

A Fuzzy Arithmetic Approach for Perishable Items in Discounted Entropic Order Quantity Model

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Abstract

This paper uses fuzzy arithmetic approach to the system cost for perishable items with instant deterioration for the discounted entropic order quantity model. Traditional crisp system cost observes that some costs may belong to the uncertain factors. It is necessary to extend the system cost to treat also the vague costs. We introduce a new concept which we call entropy and show that the total payoff satisfies the optimization property. We show how special case of this problem reduce to perfect results, and how post deteriorated discounted entropic order quantity model is a generalization of optimization. It has been imperative to demonstrate this model by analysis, which reveals important characteristics of discounted structure. Further numerical experiments are conducted to evaluate the relative performance between the fuzzy and crisp cases in EnOQ and EOQ separately.

Key Words: Discounted Selling Price, Fuzzy, Instant Deterioration, Inventory.

1. INTRODUCTION

In this paper we consider a continuous review, using fuzzy arithmetic approach to the system cost for perishable items. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in [5] and [13]. From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. For this reason, we consider the same since no researcher have discussed EnOQ model by introducing the holding cost and disposal cost as the fuzzy number. The model provides an approach for quantifying these benefits which can be substantial, and should be reflected in fuzzy arithmetic system cost. Our objective is to find optimal values of the policy variables when the criterion is to optimize the expected total payoff over a finite horizon.

In addition, the product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs, bakery items etc. suffer from depletion by decent spoilage. Lot of articles are available in inventory literature considering deterioration. Interested readers may consult the survey paper of [10], [18], [16], [15], [4] and [9] classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Products which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles are available in inventory literature considering deterioration. If this product starts to deteriorate as soon as it is received in the stock, then there is no option to provide pre-deterioration discount. Only we may give post deterioration discount on selling price.

Every organisation dealing with inventory faces a numbers of fundamental problems. Pricing decision is one of them. In the development of an EOQ system, we usually omit the case of discounting on

selling price. But in real world, it exists and is quite flexible in nature. On the other hand, in order to motivate customers to order more quantities for instant deterioration model usually supplier offers discount on selling prices. [2] developed an inventory model under continuous discount pricing. [11] studied an inventory problem under the condition that multiple discounts can be used to sell excess inventory. [14] mentioned that discount is considered temporarily for exponentially decaying inventory model. However, most of the studies except few, do not attempt to unify the two research streams: temporary price reductions and instant deterioration. This paper outlines the issue in details.

The awareness of the importance of including entropy cost is increasing everyday. Indeed, entropy is frequently defined as the amount of disorder in a system. The above consideration leads us to some important points. First, the use of entropy must be carefully planned, taking into account the multiplicity of objectives inherent in this kind of decision problem. Second, these are several economic strategies with conflicting objectives in this kind of decision making process. [12] proposed an analogy between the behaviour of production system and the behaviour of physical system. The main purpose of this research is to introduce the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement process.

In last two decades the variability of inventory level dependent demand rate on the analysis of inventory system was described by researchers like [17], [1] and [3]. They described the demand rate as the power function of on hand inventory. There is a vast literature on stock development inventory and its outline can be found in the review article by [19] where he unified two types of inventory level dependent demand by considering a periodic review model. Researchers such as [1], [16], [17], [4], [6] and [8] discussed the EOQ model assuming time value of money, demand rate, deterioration rate, shortages and so on a constant or probabilistic number or an exponential function. In this paper we consider demand as a constant function for instant deterioration model.

The paper tackles to investigate the effect of the approximation made by using the average payoff when determining the optimal values of the policy variables. The problem consists of the simultaneous optimization of fuzzy entropic EOQ and crisp entropic EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed with the help of [7] and optimum solution is obtained through LINGO software. In order to make the comparisons equitable a particular evaluation function based on discount is suggested. Numerical experiments are carried out to analyse the magnitude of the approximation error. However, a discount during post deterioration time, fuzzy system cost which might lead to a non-negligible approximation errors. The remainder of this paper is organised as follows. In section 2 assumptions and notations are provided for the development of the model. Section 3 describes the model formulation. Section 4 develops the fuzzy model. Section 5 provides mathematical analysis. In section 6, an illustrative numerical experiment is given to illustrate the procedure of solving the model. Finally section 7 concludes this article with a brief summary and provides some suggestions for future research.

