Global Stability of A Regulator For Robot Manipulators

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Abstract

In this work a regulator for robot manipulators is proposed, it has been developed considering that the equilibrium point of the closed-loop system is globally asymptotically stable in agreement with Lyapunov's direct method. The global asymptotic stability of the controlled system is analyzed. We present real-time experimental results to show the performance of the proposed regulator on a robot manipulator of direct drive with three degrees of freedom. The performance of the new control scheme is compared with respect to the popular PD Algorithm in terms of positioning error

Keywords: Regulator, Global asymptotic stability, Lyapunov function, position control, robot manipulators.

1. INTRODUCTION

The position control (also called regulation) of robot manipulators plays a fundamental role in design and analysis of the modern nonlinear robust controllers. The robot manipulators are programmed to execute a sequence of movements, such as moving to a location $(x_1, y_1, z_1)^T$ and later to move to a new location$(x_2, y_2, z_2)^T$. Some theoretical results on the stabilization of robot manipulators under bounded control actions have been reported in the open literature [1] [2].

The goal of position control is to move end-effector of the robot manipulator from any initial state to a final desired position. In an industrial robot the direct application is point-to-point control using the proportional-derivative control (PD) plus gravity compensation [1]; another used control is the proportional integral derivative control(PID) further gravity compensation and modifications of the same ones [3]. The controller design for these robots can be a linear or nonlinear model, and many of the industrial systems are nonlinear [4]-[6]. The PID requires the gravitational torque as partial component of the robot dynamics into its control law, it lacks of a global asymptotic stability proof, PID has local stability only in a closed loop with robot manipulator [9]-[15]. On the other hand, the PD has global asymptotic stability in a closed loop [7]. Finally the best feature of these controllers is that the tuning procedure to achieve global asymptotic stability reduces to select the proportional and derivative gains in a straightforward manner.

The compensation of gravity allows maintaining the desired position once a final position is reached, this requires the robots to apply the proper torque on each joint. Additionally, these regulators assume implicitly that the robot actuators are able to generate the requested torques.

However, in current manipulators robot, the actuators are constrained to supply limited torques [2],[8]. Due to these disadvantages the regulators PD and PID need to develop a control algorithm for industrial robots which does not contain their limitations, and at the same time allows performing the same or better activities carried out within the field of robot [5][16].

The control algorithms used for the control of robot manipulators should present in the equilibrium point of dynamic model global asymptotically stability, for this reason it is important that the proposed Lyapunov function candidate is positive definite and its derivative satisfies the conditions of a negative definite function [2]. To proof an appropriate performance and comply with the Lyapunov's stability criteria, in which it is established that the proposed function should be definite positively and with continuous partial derivatives, also should be considered that the candidate function fulfill the conditions of Lyapunov [2][8][9]. The Lyapunov theorem ensures that, any system that is globally asymptotically stable, must satisfy the conditions before mentioned [3][4]. Unfortunately, for a nonlinear co*nt*rol system, in order to determine a function that satisfies such conditions is in general difficult. It consists in determining functions whose derivatives along the trajectories can be rendered negative semi-definite. The proof of this result is made by the LaSalle's invariance principle [1]-[4],[19]-[27].

Kelly developed a mathematical analysis for a regulator with a polynomial function to determine its asymptotic stability [2][18]; also Meza in [17] performs a similar analysis. Sanchez and Reyes in [18] shows the analysis of a Cartesian controller and evaluate its performance by an experimental proof, yielding a good performance. Other authors, for example in [25] [26], analyze the stability of regulators, developed experimental tests and compare their results against the PD controller. In order to evaluate the performance for regulators, they only measure the Cartesian position error without considering the transitory, which could be evaluate by using other indicator, for example the norm L_2 is used to evaluate the performance along the trajectories [28].

In this paper, we introduce a position regulator for robots plus gravity compensation, motivated by the practical interest in the design of regulators and its analysis with the Lyapunov's theory, to determine that this possesses global asymptotically stability, in order to carry out its utility and performance in the position control. It is fundamental, especially in this case, where a regulator is designed with stabilizing feedback, which are expressed in terms of the first derivatives of Lyapunov's function [1],[2].

Real-time experimental results on a direct-drive robot manipulator with three degrees of freedom are presented. The proposed regulator performance to reach the desired position is good in comparison with the simple PD algorithm. In order to show its utility and performance we verify the positions errors between the initial position and final position taking into account for characteristics of our robot.

