discrete space where each pixel is assigned a scalar value, a color image can be considered as a three dimensional space where each pixel is assigned a vector whose elements represent each of the different color components.

Conventionally, in a color image, an edge could be defined by a discontinuity in a three-dimensional color space. Nevatia (1977) gave three alternatives for the definition of an edge in a color image:

i. Define a metric distance in some color space and use discontinuities in the distance to determine edges.
ii. Regard a color image as composed of three monochrome images formed by three color components respectively, and perform gray level edge detection on these three images separately. Then the edges detected in the three images might be merged by some specific procedure.

iii. Impose some uniformity constraints on the edges in the three color components to utilize all of the three color components simultaneously, but allow the edges in the three color components to be independent.

However, none of the above definitions works for all conditions. For definition i, there is no difference from the gray level edge detection, and we cannot expect more information conveyed by color; for definition ii, it will fail when three gradients for one pixel have the same strength but opposite direction (Shu-yu Zhu (1999)) for definition iii, the constraints sometimes will affect the calculation of the three color components.

Koshchan and Abidi (2005) proposed to classify edges based on a dichromatic reflections model which is commonly applied in physics-based color image processing, as five classes:

- *Object edges*, or orientation edges, arise from a discontinuity of the vector normal of a continuous surface
- *Reflectance edges* arise from a discontinuity of the reflectance of object surface
- *Illumination edges*, or shadow edges, arise from a discontinuity of the intensity of the incident lighting
- *Specular edges*, or highlight edges, arise from a special orientation between the light source, the object surface, and the observer and are due to material properties
- *Occlusion edges* are boundaries between an object and the background as seen by the observer. Occlusion edges do not represent a physical discontinuity in the scene. They exist due to a special viewing position.

However, this classification needs certain knowledge of the material properties of the objects in the scene.

There is no commonly accepted definition of an “edge” in color image. Conventionally, edges will be considered as variations in a 3-D vector fields in a color image.

2. SOME EDGE DETECTION TECHNIQUES

2.1 Introduction

Since the earliest work by Julez (1959), various approaches on edge detection have been proposed in extensive literatures in past few decades. The early edge detection methods, pioneered by Roberts (1965,) Prewitt (1970,) and Sobel (1970,) are based on the difference, or differentiation, within a small neighborhood of a pixel. Although the early edge detection methods are easy to implement, they are highly noise sensitive, and can only work for certain types of edges (e.g. step edges.)

In Marr and Hildreth’s (1980, *theory of edge detection*) work of developing a computational model of human perception, they developed an edge detection model via the zero-crossings of the image after convolution with a Laplacian of Gaussian filter due to the presence of on-center and off-center receptive fields in the human retina.

Torre and Poggio (1984) summarized that edge detection consists of two steps: a *filtering* (e.g. smoothing) step and a *differentiation* step. They also conducted a detailed analysis on the properties of filters (band-limit filters, support limited filters and filters with minimal uncertainty) and the differential operators (directional derivative, rotational invariant differentiator.)

Canny (1983, 1986) took an analytical approach to edge detection based on 1-D step edge model. He proposed three quantitative criteria: good detection (e.g. high signal/noise ratio-SNR,) good localization, and single response. Canny combined these three criteria by maximizing the product of SNR and LOC subject to the constraint of single response. He designed an optimal filer, which was shown that could be approximated by the first derivative of Gaussian. His significant contributions also include two post-processing procedures: *nonmaxima suppression* and *hysteresis*, which are critical for the quality of the edge map. Canny’s edge detector has since become the standard edge detection method, and it has been shown, in certain comparison tests, that it has the best performance among the commonly used edge detectors.

Extensive work has been done following Canny's approach/optimal filer design. The main idea behind these efforts was to design filter for certain types of 1-D signal (i.e. different types of edges,) and then properly generalize to 2-D signals. Some of these works can be found at Shen-Castan (ISEF) edge detector, Petrou et al. (1991,) and Zhiqian Wang et al. (1996) on optimal ramp edge detector. Jacob et al. (2004) proposed the design of steerable filter for feature detection based on Canny-Like criteria. Nalwa and Binford (1986) proposed a local surface fitting model for step edges by using the least square cubic fitting and Tanh function fitting.

Other edge detection methods have also developed based on different theories, such as morphology, wavelet analysis, and diffusion process:

- Nobel (1988) proposed a morphological feature detection method;
- Perona and Malick (1990) proposed an edge detection method based on the anisotropic diffusion;
- Mallat and Zhong (1992) introduced the wavelet method for edge detection via the local maxima of the wavelet transform coefficients;
- Rothwell et al. (1995) proposed a topology description of the edge detection;
- Iverson and Zucker (1995) proposed a logical/linear operator for edge detection;
- Robbins (1996), Kovesi (1997) introduced a local energy model;
- Smith and Brady (1997) proposed the SUSAN edge detector;
- Baker, Nayar and Murase (1998) proposed a parametric feature detection based on differential geometry;
- Ando (2000) proposed the edge/corner detection algorithm based on gradient covariance;
- Meer et al. (2001) proposed the edge detection algorithm with embedded confidence estimation, also known as the EDISON (**E**dge **D**etection and **I**mage **S**egmentati**on**) edge detector;
- Konishi et al. (2003) proposed an edge detection method based on statistical learning;
- Pellegrino et al. (2004) proposed an edge detection method which improved the Canny algorithm based on local energy model.

The works mentioned above only account for a small portion of the works in the edge detection area.

Some have reviewed or surveyed the above mentioned works:

- Torre and Poggio (1984) provided a detailed survey and analysis on the filter and differential operators;
- Smith and Brady (1995) provided a brief review on edge detectors in their paper regarding the SUSAN algorithm;
- Heath, Sarkar and Bowyer (1997) provided a performance evaluation on edge detection algorithm;
- Ziou and Tabbone (1998) provided a detailed survey on the gradient based edge detection techniques;
- Ando (2000) provided a brief review on current edge, corner and vertices detection methods;
- Basu (2002) provided a detailed survey on Gaussian-based edge-detection methods;

Broadly, the above approaches can be classified into several groups: gradient based methods, orientation/spatial analysis based methods, model/template fitting or matching methods, methods based on fuzzy logic, and statistical learning.

2.2 Gradient Based Methods

The gradient based methods are based on the calculation of *edge strength*. Edge strength is measured by the magnitude of the local gradient computed by a local operator, and then is used to construct the edge map, usually a binary map which distinguishes the edge points and non-edge points.

These edge detection methods can be classified by the properties of the difference operators (linear/nonlinear), and can also be characterized by the type and order of the differential operator. The gradient approaches are one of the traditional methods.

The common procedures involved in the gradient edge detection methods are: smoothing, gradient, and detection.

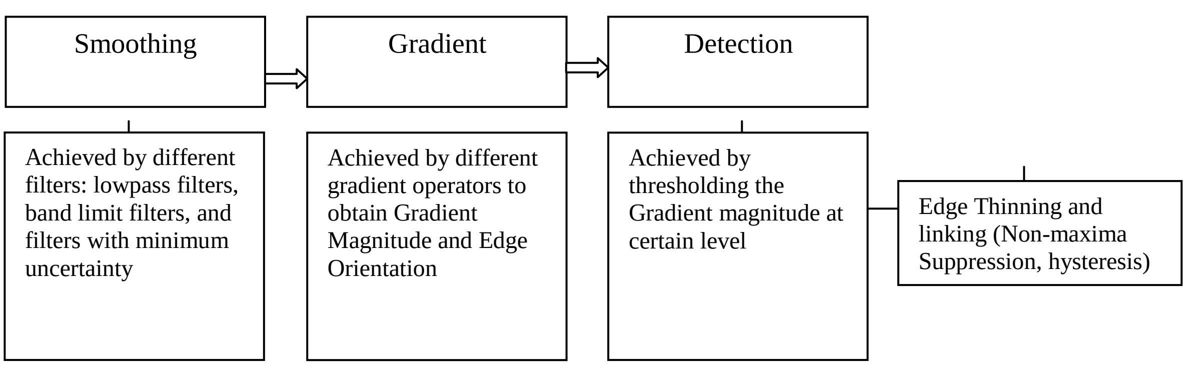
- **Smoothing** is a technique used to reduce the influence of image noise. A tradeoff between noise reduction and edge detection is that the more effectively remove the noise, the larger window and standard deviation are needed for the smoothing operator; however detailed information of image, such as corners and weak lines, may lost due to the "blurring" effect. Smoothing is also a common technique used to reduce the "aliasing" effect in signal processing. Torre and Poggio (1984) pointed out that the necessity of the filtering (smoothing) is due to the regularization for the differentiation.
- **The Gradient** is calculated by a local operator. The commonly used linear gradient operators include first derivative of Gaussian (FDOG), Laplacian of Gaussian (LoG), Difference of Gaussian (DOG). The commonly used nonlinear gradient operators include the morphological operator.

  First derivative: $\nabla I = \sqrt{I_x + I_y}, \theta = \arctan(\frac{I_y}{I_x})$;

  Second derivative: $\nabla^2 I = I_{xx} + I_{yy}$;

  Second derectional derivative along gradient direction: $\frac{\partial^2 I}{\partial^2 n} = \frac{I_x^2 I_{xx} + 2I_x I_y I_{xy} + I_y^2 I_{yy}}{I_x^2 + I_y^2}$

  The calculation of the image gradient is usually approximated by a difference.
- **Edge Detection** is the process used to identify the edge points based on the gradient magnitude and edge orientation information obtained from the gradient procedure. For the first derivative operator, edge points are detected at local maxima, for the second derivative operator, edge points are detected at the zero-crossing. Since the gradient operation also amplifies the noise contained in the image, thresholding is a common technique used to remove the effects of noise. In order to improve the quality of the edge map by suppressing the false edges and retaining the weak edges, two procedures are used: nonmaxima suppression and hysteresis, which are proposed by Canny (1986.)



The limitations with the traditional gradient methods:

- Many of these methods use a single edge model, e.g. they assume edges are step discontinuities. Thus they can't capture other local features in an image. For example, it is well known that Canny's edge detection algorithm works poorly at edge junctions.

- The calculation of the gradient (differentiation) is sensitive to noise, especially for the higher order derivatives. Thus, the evaluation of the edge strength is not accurate. Moreover, it is also well known that the gradient angle estimation is biased, because of the discrete nature of the digital images.
- These methods all rely on intermediate processing, such as thresholding, hysteresis, or possibly a thinning algorithm. For multi-scale edge detection algorithms, they have to combine the information obtained from different scales.
- Thresholding the gradient magnitude is required to remove the noise that has been amplified by the differentiation. However, there is no one unified rule or method to determine which level of threshold should be used. Little effort, compared to the design of new edge detection methods, has been placed in this area.

