Design Mathematical Tunable Gain PID-Like Sliding Mode Fuzzy Controller with Minimum Rule base

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Abstract

In this study, a mathematical tunable gain model free PID-like sliding mode fuzzy controller (GTSMFC) is designed to rich the best performance. Sliding mode fuzzy controller is studied because of its model free, stable and high performance. Today, most of systems (e.g., robot manipulators) are used in unknown and unstructured environment and caused to provide sophisticated systems, therefore strong mathematical tools (e.g., nonlinear sliding mode controller) are used in artificial intelligent control methodologies to design model free nonlinear robust controller with high performance (e.g., minimum error, good trajectory, disturbance rejection). Non linear classical theories have been applied successfully in many applications, but they also have some limitation. One of the best nonlinear robust controller which can be used in uncertainty nonlinear systems, are sliding mode controller but pure sliding mode controller has some disadvantages therefore this research focuses on applied sliding mode controller in fuzzy logic theory to solve the limitation in fuzzy logic controller and sliding mode controller. One of the most important challenging in pure sliding mode controller and sliding mode fuzzy controller is sliding surface slope. This paper focuses on adjusting the gain updating factor and sliding surface slope in PID like sliding mode fuzzy controller to have the best performance and reduce the limitation.

Keywords: Sliding Mode Fuzzy Controller, Tunable Gain, Artificial Intelligence, Robust Controller, Sliding Mode Controller, Fuzzy Logic Theory, Sliding Surface Slope

1. INTRODUCTION

The aim of science and modern technology has making an easier life. Conversely, modern life includes complicated technical systems which these systems are nonlinear, time variant, and uncertain in measurement, they need to have controlled [2]. Controller (control system) is devices that can sense data from plant to improve the plants behavior through actuation and computation. Fuzzy logic theory was used in wide range applications that fuzzy logic controller (FLC) is one of the most important applications in fuzzy logic theory. Conversely pure FLC works in many areas, it cannot guarantee the basic requirement of stability and acceptable performance [4-5].

Sliding mode controller (SMC) is one of the influential nonlinear controllers in certain and uncertain systems which are used to present a methodical solution for two main important controllers' challenges, which named: stability and robustness. Conversely, this controller is used in different applications; sliding mode controller has subsequent drawbacks i.e. chattering phenomenon, and nonlinear equivalent dynamic formulation in uncertain systems[6-12].
Although both SMC and FLC have been applied successfully in many applications but they also have some limitations. The boundary layer method is used to reduce or eliminate the chattering [1, 3, 12] and proposed method focuses on applied sliding mode controller to proposed PID error-base fuzzy logic system with minimum rule base and adjust the sliding surface slope to implement easily and avoid mathematical model base controller.
This paper is organized as follows:
In section 2, Detail of classical sliding mode controller is presented. The main subject of fuzzy logic methodology is presented in section 3. In section 4, the proposed method is presented. Modelling PUMA-560 robot manipulator formulation is presented in section 5. In section 6, the simulation result is presented and finally in section 7, the conclusion is presented.

3. CLASSICAL SLIDING MODE CONTROL

Sliding mode controller (SMC) is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. The sliding mode control law divided into two main parts [1, 3];

$$\hat{\tau} = \hat{\tau}_{eq} + \hat{\tau}_{dis} \tag{1}$$

Where, the model-based component $\hat{\tau}_{eq}$ is compensated the nominal dynamics of systems and τ_{dis} is discontinuous part of sliding mode controller and it is computed as [16-18]

$$\hat{\tau}_{dis} = K.sgn(S) \tag{2}$$

A time-varying sliding surface S is given by the following equation [18]:

$$s(x,t) = (\frac{d}{dt} + \lambda)\, e = 0 \tag{3}$$

Where λ is the constant and it is positive. To further penalize tracking error integral part can be used in sliding surface part as follows:

$$s(x,t) = (\frac{d}{dt} + \lambda)\left(\int_0^t e\, dt\right) = 0 \tag{4}$$

The main target in this methodology is keep $s(x,t)$ near to the zero when tracking is outside of $s(x,t)$. The function of $sgn(S)$ defined as;

$$sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \tag{5}$$

The K is the positive constant. One of the most important challenges in sliding mode controller based on discontinuous part is chattering phenomenon which can caused to oscillation in output. To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. This replace is caused to increase the error performance.