This paper is organized by the following form. Section 2, shows the model of the dynamics of robots and some important properties. The regulator of bounded action for position control, and its analysis to demonstrate that it has global asymptotic stability with a Lyapunov's theory is presented in Section 3. In Section 4, we present results of experimental of regulators into a three degrees-of-freedom arm, and its comparison with the control PD. Finally, we offer some conclusions in Section 5.

2. PRELIMINARIES

The dynamics model with n degrees-of-freedom of a manipulator robot with rigid links is represented by

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{\boldsymbol{q}} + g(q) \tag{1}$$

where τ is an $n \times 1$ vector of applied torque for the robot, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite inertia matrix, $C(q,\dot{q}) \in R^{n \times n}$ contains the centrifugal and Coriolis forces the size $n \times n$, $B \in R^{n \times n}$, represents the viscous friction matrix of the robot joints, $\boldsymbol{q} \in R^n$ is the vector of position, $\dot{\boldsymbol{q}} \in R^n$ is the vector of velocities of the link, $\ddot{\boldsymbol{q}} \in R^n$ is the vector of acceleration and $g(\boldsymbol{q})$ is the torque due to gravitational forces and, it is the $n \times 1$ vector, obtained as the gradient of the potential energy $U(\boldsymbol{q})$ due to gravity [1][5]:

$$g(\boldsymbol{q}) = \frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}}. \tag{2}$$

To simplify the process of analysis and compression of control law is necessary the application of the following properties of the dynamics model (1), so to facilitate the demonstration of stability condition(see [2]).

Property 1. The inertia matrix $M(\boldsymbol{q})$ is a positive definite symmetric matrix and its components are a function of $\boldsymbol{q}$, satisfies the following:

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\right] \dot{\boldsymbol{q}} = 0 \qquad \forall\, \dot{\boldsymbol{q}} \in R^n. \tag{3}$$

Property 2. Other important property of inertia matrix is:

$$M(\boldsymbol{q}) = C(\boldsymbol{q}, \dot{\boldsymbol{q}}) + C(\boldsymbol{q}, \dot{\boldsymbol{q}})^T \tag{4}$$

Property 3. The matrix centrifugal and Coriolis forces $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$, satisfies the following:

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = 0 \qquad \forall\, \boldsymbol{q} \in R^n \tag{5}$$

3. REGULATOR WITH BOUNDED ACTION FOR POSITION CONTROL

The position control problem of robot manipulators can be formulated as follows: considering the dynamics equation (1) of a robot of n degree-of-freedom, given a desired joint position q_d assumed constant, trying to determine a vector function $\boldsymbol{\tau}$, so that the position associated with the coordinates $\boldsymbol{q}$ asymptotically reaches the robot joint q_d. Formally the goal of pure position control or simply position control, is to determine $\boldsymbol{\tau}$ so that:

$$\lim_{t\to\infty} \begin{bmatrix} \tilde{q}(t) \\ \dot{q}(t) \end{bmatrix} \to 0$$

Taking into account the above, we present the design and analyses of a new control scheme for robot manipulators, the proposed regulator is an algorithm based on the energy shaping which is written in function of the potential energy, composed of a proportional and a derivative part, in both by adding the same function and the compensation of gravity. We propose the following rational saturated regulator (RSR):

$$\tau = Kp\frac{12\tilde{\boldsymbol{q}}}{\sqrt{5+6\tilde{\boldsymbol{q}}^2}} - Kv\frac{12\dot{\boldsymbol{q}}}{\sqrt{5+6\dot{\boldsymbol{q}}^2}} + g(\boldsymbol{q}) \tag{6}$$

Where Kp and Kv are the diagonal positive definite $n \times n$ matrices and so-called proportional gain and derivative gain, respectively; which are selected by the designer[2][5]. On the other hand, $\tilde{\boldsymbol{q}} \in R^n$, it is the position error between the manipulator robot's actual position and the desired position, defined as:

$$\tilde{\boldsymbol{q}} = q_d - \boldsymbol{q} \tag{7}$$

by notation is defined

$$\frac{12\tilde{\boldsymbol{q}}}{\sqrt{5+6\tilde{\boldsymbol{q}}^2}} = \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} \tag{8}$$

term of the speed is given by:

$$\frac{12\dot{\boldsymbol{q}}}{\sqrt{5+6\dot{\boldsymbol{q}}^2}} = \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix}. \tag{9}$$

