A SURVEY OF IMAGE REGISTRATION

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Abstract

In image processing, for example, when combining the information content of image, we are interested in the relationship between two or more images. The analysis of this relationship usually becomes tractable once a correspondence is set up between the images. Image registration is the task of setting up this correspondence. This paper overviews the theoretical aspects of an image registration problem. The purpose of this paper is to present a survey of image registration techniques. Registration is a fundamental task in image processing used to match two or more pictures taken, for example, at different times, from different sensors, or from different viewpoints. It geometrically aligns two images the reference and sensed images. Specific examples of systems where image registration is a significant component include matching a target with a real-time image of a scene. Various applications of image registration are target recognition, monitoring global land usage using satellite images, matching stereo images to recover shape for navigation, and aligning images from different medical modalities for diagnosis.

Keywords: Image Registration, Image Transformation, Image Mosaicing.

1. INTRODUCTION

Image registration is establishment of correspondence between images of the same scene. Many image processing applications like remote sensing for change detection, estimation of wind speed and direction for weather forecasting, fusion of medical images like PET-MRI, CT-PET etc need image registration. Image registration is a process of aligning two images acquired by same/different sensors, at different times or from different viewpoint. To register images, we need to determine geometric transformation that aligns images with respect to the reference image. The most common transformations are rigid, affine, projective, perspective and global. A variety of approaches have been published on this topic. Over the years, a large range of techniques has been developed for various types of problems. All these techniques have been independently studied for different applications. This paper organizes this research by establishing the relationship between the variations in the images and the type of registration techniques which can most appropriately be applied. All registration techniques explained in paper are useful for understanding the merits and relationships between the wide variety of existing techniques and for supporting in the selection of the most appropriate technique for a particular problem.

A widespread survey of image registration methods was published in 1992 by Brown [1]. A comprehensive survey of image registration methods is presented by Barbara Zitova and Jan Flusser [2]. They have classified the image registration techniques as area based methods and feature based methods. J. B. Antoine Maintz presented a survey of medical image registration in 1998.[3].Leszek Chmielewski presented various methods of image registration in 2001.[4].Mohammad Essadiki presented a technique for combining panchromatic and multispectral spot images [5].Subunku in his thesis presented various entropy based image registration techniques[6].S. K. Bose presented various tools for medical image registration [7].J.

Flusser used moment based approach to correct affine distortion, he has also done degraded image analysis to locate invariants in images[8,9]. Sangit Mitra and B.S. Manjunath explained various contour based approaches for multispectral image registration in their different papers[11]. Cahill, N D Williams, C.M. Shoupu propose an approach to incorporate spatial information into the estimate of entropy to improve multimodal image registration [15]. J.P.W. Plum presented another servey on medical image registration [16]. Frederik Maes and Andre Collignon apply mutual information to measure the statistical dependence or information redundancy between the image intensities of corresponding voxels in both images [17]..Xiaoxiang Wang and Jie Tian in their paper proposed a mutual information based registration method using gradient information rather than pixel intensity information [18]. Frederik Maes ,Andre present novel histogram based method for estimating and maximizing mutual information between two multimodal and possibly multiband signals [19]. J. P. Queiroz developed method for automatic registration of satellite images acquired on different dates, for both geometric and radiometric correction with respect to reference image [20]. Shannon's paper, titled "A mathematical theory of communication "is widely accepted as the origin of information theory. In this paper, Shannon used probability theory to modal information sources, i.e. data produced by a source is treated as a random variable [21]. Haim Schweitzer in his paper proposed that large collection of images can be indexed by projections on a few " eigenfeatures " , the dominant eigenvectors of the images covariance matrix [22]. Ma Debao and Liwagao introduced the new matrix characteristic methods like eigenvalues and eigenvectors and achievable accuracy is derived theoretically and verified by tests using simulated interferometric data.[23]. Haim Schweitzer demonstrated that eigenspace based algorithm registers multiple images and produces improved eigenfeatures[24].Wen Cao and Bicheng proposed PCAT (principal component analysis transform) and WPT (wavelet packet transform) for remotely sensed image fusion[25]. Yu-Te Wu, Takeo Kanade, Ching-Chung Li and Jeffrey Cohn proposed that their wavelet-based algorithm produced better motion estimates with error distributions having a smaller mean and smaller standard deviation [26]. Tarek A El-hazawi explained wavelet based image registration [27]. Hala S. Own and Aboul Ella Hassanien in their paper presents an efficient image registration technique using the Q-shift complex wavelet transform (Q-shift CWT). The experimental results proved that the proposed algorithm improves the computational efficiency and yields robust and consistent image registration compared with the classical wavelet transform [28].Azhar Quddus and Otman Basir proposed a novel, fully automatic, multistage wavelet-based image registration technique for image retrieval applications. They used multiscale wavelet representation with mutual information (MI) to facilitate matching of important anatomical structures at multiple resolutions. The proposed approach has several novel aspects including the use of MI in multistage wavelet domain [29].Stone, Harold S.,Le Moigne, Jacqueline; McGuire and Morgan proposed fast image registration by progressively registering wavelet representations at different resolutions[30].Ghazaw presented wavelet based image registration on parallel computing[31]. Jiangsheng explained image matching techniques by using Radon transform [32].

2. TRANSFORMATIONS

Image registration consists of establishing correspondence means matching of identical shapes in related image pair. This requires geometric transformation of one image into another. Change in viewpoint or relative motion between the camera and object planes introduces distortion in the features of an image e.g. a circle may appear ellipse when observed from non-fronto-parallel vantage point. However, certain features of object shape remain intact even after such transformations. These features are called invariants. The fundamental characteristic of any image registration technique is the type of spatial transformations or mapping used to properly overlay two images. The most common transformations are rigid, affine, projective, perspective, and global [1, 2].

2.1 Euclidean or Rigid Body Transformation

$$p' = Rp + t$$

$$\text{where } R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

is a rotation matrix and $t=[t_x\ t_y]^T$ is translation vector. p' and p are the transformed and original 2-D points, respectively, represented in non homogeneous coordinates $[x'\ ,y']^T$ and $[x\ ,y]^T$ respectively. Euclidean invariants are lengths and angles.

2.2 Similarity Transform

$$p' = sRp + t$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = s \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ 0 \end{bmatrix}$$

Here **s** is a scaling factor. Similarity invariants are angles, ratios of lengths, and ratios of areas.

2.3 Affine Transformation

The most commonly used registration transformation is the affine transformation which is sufficient to match two images of a scene taken from the same viewing angle but from different position. It is composed of scaling, translation, and rotation. It is global transformation which is rigid. Affine transformations are more general than rigid.

$$p' = Ap + t$$

The general 2D affine transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Angles and lengths are not preserved. Parallel lines remain parallel.

2.4 Projective Transform

This is the most general geometric transformation. Here, two 2-D points p' and p (represented in homogeneous coordinates), are related by a 3 × 3 non-singular transformation matrix (homography).

$$p' = \mu p$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective invariants include the cross ratio of four collinear points, or four concurrent lines [1, 2, 3].

3. REGISTRATION ALGORITHMS

Image registration algorithms can be classified in various ways, like based on modality, intensity or methods used for registration. Barbara Zitova and Jan Flusser [2] classified the image registration techniques as area based methods and feature based methods. Area based methods are preferably applied when in images prominent details are absent and distinctive information is provided by gray levels / colors rather by local shapes and structure. Feature based matching methods are applied when local structural information carried by image intensities are more. These methods make use of image features derived by feature extraction algorithm. Point of sharp variations such as edges, corners, contours, surfaces, point of intersection etc. what carries valuable information about images are used for matching.

