Image registration for recovering affine transformation using Nelder Mead Simplex method for optimization.

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Abstract

In computer vision system sets of data acquired by sampling of the same scene or object at different times or from different perspectives, will be in different coordinate systems. Image registration is the process of transforming the different sets of data into one coordinate system. Registration is necessary in order to be able to compare or integrate the data obtained from different measurements such as different view points, different times, different sensors etc. Image Registration is an important problem and a fundamental task in image processing technique. This paper presents an algorithm for recovering translation parameter from two images that differ by Rotation, Scaling, Transformation and Rotation-scale-Translation (RST) also known as similarity transformation. It is a transformation expressed as a pixel mapping function that maps a reference image into a pattern image. The images having rotational, scaling, translation differences are registered using correlation with Nelder-mead method for function minimization. The algorithm finds the correlation between original image and sensed images. It applies the transformation parameters on sensed images so that maximum correlation between original image and sensed images are achieved. Simulation results (Using Matlab) on images show the Performances of the method.

Keywords: Image registration, Optimization, Correlation, Affine transformation

1. INTRODUCTION

Image registration is the process of transforming the different sets of data into one coordinate system. Registration is necessary in order to be able to compare or integrate the data obtained from different measurements such as different view points, different times, different sensors etc. Image Registration is a crucial step in all image analysis tasks in which the final information is gained from the combination of various data sources like in image fusion, change detection, and multichannel image restoration .It geometrically aligns two images—the reference and sensed images. The present differences between images are introduced due to different imaging conditions, detection, and multichannel image restoration. Registration is required in remote sensing (multispectral classification, environmental monitoring, change detection, image mosaicing, weather forecasting, creating super-resolution images, integrating information into geographic information systems (GIS), in medicine (combining computer tomography (CT) and NMR data to obtain the complete information about the patient, monitoring tumor growth, treatment verification, comparison of the patient's data with anatomical atlases), and in computer vision (target localization, automatic quality control).

Image registration can be divided into main four groups according to the manner of image acquisition [2].
Different view points: Images of the same scene are acquired from different viewpoints. Example*s*: Remote sensing, shape recovery
Different times: Images of the same scene are acquired at different times, often on regular basis, and possibly under different conditions.
Examples: automatic change detection for security monitoring, maps from satellite images, remote sensing, healing therapy in medicine, monitoring of the tumor growth
Different sensors: Images of the same scene are acquired by different sensors. The aim is to Integrate of information obtained from different to gain detailed scene presentation.
Examples**:** offering better spatial resolution in Medical imaging for combination of sensors recording the anatomical body structure like magnetic resonance image (MRI), CCD image sensors, CMOS image sensors**,** Bayer sensor are different image sensors for better resolution.
Scene to model registration: Images of a scene and a model of the scene are registered. The model can be a computer representation of the scene, for instance maps or digital elevation models (DEM), another scene with similar content (another patient). The aim is to localize the acquired image in the scene/model and/or to compare them. Examples: automatic quality inspection, specimen classification.

There are two main methods for image registration.
1. Area Based methods: Area based methods are correlation like methods or template matching. These methods deal with the images without attempting to detect salient objects. W.K.Pratt [13], has given correlation techniques of image registration. Windows of predefined size or even entire images are used for the correspondence estimation. Available area based methods are: correlation-like methods, Fourier methods, Mutual information methods and optimization methods**.**
2. Feature Based methods: : Significant regions (forests and fields), lines (region boundaries, coastlines (road, lakes, mountains, rivers) or points (region corners, line intersections, points or curves with high curvature) are features here. They should be distinct, spread all over the image and efficiently detectable. They are expected to be stable in time to stay at fixed positions during the whole experiment. Two sets of features in the reference and sensed images represented by the Control points (points themselves, end points or centers of line features and centers of gravity of regions) have been detected. The aim is to find the pair wise correspondence between them using their spatial relations or various descriptors of features.

In this paper algorithm for recovering translation parameter from two images that differ by RST(Rotation-Scale-Translation).An RST transformation may be expressed as a combination of single translation, single rotation and single scale factor, all operating in the plane of the image. This in fact is a transformation expressed as a pixel mapping function that maps a reference image into a pattern image. RST is also known as geometric spatial Transformation.