Taking into account (8) and (9), we can write the RSR- regulator (6) as:

$$\tau = Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} - Kv \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix} - g(\boldsymbol{q}) \tag{10}$$

The closed-loop system equation formed by the robot dynamics (1) and structure control of energy shaping (10) generates a global stable equilibrium point in the sense of Lyapunov, such an equation expressed in terms of state variables is $[\tilde{\boldsymbol{q}}^T, \dot{\boldsymbol{q}}^T]^T$ in the following way:

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}\left[Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} - Kv \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - B\dot{\boldsymbol{q}} \right] \end{bmatrix} \tag{11}$$

In order to carry out the stability analysis, we propose the following radially unbounded positive definite function as Lyapunov function candidate:

$$V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + U_a(Kp, \tilde{\boldsymbol{q}}). \tag{12}$$

Where the first term of this Lyapunov function candidate corresponds to the kinetic energy, which is a positive definite function in $\dot{\boldsymbol{q}}$, because inertia matrix *M(**q**)* is positive definite. The second term $U_a(Kp, \tilde{\boldsymbol{q}})$ is the artificial potential energy, this term is a radially unbounded positive definite function in $\tilde{\boldsymbol{q}}$, and design Kp is a positive-definite matrix.

The term $U_a(Kp, \tilde{\boldsymbol{q}})$ in (12) is defined in the following way:

$$U_a(Kp, \tilde{\boldsymbol{q}}) = 2 \begin{bmatrix} \sqrt{\sqrt{5+6\tilde{q}_1^2} - \sqrt{5}} \\ \sqrt{\sqrt{5+6\tilde{q}_2^2} - \sqrt{5}} \\ \vdots \\ \sqrt{\sqrt{5+6\tilde{q}_n^2} - \sqrt{5}} \end{bmatrix}^T Kp \begin{bmatrix} \sqrt{\sqrt{5+6\tilde{q}_1^2} - \sqrt{5}} \\ \sqrt{\sqrt{5+6\tilde{q}_2^2} - \sqrt{5}} \\ \vdots \\ \sqrt{\sqrt{5+6\tilde{q}_n^2} - \sqrt{5}} \end{bmatrix}. \tag{13}$$

Therefore, incorporating (13) into (12), we get

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})=\frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}+2\begin{bmatrix}\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\\ \sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\\ \vdots\\ \sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\end{bmatrix}^T Kp\begin{bmatrix}\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\\ \sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\\ \vdots\\ \sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\end{bmatrix}. \tag{14}$$

To demonstrate that candidate function satisfies the Lyapunove's conditions, we have (11) which complies with the following conditions: The first term is defined positive because the inertia matrix $M(\boldsymbol{q})$ is positive definite. The second term is the artificial potential energy also is a positive definite function on the position error vector $\tilde{\boldsymbol{q}}$**.** Note that the term $\sqrt{5}$ was introduced to make $\tilde{\boldsymbol{q}}=0$, $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is zero; Therefore, the Lyapunov function candidate $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is a positive definite function in form globally and radially unbounded.

The time derivative of the Lyapunov function candidate (14) along the trajectories of the closed-loop system can be written as

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})=\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}}+\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\begin{bmatrix}\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\\ \sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\\ \vdots\\ \sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\end{bmatrix}^T Kp\begin{bmatrix}\frac{12\tilde{q}_1}{\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_1^2}}\\ \frac{12\tilde{q}_2}{\sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_2^2}}\\ \vdots\\ \frac{12\tilde{q}_n}{\sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_n^2}}\end{bmatrix}\dot{\tilde{\boldsymbol{q}}}. \tag{15}$$

Considering that the derived of the position error it is $\dot{\tilde{\boldsymbol{q}}}=-\dot{\boldsymbol{q}}$, because the desired position q_d is a constant, and substituting the value of the acceleration $\ddot{\boldsymbol{q}}$ of the equation of closed-loop (11) into (15), we have