3.1 Pixel Based Method

Cross-correlation is the basic statistical approach of registration. It is often used for template matching or pattern recognition in which the location and orientation of a template or pattern is found in picture. Cross correlation is a similarity measure or match metric. For template T and image I, where T is small compared to I, the two dimensional normalized cross-correlation function measures the similarity for each translation[1,2,3,4].

$$C(u,v) = \frac{\Sigma x\, \Sigma y T(x,y) I(x-u, y-v)}{\sqrt{[\Sigma x \Sigma y\ I\ \ 2(x-u, y-v)]}}$$

If template matches the image, then cross correlation will have it's peak at C(i ,j) . Cross correlation must be normalized since local image intensity would otherwise influence the measure.

Implementation on Dataset-1

a | b | c

FIGURE 1: (a) Original image SCOE-1. (b) translated image SCOE-2. (c) Correlation plot for SCOE 1 & SCOE 2.

Observations

Two main drawbacks of the correlation-like methods are the flatness of the similarity measure maxima (due to the self-similarity of the images) and high computational complexity. The maximum can be sharpened by preprocessing or by using the edge or vector correlation. It is used in next section.

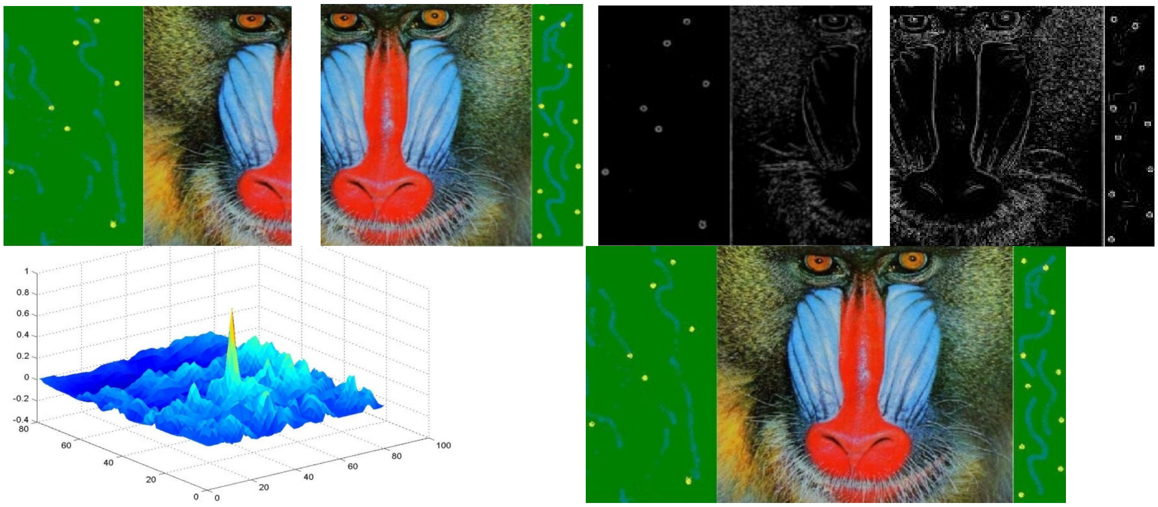
3.2 Feature Based Methods or Point Mapping Method

Feature based matching techniques do not use the grey values to describe matching entities. It makes use of image features derived by feature extraction algorithm. The purpose of feature extraction is to abstract substantial information from original data input and filter out the redundant information. Features are selected which are likely to be uniquely found in both images and more tolerant of local distortions. Computing of proper transformation depends on these features. Therefore sufficient number of features must be detected to perform calculation. After detecting features in each image, they must be matched. This technique is primary approach to register two images whose type of misalignment is unknown. This occurs if class of transformations cannot be easily categorized as translations or rigid-body movements. In this we can use landmarks and match them using general transformation. The method of point mapping consist of three stages-

- Computing features in the images
- Control points in reference image are corresponded with feature points in data image.
- Spatial mapping.

Control points for point matching play an important role in this method. Control points may be corners, line of intersections, points of locally maximum curvature on contour lines, centers of windows having locally maximum curvature and centers of gravity of closed-boundary regions [1, 2, 14]. Under this method we have computed edges of images in color vector space by using color gradients For a scalar function *f(x, y),* the gradient is a vector pointing in the direction of maximum rate of change of f at coordinates *(x ,y)* .

Implementation on Dataset 1



a	b	c	d
e	f		

FIGURE 2: (a) mar1 image adopted from MATLAB toolbox. (256 X 256). (b) mar2. (c) Features extracted from MAR-1.(d) Features extracted from MAR-2.(e) Correlation plot between (c) & (d). (f) registered mar1 and mar2

Implementation on Dataset-2
In this experiment image correction of rotated and stretched image of clown is done by point mapping method. In this method features are extracted using color gradients. Invariants are identified and projective transformation is applied on image to correct it.

a	b
c	

FIGURE 3: (a) Clown 1 original image(adopted from MATLAB toolbox).(b) : Clown 2 original image stretched and rotated.(c) Image corrected by projective transformation.

Observation
Feature based registration methods overcomes the limitation of correlation method and are often in use, particularly because of their easy hardware implementation, which makes them useful for real-time applications. Disadvantage of the these methods refers to the 'remarkableness' of the window content. There is high probability that a window containing a smooth area without any prominent details will be matched incorrectly with other smooth areas in the reference image due to its non-saliency. The features for registration should be preferably detected in distinctive parts

of the image. Windows, whose selection is often not based on their content evaluation, may not have this property.

3.3 Registration Based on High Level Features (Contour Based Image Registration)

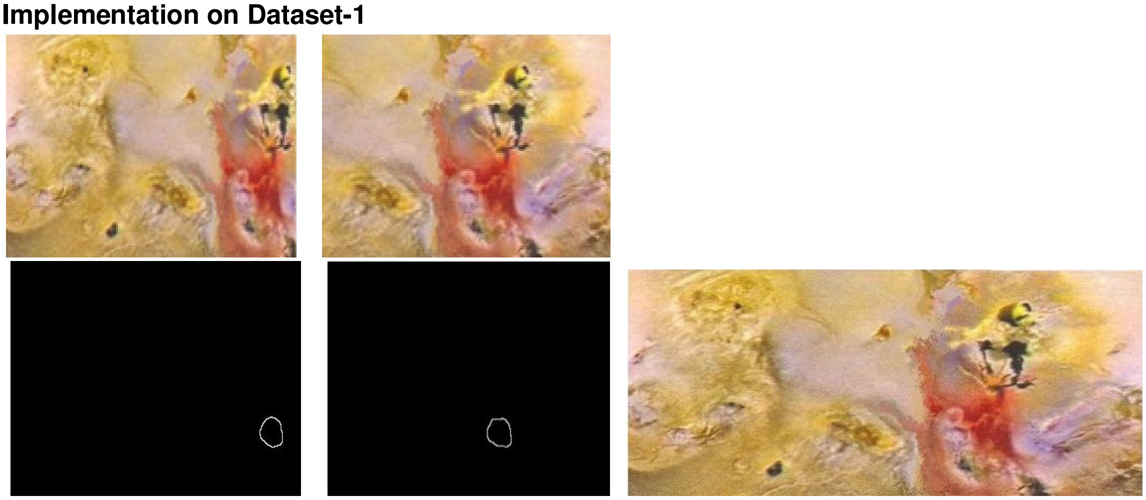
This method makes use of high statistical features for matching image feature points. For extracting regions of interest from the image color image segmentation is used. Given a set of color of interest, obtain mean of color 'm'. For segmentation classify each RGB pixel in image as having a color in the specified range or not. Euclidean distance is used to measure similarity. 'z' is an arbitrary point in RGB space and T is threshold. Euclidean distance between 'z' and 'm' is given by