2.2.1 Gaussian-based Approach

The most widely used smoothing filters are Gaussian filters. Gaussian-based edge detection methods play a very important role in edge, line and feature detection because Gaussian filters possesses some desired properties:

- Gaussian filter (Laplacian of Gaussian) is very similar to the difference of Gaussians (DOG). It is well-known in the approximation of the shape of spatial receptive fields as in the visual system of cats and has also been proposed for humans. This property provides a biological vision support for Gaussian filters.
- Gaussian filter is the only operator for which the scale-space representation of the second derivative shows that existing zero-crossings disappear when moving from finer to coarser scales, but new ones are never created (i.e. false edges are not created) for 1-D and 2-D images. It is also showed that for nonlinear directional derivatives along the gradient direction, there is no filter that does not create zero-crossing as the scale increases. This property is extremely important for multiscale edge detection, a widely used technique in edge detection. (Note: the idea of scale space is derived by Witkin (1986), and Lindberg (1997)).
- Canny's optimal filter can be well approximated by the first derivative of Gaussian (FDOG).
- The Gaussian filter is the only operator that satisfies the *uncertainty relation*

$$\Delta x \Delta w \geq \frac{1}{2}$$

$$(\Delta x, \text{and } \Delta w \text{ are variances in the spatial and frequency domains respectively.})$$

  This property allows the Gaussian operator to give the best tradeoff between the conflicting localization goals in the spatial and frequency domains simultaneously.
- 2-D Gaussian is the only *rotationally symmetric* filter that is *separable* in Cartesian coordinates.
- Derivative of Gaussian can be easily obtained by a recursive algorithm (Deriche, 1994.)

Due to Gaussian filter's attractive properties, a rich class of Gaussian-based methods has been developed in the last two decades. These works include Marr and Hildreth's Laplacian of Gaussian filter (1980) via zero-crossing detection, Canny's edge detection algorithm (1986,) which is shown and actually implemented by the first derivative of Gaussian via local maxima detection. Wikin (1983) proposed scale-space filtering, suggesting that in order to completely describe a 1-D signal over all scales, the 1-D signal should be expanded into scale-space by convolution with Gaussians over a continuum of sizes, then track the extrema through the scale-space.

Multi-scale has become a common edge detection technique. In this approach, Bergholm (1984) proposed an edge focusing algorithm that traces the edge points from a coarse-to-fine scale level. Lindeburg (1994, 1996) formalized the scale space theory and proposed an edge detection and ridge detection method with automatic scale selection (1996.) Chidiac and Ziou (1999) conducted a detailed study on classifying 1-D edge models with respect to FDOG and LOG. However, Badu (2002) pointed out that the major drawback of this approach is the need of calculating high order derivatives, which increases the computational difficulties without significantly improving the result. Due to the fact that a signal after convolution is similar to the solution of a heat equation, Perona and Malick (1990) handled the edge detection problem as anisotropic diffusion by tracing the edge points via coefficients in the heat equation through space and scale. However, backward diffusion is highly sensitive to the slightest perturbations of the initial data. Also, this method breaks down in the presence of staircase type edges. Badu (2002) pointed out that the multiscale Gaussian-based method suffers from two major problems: how to choose the size of the filter; and how to combine the edge information from different scales. The use of the Gaussian filter requires making compromises in order to give the best overall edge-detection performance.

2.2.2 Multi-scale Transformation Approach

Multi-scale analysis, as used in the Gaussian-based approach, is a standard technique in image processing. Some commonly used multi-scale transforms include Fourier Transforms, Wavelet transforms and the more recently developed Ridgelet transforms and Curvelet transforms.

The Fourier transform is widely used in signal processing. However, Fourier transformations (FT) are not suitable for edge detection, because they provide poor representations of non-stationary signals and discontinuities (Gibbs phenomena.) One remedy is to use the short time Fourier transform (STFT.) Based on the advances in theories on solving the Gibson phenomenon, Gelb and Tadmor (2003, 2009) gave some examples on the FT in edge detection. They used the "concentration" method to adapt to the local intensity's sharp changes and applied the method in brain MRI images. The concentration function takes the form

$$S_N^{\sigma}[f](x) = i\sum_{k=-N}^{N} sgn(k)\sigma(\frac{|k|}{N})\hat{f}_k e^{ikx}.$$

Where, $\sigma(\eta) = \sigma\left(\frac{|k|}{N}\right), \eta \in (0,1]$ is the concentration factor, which can take the following forms:

$$\sigma_{Gibbs}(\eta) = \frac{\pi \sin \pi\eta}{Si(\pi)},$$
$$\sigma_{Poly}(\eta) = \pi\eta,$$
$$\sigma_{Exp}(\eta) = \gamma\eta \exp\left(\frac{1}{\alpha\eta(\eta-1)}\right),$$

$$\gamma = \frac{\pi}{\int_{\epsilon}^{1-\epsilon} \exp\left(\frac{1}{\alpha\tau(\tau-1)}\right) d\tau}$$

In order to reduce the oscillations caused by the concentration factor, they employed the nonlinear enhancement by the minmod operator. The problem with this algorithm may be its computational efficiency, as well as feasibility for the brain images, where the generation of close contours is the prime purpose.

Musheng Wei et al. (2007) conducted a case study on Gelb and Tadmor's work and, through the use of simulated data, provided a robust and efficient discontinuity detection method based on polynomial filters, which are equivalent to the Fast Fourier Transform.

The Wavelet transform, which has proven to be a powerful analysis tools in signal processing, is used in digital image compression, image de-noise (Yansun Xu et al. (1994),) and edge detection as well. The Wavelet transform is a representation of signals in terms of basis functions which are obtained by dilating and translating a basic wavelet function. The wavelet transformation has the properties of *locality, multiresolution, compression, clustering, and persistence*. Compared to the traditional Fourier Transform, which is also used in signal processing, one of the advantages of wavelet transform is *locality* (i.e. the ability to locate change both in frequency domain and in time.) The development of wavelet-based edge detection algorithms shares many of the ideas of traditional edge detection methods mentioned above, such as multi-scale decomposition.

From the filter design point of view, all the gradient operators possess bandpass filter characteristics. For example, the LoG can be thought of as a bandpass filter, the bandwidth (e.g. window size) is determined by the variance of the Gaussian, and the smoothing filter is run at various bandwidths. The low-pass and high-pass filters of the wavelet transform naturally breaks a signal into similar (low-pass) and discontinuous/rapidly-changing (high-pass) sub-signals, which effectively combines the two basic properties into a single approach.

Mallat and Zhong (1992) implemented a multi-resolution Canny edge detector via local maxima of the gradient modular or the zero-crossing of the wavelet transform coefficient.

For 2-D image edge detection, Mallat followed Canny's idea by using non-maxima suppression and hysteresis. Similar to the Gaussian-based methods, this wavelet-based method also suffers from problems such as scale selection and efficient combination of edge information through different scales.

Continuing Mallat and Huwang's work in detecting singularities via local maxima of the wavelet transform coefficient, Ducottet et al. (2004) proposed an edge detection and characterization method based on the wavelet transform. They studied the three common edge types: transition edge (e.g. step edges), peak edges and line edges. They examined the evolution of the wavelet transform maxima via a wavelet maxima function (WMF.) Considering the image convolved with a first derivative of Gaussian kernel with amplitude A and σ, the WMFs for the three edge types at the wavelet transform with scale "s" are given by:

- $\text{Transition: } \mathrm{MT}_{\sigma}(s) = \frac{A}{\sqrt{2\pi}}\frac{s}{\sqrt{s^2+\sigma^2}}$.
- $\text{Peak: } \mathrm{MP}_{\sigma} = \frac{A}{\sqrt{e}}\frac{s\sigma^2}{(s^2+\sigma^2)^{3/2}}, \text{ maixma at } s = \sigma/\sqrt{2}$.
  (For fixed scale, modulus local maxima are located at a circle centered (0, 0) with radius$\sqrt{s^2+\sigma^2}$.)
- $\text{Line: } \mathrm{ML}_{\sigma} = \frac{A}{\sqrt{e}}\frac{s\sigma}{s^2+\sigma^2}, \text{ maxima at } s = \sigma$.

The edge detection and characterization are determined by calculating the local maxima at each scale following Canny's approach (e.g. nonmaxima suppression in the gradient direction) then comparing to the above edge model by least-squares fitting via chasing local maxima across different scales by a maxima tree (fine to coarse.) Ducottet et al. also argued that the localization scale "s" has to be adapted to the smoothing size and, for more blurred edges, a greater localization scale has to be used. However, they only tested their method on synthetic images and one real image. In addition, their results were not compared with other edge detection algorithms.

Another wavelet-based multi-resolution edge detection algorithm can be found in the work of Ming-Yu Shih and Din-Chang Tseng (2004.) They used a discrete wavelet transform to generate a shift-invariant gradient through a hierarchical subband system. Four decompositions (the original, right shift, down shift, and right down shift) and three scale gradients are used for their algorithm. The edges are identified by larger wavelet coefficients. The shift-invariant gradient image is generated by summing up the corresponding gradient magnitudes in four gradient images. The edge points are determined through a neighborhood logic operator and single threshold. The local maxima tracking across the scale is from a fine to coarse approach, since the finest scale edge map provides the best starting points. The end points for all extracted edges are taken at the starting points for tracking. For maxima at different scales, they assumed that there were lines between them. They also tested six wavelet filters, Haar, Daubechies, Symmlets, Biorthogonal, Coiflets, and Meyer, among which Haar generated the best results.

Liu et al. (2005) implemented wavelet-transform based edge detection in order to estimate the snowmelt onset, end and duration from the passive microwave measurements.

The Ridgelet (Candes, 1998) and Curvelet transforms (Donoho and Candes, 2000) are two other multi-scale transforms. Candes and Donoho (2005) showed that, compared to the wavelet representation in 2-D, which is isotropic, curvelets are direction sensitive and highly anisotropic, and thus are an optimally sparse representation of otherwise smooth objects. S. Dekel and A. Sherman (2009) stated that a curvelet provides an excellent time-frequency-orientation localization. They also gave a very simple example of its application in edge detection.

2.2.3 Morphology Approach

Developed by Matheron and Serra in the 1970's, mathematical morphology uses set theory in image analysis for the creation of a boundary skeleton. It is also used for pre and post image processing tasks, such as de-noising, thinning, and thickening.

Morphological gradient operators enhance variations of pixel intensity in a given neighborhood. There are two basic operations for morphological gradient operators: dilation and erosion.

- Dilation, which expands the feature, is defined as:

$$\delta_B(f) = f \oplus B = \max_{\forall [i,j] \in B} [f(m-i, n-j) + b(i,j)]$$

- Erosion, which shrinks the feature, is defined as:

$$\epsilon_B(f) = f \ominus B = \min_{\forall [i,j] \in B} [f(m-i, n-j) - b(i,j)]$$

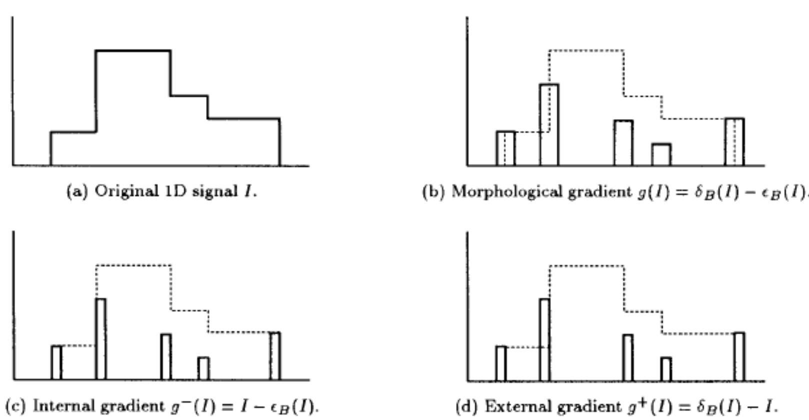
  Where f is the original image and B is a structuring element, e.g. a small neighborhood operator, which can be a square, disk or ring.