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})=\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\left[M(\boldsymbol{q})^{-1}\left[Kp\begin{bmatrix}\frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}}\\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}}\\ \vdots\\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}}\end{bmatrix}-Kv\begin{bmatrix}\frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}}\\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}}\\ \vdots\\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}}\end{bmatrix}-C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}-B\dot{\boldsymbol{q}}\right]\right]+\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}}-$$

$$\begin{bmatrix}\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\\ \sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\\ \vdots\\ \sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\end{bmatrix}^T Kp\begin{bmatrix}\frac{12\tilde{q}_1}{\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_1^2}}\\ \frac{12\tilde{q}_2}{\sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_2^2}}\\ \vdots\\ \frac{12\tilde{q}_n}{\sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_n^2}}\end{bmatrix}\dot{\boldsymbol{q}} \tag{16}$$

solving the suitable operations, we have:

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} - \dot{\boldsymbol{q}}^T Kv \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix} - \dot{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T B\dot{\boldsymbol{q}} -$$

$$\begin{bmatrix} \sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}} \\ \sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}} \\ \vdots \\ \sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}} \end{bmatrix}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{\sqrt{5+6\tilde{q}_1^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{\sqrt{5+6\tilde{q}_2^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{\sqrt{5+6\tilde{q}_n^2}-\sqrt{5}}\ \sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} \dot{\boldsymbol{q}} \tag{17}$$

using property 1 in the third term and fourth term

$$-\dot{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} \equiv 0. \tag{18}$$

Let us define w as: $w = \sqrt{5+6\tilde{q}_i^2 - \sqrt{5}}$, where $i = 1,2 \dots n$, and substituting w inside the fifth term of (17) together with (18)

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} - \dot{\boldsymbol{q}}^T Kv \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix} - \dot{\boldsymbol{q}}^T B\dot{\boldsymbol{q}} - \begin{bmatrix} \sqrt{w_1} \\ \sqrt{w_2} \\ \vdots \\ \sqrt{w_n} \end{bmatrix}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{w_1}\ \sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{w_2}\ \sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{w_n}\ \sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} \dot{\boldsymbol{q}} \tag{19}$$

The fourth term of (19) can be written as:

$$\begin{bmatrix} \sqrt{w_1} \\ \sqrt{w_2} \\ \vdots \\ \sqrt{w_n} \end{bmatrix}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{w_1}\ \sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{w_2}\ \sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{w_n}\ \sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} \dot{\boldsymbol{q}} = \begin{bmatrix} \sqrt{w_1} \\ \sqrt{w_2} \\ \vdots \\ \sqrt{w_n} \end{bmatrix}^T Kp \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{w_1}\ \sqrt{5+6\tilde{q}_1^2}} & 0 & \cdots & 0 \\ 0 & \frac{12\tilde{q}_2}{\sqrt{w_2}\ \sqrt{5+6\tilde{q}_2^2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{12\tilde{q}_n}{\sqrt{w_n}\ \sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} \dot{\boldsymbol{q}} \tag{20}$$

Note that, the matrix Kp is a diagonal positive definite matrix, and as the product of diagonal matrices is commutative, with which simplify the expression being (20) as:

$$\begin{bmatrix} \sqrt{w_1} \\ \sqrt{w_2} \\ \vdots \\ \sqrt{w_n} \end{bmatrix}^T \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{w_1}\ \sqrt{5+6\tilde{q}_1^2}} & 0 & \cdots & 0 \\ 0 & \frac{12\tilde{q}_2}{\sqrt{w_2}\ \sqrt{5+6\tilde{q}_2^2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{12\tilde{q}_n}{\sqrt{w_n}\ \sqrt{5+6\tilde{q}_n^2}} \end{bmatrix} Kp\dot{\boldsymbol{q}} = \begin{bmatrix} \frac{12\tilde{q}_1}{\sqrt{5+6\tilde{q}_1^2}} \\ \frac{12\tilde{q}_2}{\sqrt{5+6\tilde{q}_2^2}} \\ \vdots \\ \frac{12\tilde{q}_n}{\sqrt{5+6\tilde{q}_n^2}} \end{bmatrix}^T Kp\dot{\boldsymbol{q}}. \tag{21}$$

Substituting (21) into (19), finally we obtain the result the time derivative of Lyapunov candidate function

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T Kv \begin{bmatrix} \frac{12\dot{q}_1}{\sqrt{5+6\dot{q}_1^2}} \\ \frac{12\dot{q}_2}{\sqrt{5+6\dot{q}_2^2}} \\ \vdots \\ \frac{12\dot{q}_n}{\sqrt{5+6\dot{q}_n^2}} \end{bmatrix} - \dot{\boldsymbol{q}}^T B\dot{\boldsymbol{q}} \ \leq 0 \quad (22)$$