2. GEOMETRIC SPATIAL TRANSFORMATION (RST TRANSFORMATION):

Let us consider an image function f defined over a *(w, z)* coordinate system, undergoes geometric distortion to produced an image g defined over an *(x,y)* coordinate system. This transformation may be expressed as

$$(x,y)= T\{(w,z)\} \qquad (1)$$

Rotation

The new coordinates of a point in the *x-y* plane rotated by an angle θ around the *z*-axis can be directly derived through elementary trigonometry [7].
Here *(x,y) =T{(w,z)},* where *T* is the rotation transformation applied such as

$$\begin{aligned} x &= w\cos\theta - z\sin\theta \\ y &= w\sin\theta + z\cos\theta \end{aligned} \qquad (2)$$

which can be represented in matrix form as follows.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix} \qquad (3)$$

Scaling

If the *x*-coordinate of each point in the plane is multiplied by a positive constant S_x, then the effect of this transformation is to expand or compress each plane figure in the *x*-direction.
If $0 < S_x < 1$, the result is a compression; and if $S_x > 1$, the result is an expansion. The same can also be done along the *y*-axis. This class of transformations is called scaling [7].
Here *(x,y) =T{(w,z)}*, where *T* is the Scaling transformation applied such as

$$\begin{aligned} x &= S_x w \\ y &= S_y z \end{aligned} \qquad (4)$$

The above transformation can also be written in matrix form

$$S_x = \begin{bmatrix} s_x & 0 \\ 0 & 1 \end{bmatrix} , \qquad S_y = \begin{bmatrix} 1 & 0 \\ 0 & s_y \end{bmatrix} \qquad (5)$$

Translation

A translation is defined by a vector *T=(dx,dy)* and the transformation of the Coordinates is given simply by

$$\begin{aligned} x &= w + dx \\ y &= z + dy \end{aligned} \qquad (6)$$

The above transformation can also be written in matrix form

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} w \\ z \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix} \qquad (7)$$

3. ALGORITHM FOR RECOVERING ROTATION, SCALING AND TRANSLATION

The images are having rotational, scaling, translation differences are registered are registered using correlation with Nelder-mead method [9] for function minimization. The minimum value of correlation between original and sensed images are found by optimization method. The Nelder-Mead algorithm is most widely used methods for nonlinear unconstrained optimization.[9]. The Nelder-Mead method attempts to minimize a scalar valued nonlinear function of n real variables using only function values, without any derivative information. The method approximately finds a locally optimum solution to problem with n variables when the objective function varies smoothly.
The Nelder-Mead algorithm [8] was proposed as a method for minimizing a real-valued function f(x) for x Є R^n . Four scalar parameters must be specified to define a complete Nelder-Mead method: coefficients of reflection (ρ), expansion (χ), contraction (ϒ) and shrinkage(σ).

The following algorithm finds the correlation between original image and sensed images. It applies the transformation parameters on sensed images so that maximum correlation between original image and sensed images are achieved.
If *A(m,n)* is reference image and *B(m,n)* is sensed image and then correlation coefficient between *A* and *B* is found by

$$r = \frac{\sum_m \sum_n (A_{mn} - A_o)(B_{mn} - B_o)}{\sqrt{\sum_m \sum_n (A_{mn} - A_o)^2 \sum_m \sum_n (B_{mn} - B_o)^2}} \ldots\ldots\ldots (8)$$

Where r is correlation coefficient which value should be between 0 to 1. Minimum value of r shows the dissimilarity of image and for the same images it will have value 1.
A_0 and B_0 represent mean of Image A and B respectively.

Algorithm:

(1) Acquire the sensed image, It is assumed that the acquire image differs by rotation, scaling and translation as compared to reference image. As a preprocessing process ,the spatial filter is applied to a sensed image.