$$B(t) = \{x, |S(t)| \le \emptyset\}; \emptyset > 0 \tag{6}$$

Where ∅ is the boundary layer thickness. Therefore, to have a smote control law, the saturation function $Sat(S/_{\emptyset})$ added to the control law: Suppose that τ_{sat} is computed as [16-18]

$$\hat{\tau}_{sat} = K.sat\left(S/_{\emptyset}\right) \tag{7}$$

Where $Sat\left(S/_{\emptyset}\right)$ can be defined as

$$sat\left(S/_{\emptyset}\right) = \begin{cases} 1 & (S/_{\emptyset} > 1) \\ -1 & \left(S/_{\emptyset} < 1\right) \\ S/_{\emptyset} & (-1 < S/_{\emptyset} < 1) \end{cases} \tag{8}$$

Moreover by replace the formulation (7) in (1) the control output is written as;

$$\hat{\tau} = \hat{\tau}_{eq} + K.sat\left(S/_{\emptyset}\right) = \begin{cases} \tau_{eq} + K.sgn(S) & , |S| \ge \emptyset \\ \tau_{eq} + K.S/_{\emptyset} & , |S| < \emptyset \end{cases} \tag{9}$$

4. PID FUZZY LOGIC CONTROLLER

A PID fuzzy controller is a controller which takes error, integral of error and derivative of error as inputs. Fuzzy controller with three inputs is difficult to implementation, because it needs large number of rules, in this state the number of rules increases with an increase the number of inputs or fuzzy membership functions [4-5, 24-31]. In the PID FLC, if each input has 7 linguistic variables, then $7 \times 7 \times 7 = 343$ rules will be needed. The proposed PID FLC is constructed as a parallel structure of a P+D sliding surface slope and P+I+D sliding surface slope, and the output of the PID FLC is formed by adding the output of two fuzzy control blocks. This work will reduce the number of rules needed to $7 \times 7 = 49$ rules only.

This controller has two inputs (S_1, S_2) and one output (τ_{fuzzy}). The inputs are first sliding surface (S_1) which measures by the equation (3), the second sliding surface (S_2) which measures by the equation (4). For simplicity in implementation and also to have an acceptable performance the triangular membership function is used. The linguistic variables for first sliding surface (S_1) are; Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and it is quantized in to thirteen levels represented by: -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 the linguistic variables for second sliding surface (S_2) are; Fast Left (FL), Medium Left (ML), Slow Left (SL),Zero (Z), Slow Right (SR), Medium Right (MR), Fast Right (FR), and it is quantized in to thirteen levels represented by: -6, -5, -0.4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 and the linguistic variables to find the output are; Large Left (LL), Medium Left (ML), Small Left (SL), Zero (Z), Small Right (SR), Medium Right (MR), Large Right (LR) and it is quantized in to thirteen levels represented by: -85, -70.8, -56.7, -42.5, -28.3, -

14.2, 0, 14.2, 28.3, 42.5, 56.7, 70.8, 85. Design the rule base of fuzzy inference system can play important role to design the best performance of sliding mode fuzzy controller, this paper focuses on heuristic method which, it is based on the behavior of the control systems.
The complete rule base for this controller is shown in Table 1. Rule evaluation focuses on operation in the antecedent of the fuzzy rules in sliding mode fuzzy controller. Max-Min aggregation is used in this work which the calculation is defined as follows;

$$\mu_U(x_k, y_k, U) = \mu_{\cup_{i=1}^{r} FR^i}(x_k, y_k, U) = max\left\{min_{i=1}^{r}\left[\mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U)\right]\right\} \quad (10)$$