Using the fact that the Lyapunov function candidate (14) is a globally positive definite function and its time derivative is a globally negative semi-definite function, we conclude that the equilibrium of the closed-loop system (11) is stable. Finally, we can use the LaSalle's invariance principle to demonstrate the global asymptotic stability of the equilibrium. Toward this end, let us defined the set Ω as:

$$\Omega = \left\{ \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} \epsilon\, R^{2n} : \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = 0 \right\} \quad (23)$$

$$\Omega = \{\tilde{\boldsymbol{q}}\ \epsilon\ R^n,\ \ \dot{\boldsymbol{q}} = 0\ \epsilon\ R^n\} \quad (24)$$

The unique invariant is $[\tilde{\boldsymbol{q}}^T, \dot{\boldsymbol{q}}^T]\ \epsilon\ R^{2n}$. We conclude that this equilibrium is globally asymptotically stable.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The algorithm RSR is experimentally tested in an experimental platform, which consists of a three degree-of freedom direct-driver robot manipulator, designed and built at The Benemerita Universidad Autónoma de Puebla to research robot control algorithms. Figure 1 shows the manipulator robot. It is a direct-drive manipulator robot that consists of links made of 6061 aluminum actuated by brushless direct drive servo actuator from Parker Compumotor to drive the joints without gear reduction (the motors characteristics used in the experimental robot are on the Table 1).

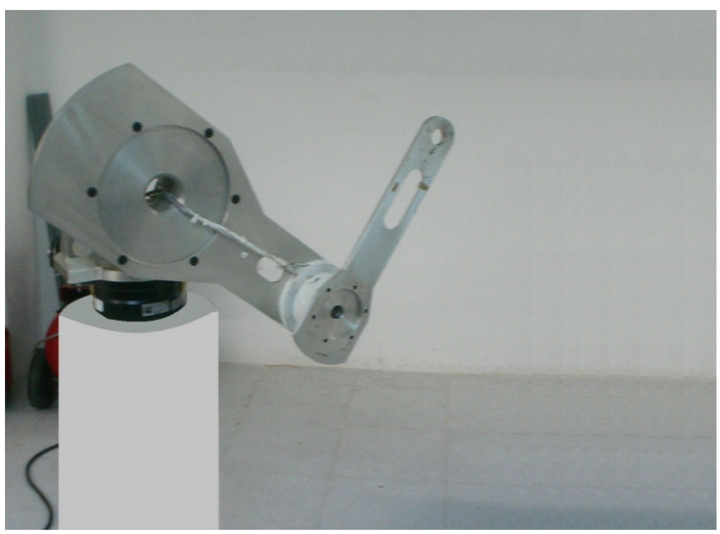

FIGURE 1: Robot Manipulator.

In this robot manipulator we recall the equation of the control law (10) to apply the torque in each joint to produced the movement in every link of the robot. The proportional gains were chosen such that $\tau < \|\tau_{max}\|$, where τ_{max} represents the maximum applied torque of the ith joint (see limits of actuators in Table 1).

The empirical procedure that was used to select the tunning of the proportional gain is given by: $Kp_i = 80\%\tau_{imax}/q_{di}$, after several experimental tests and considering that the best time response without overshoot the minimum steady-state position error were obtained without reach the saturation zone of the actuators torques.

Link	Model	Torque [Nm]	p/rev
Base	DM-1015B	15	1024000
Shoulder	DM-1050A	50	1024000
Elbow	DM-1004C	4	1024000

TABLE 1: Servo actuators of the robot manipulator.

The proportional and derivative gains were selected as:

;

The initial conditions for the joint positions of the robot manipulator were defined as:

Link one (Base) degrees; Link two (Shoulder) degrees; Link three (Elbow) degrees.

The initial conditions for the joint velocities are:

Base joint degrees/sec; Shoulder joint degrees/sec; Elbow joint degrees/sec.

The desired final position was definite as: *degrees*, *degrees* and *degrees*, considered time in the experiment to that the robot arrive to the final positions was seconds.

The experimental results of RSR-regulator are depicted in Figures 2-3. We analyze the acting of the regulator in function of the position errors and the applied torque .