$$D(z,m) = \|z - m\| = [(z-m)^T(z-m)]^{1/2}$$

$$= [(z_{R}-m_R)^2 + (z_G - m_G)^2 + (z_B - m_B)^2]^{1/2}$$

Here RGB denotes RGB components. The locus of points such that D (z, m) <=T is a solid sphere of radius T and point contained within, or on the surface of the sphere satisfy specified color criterion. Coding these two sets of points in the image with black and white produces a binary, segmented image. After segmentation remove noise by 'Gaussian' filter. Threshold blurred image and then obtain contour of image. For registration of jup1 and jup2 contours of red colors are extracted by above said method .Centroid of contours i.e. (x, y) coordinates of centre of gravity of jup2 are corresponded with centroid of jup1 contour. Similar procedure is repeated to extract contours of berry images .Angle between x-axis and major axis of contour is calculated to find angle of rotation .

Implementation on Dataset-1



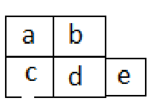


FIGURE 4 : (a) original image jup-1 (256 X256).(b) translated image jup-2(256 X256). (c) Counter extracted from jup-1(d) Counter extracted from jup-2(e) Mosaic of jup-1 and jup-2

Implementation on Dataset-2

a b

FIGURE 5: (a) Original image Berry1(256 X256). (b) Berry2 rotated by 90 deg. (256 X256).

Actual angle of rotation = 90 deg, Calculated angle of rotation = 88.85 deg.

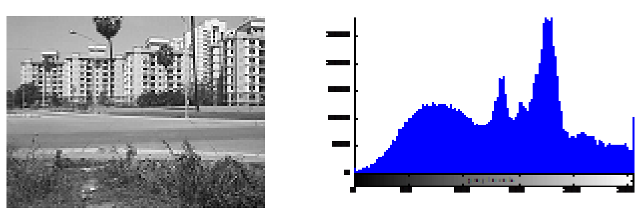
Observations:
Feature based methods do not use the gray values for matching and hence overcomes the limitations of spatial methods. Feature based method filter out the redundant information. Accuracy of this method is more but the limitation is , it is manual and slow.

3.4 Multimodal Image Registration Using Mutual Information
Multispectral image registration is also called as multimodal analysis. Images of the same scene are acquired by different sensors. The aim is to integrate the information obtained from different source streams to gain more complex and detailed scene representation. Different types of application are, in remote sensing fusion of information from sensors with different characteristics like panchromatic images, offering better spatial resolution, color/multispectral images with better spectral resolution, or radar images independent of cloud cover and solar illumination. Medical imaging applications are, combination of sensors recording the anatomical body structure like MRI, ultrasound or CT with sensors monitoring functional and metabolic body activities like PET, SPECT. Result can be applied, in radiotherapy and nuclear medicine.Registration of multispectrul / multisensor images is a challenging area. In general such images have different gray level characteristics and simple techniques such as those based on area correlation cannot be applied directly. This section is an attempt to solve this difficult problem by employing a basic concept from information theory.

Entropy of image
For a high dimensional discrete random variable $X = (X_1, X_2, ---, X_d)$ that has a probability mass function of $P(x_1, x_2, \cdots, x_d)$, the entropy is

$$H(X) = \sum_{x, x_2, \cdots, x_d} P(x_1, x_2, \cdots, x_d) \log \frac{1}{P(x_1, x_2, \cdots, x_d)}$$

From the properties of entropy, entropy doesn't depend on value of random variable, but only depends on distribution. This is explained with following example.
E.g. consider image shown in following figure 6.

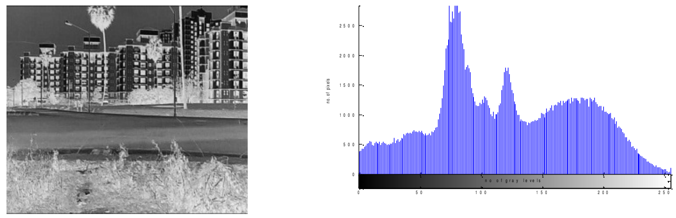


a b

FIGURE 6: (a)The image of size 500 X 500 with 256 gray levels. (b)Histogram of image.

Entropy (gray image) = 7.7540

By applying bijective mapping to the intensity values, we get synthetic image as shown below in figure 7.



a	b

FIGURE 7: (a) Synthetic image of size 500X500 (left) with 256 gray levels.(b)Histogram of image

Synthetic Entropy (synthetic image) =7.7549

From Figure 6 and 7, it is easy to see that entropy of image does not change even if histogram has changed. Even after randomly shuffling pixels of image, entropy of image remains same. From above discussion it is clear that natural images contain less uncertainty. In this type of images pixel intensity values depend on neighboring pixels. In other words, in natural image the value of pixel is likely to be close to some of its neighbors. Hence this dependency reduces the total entropy [6].

Entropy as Alignment Measure

The objective of using entropy as alignment measure is simple. Corresponding features extracted from the images should become statistically more dependent with better alignment. This is explained with following example.

a	b	c	d
e	f	g	h

FIGURE 8: (a) Subject. (b) subject rotated by 5^0 .(c) Subject rotated by 20^0 .(d) Subject with rotated neck. (e) Joint histogram a Vs a. (f) Joint histogram a Vs b. (g) Joint histogram a Vs c. (h) Joint histogram a Vs d .

In Figure 8, scatter plots (joint histogram) display pixel intensity value pairs from all images. Notice that, since in Figure e. images are aligned, pixel samples cluster around x-y line. At bad alignment, samples are scattered, i.e. joint histogram is more dispersed. This indicates that if alignment is good joint distribution tends to be sharper with peak at good alignment. This illustrates misregistration measured by dispersion of 2-D histogram of image intensities[6].