(2) Apply rotational, scaling and translational transformation on sense image by placing the initial value for θ, *dx* , *dy* , S_x and S_y in equation *(3)* and *(7)*. Find the correlation coefficient *r* between reference and sensed image using equation *(8)*.
(3) Apply NELDER-MEAD simplex method for optimization in step *2*, which gives RST transformation parameters when the maximum correlation occur between sensed image and reference image.

(4) Repeat the process by changing the value of θ, *dx* , *dy* , S_x and S_y till optimum value of *r* is obtained.

(5) Step *4* gives the value of θ, *dx,dy,* S_x and S_y for the maximum correlation. This obtained transformation parameters are applied to sensed image using equation *(3)* and *(7)* to get the aligned image.

(6) If $r_{max} < r_{threshold}$, obtained θ, *dx dy,* S_x and S_y in step *4* will be taken as initial transformation parameters and obtained aligned image in step *5* will be taken as sensed image and repeat steps *2* to *5*.

If $r_{max} \geq r_{threshold}$, the aligned image obtained in step *5* is the registered image.

4. RESULTS

In figure 1, the original images of different sizes are shown .

Image 1

Image 2

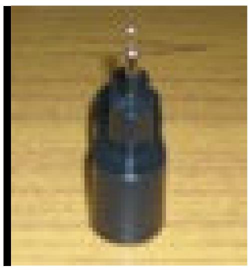


Image 3

Image 4

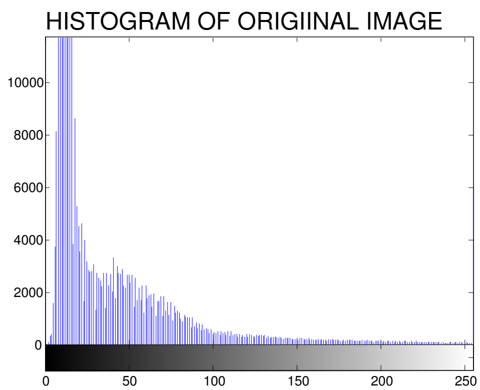
FIGURE 1 Original Images

Table 1 Correlation of sensed images differ by only rotation from original image

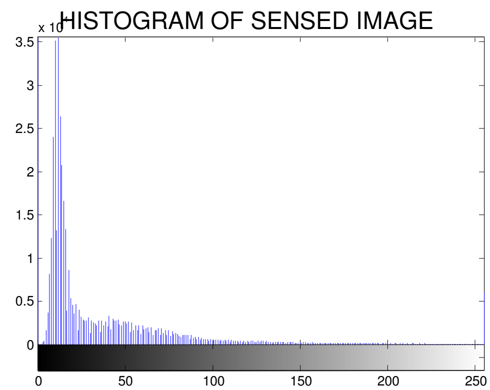
Sr. No	Original Images	Sensed image(rotated by θ angle from original)	Maximum Correlation r
1.	Image1.jpg (576 x 720)	5°	0.9996
		10°	0.9996
		15°	0.9996
		20°	0.9996
		25°	0.9997
		30°	0.9997
		35°	0.9997
		40°	0.9997
2.	Image2.jpg (113x 150)	5°	0.9811
		10°	0.9869
		15°	0.9879
		20°	0.9883
		25°	0.9890
		30°	0.9891
		35°	0.9893
3.	Image3.jpg (245 x326)	5°	0.9967
		10°	0.9970
		15°	0.9973
		20°	0.9974
		25°	0.9976
		30°	0.9976
		35°	0.9978
		40°	0.9978
		45°	0.9989