The most widely used morphological gradient operator is the Beucher gradient operator, which can be defined in the discrete case as:

$$g(f) = \delta_B(f) - \epsilon_B(f)$$

The Beucher gradient is a general-purpose gradient with good properties of symmetry and a good compromise between thinness and noise immunity. However, in some applications the result may be too thick and thinning gradients must be used.

Other morphological operators include the internal gradient (Dilation residual), external gradient (erosion residual), regularized gradient, directional gradients, and thinning and thickening gradients (Rivest et al. *morphological gradients*). Graphs for the internal gradient and external gradient are as follows:



(Rivest et al. *morphological gradients,* figure 3).

Nobel (1988) investigated mathematical morphology in image feature detection for features such as boundaries, ridges and corners. A more recent study is provided by Zhao Yu-qian et al. (2005,) in which they applied a reduced noise morphological operator to medical images. Roushdy (2006) conducted a comparative study showing that morphological filtering can be applied as preprocess to filter noise.

Smith and Brady (1995, 1997) proposed SUSAN (Smallest Univalue Segment Assimilating Nucleus) edge detection algorithm, which is frequently cited in recent literature. This non-linear technique indexes a circular mask-Univalue Segment Assimilating Nucleus (USAN) over the image and at each location determines the area of the mask having similar intensity values to the central pixel value, referred as the nucleus. The locations in the image where the USAN is locally at a minimum, mark the positions of step and line features. Significant features for this detector include tolerance to noise and running very fast. However, the detector is not invariant to image contrast because it requires the setting of a threshold which is used to decide whether or not elements of the mask are ‘similar’ to the central pixel value, thus determining the minimum edge contrast that can be detected.

Recent advances include: Xiaoxin Guo et al. (2005) proposed an adaptive Edge Detector based on combination of morphological operator and LoG; Xiangzhi Bai and Fugen Zhou’s (2007) edge detector on contour based morphological operator; Li Ding and Han Chongzhao (2008) proposed an edge detector based on morphological operator and rough set theory.

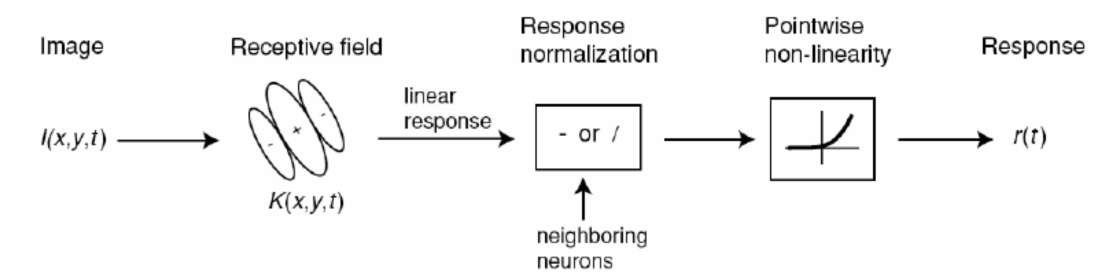
2.2.4 Topology Approach
Rothwell et al. (1997) proposed an edge detection method based on a complete description of topology. They pointed out that the traditional edge detection methods, such as Canny, focused on the geometric description of image, such as localization and signal to noise ratio performance. However, such edge detectors usually perform poorly at junctions mainly due to unreliable edge orientation estimation. Thus these methods can’t provide complete and reliable topological descriptions. They stated that the topology of the contour description should be as close as possible to the projection of the 3-D scene topology. To achieve this, they followed Canny’s approach for filtering and sub-pixel location. Then, instead of hysteresis, they applied dynamic thresholding based on the edge point candidates passed through a pre-set threshold.

Bergevin and Bubel (2004) proposed a study on detection and characterization of junctions in 2-D images. They gave a good summary on junction detection. They also proposed an interesting junction detection algorithm based on topology, which provided a good location of junctions. Although their algorithm is computation intensive, the authors stated that the accuracy can be reduced in order to make the detection more efficient, depending on the requirement of the application.

2.3 Orientation/Spatial Analysis Based Methods
Considering the spatial character of edges in 2D images, this approach emphasizes on the orientation of a gradient rather than its strength. This approach includes Zuniga and Haralick’s directional derivative operators (1987) and Gregson’s angular dispersion operator (1993.) These methods rely on the coherency of the gradient direction near edges, hence they are invariant with respect to grayness dimension. The problem with these methods is their difficulty to extract reliable directions from noisy images, where the orientation estimation (for directional derivative) is usually biased.

Freeman and Adelson (1991) proposed the design of steerable filters and steerable pyramids, which use different filters that are tuned in different directions in order to extract image features. Simoncelli and Freeman (1995) also proposed a steerable pyramid, a flexible architecture for multi-scale derivative computation.

The Gabor filter is one of the commonly used band-limit filters. It is often used in texture discrimination, face recognition, handwriting recognition, and voice/speech recognition. The use of the Gabor filter in computer vision has sound biological support. Jones and Palmer (1987) showed that the real part of complex Gabor functions fits well with the receptive field weight functions found in simple cells in the cat striate cortex. Hubel and Weisel (1965,) and Olshausen and Field (2005) showed that in the visual system of higher mammals, the first stage of processing occurring in the visual area V1, consists of the convolution of the retinal image with filters having different orientation, size and shape (see the following graph.) The Gabor filter can be thought of as the "time-frequency view" point of signal representation. The Gabor filter can also be tuned spatially to certain directions and uses quadratic pairs, even and odd, of Gabor functions. The outputs from the odd and even Gabor functions can be combined to provide local frequency information. Kovesi (1997) and Pellegrino (2004) provided application of Gabor filters in edge detection.



(Model of human low-level vision system, graph taken from Olshausen and Field, 2005)

Another spatial analysis approach that has gained more attention is the phase congruency or local energy feature edge detection model. Marrone and Owens (1987) pointed out that image features occur at points of high congruency in the phase domain of an image signal, e.g. feature information is encoded at points where the phase angle deviation of the components of the frequency representation is small. This idea was further developed by Venkatesh and Owens (1989,) Robbins (1996,) Owens and Ross (1996,) and Kovesi (1997, 2002.)

Phase congruency is derived from the frequency representation of the image via the Fourier transform:

$$F(x) = \sum A_n \sin(2\pi n x + \emptyset_n)\,,\ \emptyset_n - \text{phase offset of the nth component}, A_n \text{ is amplitude}$$

At points of maximal phase congruency, there is order in the image data and thus such points are high in information. The phase congruency is also proportional to local energy, therefore the local maxima in the phase congruency correspond to local maxima in the local energy. The local energy of a 1-D signal can be defined as the square root of the sum of squares of the signal convolved with a quadrature pair of filters, consisting of an even and an odd symmetric filter that have zero mean and are orthogonal.

Although Marrone and Owens (1987) showed that two orthogonal orientations are sufficient to give a global measure of local energy, Kovesi (1997, 2002) implemented the local energy model via wavelet and Gabor filters in 12 directions (30 degrees.)

The advantages of the local energy approach are:

- The local energy model does not have specified local feature model, e.g. does not require prior knowledge of the image. This seems to provide a universal approach to detect edges and other local features
- The local energy is based on the spatial frequency domain, and thus is invariant to intensity.

The applications of the local energy model can be found at Zaafouri et al. (2010) on satellite images which contain edge, lines and other complex local features, and Linguraru et al. (2003) on the MR images in the analysis of the human brain.

2.4 Model/Template Fitting or Matching Methods

This approach is based on the fitting or matching of particular models to the image field. It includes Ronsenweld's compass gradient template (1972,) Haralik's method of fitting a cubic facet and detecting a zero crossing of the second order derivative (1984,) Nalwa and Binford's method of fitting a one-dimensional surface to hyperbolic tangential functions (1986,) Chen and Yang's method of fitting a B-spline with regularization (1995,) Baker et al.'s parametric feature detection based on the parametric manifold (1998,) and Meers et al.'s method of edge detection with embedded confidence estimation (2001.) The Hough Transform (Duda and Hart, 1972) and the generalized Hough Transform can also be classified as the model matching approach.

It is worth mentioning that there is a rich class of techniques called deformable template matching, which are used in image segmentation, such as face recognition, object matching, and medical image construction. An example can be found in Kass et al.'s (1988) Snakes-active contour models. A snake is an energy-minimizing spline (e.g. salient image contours) guided by external constraint forces and influenced by image forces that pull it toward features such as lines and edges.

2.5 Edge Detection based on Other Theories

There is still a huge amount of edge detection algorithms that can't be classified in the above three groups, such as methods based on fuzzy logic and statistical learning. Here are some examples that we encountered during our research. The problem with these methods may be simply that they are less noticed.

Konishi et al. (2003) provided an edge detection algorithm based on statistical learning. They used pre-segmented images to learn the probability distributions of filter responses conditioned on whether they are evaluated on or off an edge. Edge detection is formulated as a discrimination task specified by a likelihood ratio test on the filter responses. This approach emphasizes the necessity of modeling the image background (the off-edges.)

Karen and Wharton (2008) proposed a logarithmic edge detection algorithm, which is insensitive to the change of image intensity based on Parameterized Logarithmic Image Processing (PLIP.)

Alshennawy et al. (2009) proposed an edge detection algorithm based on the fuzzy logic technique.

Arandigia et al. (2009) proposed an edge detection algorithm, the ENO-EA edge detector, which is insensitive to the changes of illumination, based on ENO-EA (Essentially-Non-Oscillatory Edge-Adapted) reconstruction.

Ganguly et al. (2009) proposed an edge detection method based on a K-means algorithm using the artifacts inputs-mean, standard deviation, entropy and busyness, obtained through a 3 by 3 window calculated from the image.

3. SOME COMMONLY USED EDGE DETECTION ALGORITHMS

Some commonly used edge detection algorithms include Canny (1986,) Shen-Castern (1986,) Nalwa-Binford (1986,) Bergholm (1984,) Iverson-Zucker (1995,) Rothwell (1995,) Edison (2001,) SUSAN (1997,) and the Hough Transform. The reason we chose these algorithms is because they have been tested and evaluated by other authors. Although these authors may use different criterion to evaluate these algorithms, we can still have some idea as to how well, and under what conditions, these algorithms work. The following table summarizes these algorithms.