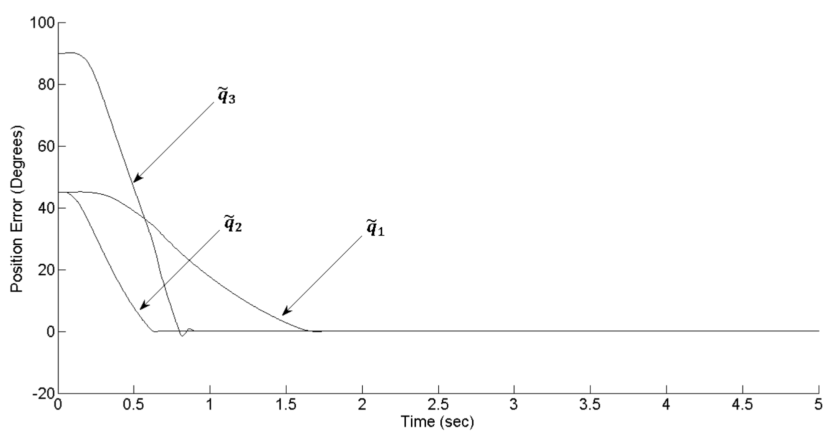

FIGURE 2: Position error of regulator RSR.

Figure 2 shows that, as time evolves, the position errors tend to zero, in agreement with position control objective defined in equation (10), in this case we can see with regard to the tracking position error in the exponential regulator was of approximately 0.035156±0.0087 *degree* in 1.757517*sec*. = 0.086329 ± 0.0074 *degree* in 0.672499*sec*. 0.123596 ± 0.0269 *degree* in 0.902499*sec*. The position is maintained until the end of the experiment. It can be concluded that the robot system moves fast and practically without overshoot toward the reference position .

Figure 3 shows that the applied torque for each joint remain within the prescribed limits of these actuators (see Table 1), and the magnitude of the torques stay inside physics limits in steady-state. Each joint has a smooth moving, free from irregularities; in stationary state the magnitude of the base torque is: 2.497 *Nm*; for shoulder joint 34.775 *Nm*; and elbow joint 2.963 *Nm*. Also it is observed that the applied torque for the link two ( ) is higher than the other two links, this is because of it corresponds to the shoulder joint, which has to support the weight of the arm, causing high oscillations of the applied torque during the first instants of time of the movement, and later decrease until a value of 11.11396±1.23*Nm*; the applied torque by the servos to the links one and three are approximately constant during the beginning of the movement, later on they present smaller oscillations to locate to the links in their final position. Torque oscillates among -0.61233*Nm* and 0.920216*Nm*; among -0.11133*Nm* and 1.524700*Nm*. The value of the applied torque to the shoulder and elbow links do not decrease to zero because they have to stay in the specified position, for such a reason the servo actuators apply a torque to compensate the effects of the force of gravity that it acts on them.

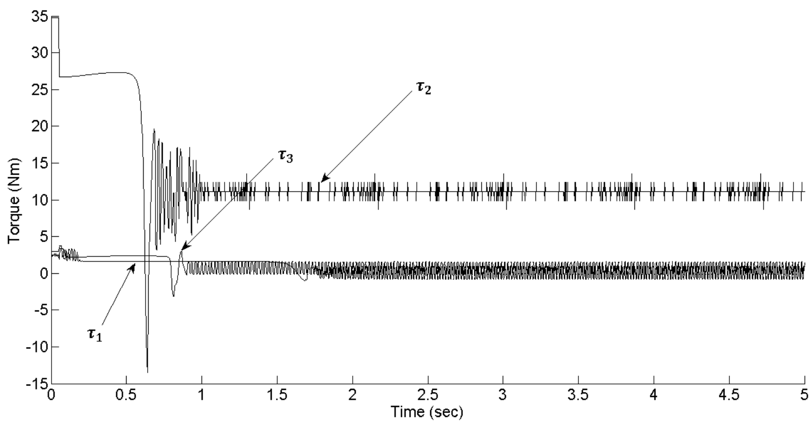

FIGURE 3: Applied torque to the joints of the manipulator.

In order to compare the performance of the RSR-regulator presented here, develop experimental tests with the known PD-regulator

(25)

The tunning-up for proportional and derivative gains were selected as:

; .

Figure 4 shows that the position error of PD regulator falls with a tendency to zero, however, this presents a broad overshoot and when the experiment concludes his position error was several degrees −0.72399 ± 0.000275 *degrees*, 5.47241 ± 0.00386 *degrees*, 4.466492 ± 0.001098 *degrees*.