(a) Original Image (b) Sensed Image (c) Registered Image

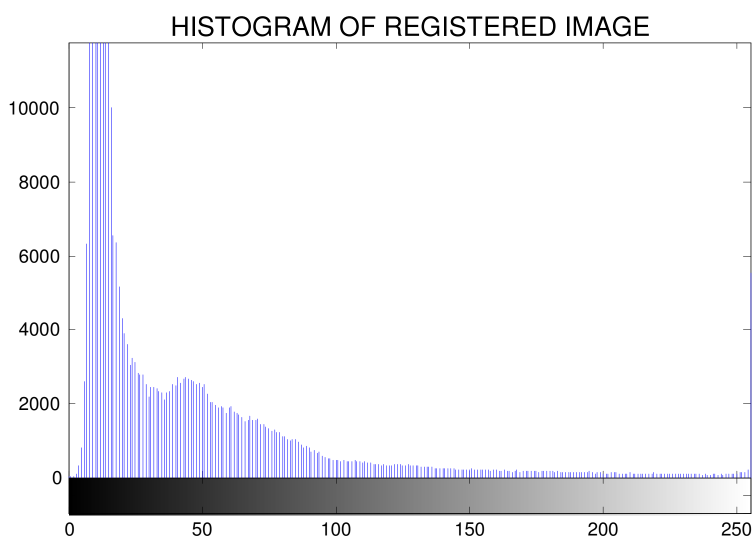
FIGURE 2 Registration of Sensed image rotated by 25 degree from original**.**



(A)



(B)

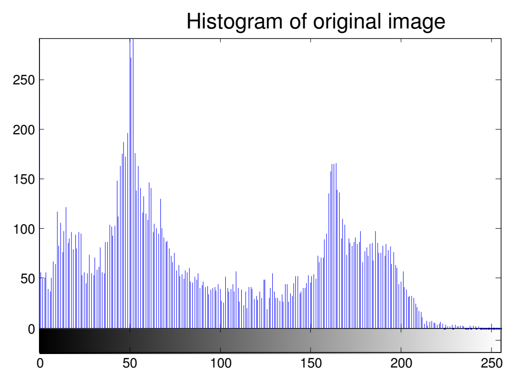


(C)

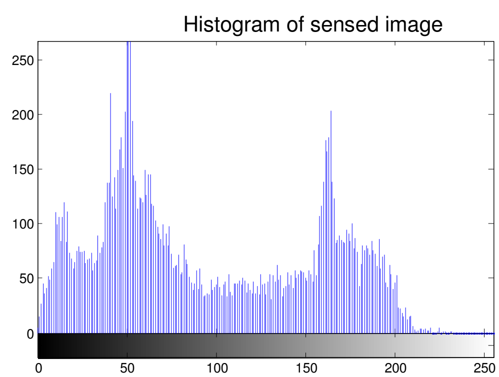
FIGURE 3 (A) Histogram of Original Image (image1.jpg)
(B) Histogram of Sensed Image (differ by rotation of 15 degree from original)
(C)Histogram of registered image

TABLE 2. Correlation of sensed images differ by only scaling from original image

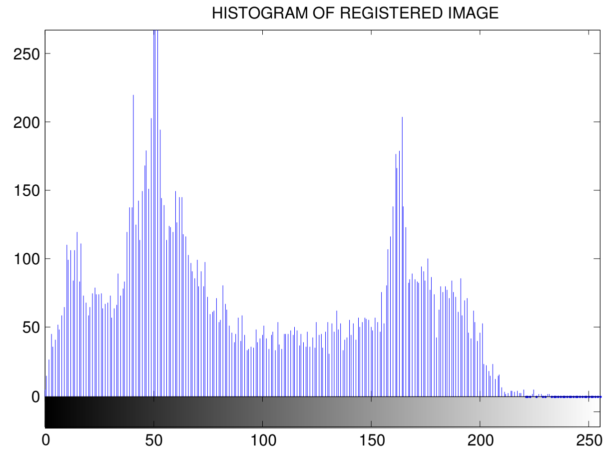
Sr. No	Original Images	Sensed image (scaled by *S* factor from original)	Size of sensed Image	Maximum Correlation
1.	Image1. jpg (576x720)	0.6	345 x 432	0.9972
		0.7	403 x 503	0.9971
		0.8	460 x 576	0.9989
		0.9	518 x 648	0.9968
		1.1	633 x 792	0.9966
		1.2	691 x 864	0.9967
		1.3	748 x 936	0.9968
		1.4	806 x 1007	0.9955
		1.5	864 x 1080	0.9973
2.	Image2. jpg (113x150)	0.6	67 x 90	0.9044
		0.7	79 x 105	0.9676
		0.8	90x120	0.9733
		0.9	101x135	0.9843
		1.1	124x165	0.9893
		1.2	135x180	0.9764
		1.3	146x195	0.9949