The last step to design fuzzy inference in sliding mode fuzzy controller is defuzzification. In this design the Center of gravity method (COG) is used and calculated by the following equation;

$$COG(x_k, y_k) = \frac{\sum_i U_i \sum_{j=1}^{r} \cdot \mu_u(x_k, y_k, U_i)}{\sum_i \sum_{j=1}^{r} \cdot \mu_u(x_k, y_k, U_i)} \quad (11)$$

Decrease the overshoot → S_2

$S_1 \backslash S_2$	FL	ML	SL	Z	SR	MR	FR
NB	LL	LL	LL	ML	SL	SL	Z
NM	LL	ML	ML	ML	SL	Z	SR
NS	LL	ML	SL	SL	Z	SR	MR
Z	LL	ML	SL	Z	SR	MR	LR
PS	ML	SL	Z	SR	SR	MR	LR
PM	SL	Z	SR	MR	MR	MR	LR
PB	Z	SR	SR	MR	LR	LR	LR

S_1 — Decrease the rise time

TABLE 1: Modified Fuzzy rule base table

This table used to describe the dynamics behavior of sliding mode fuzzy controller. Table 2 is shown the COG deffuzzification lookup table in fuzzy logic controller. These output values were obtained by mathematical on line tunable gain adjustment to reach the best performance in sliding mode fuzzy controller.

$\overrightarrow{S_2}$ / $S_1 \downarrow$	**Membership Function**												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-6	-85	-84.8	-84.8	-84	-82.1	-81	-79	-71	-68	-65	-62	-60	-54
-5	-84.8	-84	-82	-80	-78	-77	-74	-70	-64	-60	-56	-54	-47
-4	-78	-73	-70	-68	-64	-61	-60	-57	-55	-50	-47	-40	-38
-3	-70	-60	-58	-51	-42	-38	-34	-33	-31	-29	-28.4	-28.1	-28
-2	-50	-48	-45	-40	-38	-34	-32	-30	-28	-26	-25	-21	-20
-1	-30	-25	-21	-18	-16	-14	-10	-9	-8	-7	-6.8	-6	-5
0	-10	-8	-6	-1	2	3	6	7	8	10	12	15	17
1	15	18	21	22	23	25	27	28	29	30	30.5	30.8	31
2	29	29.8	31	33	34	34.6	35	35.2	36	37	38	39	42
3	40	41	42	43	45	45	46	46.3	46.8	47	48	51	52
4	48	49	50	52	53	55	56	57	58	59	60	61	63
5	60	61	62	63	64	66	67	68	68.5	69	70	70.8	71
6	66	68.7	68.9	70	72	74	75	77	78	79	81	83	84

TABLE 2 : COG lookup table in fuzzy sliding mode controller

5. THE PROPOSED METHOD

Sliding mode controller has two main parts: equivalent controller, based on dynamics formulation and sliding surface saturation part based on saturation continuous function to reduce or eliminate the chattering [19-23]. Reduce or eliminate the chattering regarding to reduce the error is play important role in this research. Applied sliding mode controller in fuzzy logic method have been proposed by several researchers [19-23] but in proposed method the new PID method with 49 rules is implemented and adjust by on line mathematical method. SMFC is fuzzy controller based on sliding mode method to easy implementation, stability, and robustness. A block diagram for sliding mode fuzzy controller is shown in Figure 1.