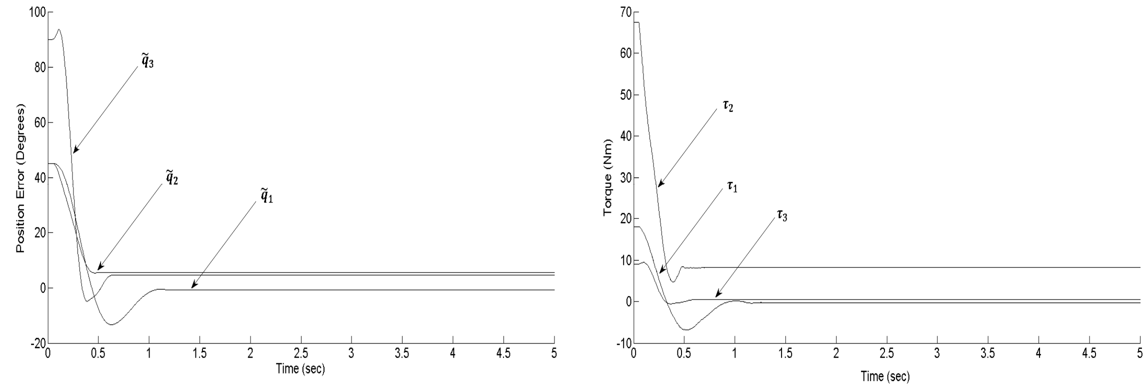

FIGURE 4: Position error and applied torque by means of the control PD.

As we can appreciate the values of the torque applied by the PD-regulator (see Figure 4), they are bigger than the maximum torque values that can give the servo actuators according to Table 1; therefore they work in the saturation condition during the first instants of the robot's displacement. Applied torque by the PD when the robot manipulator approaches to the desired position stays approximately constant to maintain the position.

Previous results do not allow us to properly compare the performance of proposed regulator and PD-regulator. With this propose, the L_2 norm criterion has error useful. The scalar value L_2 norm of position error is given by:

$$L_2 \qquad - \tag{26}$$

Where is the duration of the experimental test, in our case, 5 seconds. A small L_2 value represents a better regulator performance.

The performance result of regulators is shown in Figure 5. It can be observed that the smaller L_2 norm corresponds to RSR-regulator, considering these results, we can conclude that the RSR-regulator had the best performance when compared whit PD-regulator using the scalar-valued L_2 norm criterion.

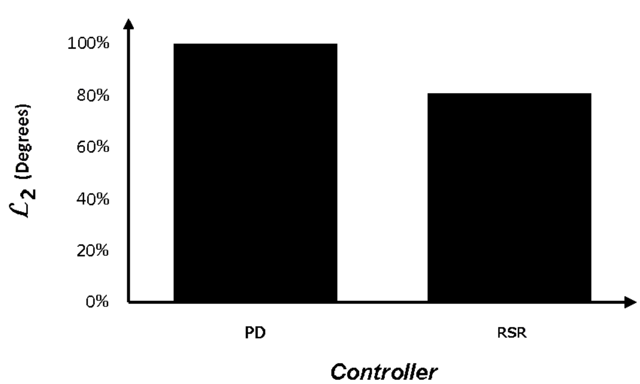

FIGURE 5: Performance index of regulators.

5. CONCLUSIONS & FUTURE WORK

In this paper, we have presented a simple regulator (RSR) to solve the position control problem of robot manipulators, motivated by the practical interest of relying on control algorithms that preserve global asymptotic stability. The RSR-regulator is analyzed, demonstrating that the origin of the state space is asymptotically stable in Lyapunov's sense.

We have developed experiments on a direct-drive robot system of 3 degrees-of freedom, that demonstrate the stability and performance of the RSR regulator. We have shown that, for desired position and under the design guidelines, the requested torques remain within the prescribed limits of the actuators, guaranteeing their correct operation during the experiment, and the steady-state position errors are inside an interval around zero, as shown from the results of the experiments that are carried out. The L_2 norm provided a suitable index used to compare the performance of the RSR-regulator with the PD-regulator under de same conditions, which for the case presented herein, it showed that the RSR-regulator has a better performance. A future work is the generation with auto-tuning control algorithms that enable better performance and reduce time tuning the gains.

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