(A)



(B)



(C)

FIGURE 4 **(A)** Histogram of original image (Image2.jpg 113 x150)
(B) Histogram of sensed image (Image2. jpg 90x120) scaled by 0.8 from original.
(C) Histogram of Registered image (image2.jpg 113 x150)

TABLE 3 Correlations of sensed images differ by only translation from original image**.**

Sr. No	Original Images	Sensed image (transformed by dx and dy from original		Maximum Correlation
		dx	**dy**	
1.	Image1. jpg (576x720)	5	5	1
		10	25	1
		-3	-4	1
		-20	15	1
		50	-50	1
		120	70	1
		-90	0	1
2.	Image2. jpg (113x150)	0	100	1
		34	-7	1
		-200	12	1
		-130	-78	1
		56	90	1
		198	-198	1
3.	Image3. jpg (245x326)	34	-34	1
		12	200	1
		-8	21	1
		-65	-90	1
		-75	180	1
		75	-180	1
4.	Image4. jpg (113x150	0	100	1
		34	7	1
		-200	-12	1
		-130	78	1
		56	90	1
		198	-198	1
		-8	21	1
		75	-180	1

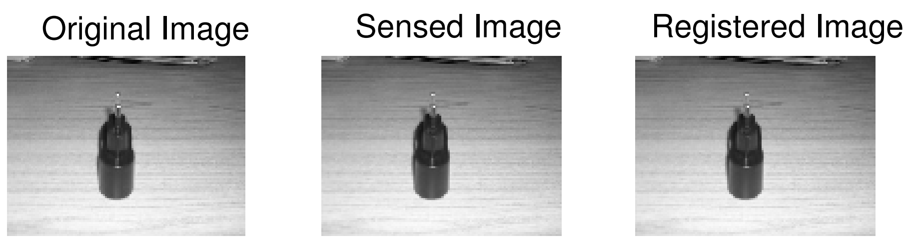


FIGURE 5: Sensed image is translated by *dx*= 34 , *dy*=-34

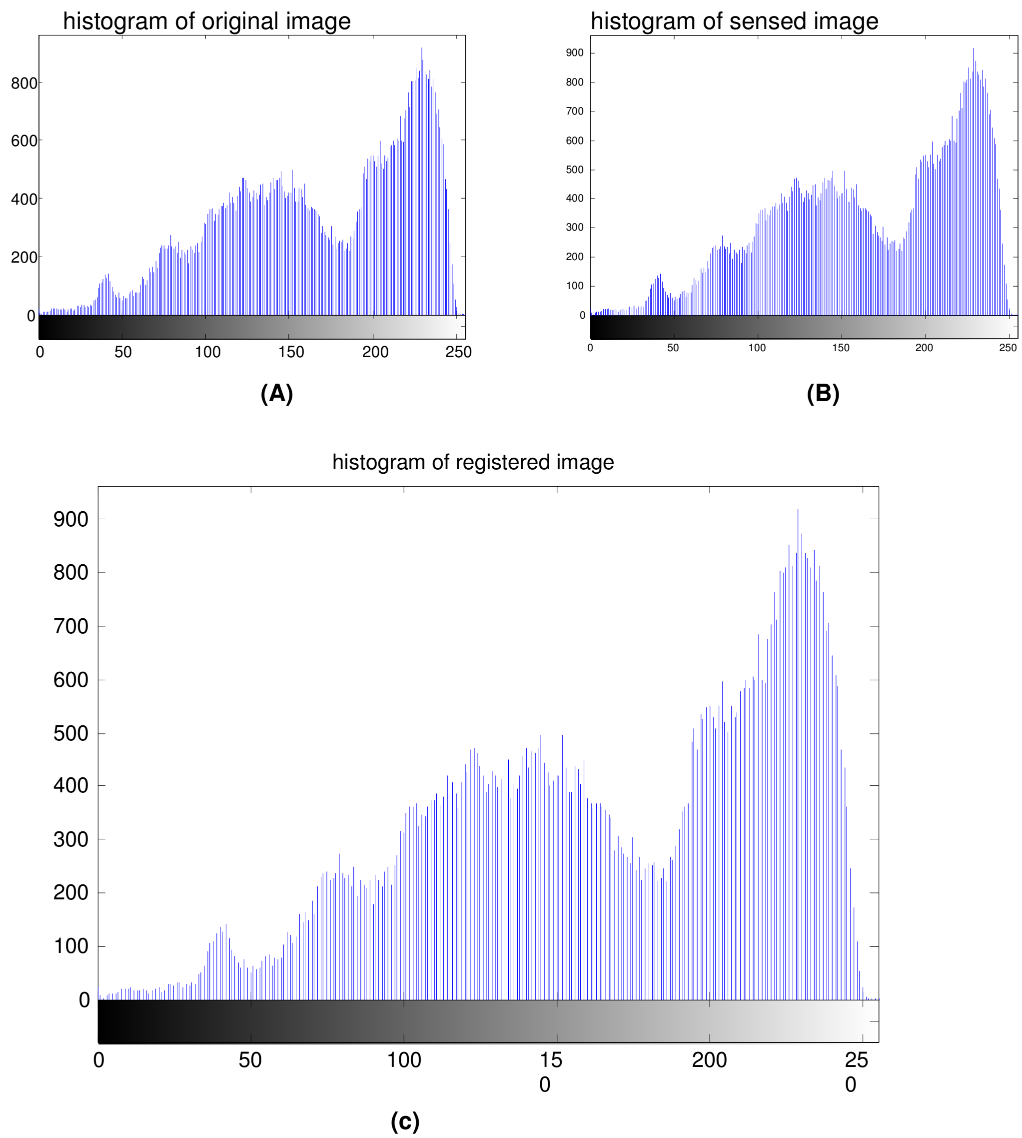


FIGURE 6 (A)Histogram of original image(Image3.jpg)
(B)Histogram of translated sensed image(*dx*=-15, *dy*=20)
(C)Histogram of registered image (image3.jpg)

FIGURE 7 Registration of sensed images differ by rotation, scaling and translation from original **image.**

For FIGURE 7 Size of original image: 427 x 317
Size of sensed image: 423 x 319
Sensed image rotated by original : 3
Correlation : 0.9909

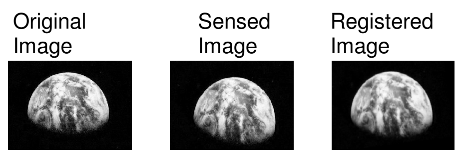


FIGURE 8 Registration of sensed images differ by rotation, scaling and translation from original image.

For FIGURE 8 Size of image: 384 x 276
Size of sensed image: 382 x 255
Sensed image rotated by original : 7
Correlation : 0.9602