	Description	Parameter Setting
Canny	"Standard method"	Three parameters: sigma-Gaussian filter, low and high hysteresis thresholds
Nalwa and Binford	Surface fitting	
Bergholm	Scale space filtering	
Iverson-Zucker	Improves linear edge detection by including logical checks	Modified Inverson-Zucker algorithm allowed user to specify three parameters: direction parameter (4-10), low and high hysteresis thresholds
Rothwell	Dynamic thresholding	Three parameters: the smoothing amount-sigma, the edge threshold, and a parameter, alpha, that adapts the edge threshold to increase the pixels that are near other edges.
EDISON (Edge Detection and Image Segmentation, Meer and Georgescu 2001)	Template-matching /Statistical approach	Nine parameters
SUSAN	Morphology approach	Size of USAN, t-constrast, g-threshold parameters for USAN
Hough Transform	Template-matching	End point setting for finite features

4. EDGE DETECTION ALGORITHMS FOR SPECIAL TASKS

Because of the geometric and photometric complexity of real images, there is no one edge detection algorithm that will work for all the images. In fact, common edge detectors experience various problems near junctions and low-contrast or high contour intensity areas in 2-D images. Thus, different edge detection algorithms have been developed for special tasks, such as interest point and corner detection, as well as face detection.

4.1 2-D Image Feature Detection Algorithms

The most common approach for 2-D edge detection is to treat image as an intensity surface and use derivatives and curvature measurements of the surface to detect 2-D image features. Beaudet (1978) suggested that saddle points on the intensity surface were candidates for corner features via the determinant of the Hessian matrix. Deschler and Nagel (1982) also used Beaudet's Gaussian kernel to determine the "Gaussian curvature."

Moravec (1977, 1979) developed a method of 2-D feature detection without an explicit model. Instead, he looked for "points of interest" where there was a small area of large intensity variation. Following this approach, Harris and Stephens (1988) estimated the autocorrelation from the first derivatives of the image using a Gaussian convolution kernel. Harris' corner edge detector, also known as the Plessey operator, is one of the most commonly used corner detectors. Heitger et al. (1992) proposed a new approach based on simulating cortical simple, complex and end-stopped cells in biological visual systems. Odd and even symmetrical orientation selective filters are combined to estimate the local energy of 1-D image features, and differentiation along certain orientations is used to detect 2-D features. Forstner (1994) proposed a 2-D feature detection method based on the analysis of the local gradient field at an image point. Smith and Brady (1997) proposed a SUSAN detector.

An earlier detailed survey can also be found in Robbins's PhD thesis (1996) regarding local energy detection. Another detailed review can be found in Schmid et al. (2000.)

	Tested Algorithms
Schmid et al. (2000)	Harris (Harris and Stephens, 1988)
	Improved Harris
	Cottier (1994)
	Horaud (1990)
	Heitger (1992)
	Forstner (1994)
Cooke et al. (2006)	Kitchen-Rosenfeld detector
	Paler detector
	Harris detector
	SUSAN detector
	Modified local energy detector
	Generalized Hough Transform
	Shi-Tomasi detector (Template matching based corner detector)

4.2 Feature Detection

Mikolajczyk et al. (2006) provided a comparison of affine region detectors. They studied 6 affine covariant region detectors:

- Harris Affine detector
- Hessian Affine detector
- MSER (maximally stable extrema region)
- An edge-based region detector
- An intensity extrema-based region detector
- An entropy-based region detector

Tyutelaar et al. (2007) gave an exhaustive local invariant feature detection survey, which included 266 relevant research papers in this area.

In general, good feature detectors shall have following properties: *Repeatability, Distinctiveness, Informativeness, Locality, Quantity, Accuracy, and Efficiency.* Based on the underlying theories, current feature detectors can be categorized as:

- Contour Curvature Based Methods
- Intensity Based Methods
- Biologically Plausible Methods
- Color-based Methods
- Model-based Methods
- Toward Viewpoint Invariant Methods
- Segmentation-based Methods
- Machine learning-based Methods

According to different types of features, respective detectors can be categorized as: *corner detectors, blob detectors,* and *region detectors.*

- *Corner:* detected points correspond to the points in the 2-D image with high curvature. They do not mean the projections of 3-D corners.
- *Blob:* local regions correspond to a single object or part thereof. A well-known example is the blobworld system proposed by Carson et al. (2002.)
- *Region:* can be considered as a larger local feature than a blob.

Corner Detectors	Harris Detector (based on auto-correlation matrix)	Intensity Based
	SUSAN Detector	Morphological Approach
	Harris-Laplace (Harris-Affine)	Scale Invariant
	Edge-based Regions	Affine invariant
Blob Detectors	Hessian Detector	Intensity based (second derivative of Gaussian)
	Hessian-Laplace	Intensity based (second derivative of Gaussian)
	Salient region	Information theory
	DoG (difference of Gaussian)	Aimed for efficiency
	SURF (speeded-up robust feature)	
Region Detectors	Intensity-based region	Intensity based/small region
	MSER (Maximally stable extremal region)	Intensity based/small region
	Segmentation based method (Superpixel)	

Tyutelaar et al. (2007) also provided evaluations of feature detectors, which will be discussed in section 6.

4.3 Face Detection

A good survey of face detection can be found in Ming-Hsuan Yang (2002,) containing 181 reference papers. Another survey on face detection can be found in Hjelma et al. (2001,) containing 224 reference papers.

A brief summary of the methods is as follows (Ming-Hsuan Yang (2002)):

TABLE 1
Categorization of Methods for Face Detection in a Single Image

Approach	Representative Works
Knowledge-based	
	Multiresolution rule-based method [170]
Feature invariant	
– Facial Features	Grouping of edges [87] [178]
– Texture	Space Gray-Level Dependence matrix (SGLD) of face pattern [32]
– Skin Color	Mixture of Gaussian [172] [98]
– Multiple Features	Integration of skin color, size and shape [79]
Template matching	
– Predefined face templates	Shape template [28]
– Deformable Templates	Active Shape Model (ASM) [86]
Appearance-based method	
– Eigenface	Eigenvector decomposition and clustering [163]
– Distribution-based	Gaussian distribution and multilayer perceptron [154]
– Neural Network	Ensemble of neural networks and arbitration schemes [128]
– Support Vector Machine (SVM)	SVM with polynomial kernel [107]
– Naive Bayes Classifier	Joint statistics of local appearance and position [140]
– Hidden Markov Model (HMM)	Higher order statistics with HMM [123]
– Information-Theoretical Approach	Kullback relative information [89] [24]

[Note: More recent advances in face recognition (2004) can be found at his website: http://vision.ai.uiuc.edu/mhyang]

5. COLOR EDGE DETECTION ALGORITHMS

In 1977, Ramakant Nevatia of USC published the first journal paper on color edge detection. Since then a large number of conference papers have been written. The fundamental difference between color images and gray-level images is that, in a color image, a *color vector* (which generally consists of three components) is assigned to a pixel, instead of having a scalar assigned to each pixel as in gray-level images.

While edge detection in gray-level images is a well-established area, edge detection in color images has not received the same attention. The challenges for edge detection in color images are:
The existence of hue, saturation, and illumination in color images makes it quite difficult to give a proper definition of “edge” in color images which is closely related to the human perception. For example, the boundary between two regions with the same contrast but different hue will be perceived as an ‘edge’, where the traditional monochromatic edge detection algorithm will fail. The presence of hue, saturation, shading, shadows, transparencies and highlights makes it hard to accurately locate the boundaries, e.g. edges.

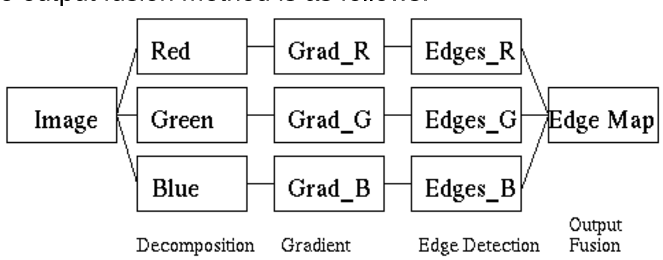
- The traditional gradient magnitude estimation and edge orientation estimation are more complex both in vector space and in color spaces. For example, for different color channels, the estimated gradient may have the same magnitude but point in different directions. For some color spaces, such as HSI, obtained through a nonlinear transformation from RGB space, the problem is the singularity at the axis of the color cylinder where R=G=B=0 or saturation=0. In addition, Hue is unstable at low saturations and has non uniform effects on noises.
- The existence of different and complex color spaces, such as HSI and CIE L*u*v, makes it quite difficult to develop a general color edge detector suitable for all color spaces. A color edge detection algorithm is highly related to the color space that it is applied.

A comparative study on color edge detection can be found at Koschan (1995.) Zhu et al. (1999) conducted a comprehensive analysis on edge detection in color image. Koschan (2005) provided a brief survey on the vector-valued techniques on detection and classification of edges based on a dichromatic reflection model (DRM.) A more detailed review can also be found in Sarif Kumar Naik and C. A. Murthy (2005.)

Color edge detection techniques can be separated into two classes: *Monochromatic-based techniques* and *Vector value-based techniques. Monochromatic-based techniques* treat information from individual color channels separately and then combine the results. This approach can be further classified into *output fusion methods* and *multidimensional gradient methods*.

The output fusion methods, which appear to be the most popular, perform edge detection three times, once each for red, green, and blue, and the output is fused together to form one edge map. Nevatia (1977) designed the first output fusion method based on Hueckel’s edge detector. Shiozaki (1986) found entropy in each component using a local entropy operator and merged three values for color edge detection.

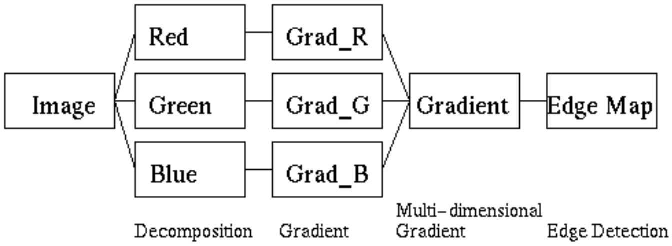
The procedure for the output fusion method is as follows.



(Graphs are taken from Mark Ruzon’s website http://ai.standford/~ruzon/compass/color.html)

The multidimensional gradient methods calculate the gradient in 3 different channels and then combine these gradients to generate the multi-dimensional gradient, which is used to generate the edge map. Fan et al. (2001) proposed a method where they find edges in the YUV color space. Edge magnitudes are found individually and thresholded within each component and then merged. (Note: this classification can be found in Ruzon and Tomasi (2001).)

The procedures for multidimensional methods are as follows.