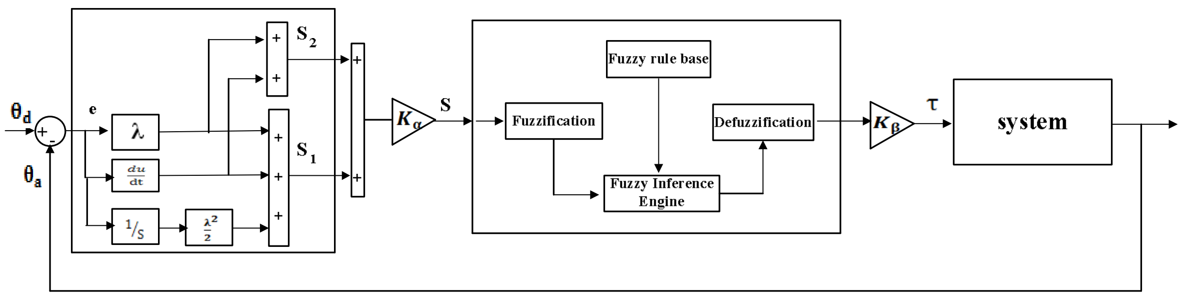

FIGURE 1: Sliding Mode Fuzzy Control (SMFC).

The system performance in this research is sensitive to the sliding surface slope λ input and output gain updating factor K_α & K_β for sliding mode fuzzy controller. Sliding surface slope can change the response of the output if large value of λ is chosen the response is very fast but the system is very unstable and conversely, if small value of λ considered the response of system is very slow but the system is very stable. Determine the optimum value of λ for a system is one of the most important challenging works in SMFC. For nonlinear, uncertain, and time-variant plants on-line tuning method can be used to self adjusting all coefficients. To keep the structure of the controller as simple as possible and to avoid heavy computation, a new supervisor tuner based on updated by a new coefficient factor kn is presented. In this method the supervisor part tunes the output scaling factors using gain online updating factors. The inputs of the supervisor term are error and change of error $(e, \dot{e})$ and the output of this controller is U, which it can be used to tune sliding surface slope, λ.

$$\boldsymbol{k_n = e^2 - \frac{(r_v - r_{vmin})^5}{1+|e|} + r_{vmin}} \tag{12}$$

$$r_v = \frac{\big(de(k) - de(k-1)\big)}{de(.)} \tag{13}$$

$$de(.) = \begin{cases} de(k); & if\ de(k) \geq de(k-1) \\ de(k-1) & if\ de(k) < de(k-1) \end{cases}$$

In this way, the performance of the system is improved with respect to the SMFC controller. So the new coefficient is calculated by;

$$\boldsymbol{\lambda_{new} = \lambda_{old} \times K_n} \tag{14}$$

$$\boldsymbol{K_{\alpha_{new}} = K_{\alpha_{old}}} \times K_n \tag{15}$$

5. APPLICATION

This method is applied to 3 revolute degrees of freedom (DOF) robot manipulator (e.g., first 3 DOF PUMA robot manipulator). The equation of an *n-DOF* robot manipulator governed by the following equation [1, 3]:

$$\boldsymbol{M(q)\ddot{q} + N(q,\dot{q}) = \tau} \tag{17}$$

Where $\boldsymbol{\tau}$ is actuation torque, $\boldsymbol{M(q)}$ is a symmetric and positive define inertia matrix, $N(q,\dot{q})$ is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form:

$$\boldsymbol{\tau = M(q)\ddot{q} + B(q)[\dot{q}\,\dot{q}] + C(q)[\dot{q}]^2 + G(q)} \tag{18}$$

Where the matrix of coriolios torque is $\boldsymbol{B(q)}$, $\boldsymbol{C(q)}$ is the matrix of centrifugal torques, and $\boldsymbol{G(q)}$ is the vector of gravity force. The dynamic terms in equation (15) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component $\boldsymbol{\ddot{q}}$ influences, with a double integrator relationship, only the joint variableq_i, independently of the motion of the other joints. Therefore, the angular acceleration is found as to be[3]:

$$\boldsymbol{\ddot{q} = M^{-1}(q).\{\tau - N(q,\dot{q})\}} \tag{19}$$

This technique is very attractive from a control point of view. This paper is focused on the design mathematical tunable gain model free PID-like sliding mode fuzzy controller for PUMA-560 robot manipulator based on [13-15].