Mutual Information to Describe Dispersive Behavior of 2-D Histogram

Mutual information measures statistical dependence between two random variables. Mutual Information criteria presented here states that, mutual Information of image intensity values of corresponding voxel pairs is maximum if images are geometrically aligned. Let A and B represent random variables and P_A (a) and P_B (b) represents its marginal probability distributions. Let $P_{AB}(a,b)$ represents joint probability distribution

then $A \& B$ are independent if $P_{AB}(a,b) = P_A(a) * P_B(b)$ Mutual Information I (A, B) is given by

$$I(A,B) = \sum_{a,b} P_{AB}(a,b) \log[P_{AB}(a,b) / \{P_A(a) \cdot P_B(b)\}]$$

Mutual Information is related to entropy by following equations.

$$I(A,B) = H(A) + H(B) - H(A,B)$$

Where $H(A,B)$ is joint entropy and $H(A/B) \& H(B/A)$ are conditional entropy

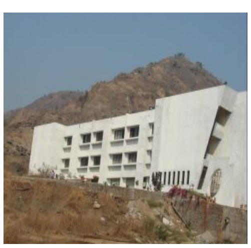
$$H(A) = -\sum_{a} P_A(a) \cdot \log P_A(a)$$

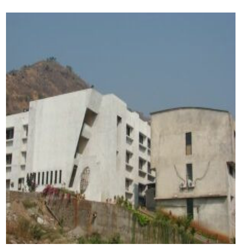
$$H(A/B) = -\sum_{a,b} P_{AB}(a,b) \cdot \log P_{AB}(a,b)$$

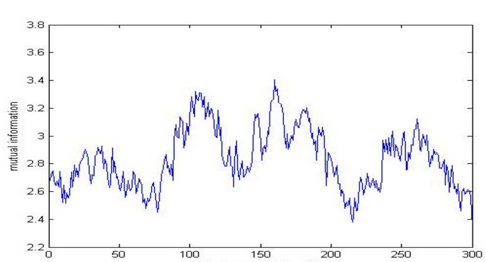
$$H(A/B) = -\sum_{a,b} P_{AB}(a,b) \cdot \log P_{A/B}(a/b)$$

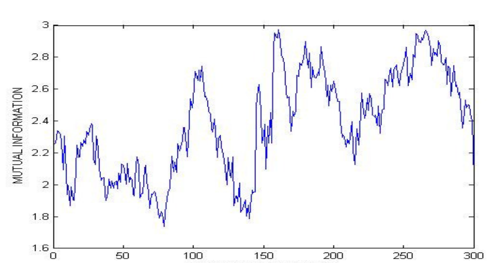
Consider that x & y is image intensity values of a pair of corresponding voxel in two images which are to be registered. Intensities x and y are related through the geometric transformation Tα defined by registration parameter α. The MI registration criteria states that the images are geometrically aligned by the transformation Tα for which I(A,B) is maximum. This is explained in following example. Figure 9 (d) and (e), shows 2-D histogram of image intensity values in non-registered and registered position. We observe that at good alignment i.e. at x=160, joint distribution is sharper and shows peak. Mutual Information between SCOE 1 & SCOE 2 is 3.4026 and matching point is at 160. Similarly Mutual Information between SCOE 1 & SCOE 3 is 2.97and matching point is at 161.

Implemetation on Dataset-1





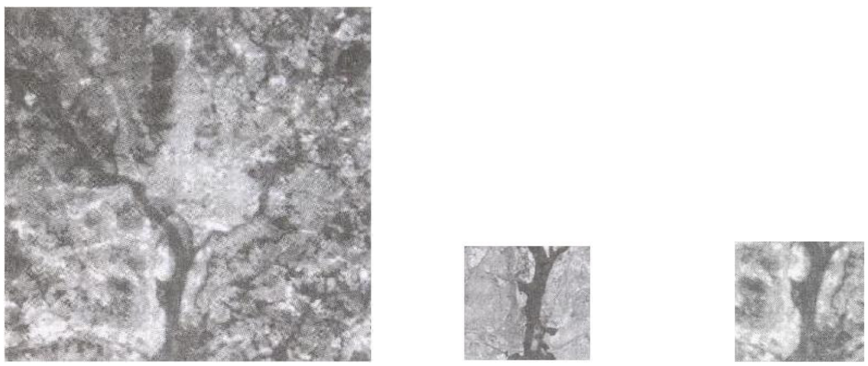




a	b	c
d	e	
f	g	

FIGURE 9 : (a-c)Sample images SCOE-1, SCOE-2 and SCOE-3.(d) I (A, B) as a function of mis registration for SCOE 1 and SCOE 2.(e) I (A, B) as a function of misregistration for SCOE 1 and SCOE 3.(f) mosaic of SCOE1 and SCOE2.(g) mosaic of SCOE1 and SCOE3 in multiple spectrums

Implementation on Dataset-2

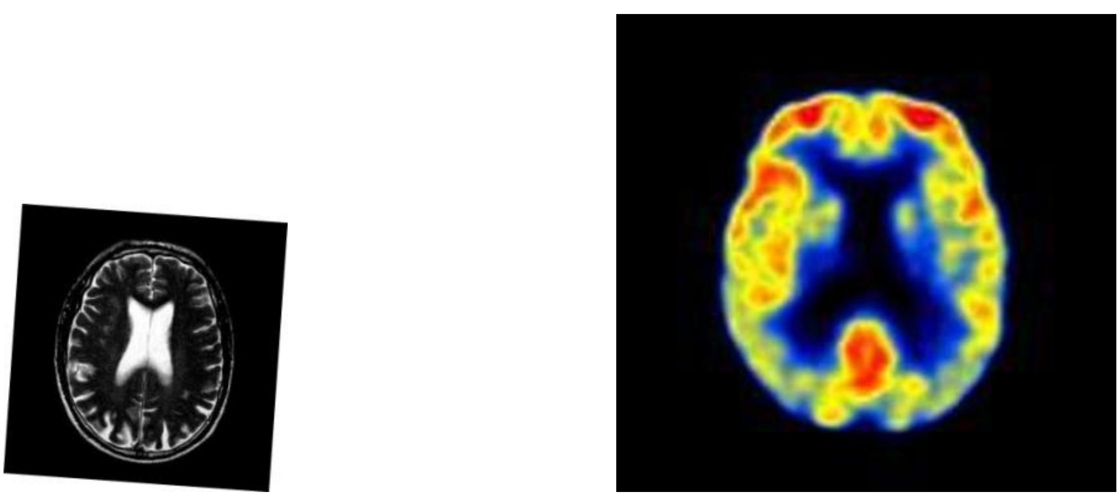


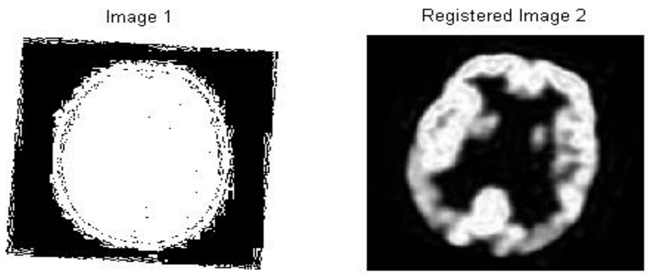
a	b	c

FIGURE 10: (a) Image in thermal infrared bands (450 X 450). (b)Template in near infrared (150 X 150).(c)Matched template (150 X 150).

For images a, b and c, Template matched at x = 124 and Y = 300
Mutual information = 0.8522

Implementation on Dataset 3 to Correct Rotation and Scale





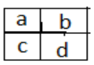


FIGURE 11: (a) MRI image (230 X 230) rotated 4^{0}.(b) CT image (512 X 512).(c-d)Registered CT image(233 X 233)

Implementation on Dataset 4 to Correct Rotation and Scale

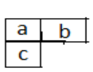


FIGURE 12: (a) Prutha image (230 X 230) .(b) Prutha image (512 X 512) rotated by 6^{0}.(d)Registered Prutha image(231 X 231)

Observation

It is observed that proposed method yields a more accurate registration than any other registration method. But this method has its own limitations. When images are of low resolution, when images contain little information, or when the region of overlap is small then mutual information result in mis-registration .It has one more limitation of speed , when time is an important constraint one cannot use this method . Although it has some limitations entropy and mutual information are best approaches for multimodal image registration.