(Graphs are taken from Mark Ruzon's website http://ai.standford/~ruzon/compass/color.html)

Vector value-based techniques treat the color information as color vectors in a vector space provided with a vector norm. Cumani (1991) proposed an extension of the second-order directional derivative in color images via zero crossings of the image surface. Koschan (2005) pointed out that Cumani's algorithm cannot be easily used for HIS, CIELUV or CIELAB color spaces since it always assumes that a Euclidian metric exists for an n-dimensional space. Yang and Tsai (1996) tried to find, for each 8 by 8 image block, the best axis in color space on which to project the image data, creating a single-band image. Another method is presented by Trahanias and Venetsanapoulos (1996,) who used vector order statistics to compute a variety of statistical measures of edge detection. Inspired by the morphological approach in gray-level images, the scheme is to detect and combine the minima and maxima of the image function. However, there is no "min-max" operator for vectors and since the ordering of vector-valued data can't be uniquely defined, a number of ways have been proposed. These ways include marginal ordering (M-ordering), reduced aggregate ordering (R-ordering), partial ordering (P-ordering) and conditional ordering (C-ordering.) Zhu et al. (1999) pointed out that R-ordering reduces each multichannel variable to a scalar value according to a distance criterion.

5.1 Color Variants of The Canny Operator

Kanade (1987) adopted the Canny approach into color edge detection. In RGB space, the pixel value is the vector C= (R, G, B.) The variation of C, image function at any point (x, y) is given by the Jacobian Matrix J,

$$J = \begin{pmatrix} R_x & R_x \\ G_x & G_x \\ B_x & B_x \end{pmatrix} = (C_x, C_y), \text{where } R_x = \frac{\partial R}{\partial x}$$

The direction along the largest change is represented by the eigenvector of $J^T J$ corresponding to the largest eigenvalue. The orientation of a color edge is determined by

$$\tan(2\theta) = \frac{2C_x C_y}{\|C_x\|^2 - \|C_y\|^2}.$$

The magnitude m of a color edge is determined by

$$m^2 = \|C_x\|^2 \cos^2(\theta) + 22C_x C_y \sin(\theta)\cos(\theta) + \|C_y\|^2 \sin^2(\theta).$$

To combine the information from 3 color channels, different norms can be applied in above equation, including the L1 norm (sum of absolute values), L2 norm (Euclidian norm) or L∞ norm (maximum of absolute values).

5.2 Cumani Operator

Cumani (1991) proposed a method based on the second partial derivative of the image function. The squared local contrast for pixel (x, y), **S(p; n)** is defined as the directional derivative in the direction of unit vector $n = (n_1, n_2)$,

$$S(p;n) = Kn_1^2 + 2Fn_1, n_2 + Hn_2^2$$

$$K = \sum_{i=1}^{3} \frac{\partial C_i}{\partial x}\frac{\partial C_i}{\partial x}, F = \sum_{i=1}^{3} \frac{\partial C_i}{\partial x}\frac{\partial C_i}{\partial y}, H = \sum_{i=1}^{3} \frac{\partial C_i}{\partial y}\frac{\partial C_i}{\partial y},$$

The eigenvalues of the matrix

$$A = \begin{pmatrix} K & F \\ F & H \end{pmatrix}$$

coincide with the extreme values of **S(p; n)** and are obtained if **n** is the corresponding eigenvector.

The eigenvalue with the strongest magnitude and the normal vector are given by

$$\lambda_+ = \frac{K+H}{2} + \sqrt{\frac{(K+H)^2}{2} + F^2}, n_+ = (\cos\theta_+, \sin\theta_+),$$

$$\theta_+ = \begin{cases} \frac{\pi}{4}, (k-H) = 0 \text{ and } F > 0 \\ \frac{-\pi}{4}, (k-H) = 0 \text{ and } F < 0 \\ \text{undefined}, K = F = H = 0 \\ \frac{1}{2}\arctan\left(\frac{2F}{K-H}\right), \text{otherwise.} \end{cases}$$

Thus, the first derivative along the unit vector n can be defined as

$$D_s(p;n_+) = \nabla\lambda_+ \, n_+ = K_x n_1^3 + (K_y + 2F_x)n_1^2 n_2 + (H_x + 2F_y)n_2^2 n_1 + H_y n_2^3.$$

The edge can be detected by finding the zero-crossings of $D_s(p;n_+)$.

The Cumani operator always assumes that a Euclidian metric exists, and thus the technique can't be easily used for HIS or CIE L*u*v color spaces.

5.3 Operators based on Vector Order Statistics

Let image vectors in a window W be denoted by $x_i, i = 1,2 \dots, n$ and let $D = (x_i, x_j)$ be a measure of distance between vectors x_i and x_j. The reduced scalar quantity associated with x_i is defined as

$$d_i = \sum_{k=1}^{n} D(x_i, x_j), i = 1,2,\dots n.$$

The arrangement of the d_i in ascending order corresponds to the same ordering of

$$x^{(1)} \le x^{(2)} \le \cdots \le x^{(n)}.$$

In an ordered sequence $x^{(1)}$ is the vector median and high rank vectors are considered as outliers.

The vector range (VR) is the simplest color edge detector, however it is sensitive to noise.

$$VR = D\left(x^{(1)}, x^{(n)}\right).$$

The vector dispersion edge detector (VDED) is a linear combination of the ordered vectors

$$VDED = \left\| \sum_{i=1}^{n} \alpha_i \, x^{(i)} \right\|.$$

The coefficient can be chosen to reduce noise.

One proposed class of operators is the minimum vector dispersion (MVD) detector, defined as

$$MVD = \min_j \left\{ D\left[x^{(n-j+1)}, \sum_{i=1}^{l} \frac{x^{(i)}}{l} \right] \right\} \; j = 1,2, \ldots, k \; k \text{ and } l < n.$$

The choice of k and l depend on n, the size of the window. These two parameters control the compromise between noise reduction and computational complexity.

Another family of operators is the nearest-neighbor vector rank (NNVR), a generalized VDED.

$$NNVR = D[x^{(n)}, \sum_{i=1}^{n} w_i \, x^{(i)})].$$

The operator is defined as the distance between the outlier and the weighted sum of all ranked vectors. The weight w_i is determined adaptively for each location/pixel. Usually the weight coefficients are non-negative and sum to 1. The MVD and NNVR can also be combined. The L2 norm (Euclidean norm) is most commonly used.

6. EVALUATION OF EDGE DETECTION ALGORITHMS

Since there is no single edge detection algorithm that will perform well under all conditions, there is no single rule or criterion that can be used to evaluate the existing edge detection algorithms. As Song Wang et al. (2005) pointed out, “A major dilemma in edge-detection evaluation is the difficulty to balance the objectivity and generality: a general-purpose edge-detection evaluation independent of specific applications is usually not well defined, while an evaluation on a specific application has weak generality.”

6.1 Evaluation of Commonly Used Edge Detection Algorithms

The edge-detection evaluation methods can be categorized in several ways.

- *Subjective and Objective methods*. The former uses the human visual observation and decision to evaluate the performance of edge detection. Given the inherent inconsistency in human perception, subjective evaluation results may exhibit a large variance for different observers. In objective methods, quantitative measures are defined based solely on images and the edge-detection results.
- *Ground truth*. With ground truth classification, edge detection methods can be quantitatively evaluated in a more credible way. Without the ground truth, some local coherence information is usually used to measure the performance.
- *Test images*: synthetic-image-based methods and real-image-based methods.

More detailed discussions on various edge detectors and edge-detection evaluation methods can be found in Salott et al. (1996.) and Health et al. (1997.)

Heath et al. (1997) compared five edge detection algorithms: Canny, Nalwa, Iverson, Bergholm, and Rothwell. They used 28 images categorized by man-made vs. natural, and textured vs. non-textured. Their evaluation is based on the subjective evaluation of edge images by people, followed by statistical analysis. The test results showed the following ranking.

Fixed Parameter	(Canny, Nalwa) < Bergholm
Adapted Parameter	(Iverson, Nalwa) < (Rothwell, Bergholm, Canny)

They suggested that the choice of edge detection algorithm may depend on its application. For example, high level processing may prefer Canny's algorithm which will provide better quality by adjusting the parameter manually.

Bowyer et al. (1999) compared eight edge detectors: Sobel, Canny, Bergholm, Sarkar and Boyer, Heitger, Rothwell, Black, and SUSAN. They used 20 images and ground truth was manually created for each image. The performance of these edge detectors was evaluated by receiver-operating-characteristic (ROC) curves. Their findings were:

- Canny and Heiteger edge detector outperform the other detectors.
- All these edge detectors are not able to detect certain features, which provide evidence against the development of a hybrid edge detector.

Song Wang et al. (2005) evaluated five edge detectors: Sobel, LoG, Canny, Rothwell, and Edison. They collected 1030 true images and created the ground truth of the object boundary manually. The performance of the edge detectors was evaluated by the detection of closed boundaries of objects in these images. Their main findings are:

- The overall performances of the five edge detectors are very similar.
- The selection of the detector parameters has significant impact on the final performance.
- The evaluated edge detectors do complement each other.

Hence, their suggested the following future research directions: improving image-dependent-detector-parameter selection and boosting performance with hybrid edge detectors.

Argialas (2005) evaluated eight edge detectors: Canny, Rothwell, Black, SUSAN, Iverson-Zucker, Bezdek, Edison, and Generalized Hough transform (Fitton and Cox, 1998.) They used two satellite images of a geothermal terrain. The performance was evaluated by the Rosenfeld evaluation metric (local edge coherence) and the Pratt evaluation metric (function of the distance between correct and measured edge positions.) Their findings are:

- The Canny edge detector performs best with the Rothwell algorithm to others.
- Hough transform suffers from the localization problem. It can only provide straight lines which can't capture the true, detailed features of the image, and it is also hard to set the starting and stopping criterion.

6.2 Evaluation of Corner, Interest Point, Feature Detection Algorithms

Schimd (2000) summarized the evaluation methods for feature detectors as:

- *Ground-Truth Verification*
- *Visual Inspection*
- *Localization Accuracy*
- *Theoretical Analysis*
- *A specific task*

Evaluation results of Tyutelaar et al. (2007) are listed below.

Feature Detector	Corner	Blob	Region	Rotation invariant	Scale invariant	Affine invariant	Repeatability	Localization accuracy	Robustness	Efficiency
Harris	√			√			+++	+++	+++	++
Hessian		√		√			++	++	++	+
SUSAN	√			√			++	++	++	+++
Harris-Laplace	√	(√)		√	√		+++	+++	++	+
Hessian-Laplace	(√)	√		√	√		+++	+++	+++	+
DoG	(√)	√		√	√		++	++	++	++
SURF	(√)	√		√	√		++	++	++	+++
Harris-Affine	√	(√)		√	√	√	+++	+++	++	++
Hessian-Affine	(√)	√		√	√	√	+++	+++	+++	++
Salient Regions	(√)	√		√	√	(√)	+	+	++	+
Edge-based	√			√	√	√	+++	+++	+	+
MSER			√	√	√	√	+++	+++	++	+++
Intensity-based			√	√	√	√	++	++	++	++
Superpixels			√	√	(√)	(√)	+	+	+	+

The above detectors are categorized into 4 groups (row 1 to row 4) by their invariance: *rotation, similarity, affine and prospective*.