6. RESULTS

PD sliding mode fuzzy controller (PD-SMFC) and mathematical tuneable gain model free PID-like sliding mode fuzzy controller (GTSMFC) were tested to compare response trajectory. In this simulation the first, second, and third joints are moved from home to final position without and with external disturbance. Trajectory performance, chattering phenomenon and disturbance rejection are compared in these two controllers. These systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude which the sample time is equal to 0.1. This type of noise is used to external disturbance in continuous and hybrid systems.

Tracking performances: Figure 2 shows the tracking performance in GTSMFC and SMFC without disturbance for Step trajectory.

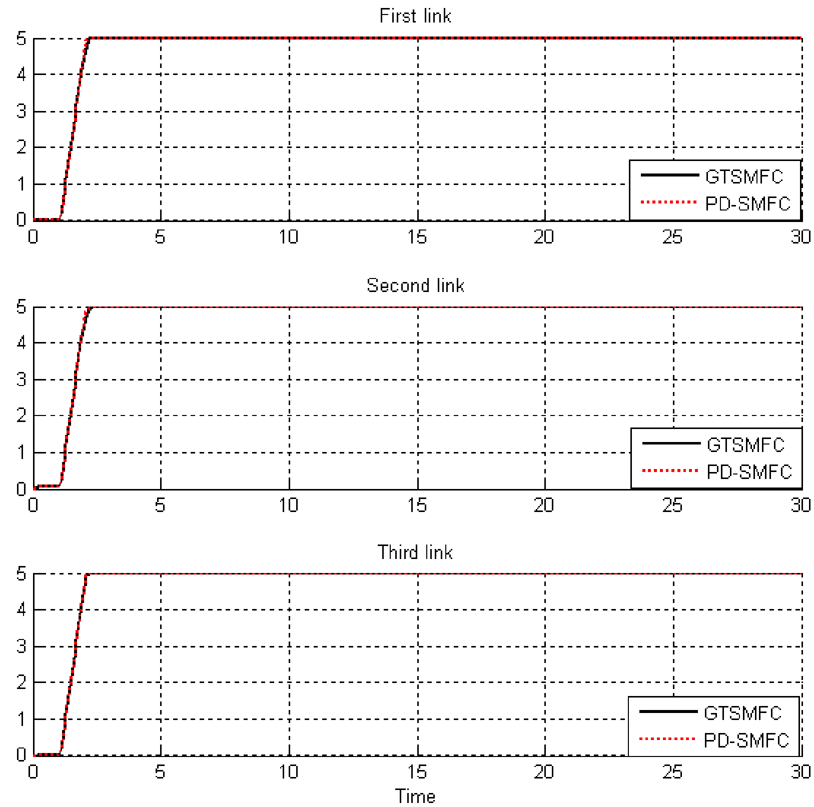

FIGURE 2 : Step GTSMFC and SMFC for First, second and third link trajectory without any disturbance.

By comparing, Figure 2, in GTSMFC and SMFC, both of them have the same overshoot **(1%)** the GTSMFC and SMFC's rise time are 0.483 Sec.

Disturbance Rejection

Figure 3 is indicated the power disturbance rejection in GTSMFC and SMFC. A band limited white noise with predefined of 40% the power of input signal is applied to these controllers; it found slight oscillations in SMFC's trajectory responses.