3.5 Image Registration in Frequency Domain

Correlation theorem has one useful property. Correlation theorem states that, the Fourier transform of the correlation of two images is the product of Fourier transform of one image and complex conjugate of Fourier transform of other. The Fourier transform of an image f(x,y) is a complex function, each function value has real part $R(\omega x,\omega_y)$ and an imaginary part $I(\omega_x,\omega_y)$at each frequency (ω_x,ω_y) of frequency spectrum.

$$F(\omega_x \omega_y) = |F(\omega_x \omega_y)| e^{-\varphi(\omega_x \omega_y)}$$

Where $|F(\omega_x \omega_y)|$ is magnitude , and $\varphi(\omega_x \omega_y)$ *is* phase angle

$$|F(\omega_x \omega_y)|^2 = R^2(\omega_x \omega_y) + I^2(\omega_x \omega_y)$$

$$\varphi(\omega_x \omega_y) = \tan^{-1}\left[\frac{I(\omega_x \omega_y)}{R(\omega_x \omega_y)}\right]$$

Cross power spectrum of two images is defined as

$$F(\varphi_x \varphi_y) = \frac{F1(\varphi_x \varphi_y) F2^*(\varphi_x \varphi_y)}{|F1(\varphi_x \varphi_y) F2^*(\varphi_x \varphi_y)|}$$

Shift theorem guarantees that phase of cross power spectrum is equivalent to the phase difference between the images. If we represent the phase of cross power spectrum in it’s spatial form, i.e. by taking the inverse Fourier transform of the representation in the frequency domain, then we will have a function which is an impulse, that is approximately zero everywhere except at displacement which is needed to optimally register two images. Above method is used to register images having only translation.

Implementation on Dataset-1 to Correct Translation

a	b	c

FIGURE 13: (a) Edges extracted from SCOE-1. (b) Edges extracted from SCOE-2. (c) cross power spectrum for SCOE 1 & SCOE 2 in frequency domain.

Observation

It is observed that in frequency based method accuracy is more than correlation method but less as compared to other methods. But if we extract image features and then apply Fourier method accuracy increases. In frequency domain it should be noted that some form of interpolation must be used[1,2].

3.6 Wavelet Transform Based Image Registration

Registration of image using wavelets is being discussed in this section. Image Registration using wavelets has already been studied by many researchers. In this step feature that is extracted using wavelet or other transforms used as input and correspondence between these features will be recognized. To find out correspondence between two images so that transform model can be calculated using these correspondence points is very important part of overall problem. Cross correlation or Mutual Information could be used as a measure of similarity

Wavelet transform decomposes an image into various sub images based on local frequency content. Using discrete wavelet transform (DWT), a function f*(t)* can be represented by

$$f(t) = \sum_{j,k} a_{j,k} \psi_{jk}(t)$$

Where $a_{j,k}$ are wavelet coefficients , $\Psi_{j,k}(t)$ are basis function , j is scale , k is translation of mother wavelet Ψ(t). Two dimensional DWT can be obtaine by applying DWT across rows and columns of an image.The two dimensional DWT of image f(x,y)is

$$f(x,y) = \sum_{j,k} C_{J_0}(k,l)\phi_{j,k,l}(x,y) + \sum_{S=H,V,D}\sum_{J=J_0}^{\infty}\sum_{k,l} D_j^S[k,l]\psi_{j,k,l}^S(x,y)$$

Where C_{J0} is approximation coefficient, $\varphi_{j,k,l}$(x,y) is scaling function, D_j^S is set of detail coefficients and $\Psi^S_{j,k,l}$ is set of wavelet function. The DWT coefficients are computed by using a series of low pass filter h[k], high pass filters g[k] and down samplers across both rows and columns. The results are the wavelet coefficient the next scale. The filter bank approach to calculate two dimensional dyadic DWT is shown in Figure 14. The wavelet coefficients are of smaller spatial resolution as they go from finer scale to coarser scale. The coefficients are called the approximation (A), horizontal detail (H), vertical detail (V) and diagonal detail (D) coefficient.

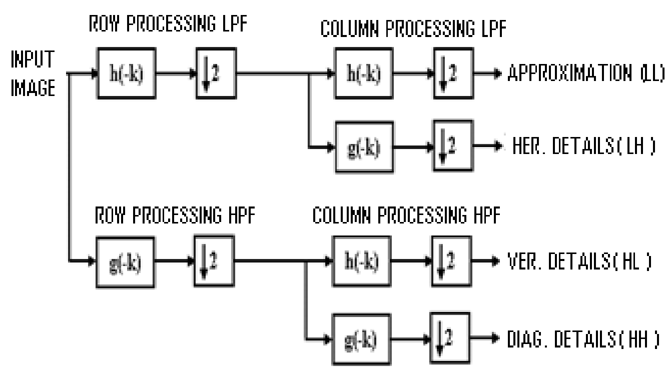


FIGURE 14: Two-dimensional orthogonal wavelet decomposition

In wavelet transformation due to sampling, the image size is halved in both spatial directions at each level of decomposition process thus leading to a multi-resolution signal representation. The decomposition and reconstruction of wavelet pyramid of source images are based on Mallet's theories[25,26,27,28,29,30,31].

a	b
c	d

FIGURE 15: (a)Original image SCOE-1(256X256).(b)Translated image SCOE-2(256X256).(c)Wavelet decomposition at level 2 by Harr wavelet for SCOE-1.(d) Wavelet decomposition at level 2 by Harr wavelet for SCOE-2.

Mutual Information as a Measure of Similarity

Origin of MI is from information theory, MI is an entropy-based concept and denotes the amount of information that one variable can offer to the other. Mutual information measures statistical dependence between two random variables. Mutual Information criteria presented here states that, mutual Information of image intensity values of corresponding voxel pairs is maximum if images are geometrically aligned. Let A and B represent random variables and P_A (a) and P_B (b) represents its marginal probability distributions. Let $P_{AB}(a,b)$ represents joint probability distribution then A & B are independent if $P_{AB}(a,b) = P_A(a) * P_B(b)$

Mutual Information I (A, B) is given by

$$I(A,B) = \sum_{a,b} P_{AB}(a,b)\log[P_{AB}(a,b)/\{P_A(a).P_B(b)\}]$$

Mutual Information is related to entropy by following equations.

$$I(A,B) = H(A) + H(B) - H(A,B)$$

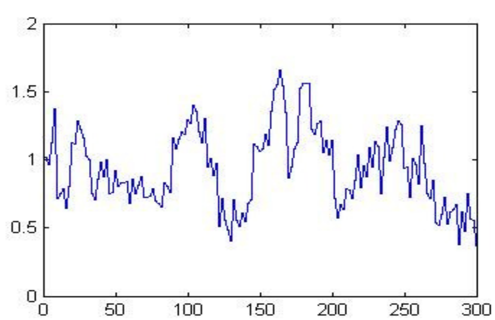


FIGURE16: I (A, B) as a function of mis registration for SCOE 1 and SCOE 2

Observations

It is observed that combinational approach of wavelet and mutual information gives better results as compared to wavelet - correlation combination and Fourier based image registration. Even wavelet mutual information combination can be used in case of multimodal image registration.If we involve mutual information as similarity criteria, speed decreases and hence we can not use this combination in applications where speed is required.