7. CONCLUSION/OBSERVATION

There are many fruitful researches in the field of local feature detection algorithms. The effectiveness of these algorithms is problem dependent. Therefore, we can expect more procedures to be developed. It is interesting to the authors to notice that, after nearly 20 years Canny proposed his edge detection algorithm, there were few other edge detection algorithms surpassed Canny's method in general.

One direction of future research, perhaps, is to develop a "data mining" type of automated system that works for a general type of images. With the creation of these systems, more meaningful applications will follow and more values will be created by the advances of the researches and technology.

8. REFERENCES

[1] Aghababaie, Majid and Gholamali Rezairad, "Neural Network Implementation of Image Edge Detectors," Fifth International Conference on Computer Graphics, Imaging and Visualization, August 2008.

[2] Alshennawy, Abdallah A. and Ayman A. Aly, "Edge Detection in Digital Images Using Fuzzy Logic Technique," Proceedings of World Academy, of Science, Engineering and Technology Volume 41, May 2009.

[3] Alvarez, Luis, Pierre-Louis Lions, and Jean-Michel Morel, "Image Selective Smoothing and Edge Detection by Nonlinear Diffusion. II," SIAM Journal on Numerical Analysis, Vol. 29, No. 3, p.845-866, June 1992.

[4] Ando, S., "Image Field Categorization and Edge/Corner Detection from Gradient Covariance," IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 22(2), pp. 179-190, 2000.

[5] Arandiga, F., A. Cohen, R. Donat and B. Matei, "Edge Detection Insensitive to Changes of Illumination in the Image," Image and Vision Computing, In Press, Corrected Proof, Available online 19 September 2009.

[6] Archibald, Rick, Anne Gelb and Jungho Yoon, "Determining the locations and discontinuities in the derivatives of functions," Applied Numerical Mathematics 58 (2008) 577–592.

[7] Argialas, D.P. and O.D. Mavrantza, "Comparison of Edge Detection and Hough Transform Techniques for the Extraction of Geologic Features," Geo-Imagery Bridging Continents XXth ISPRS Congress, Istanbul, Turkey, Commission 3, pp. 790-795, 2004.

[8] Aw, Y.K., R. Owens and J. Ross, “A Catalogue of 1-D Features in Natural Images,” CVGIP – Graphical Models and Image Processing, 56(2):173–181, March 1994.

[9] Babaud, J., A.P. Witkin, M. Baudin, and R.O. Duda, “Uniqueness of the Gaussian Kernel for Scale-Space Filtering,” IEEE Transactions of Pattern Analysis and Machine Intelligence, 8(1):26-33, Jan 1986.

[10] Bai, Xiangzhi and Fugen Zhou, “Edge Detection Based on Mathematical Morphology and Iterative Thresholding,” Computational Intelligence and Security, Springer-Verlag Berlin, 2007, 4456, pp.953-962.

[11] Baker, Simon, Shree K. Nayar, and Hiroshi Murase, “Parametric Feature Detection,” International Journal of Computer Vision 27(1): 27-50, 1998.

[12] Bao, P., L. Zhang, and X. Wu, “Canny Edge Detection Enhancement by Scale Multiplication,” IEEE Transactions on Pattern Analysis and Machine Intelligence, 27, 9, September 2005, 1485–1490.

[13] Basu, Mitra, “Gaussian-Based Edge-Detection Methods—A Survey,” IEEE Transactions on Systems, Man, and Cybernetics—Part C: Applications and Reviews, Vol. 32, No. 3, August 2002.

[14] Beaudet, P.R., “Rational invariant image operators.” Fourth International Conference on Pattern Recognition, pp. 579-583, 1978.

[15] Bergevin, Robert, and Annie Bubel, “Detection and Characterization of Junctions in a 2D Image.” Computer Vision and Image Understanding 93(3): 288-309 (2004).

[16] Bergholm, F., “Edge Focusing,” IEEE Transactions on Pattern Analysis and Machine Intelligence, 9(6):726-741, Nov 1987.

[17] Berman, Zeev and John S. Baras, “More About Wavelet Maxima Representations,” IEEE International Conference on Acoustics, Speech, and Signal Processing, Vol 5., 1993.

[18] Berman, Zeev and John S. Baras, “Properties of the Multiscale Maxima and Zero-crossings Representations.” IEEE Transactions on Signal Processing 41, No.12, 3216-3231 (1993).

[19] Bowyer, Kevin et al., “Edge Detector Evaluation Using Empirical ROC Curves,” CVIU(84), No. 1, October 2001, pp. 77-103.

[20] Candès, E. J., “Ridgelets: Theory and Applications.” Ph.D. Thesis, Technical Report, Department of Statistics, Stanford University, 1998.

[21] Candès, E. J. and D. L. Donoho. “Continuous Curvelet Transform: I. Resolution of the Wavefront Set,” Appl. Comput. Harmon. Anal. 19 162-197, 2003.

[22] Candès, E. J. and D. L. Donoho. “Continuous Curvelet Transform: II. Discretization and Frames.” Appl. Comput. Harmon. Anal. 19 198-222, 2003.

[23] Canny, John, “A Computational Approach to Edge Detection,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. PAMI-8, No. 6, November 1986.

[24] Canny, John, “Finding edges and lines in image”. Master’s thesis, MIT, 1983.

[25] Carson, Chad and Hayit Greenspan, “Blobworld: Image Segmentation Using Expectation-Maximization and Its Application to Image Querying,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 24, No. 8, August 2002.

[26] Catté, Francine, Pierre-Louis Lions, Jean-Michel Morel, and Tomeu Coll, “Image Selective Smoothing and Edge Detection by Nonlinear Diffusion,” SIAM Journal on Numerical Analysis, Vol. 29 No. 1, p.182-193, Feb. 1992.

[27] Chen, G. and Y.H.H. Yang, “Edge Detection by Regularized Cubic B-spline Fitting,” IEEE Trans. Systems Man Cybernet. 25, 636–643, 1995.

[28] Chidiac, H., and D. Ziou, “Classification of Image Edges”, In Proceedings of the Vision Interface’99, Troise-Rivieres, Canada, pp. 17-24, 1999.

[29] Conci, A. and L.H. Monteiro, “Multifractal Characterization of Texture Based Segmentation,” Proceeding of IEEE International Conference on Image Processing, Vancouver, Canada, 1, 2000, pp. 792-795.

[30] Cooke, T., F. Jakel, C. Wallraven, and H.H. Bulthoff, “Multimodal Similarity and Categorization of Novel, Three-Dimensional Objects.” Neuropsychologia, 2007. 45(3): p. 484-495.

[31] Cumani, A., “Edge Dection in Multispectral Images,” CVGIP: Graphical Models and Image Processing, vol. 53, no. 1, 1991, pp. 40-51.

[32] Cvetkovic Z. and M. Vetterli, "Wavelet Extrema and Zero-crossings Representations: Properties and Consistent Reconstruction," IEEE International Conference on Acoustics, Speech, and Signal Processing, Vol. 6, pp.117-120, 1994.

[33] Dekel, S. and A. Sherman, “Curvelets: A Low-Level Framework for Computer vision,” Preprint, GE Healthcare, 2008.

[34] Demigny, D. and T. Kamle, “A Discrete Expression of Canny’s Criteria for Step Edge Detector Performances Evaluation,” IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-19, 11, November 1997, 1199–1211.

[35] Deriche, R., “Fast Algorithms for Low-level Vision,” IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI, 12(A), 1990, pp. 78-87.

[36] Deschênes, François, and Djemel Ziou, “Detection of Line Junctions and Line Terminations Using Curvilinear Features.” Pattern Recognition Letters 21(6-7): 637-649 (2000).

[37] Dreschler, L., and H. Nagel, “Volumetric Model and 3-D Trajectory of a Moving Car Derived from Monocular TV-frame Sequence of a Street Scene,” Proceedings of the International Joint Conference on Artificial Intelligence, Vancouver, Canada, 1981, pp. 692–697.

[38] Ducottet, Christophe, T. Fournel, and Cécile Barat, “Scale-Adaptive Detection and Local Characterization of Edges Based on Wavelet Transform,” Signal Processing 84(11): 2115-2137 (2004).

[39] Duda, R.O. and P.E. Hart, “Use of the Hough transform to detect lines and curves in pictures,” CACM 12, 1, January 1972, 11-15.

[40] Eckhoff, Knut S., “On a High Order Numerical Method for Functions with Singularities,” Mathematics of Computation, Vol. 67, No. 223 (Jul., 1998), pp. 1063-1087.

[41] Engelberg, Shlomo and Eitan Tadmor, “Recovery of Edges from Spectral Data with Noise — A New Perspective,” Siam J. Number Anal. Vol. 46, No. 5, pp. 2620–2635.

[42] Fan, J., D.K.Y. Yau and W.G. Aref, “Automatic Image Segmentation by Integrating Color-Edge Extraction and Seeded Region Growing,” IEEE Trans. On Image Processing, Vol. 10, No. 10, pp. 1454-1466, 2001.

[43] Fitton, N.C. and S.J.D. Cox, “Optimizing the Application of the Hough Transform for Automatic Feature Extraction from Geoscientific Images,” Comput. Geosci., 24(10), 933–951, 1998.

[44] Forstner, W., “A Framework for Low Level Feature Extraction.” Computer Vision - ECCV '94, J.O. Eklundh eds, LNCS 802, Springer, vol. II, pp. 383-394, 1994.

[45] Freeman, W.T. and E.H. Adelson, “The Design and Use of Steerable Filters," IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol. 13:9, pp. 891-906, 1991.

[46] Frei, W. and C. Chen, “Fast Boundary Detection: A Generalization and a New Algorithm,” IEEE Trans. Computers, C-26, 10, October 1977, 988–998.

[47] Gang, Liu, Lu Xueqin and Huang Guohong, “A Steerable Filter Bank for Image Fusion,” December 2008, pp. 574-577.

[48] Ganguly, Debashis, Swarnendu Mukherjee, Kheyali Mitra, and Partha Mukherjee, "A Novel Approach for Edge Detection of Images," International Conference on Computer and Automation Engineering, pp.49-53, 2009.

[49] Gelb, Anne and Dennis Cates, “Segmentation of Images from Fourier Spectral Data,” Communications in Computational Physics, Vol. 5, No. 2-4, pp. 326-349.

[50] Gelb, Anne and Eitan Tadmor, “Detection of Edges in Spectral Data II. Nonlinear Enhancement,” SIAM Journal on Numerical Analysis, Vol. 38, No. 4 (2001), pp. 1389-1408.

[51] Gevers, Theo, “Reflectance-based Classification of Color Edges,” Ninth IEEE International Conference on Computer Vision (ICCV'03) - Volume 2, pp. 856, October 2003.

[52] Gottlieb, David and Chi-Wang Shu, “On the Gibbs Phenomenon and its Resolution,” SIAM Review, Vol. 39, No. 4 (Dec., 1997), pp. 644-668.

[53] Gregson, P.H., “Using Angular Dispersion of Gradient Direction for Detecting Edge Ribbons,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 15, no. 7, pp. 682-696, 1993.