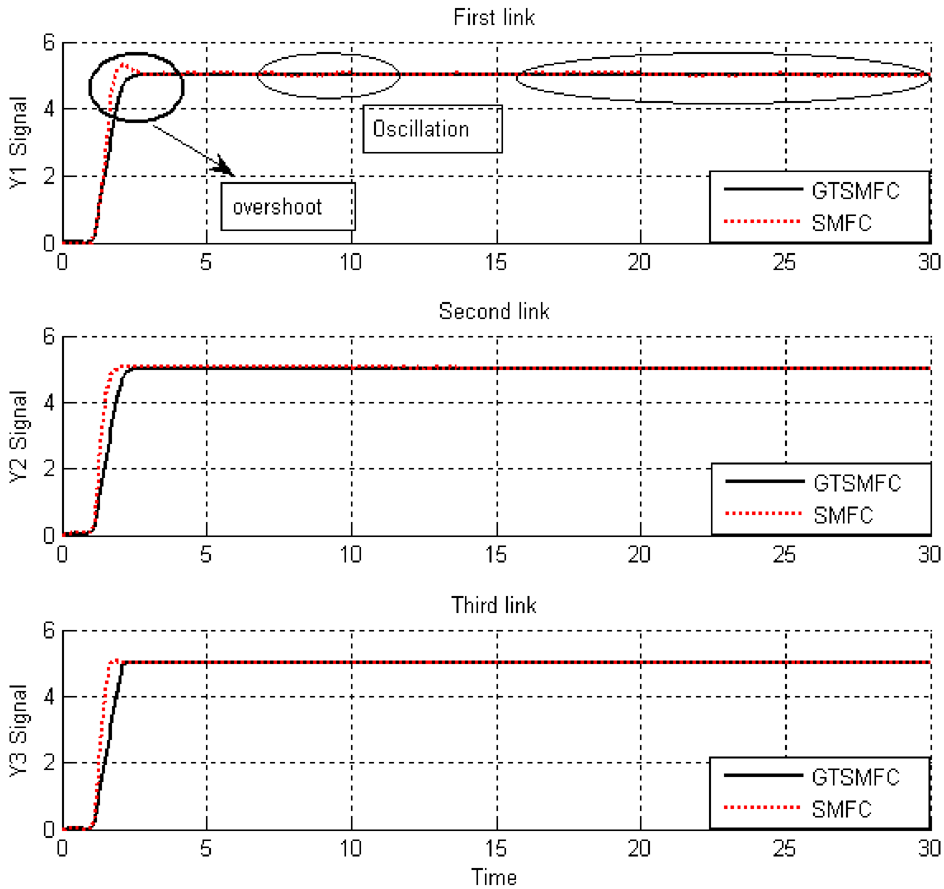

FIGURE 3 : Step GTSMFC and SMFC for First, second and third link trajectory with external disturbance**.**

Among above graph, relating to trajectory with external disturbance, SMFC has slightly fluctuations. By comparing overshoot; GTSMFC's overshoot **(1%)** is lower than SMFC's **(2.2%).**

Chattering Phenomenon

As mentioned in previous, chattering play important roles in sliding mode controller which one of the major objectives in this research is reduce or remove the chattering in system's output with uncertainty and external disturbance. Figure 4 has shown the power of boundary layer (saturation) method and online mathematical gain tuning methodto reduce the chattering in GTSMFC and also SMFC.