3.7 Hotelling Transform Based Image Registration

In this section automatic method of template matching using Hotelling transform as well as Hotelling transform for image alignment is presented. The proposed method is validated on pair of remotely sensed scenes and medical images.

Principal Component Analysis

In this section we present Hotelling Transform (Principal Component Analysis) for template matching and image alignment to register images. The proposed method is validated on pair of remotely sensed scenes and medical images. Proposed method for template matching is compared with other algorithms.

Principal component analysis is one of the most frequently used dimension reduction method. Principal component analysis also called as Hotelling Transform. Hotelling Transform is based on stastical properties of vector representations. The material discussed here can be used as the basis for describing sets of images, that are registered spatially, but their corresponding pixel values are different. If we have 'n' component images having different pixel values, These images can be treated as a unit by expressing each group of 'n' corresponding pixels as a vector. Let x_1 , x_2 ----,x_n are values of first pixel in each of the n' images then 'n' elements can be expressed as follow

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Where, x is column vector. This one vector represents one commen pixel in all two image. If images are of size P X Q, there will be total of N=P*Q such n- dimensional vectors.

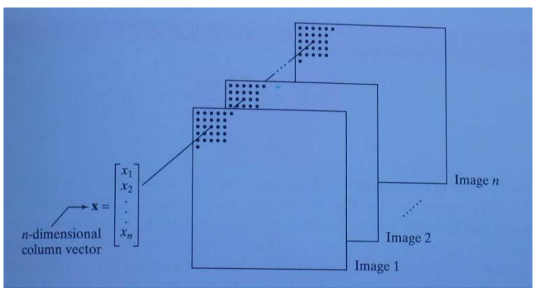


FIGURE 17: Forming a vector from corresponding pixels in a stack of images of the same size

We can assume these vectors as random quantities,
Mean vector of population is

$$m_x = E[x]$$

The covariance matrix of vector population is

$$C_x = E\{(x - m_x)(x - m_x)^T\}$$

Element C_{ij} of C_x is the variance of xi , the i th component of x vectors and element C_{ij} is covariance between components xi and xj .The matrix $C_x$is symmetric and real. For a sample of N vectors from a random population, the mean vector and covariance matrix can be given by expression

$$m_x = \frac{1}{N}\sum_{k=1}^{N} x_k$$

Thus covariance matrix can be estimated as

$$C_x = \frac{1}{N}\sum_{k=1}^{N} x_k x_k^T - m_x m_x^T$$

Since Cx is real and symmetric, it is possible to find a set of N ortho-normal eigenvectors. Let ei and λi be eigenvectors and corresponding eigenvalues of Cx where i = 1,2,---,N . 'A' is a matrix whose rows are eigenvectors of covariance matrix Cx . Then A is ordered so that the first row of A is eigenvectors corresponding to the largest eigenvalue , and last row corresponds its smallest eigen value. If we use A as transformation matrix to map the x's into y. Then y is given by

$$Y = A(x - m_x)$$

Above expression of 'y ' is called Hotelling Transform or Principal Component Transform. Alternatively, the Hotelling Transform can be viewed as the discrete version of the Karbunen-Loeve transform (KLT).This transform has some useful properties . Mean of 'y' vector is zero i.e. $m_y = E(y) = 0$

Covariance matrix of y's is estimated as

$$C_y = AC_xA^T$$

It is observed that C_y is a diagonal matrix.Elements of C_y along main diagonal are eigenvalues of C_x.

$$C_y = \begin{bmatrix} \lambda_1 & \cdots\cdots & 0 \\ \cdots & \lambda_2 \cdots & \cdots \\ \cdots & \cdots\cdots & \cdots \\ \cdots & \cdots\cdots & \cdots \\ \cdots & \cdots\cdots & \lambda_n \end{bmatrix}$$

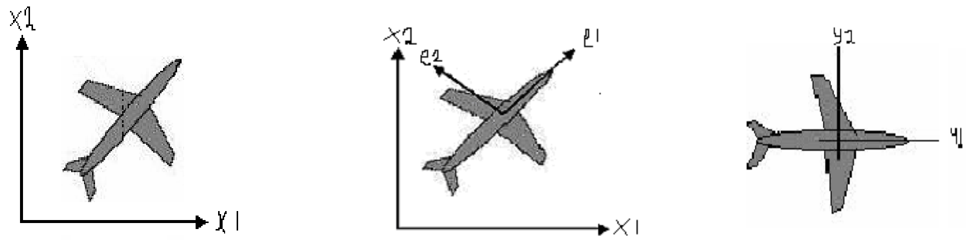
λ_j are eigenvalues of C_x . The important property of Hotelling Transform is we can reconstruct vector x from vector y.

$$\hat{x} = A^T y + m_x$$

The error between vector x and reconstructed vector $\hat{x}$ can be minimized by using the eigenvectors corresponding to the largest eigenvalues. The fact that zero values are everywhere except along the main diagonal in C_y indicates that the components of the transformed vector y are uncorrelated i.e. the correlation previously existing between the different components of random vector $\hat{x}$has been removed in transformed domain. Therefore, if the input is split into blocks and Hotelling Transform is applied block wise, the coding may be more efficient since the data in the transformed block are uncorrelated.

Geometrical Interpretation of Hotelling Transform (PCA)

Here we are giving a geometrical interpretation of transform coding. For this, we use 2-D vectors instead on N-D vectors. Hotelling Transform can also be used to align region or boundaries with the eigenvectors of the object. We have to form two-dimensional vectors from the coordinates of the boundary or region. Figure 18(a) shows that vectors are formed from the coordinates of the pixel in the object or we can use coordinates of points on the boundary. The resulting vectors are treated as 2-D population of random vectors. Let each pixel in the object is a 2-D vector x-(a,b)^T, where a and b are coordinate values of that pixel with respect to x1 and x2 axis.



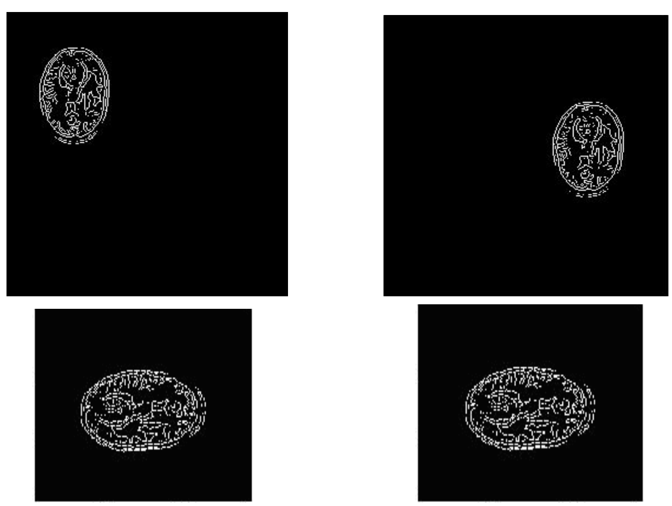
a b c

FIGURE 18: (a) Arbitrary object in x1-x2 coordinate system. (b) origine of new coordinate system coordinate system .(c) After Hotelling transform, the object is aligned with its principle axes.