[54] Grillenzoni, Carlo, “Statistics for Image Sharpening,” Statistica Neerlandica (2008) Vol. 62, nr. 2, pp. 173–192.

[55] Guo, Xiaoxin, Zhiwen Xu, Yinan Lu, and Yunjie Pang, “An Adaptive Edge Detector Using Soft Mathematical Morphology,” cit, pp.603-607, Fifth International Conference on Computer and Information Technology, 2005.

[56] Hall, Peter et al., “Tracking Edges, Corners and Vertices in an Image,” Board of the Foundation of the Scandinavian Journal of Statistics 2007.

[57] Haralick, R.M., “Digital Step Edges from Zero Crossings of Second Directional Derivatives.” IEEE Transactions on Pattern Analysis and Machine Intelligence 6, 58–68, 1984.

[58] Harris, C. and M. Stephens, “A Combined Corner and Edge Detector,” Proc. of the 4th ALVEY Vision Conference, pp 147-151, University of Manchester, England, 1988.

[59] Health, Michael D. et al., “A Robust Visual Method for Assessing the Relative Performance of Edge-Detection Algorithms,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 19, No. 12, December 1997.

[60] Heitger, F., L. Rosenthaler, R. von der Heydt, E. Peterhans, and O. Kubler, “Simulation of neural contour mechanisms: from simple to end-stopped cells.” Vis. Res., 32:963–981,

1992.

[61] Heric, D. and B. Potocnik, “Multiscale Edge Detector,” 48th International Symposium ELMAR-2006 Focused on Multimedia Signal Processing and Communications.

[62] Hermosilla, T., E. Bermejo, A. Balaguer and L.A. Ruiz, “Non-linear Fourth-order Image Interpolation for Subpixel Edge Detection and Localization,” Image and Vision Computing, Volume 26, Issue 9, 1 September 2008, Pages 1240-1248.

[63] Hjelmås, E., “Face Detection: A Survey.” Computer Vision and Image Understanding, 83(3), 236-274, 2001.

[64] Hubel, D.H. and T.N. Wiesel, “Receptive Fields and Functional Architecture in the Cat’s Visual Cortex,” Journal of Neuroscience, 160, 106–154, 1962.

[65] Iverson, L., and S.W. Zucker, “Logical/Linear Operators for Image Curves,” IEEE Trans. Pattern Analysis and Machine Intelligence, 17(10): 982-996, 1995.

[66] Jacob, Mathews and Michael Unser, “Design of Steerable Filters for Feature Detection Using Canny-Like Criteria,” Transactions on Pattern Analysis and Machine Intelligence, August 2004, pp. 1007-1019.

[67] Jacquey, Florence, Frédéric Comby, and Olivier Strauss, “Fuzzy Edge Detection for Omnidirectional Images,” Fuzzy Sets and Systems 159 (2008) 1991–2010.

[68] Jiang, Wei et al., “Gabor Wavelets for Image Processing,” 2008 ISECS International Colloquium on Computing, Communication, Control, and Management.

[69] Jones, J.P. and L.A. Palmer, “An Evaluation of the Two Dimensional Gabor Filter Model of Simple Receptive Fields in Cat Striate Cortex,” J. Neurophysiology 58, 1233–1258, 1987.

[70] Jones, Ronald and Imants Svalbe, “Algorithms for the Decomposition of Gray-Scale Morphological Operations,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 11, No. 7, July 1989.

[71] Julez, B., “A Method of Coding TV Signals Based on Edge Detection,” Bell System Technology, Vol. 38, No. 4, pp. 1001-1020, 1959.

[72] Kanade, T., “Image Understanding Research at CMU,” Proc. Image Understanding Workshop, vol. II, 1987, pp. 32–40.

[73] Kass, M, A. Witkin and D. Terzopoulos, “Snakes: Active Contour Models,” International Journal of Computer Vision, 1(4): 321-331, 1988.

[74] Konishi, Scott, Alan L. Yuille, James M. Coughlan, and Song Chun Zhu, “Statistical Edge Detection: Learning and Evaluating Edge Cues,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 25, No. 1, January 2003.

[75] Koplowitz, Jack and Vito Greco, “On the Edge Location Error for Local Maximum and Zero-Crossing Edge Detectors,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 16, No. 12, December 1994.

[76] Koren, Raz and Yitzhak Yitzhaky, “Automatic Selection of Edge Detector Parameters Based on Spatial and Statistical Measures,” Computer Vision and Image Understanding 102 (2006) 204–213.

[77] Koschan, A., "A Comparative Study On Color Edge Detection," Proc. 2nd Asian Conference on Computer Vision ACCV'95, Singapore, Vol. III, pp. 574-578, December

1995.

[78] Koschan, A. and M. Abidi, “Detection and Classification of Edges in Color Images,” IEEE Signal Processing Magazine, 22, 1, January 2005, 64–73.

[79] Kovesi, P., “Edges Are Not Just Steps,” Proc. 5th Asian Conf. Computer Vision, Mebourne, Australia, Jan. 2002, pp. 822–827.

[80] Kovesi, P., “Symmetry and Asymmetry from Local Phase,” in AI’97, Tenth Australian Joint Conference on Artificial Vision, Proceedings – Poster Papers, Perth, 185–190, 1997.

[81] Lee, Cheng-Hsiung and P. W. Huang, “Classification for Pathological Prostate Images Based on Fractal Analysis,” Congress on Image and Signal Processing, Vol. 3, pp.113-117, 2008.

[82] Lee, D., “Coping with Discontinuities in Computer Vision: Their Detection, Classification, and Measurement,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 321-344, Vol. 12, No. 4, April 1990.

[83] Li, Ding and Han Chongzhao, “Research on Edge Detection of Gray-scale Image based on Mathematical Morphology Algorithm and Rough Sets,” 2008 International Conference on Computer and Electrical Engineering.

[84] Li, Jun, “A Wavelet Approach to Edge Detection,” Master’s Thesis Sam Houston State University, 2003.

[85] Lim, Dong Hoon, “Robust Edge Detection in Noisy Images,” Computational Statistics & Data Analysis 50 (2006) 803 – 812.

[86] Lim, Dong Hoon, “Robust Rank-order Test for Edge Detection in Noisy Images,” Nonparametric Statistics Vol. 18, No. 3, April 2006, 333–342.

[87] Lindeberg, Tony, “Edge Detection and Ridge Detection with Automatic Scale Selection,” Department of Numerical Analysis and Computing Science, Royal Institute of Technology, S-100 44, Stockholm, Sweden, Jan 1996.

[88] Lindeberg, Tony, “Scale-space'', Encyclopedia of Computer Science and Engineering (Benjamin Wah, ed), John Wiley and Sons, Volume 4, pages 2495-2504, Hoboken, New Jersey, Jan 2009.

[89] Linguraru, MG, MÁG Ballester, and N. Ayache, “Medical Image Computing and Computer-Assisted Intervention” (MICCAI) Vol. 2879. Montréal, Canada: Nov 15-18, 2003. A multiscale feature detector for morphological analysis of the brain; pp. 738–745.

[90] Liu, H., L. Wang and K.C. Jezek, “Wavelet Transform Based Edge Detection Approach to Derivation of Snowmelt Onset, End and Duration from Satellite Passive Microwave Measurements,” International Journal of Remote Sensing 26, 4639–4660, doi:10.1080/01431160500213342 (2005).

[91] Lu, YI and Ramesh C. Jain, “Behavior of Edges in Scale Space,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 11, Issue 4, April 1989.

[92] Lu, Yi and Ramesh C. Jain, “Reasoning About Edges in Scale Space," IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 14, no. 4, April, 1992, PP. 450-468.

[93] Mallat, S., “Wavelets for a vision," Proceedings of the. IEEE, Vol. 84, No. 4, pp. 604 614, 1996.

[94] Mallat, Stephane G., “A Theory for Multiresolution of Signal Decomposition: The Wavelet Representation,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 24, No. 8, August 2002.

[95] Mallat, Stephane and Sifen Zhong, “Characterization of Signals from Multiscale Edges,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 14, No. 7, July 1992.

[96] Mallat, Stephane and Wen Liang Hwang, “Singularity Detection and Processing with Wavelet,” IEEE Transactions on Information Theory, Vol. 38, No. 2, March 1992.

[97] Marr, D and E. Hildreth, “Theory of Edge Detection,” Proceedings of the Royal Society of London. Series B, Biological Sciences, Vol. 207, No. 1167. (Feb. 29, 1980), pp. 187-217.

[98] Mansoori, Eghbal G. and Hassan J. Eghbali, “Heuristic Edge Detection Using Fuzzy Rule-Based Classifier,” Journal of Intelligent & Fuzzy Systems 17 (2006) 457–469.

[99] Medina-Carnicer, R. and F.J. Madrid-Cuevas, “Unimodal Thresholding for Edge Detection,” Pattern Recognition 41 (2008) 2337–2346.

[100] Medina-Carnicer, R. et al., “On Candidates Selection for Hysteresis Thresholds in Edge Detection,” Pattern Recognition, Volume 42, Issue 7, July 2009, Pages 1284-1296.

[101] Meer, P. and B. Georgescu, “Edge Detection with Embedded Confidence,” IEEE Transactions on Pattern Analysis and Machine Intelligence., PAMI-23, pp. 1351-1365, 2001.

[102] Mikolajczyk, Krystian, Tinne Tuytelaars, Cordelia Schmid, Andrew Zisserman, Jiri Matas, Frederik Schaffalitzky, Timor Kadir, and Luc J. Van Gool, “A Comparison of Affine Region Detectors.” International Journal of Computer Vision 65(1-2): 43-72, 2005.

[103] Moravec, H.P., “Towards Automatic Visual Obstacle Avoidance,” Proc 5th International Joint Conference on Artificial Intelligence, pp. 584, 1977.

[104] Moravec, H.P., “Visual Mapping by a Robot Rover,” Proc. 6th International Joint Conference on Artificial Intelligence, pp. 598-600, 1979.

[105] Morrone, M.C. and R.A. Owens, “Feature Detection from Local Energy,” Pattern Recognition Letters, 6:303–313, 1987.

[106] Naik, Sarif Kumar and C. A. Murthy, “Standardization of edge magnitude in color images,” IEEE Transactions on Image Processing 15(9): 2588-2595, 2006.

[107] Nalwa, V.S. and T.O. Binford, “On Detecting Edges,” IEEE Transactions on Pattern Analysis and Machine Intelligence, 8:699-714, 1986.

[108] Nevatia, R., “A Color Edge Detector and Its Use in Scene Segmentation.” IEEE Trans. Syst., Man, Cybern. 7 (1977), pp. 820–826.

[109] Ng, Jeffrey and Anil A. Bharath, “Extrapolative Spatial Models for Detecting Perceptual Boundaries in Colour Images,” International Journal of Computer Vision 73(2), 179–194, 2007.

[110] Nielsen, Mads, Luc Florack, and Rachid Deriche. "Regularization, Scale Space, and Edge Detection Filters". Journal on Mathematical Imaging and Vision, 7(4):291-307, 1997.