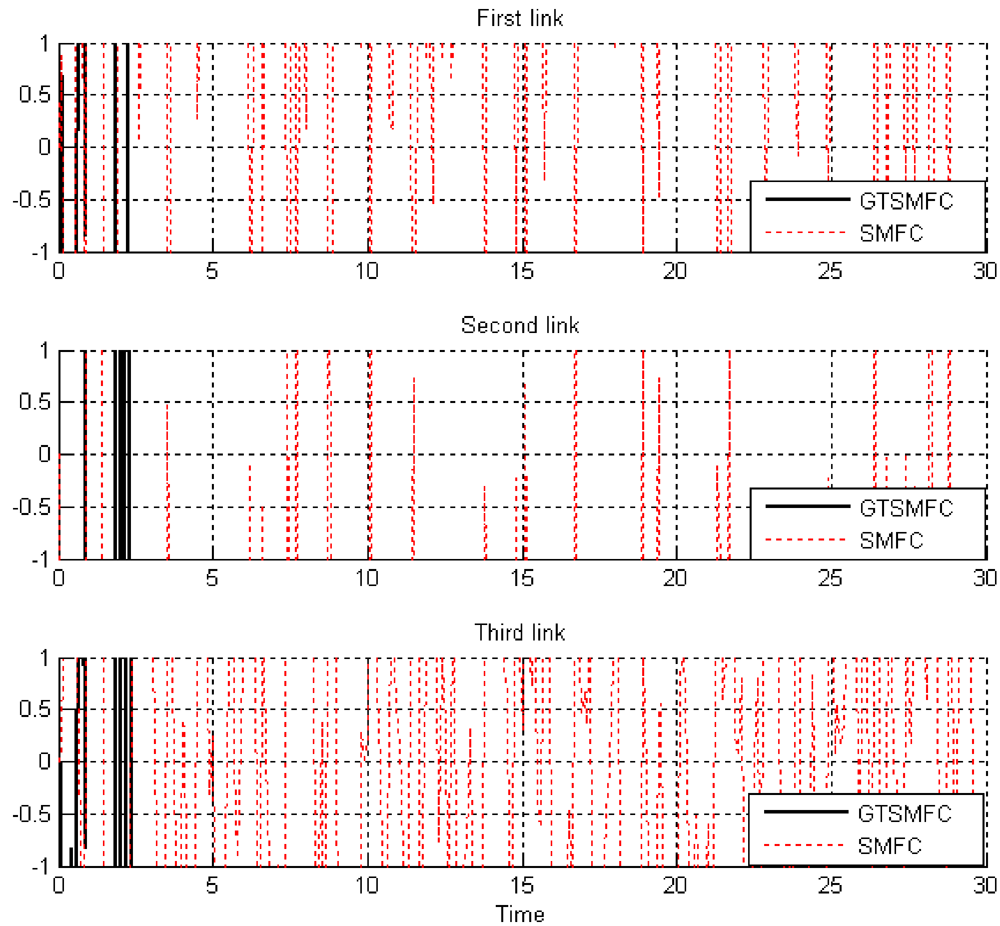

FIGURE 4: GTSMFC Vs. SMFC chattering with disturbance

7. CONCLUSION

Refer to the research, adjusting the gain updating factor and sliding surface slope in PID like sliding mode fuzzy controller design and application to robot manipulator has proposed in order to design high performance nonlinear controller in the presence of uncertainties. Regarding to the positive points in fuzzy logic controllers in uncertain systems, sliding mode controller which it has stability and robustness and on line tunable gain to tune the coefficient in structure and unstructured uncertain system the output responses have improved. Sliding mode controller by adding to the proposed PID fuzzy logic method with minimum rule base has covered negative points in pure fuzzy logic method and sliding mode methodology. Obviously the methodology of online tuning is the main goal in this research which most of researcher used fuzzy logic or neural network to adjust the parameters but in this method we used mathematical methodology that it is model free.

REFERENCES

[1] T. R. Kurfess, *Robotics and automation handbook*: CRC, 2005.

[2] K . Ogata, *Modern control engineering*: Prentice Hall, 2009.

[3] B. Siciliano and O. Khatib, *Springer handbook of robotics*: Springer-Verlag New York Inc, 2008.

[4] L. Reznik, *Fuzzy controllers*: Butterworth-Heinemann, 1997.

[5] S. Mohan and S. Bhanot, "Comparative study of some adaptive fuzzy algorithms for manipulator control," *International Journal of Computational Intelligence,* vol. 3, pp. 303–311, 2006.

[6] O. Kaynak, "Guest editorial special section on computationally intelligent methodologies and sliding-mode control," *IEEE Transactions on Industrial Electronics,* vol. 48, pp. 2-3, 2001.

[7] Shahnazi R., H. Shanechi, N. Pariz. "Position control of induction and servomotors: A novel adaptive fuzzy PI sliding mode control". IEEE Conferences on power engineering, 2006, P.P. 1-9.

[8] Medhaffar H., N. Derbel, and T. Damak. "A decoupled fuzzy indirect adaptive sliding mode controller with application to robot manipulator". Int. Journal on modeling, identification and control, 1(1), 23-29, 2006.