These vectors are used to calculate mean vector mx and covariance matrix *Cx*. Then the effect of using equation of Hotelling Transform is to establish a new coordinate system. The origin of this new coordinate system is at the centroid of the population and its axes are in the direction of eigenvectors of *Cx*. It is shown in figure 18 (b). It is observed that transformation in equation *y=A(x-mx)* is a rotation and translation that aligns the object with eigenvectors. Two eigenvectors are perpendicular.The '*y*' axis is also called as eigen axis. The ability of Hotelling Transform to align the object with its principal axis provides a reliable means for removing the effects of rotation. This is a rugged alignment procedure which uses all coordinates of object (region or boundary) to compute the transformation matrix and aligns the data in the direction of its principal spread [15].

Implementation on Dataset-1 for Image Alignment

In this experiment, Hotelling Transform is applied to binary images .Fig. 19 (a) shows gradient image of MRI of head and fig 19 (b) shows same image but head is at translated and rotated position. .



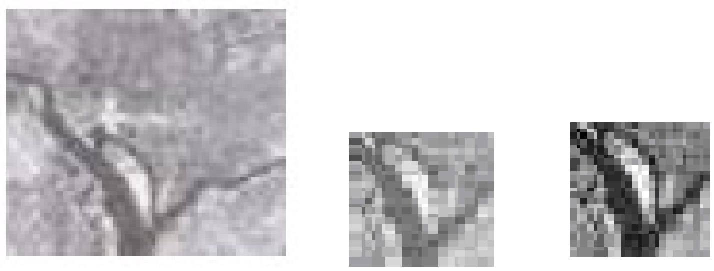
a b
c d

FIGURE 19: (a) MRI image of brain (400X400). (b) Translated and rotated version of same image (400X400). (c) Hotelling transform of the image a . (d). (Hotelling Transform of the images b.

Figure 19(c,d) shows Hotelling Transform of MRI image and its rotated version. It can be seen that Hotelling Transform automatically positioned the two images in such a way that corresponding points are practically aligned. This removes rotational effects. Translational effects are also removed since the object is centered on its mean. After this alignment images can be fused to get complete information for diagnosis. In this section we have given background theory of principal component analysis and discussed implementation details. It is observed that if we use eigenvalues and eigenvectors for registration of images accuracy is more precise than that of other methods. We can use his method for template matching**.** It is observed that Hotelling Transform can be used to align region or boundaries with the eigenvectors of object but this method can be applied only to regions and boundaries. In terms of future work, effort could be directed at the problems of an unknown image scaling and unknown global rotation between the target and reference images. Both are outstanding issues requiring a solution and should prove useful for multimodal images. There are possible solutions to the problem that could be investigated in future work.

Implentation on Dataset -2 for Template Matching

Template matching is the process of finding the location of sub image, called a template, inside an image. Template matching involves comparing a given template with windows of the same size in an image and identifying the window that is most similar to the template. The accuracy of a template matching process depends on the accuracy of metric used to determine the similarity between a template and a window. There are many similarity measures known to produce best results .Hotelling Transform can be used for template matching. It is observed that when template match with the window of the same size in an image then eigenvector corresponding to that location shows largest eigenvalue and eigenvalue of second eigenvector shows minimum value.



a b c

FIGURE 20: (a) Sample image (50 X 50). (b) Template (25 X 25). (c) Matched template (25 X 25).

4. COMPARATIVE STUDY OF IMAGE REGISTRATION METHODS.

All methods explained in the paper are implementated on the three datasets . This made comparison of all the methods easy.

Implentation on Dataset-1 on Dataset 1

Main application of image registration is in remote sensing for image mosaicking of surveyed area for monitoring of global land usage; landscape planning etc. A camera typically has a limited field of view. A lens with a wide field of view incurs substantial distortion. In addition, capturing the entire scene with the limited camera resolution compromises the image quality. Image mosaicing algorithms register or stitch a sequence of images into a composite image.

FIGURE 21: Sample images and mosaic of VR images from H1 to H 16

Observations

Location of maximum match point is shown in table

TABLE I: Location of Maximum Match using Different Methods

S.N.	Image combination	Correlation Method	Fourier Transform Method	Mutual Information Method	Principal Component Analysis	Wavelet Transform Method	Feature Based Method
1	**H1-H2**	63	62	65	62	59	61
2	**H2-H3**	110	111	113	112	113	114
3	**H3-H4**	62	63	57	62	63	63
4	**H4-H5**	79	77	78	80	80	82
5	**H5-H6**	147	147	150	147	148	150
6	**H6-H7**	93	92	89	93	93	95
7	**H7-H8**	72	74	73	74	74	75
8	**H8-H9**	73	74	73	74	74	74
9	**H9-H10**	88	88	90	89	88	90
10	**H10-H11**	83	84	84	83	83	82
11	**H11-H12**	79	79	82	80	80	80
12	**H12-H13**	81	81	78	81	81	82
13	**H13-H14**	85	84	89	89	83	89
14	**H14-H15**	128	126	129	128	125	129

TABLE II: Location of Maximum Match using Different Methods

S.N.	Image combination	Correlation Method	Fourier Transform Method	Mutual Information Method	Principal Component Analysis	Wavelet Transform Method	Feature Based Method
1	**H1-H3**	169	167	171	169	145	168
2	**H2-H4**	171	172	167	170	165	170
3	**H3-H5**	136	133	140	136	133	135
4	**H4-H6**	223	233	235	223	224	224
5	**H5-H7**	231	237	240	232	233	232
6	**H6-H8**	161	158	150	160	162	162
7	**H7-H9**	141	140	130	140	141	140
8	**H8-H10**	158	156	160	159	155	157
9	**H9-H11**	143	143	130	142	142	142
10	**H10-H12**	163	162	150	164	163	164
11	**H11-H13**	160	161	132	159	158	159
12	**H12-H14**	165	165	165	165	163	164
13	**H13-H15**	210	201	210	210	210	209