In the results by observing the histogram of Image1.jpg we can see that it is having narrow range of gray level so in the NM simplex methods it also goes to the expansion along with reflection. So it will take more time to find the value at which the maximum correlation achieved. In histogram of Image2.jpg and Image3.jpgwe can observe that pixels are having all range of gray level(0-255), so in the NM simplex method within reflection only maximum correlation is achieved. It will not go to expansion so it takes less time to compare to Image1.jpg. As shown in TABLE 1, this method gives the registration of rotated sensed image for maximum 45 degree. From TABLE 2 it can seen that registration can be achieved if the sensed image is scaled between 0.5 to 1.5 times original image. TABLE 3 shows the result in which the sensed image is translated and using this method we can get the registration for any translation parameters. Figures 3,4 and 6 show the histograms of original, sensed and registered images. Histograms of registered images are same as original images.

5. CONCLUSIONS

In this paper authors have presented a technique for registering the images differ by rotation, scaling and translation. The techniques presented can be applied a wide class of problems involving determination of correspondence between two images related by an similarity and affine transformation. This algorithm is useful for images taken from same sensor and which are misaligned by small transformation such as scaling, rotation or translation. But limitation of this algorithm is inability to register the dissimilar images, having different information. Here different optimization algorithm is implemented for maximization of correlation among which NM simplex method gives the best result.

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