[9] Shahnazi R., H. Shanechi, N. Pariz. "Position control of induction and servomotors: A novel adaptive fuzzy PI sliding mode control". IEEE Journals on energy conversions, 23(1), 138-147, 2008.

[10] Weng C. C., W. S. Yu. "Adaptive fuzzy sliding mode control for linear time-varying uncertain systems". IEEE conference on fuzzy systems, 2008, P.P: 1483-1490.

[11] Yu Z. X. "Adaptive sliding mode-like fuzzy logic control for nonlinear systems". Journal of communication and computer, 6(1), 53-60, 2009.

[12] Piltan, F., et al. "Design sliding mode controller for robot manipulator with artificial tunable gain". Canaidian Journal of pure and applied science, 5 (2), 1573-1579, 2011.

[13] B. S. R. Armstrong, "Dynamics for robot control: friction modeling and ensuring excitation during parameter identification," 1988.

[14] B. Armstrong*, et al.*, "The explicit dynamic model and inertial parameters of the PUMA 560 arm," 2002, pp. 510-518.

[15] P. I. Corke and B. Armstrong-Helouvry, "A search for consensus among model parameters reported for the PUMA 560 robot," 2002, pp. 1608-1613.

[16] V. Utkin, "Variable structure systems with sliding modes," *Automatic Control, IEEE Transactions on,* vol. 22, pp. 212-222, 2002.

[17] R. A. DeCarlo*, et al.*, "Variable structure control of nonlinear multivariable systems: a tutorial," *Proceedings of the IEEE,* vol. 76, pp. 212-232, 2002.

[18] K. D. Young*, et al.*, "A control engineer's guide to sliding mode control," 2002, pp. 1-14.

[19] C. C. Weng and W. S. Yu, "Adaptive fuzzy sliding mode control for linear time-varying uncertain systems," 2008, pp. 1483-1490.

[20] M. Ertugrul and O. Kaynak, "Neuro sliding mode control of robotic manipulators," *Mechatronics,* vol. 10, pp. 239-263, 2000.

[21] P. Kachroo and M. Tomizuka, "Chattering reduction and error convergence in the sliding-mode control of a class of nonlinear systems," *Automatic Control, IEEE Transactions on,* vol. 41, pp. 1063-1068, 2002.

[22] Y. Li and Q. Xu, "Adaptive Sliding Mode Control With Perturbation Estimation and PID Sliding Surface for Motion Tracking of a Piezo-Driven Micromanipulator," *Control Systems Technology, IEEE Transactions on,* vol. 18, pp. 798-810, 2010.

[23] B. Wu*, et al.*, "An integral variable structure controller with fuzzy tuning design for electro-hydraulic driving Stewart platform," 2006, pp. 5-945.

[24] L. A. Zadeh, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic," *Fuzzy Sets and Systems,* vol. 90, pp. 111-127, 1997.

[25] J. Zhou and P. Coiffet, "Fuzzy control of robots," 2002, pp. 1357-1364.

[26] S. Banerjee and P. Y. Woo, "Fuzzy logic control of robot manipulator," 2002, pp. 87-88.

[27] K. Kumbla*, et al.*, "Soft computing for autonomous robotic systems," *Computers and Electrical Engineering,* vol. 26, pp. 5-32, 2000.

[28] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller. I," *IEEE Transactions on systems, man and cybernetics,* vol. 20, pp. 404-418, 1990.

[29] R. J. Wai*, et al.*, "Implementation of artificial intelligent control in single-link flexible robot arm," 2003, pp. 1270-1275.

[30] R. J. Wai and M. C. Lee, "Intelligent optimal control of single-link flexible robot arm," *Industrial Electronics, IEEE Transactions on,* vol. 51, pp. 201-220, 2004.

[31] M. B. Menhaj and M. Rouhani, "A novel neuro-based model reference adaptive control for a two link robot arm," 2002, pp. 47-52.