Implentation on Dataset-2

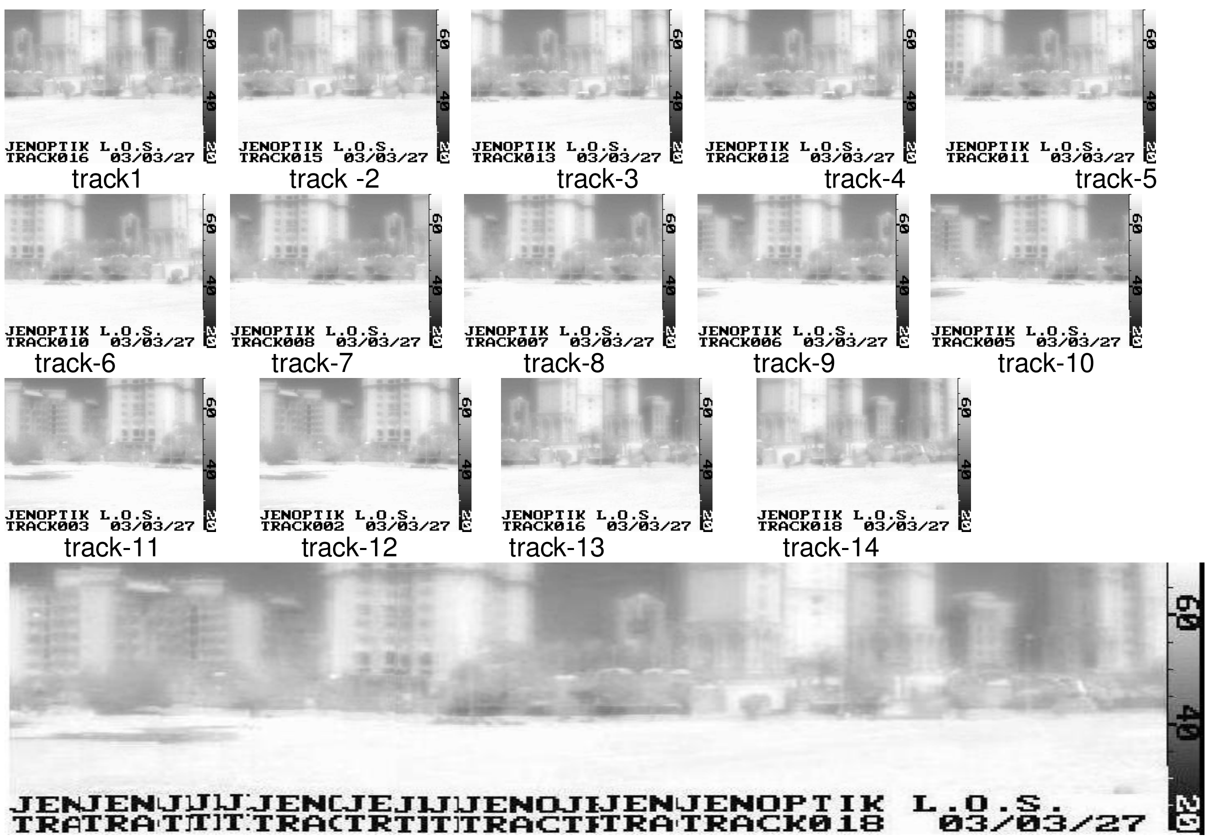


FIGURE 22: Sample images in infrared band and mosaic of IR images from track-1 to track-14

Observation
Location of maximum match point is shown in table.

TABLE III : Location of Maximum Match using Different Methods

S.N.	Image combination	Correlation Method	Fourier Transform Method	Mutual Information Method	Principal Component Analysis	Wavelet Transform Method	Feature Based Method
1	**T1-T2**	44	45	46	45	46	44
2	**T2-T3**	51	50	48	49	50	50
3	**T3-T4**	19	21	22	20	20	20
4	**T4-T5**	22	21	21	20	19	20
5	**T5-T6**	20	20	21	20	22	20
6	**T6-T7**	57	59	50	57	58	58
7	**T7-T8**	30	32	32	30	34	31
8	**T8-T9**	22	24	20	24	22	23
9	**T9-T10**	20	20	11	19	22	18
10	**T10-T11**	60	63	70	61	60	62
11	**T11-T12**	28	28	30	27	28	27
12	**T12-T13**	50	50	51	51	50	49
13	**T13-T14**	27	27	28	27	28	27

TABLE IV: Location of Maximum Match using Different Methods

S.N.	Image combination	Correlation Method	Fourier Transform Method	Mutual Information Method	Principal Component Analysis	Wavelet Transform Method	Feature Based Method
1	**T1-T3**	94	95	99	95	96	94
2	**T2-T4**	71	71	71	71	70	70
3	**T3-T5**	41	41	41	41	39	41
4	**T4-T6**	41	40	41	41	40	40
5	**T5-T7**	77	78	70	77	78	77
6	**T6-T8**	90	90	98	89	92	90
7	**T7-T9**	56	56	58	56	55	55
8	**T8-T10**	40	43	40	43	42	41
9	**T9-T11**	82	82	75	81	82	81
10	**T10-T12**	90	90	83	89	88	90
11	**T11-T13**	78	81	90	78	77	77
12	**T12-T14**	77	77	77	77	77	77

Implentation on Dataset-3

An arbitrary set of images of Saraswati College of Engg., Kharghar, Navi Mumbai were collected, using a panoramic set up. To capture the images the camera was mounted on a leveled tripod. While capturing the images camera was operated in manual mode where all camera parameters like aperture, shutter speed, focal length were constant. Since images are taken from different planes, the topology of mosaic was unknown. The motion between the images was unknown and was not assumed to be constant.


SCOE-1


SCOE-2


SCOE-3


SCOE-4


SCOE-5


SCOE-6

FIGURE 23: Sample images of SCOE and mosaic of images from SCOE1 to SCOE6

Observation

TABLE V : Location of maximum match

s. n.	Combination of images	Pixel based method		Fourier based method		Feature based method		Mutual Information based method		Principal component analysis	
		x-translation	y-translation	x-translation	y-translation	x-translation	y-translation	x-translation	y-translation	x-translation	y-translation
1	SCOE1 - SCOE2	157	-2	157	-5	156	-2	160	-2	159	-2
2	SCOE1 - SCOE2	163	35	164	35	163	35	163	35	164	35
3	SCOE1 - SCOE2	220	15	223	17	219	16	221	16	222	17
4	SCOE1 - SCOE2	163	10	161	5	162	10	161	09	162	10
5	SCOE1 - SCOE2	120	08	118	04	118	08	118	07	119	08

5. CONCLUSION

Various methods are reported in literature to register images which are in same band. In pixel based method cross correlation is used as similarity measure. It is observed that in natural images like buildings or scenery, correlation method shows match at multiple points. The feature based method makes use of features like point of intersection, edges, corners, centers of contours etc. for matching sample template with reference image. But this method is manual and hence time consuming. The method combining image features with correlation method have many advantageous properties of both feature-based and intensity based. It overcomes the limitation of intensity based method. Contour based methods do not use the gray values for matching and hence overcomes the limitations of spatial methods. Feature based method filter out the redundant information. Accuracy of this method is more but the limitation is, it is manual and slow. In frequency based method accuracy is more than correlation method but less as compared to other methods. But if we extract image features and then apply Fourier method accuracy increases. In frequency domain it should be noted that some form of interpolation must be used. These are some of the conclusions about methods used for registration of images which are in same spectral band. Image registration is difficult when images are obtained through different sensor types. Mutual Information, Hotelling Transform, Fuzzy logic are some of the approaches that can be used for multimodal image registration. For three different set of images SCOE1 to SCOE6, VR images form H1 to H15 and Infrared images track1 to track14 we carried out some of the popular algorithms and compared on the basis of point of match. Table I to Table V show the comparison. Combinational approach of wavelet and mutual information gives better results as compared to wavelet - correlation combination and Fourier based image registration. Even wavelet mutual information combination can be used in case of multimodal image registration. If we involve mutual information as similarity criteria, speed decreases and hence we can not use this combination in applications where speed is required. If we use eigen values and eigenvectors for registration of images accuracy is more precise than that of other methods. We can use this method for template matching**.** It is observed that Hotelling Transform can be used to align region or boundaries with the eigenvectors of object but this method can be applied only to regions and boundaries. It is observed that method based on information theory yields a more accurate registration than any other registration method. But this method has its own limitations. When images are of low resolution, when images contain little information, or when the region of overlap is small then mutual information result in mis-registration .It has one more limitation of speed , when time is an important constraint one cannot use this method . Although it has some limitations entropy and mutual information are best approaches for multimodal image registration.

6. FUTURE SCOPE

A major challenge in the current literature is to perform population registration on large collections of data sets. Currently available tools pre selects a reference dataset that is template and registers in a pair wise fashion. The computational complexity and accuracy of this approach can be eliminated by performing a simultaneous registration on the whole population. The methods, we have explored in this paper have desirable computational speed for achieving population registration. An immediate next step would be to investigate this open problem that may lead to a significant contribution.

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