[111] Noble, J.A., “Finding Corners,” Image and Vision Computing, 6(4):121-128, 1988.

[112] Olshausen, B. and D. Field, “How Close Are We to Understanding V1?” Neural Comput.

17, 1665-1699, 2005.

[113] Panetta, Karen A., Eric J. Wharton and Sos S. Agaian, “Logarithmic Edge Detection with Applications.” JCP 3(9): 11-19, 2008.

[114] Park, Jiyoung et al, “Using structured light for efficient depth edge detection,” Image and Vision Computing, Volume 26, Issue 11, 1 November 2008, Pages 1449-1465.

[115] Pellegrino, F.A., W. Vanzella and V. Torre, “Edge Detection Revisited,” IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernet. 34(3), 1500–1518, 2004.

[116] Perona, Pietro and Jitendra Malik, "Scale Space and Edge Detection Using Anisotropic Diffusion," IEEE Transactions on Pattern Analysis and Machine Intelligence, 12(7), July 1990, 629-639.

[117] Petrou, M. and J. Kittler, “Optimal Edge Detectors for Ramp Edges,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 13, No. 5, May 1991, 483–491.

[118] Pratt, William K., “Digital Image Processing,” PIKS Inside, Third Edition. 2001.

[119] Prewitt, J., “Object Enhancement and Extraction,” Picture Processing and Psychopictorics (B. Lipkin and A. Rosenfeld, Ed.), NY, Academic Pres, 1970.

[120] Rao, K. Raghunath and Jezekiel Ben-Arie, “Optimal Edge Detection Using Expansion Matching and Restoration,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 16, No.12, December 1994.

[121] Rivest, J.F., P. Soille, and S. Beucher, “Morphological Gradients,” Journal of Electronic Imaging, 2 (4): 326-336, October 1993.

[122] Robbins, B., and R. Owens, “2D Feature Detection Via Local Energy,” Image and Vision Computing, Vol. 15, pp. 353-368, 1997.

[123] Roberts L.G., “Machine Perception of Three-Dimensional Solids,” In J.T. Tippett (Ed.), Optical and Electro-Optical Information Processing (Ch. 9, pp. 159-197). Cambridge, Massachusetts: MIT Press, 1965.

[124] Rosenfeld, A., M. Thurston and Y.H. Lee, “Edge and Curve Detection: Further Experiments,” IEEE Transactions on Computers, Vol. C-21, No. 7, pp. 677-715, July 1972.

[125] Rothwell, C., Mundy J., Hoffman W., and Nguyen V.-D., “Vision by Topology” Technical Report No. 2444, INRIA, 1994.

[126] Roushdy, M., “Comparative Study of Edge Detection Algorithms Applying on the Grayscale Noisy Image Using Morphological Filter,” ICGST, International Journal of Graphics, Vision, and Image Processing, Vol. 6, Issue 4, pp. 17-23, Dec. 2006.

[127] Ruzon, Mark A., Carlo Tomasi, “Edge, Junction, and Corner Detection Using Color Distributions,” IEEE Trans. Pattern Anal. Mach. Intell. 23(11): 1281-1295, 2001.

[128] Salotti, Marc, Fabrice Bellet and Catherine Garbay, “Evaluation of Edge Detectors: Critics and Proposal,” Workshop on Performance Characterization of Vision Algorithms, 1996.

[129] Schmeelk, J., “Wavelet Transforms and Edge Detectors on Digital Images,” Mathematical and Computer Modeling 41(2005) 1469-1478.

[130] Schmid, C., R. Mohr, and C Bauckhage. “Evaluation of Interest Point Detectors,” International Journal of Computer Vision, 2000, 37(2), pp. 151–172.

[131] Seo, Kyungryong and Soo-Young Lee, “Edge Detection Using Orientation-Independent Operators in Large-Scale Sensor Networks,” Future Generation Communication and Networking - Volume 2,December 2007.

[132] Shen, J. and S. Castan, “An Optimal Linear Operator for Step Edge Detection Computer Vision,” Graphics and Image Processing, Vol. 54, pp. 112–133, 1992.

[133] Shi, X., A.L. Ribeiro Castro, R. Manduchi and R. Montgomery, "Rotational Invariant Operators Based on Steerable Filter Banks", IEEE Signal Processing Letters, 13(11):684-87, November 2006.

[134] Shih, Ming-Yu and Din-Chang Tseng, “A Wavelet-Based Multiresolution Edge Detection and Tracking,” Image Vision Comput. 23(4): 441-451 (2005).

[135] Shiozaki, A., “Edge Extraction Using Entropy Operator,” Computer Vision, Graphics and Image Processing 36(1): 1–9, 1986.

[136] Simoncelli, E. P. and W. T. Freeman, “The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation,” 2nd Annual IEEE Intl. Conference on Image Processing, Washington, DC. October, 1995.

[137] Smith, S. M. and J. M. Brady, "SUSAN - A New Approach to Low Level Image Processing.". International Journal of Computer Vision 23 (1): 45–78, 1997.

[138] Sobel, I., “Camera Models and Perception,” Ph.D. Thesis, Stanford University, Stanford, CA, 1970.

[139] Starck, J. L., E. J. Candès and D. L. Donoho. “The Curvelet Transform for Image Denoising.” IEEE Transactions on Image Processing, 11 670—684, 2000.

[140] Sumengen, Baris and B. S. Manjunath, "Multi-scale Edge Detection and Image Segmentation," Proc. European Signal Processing Conference (EUSIPCO), Sep. 2005.

[141] Tagare, H. D. and R. J. P. deFigueiredo, “On the Localization Performance Measure and Optimal Edge Detection,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 12, No. 12, December 1990, 1186–1190.

[142] Torre, V. and T. Poggio, “On Edge Detection,” Massachusetts Institute of Technology Artificial Intelligence Laboratory A.L. Memo 768 August 1984.

[143] Trahanias, P.E., and A.N. Venetsanopoulos, “Vector Order Statistics Operators as Color Edge Detectors,” SMC-B(26), No. 1, February 1996, pp. 135-143.

[144] Tuytelaars, Tinne and Krystian Mikolajczyk, "Local Invariant Feature Detectors: A Survey," Foundations and Trends in Computer Graphics and Vision, Vol. 3, No. 3, pp 177-280, 2008.

[145] Varma, Manik and Rahul Garg, “Locally Invariant Fractal Features for Statistical Texture Classification,” iccv, pp.1-8, 2007 IEEE 11th International Conference on Computer Vision, 2007.

[146] Venkatesh, S., and R. Owens, “On the Classification of Image Features,” Pattern Recognition Letters, 11:339–349, 1990.

[147] Wang, Gang, Liang Xiao and Anzhi He, “A Sub-Pixel Multifractal Method for the Image Segmentation,” Fourth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD 2007) Vol.4, pp.566-572, 2007.

[148] Wang, Song, Feng Ge and Tiecheng Liu, “Evaluating Edge Detection Through Boundary Detection,” Department of Computer Science and Engineering, University of South Carolina, Columbia, SC 29208, USA, 2006.

[149] Wang, Xin, “Laplacian Operator-Based Edge Detectors,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 29, No. 5, May 2007.

[150] Wang, Xuewen, Xiaoqing Ding, and Changsong Liu, “Gabor Filters-based Feature Extraction for Character Recognition,” Pattern Recognition, Volume 38, Issue 3, March 2005, Pages 369-379.

[151] Wang, Zhiqian et al. “Optimal Ramp Edge Detection Using Expansion Matching,” IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume 18, No. 11, November 1996, 1092–1097.

[152] Wei, Musheng et al., “Detection of Edges from Spectral Data: New Results,” Applied and Computational Harmonic Analysis, Volume 22, Issue 3, May 2007, Pages 386-393.

[153] Wilson, R. and A.H. Bhalerao, “Kernel Designs for Efficient Multiresolution Edge Detection and Orientation Estimation,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 384-390, March 1992.

[154] Witkin, Andrew P., “Scale-Space Filtering,” International Joint Conferences on Artificial Intelligence, 1983: 1019-1022.

[155] Wu, Yong, Yuanjun He, and Hongming Cai, “Optimal Threshold Selection Algorithm in Edge Detection Based on Wavelet Transform,” Image and Vision Computing 23 (2005) 1159–1169.

[156] Xu, Yansun, et al., “Wavelet Transform Domain Filters: A Spatially Selective Noise Filtration Technique,” [J]. IEEE Trans. Image Processing, 1994, 3 (6): 747-758.

[157] Yang, C. K. and W. H. Tsai, "Reduction of color space dimensionality by moment-preserving thresholding and its application for edge detection in color images," Pattern Recognition Letters, Vol. 17, pp. 481-490, 1996.

[158] Yang, Ming-Hsuan, David J. Kriegman, and Narendra Ahuja, “Detecting Faces in Images: A Survey.” IEEE Trans. Pattern Anal. Mach. Intell. 24(1): 34-58, 2002.

[159] Yingjie, Zhang and Ge Liling, “New Edge Detection Approach by Using Linear Filtering,” Third International IEEE Conference on Signal-Image Technologies and Internet-Based System, December 2007.

[160] Yu-quian, Zhao, Gui Wei-hua, Chen Zhen-cheng, Tang Jing-tian, and Li Ling-yun, “Medical Images Edge Detection Based on Mathematical Morphology,” IEEE Engineering in Medicine and Biology Society, 27th Annual International Conference, Pages 6492-6495, 01-04 Sept. 2005.

[161] Yullie, Alan L. and Tomaso A. Poggio, “Scaling Theorems for Zero Crossings” IEEE PAMI, 8, 1986.

[162] Zaafouri, Ahmed, Mounir Sayadi and Farhat Fnaiech, “Edge Segmentation of Satellite Image using Phase Congruency Model,” World Academy of Science, Engineering and Technology, Issue 61, January 2010.

[163] Zhai, Lei, Shouping Dong, and Honglian Ma, “Recent Methods and Applications on Image Edge Detection,” 2008 International Workshop on Education Technology and Training & 2008 International Workshop on Geoscience and Remote Sensing, December 2008.

[164] Zhang, J., M. Marszalek, S. Lazebnik and C. Schmid, “Local Features and Kernels for Classification of Texture and Object Categories: A Comprehensive Study,” International Journal of Computer Vision, Volume 73, Issue 2, June 2007.

[165] Zhu, Shu-Yu, K.N. Plataniotis and A.N. Venetsanopoulos, “A Comprehensive Analysis of Edge Detection in Colour Image Processing,” Optical Engineering, Vol. 38, No. 4, pp. 612-625, April 1999.

[166] Ziou, D. and S. Tabbone, “Edge Detection Techniques –An Overview,” TR-195, Département de Math et Informatique, Universitéde Sherbrooke, Québec, Canada, 1997, pp. 1-41.

[167] Zuniga, O.A. and R.M. Haralick, “Integrated Directional Derivative Gradient Operator,” IEEE Transactions on Systems, Man, and Cybernetics SMC-17 (3) 508-517, 1987.