TABLE-1: Major Characteristics of Inventory Models on Selected Researches.

| Author(s) and published Year | Structure of the Model | Deterioration | Inventory Model Based on | Discount allowed | Demand | Back-logging allowed |
|------------------------------|------------------------|-------------------------|--------------------------|------------------|--------------------|----------------------|
| Mahata et al. (2006) | Fuzzy | Yes (constant) | EOQ | No | Constant | No |
| Panda et al. (2009) | Crisp | Yes (constant) | EOQ | Yes | Stock dependent | Yes (partial) |
| Jaber et al. (2008) | Crisp | Yes (on hand inventory) | EnOQ | No | Unit selling price | No |
| Vujosevic et al. (1996) | Fuzzy | No | EOQ | No | Constant | No |
| Skouri et al. (2007) | Crisp | Yes (Weibull) | EOQ | No | Ramp | Yes (partial) |
| Present paper (2010) | Fuzzy | Yes (constant) | EnOQ | Yes | Constant | No |

2. NOTATIONS AND ASSUMPTIONS

Notations

| | | |
|--------|---|--|
| C0 | : | set up cost |
| c | : | per unit purchase cost of the product |
| s | : | constant selling price of the product per unit ($s > c$) |
| h | : | holding cost per unit per unit time |
| d | : | disposal cost per unit. |
| r | : | discount offer per unit after deterioration. |
| Q1 | : | order level for post deterioration discount on selling price with instant deterioration. |
| Q2 | : | order level for no discount on selling price with instant deterioration. |
| T1, T2 | : | cycle lengths for the above two respective cases. |

Assumptions

Replenishment rate is infinite.

The deterioration rate θ is constant and ($0 < \theta < 1$)

3. Demand is constant and defined as follows.

$$R(I(t)) = a$$

Where $a > 0$ is the demand rate independent of stock level.

$r, (0 \leq r \leq 1)$ is the percentage discount offer on unit selling price during instant deterioration.

$\alpha = (1-r)^{-n}$ ($n \in R$ the set of real numbers) is the effect of discounting selling price on demand during deterioration. α is determined from priori knowledge of the seller with constant demand.

5. The entropy generation must satisfy $S = \frac{d\sigma(t)}{dt}$ where, $\sigma(t)$ is the total entropy generated by time t and S is the rate at which entropy is generated. The entropy cost is computed by dividing the total commodity flow in a cycle of duration T_i . The total entropy generated

$$\sigma(T_i) = \int_0^{T_i} S dt, S = \frac{R(I(t))}{s} = \frac{a}{s}$$

over time T_i as

Entropy cost per cycle is

$$EC(T_i) = (EC) \text{ With deterioration} = \frac{Q_i}{\sigma(T_i)} \quad (i=1,2)$$

3. MATHEMATICAL MODEL

At the beginning of the replenishment cycle the inventory level raises to Q_1 . As the time progresses it is decreased due to instantaneous stock with constant demand. Ultimately inventory reaches zero level at T_1 . As instant deterioration starts from origin, $r\%$ discount on selling price is provided to enhance the demand of decreased quality items. This discount is continued for the rest of the replenishment cycle. Then the behaviour of inventory level is governed by the following system of linear differential equation.

$$\frac{dI(t)}{dt} = -[\alpha a + \theta I(t)] \quad 0 \leq t \leq T_1 \quad (1)$$

with the initial boundary condition

$$\left. \begin{array}{l} I(0) = Q_1 \\ \text{and } I(T_1) = 0 \end{array} \right\} 0 \leq t \leq T_1$$

Solving the equations,

$$I(t) = \frac{a\alpha}{\theta} [e^{\theta(T_1-t)} - 1] \quad 0 \leq t \leq T_1 \quad (2)$$

$$Q_1 = \frac{a\alpha}{\theta} [e^{\theta T_1} - 1] \quad (3)$$

Holding cost and disposal cost of inventories in the cycle is,

$$HC + DC = (h + \theta d) \int_0^{T_1} I(t) dt$$

Purchase cost in the cycle is given by $PC = cQ_1$.

Entropy cost in the cycle is

$$EC = \frac{Q_i \text{ with deterioration}}{\sigma(T_1)} = \frac{Q_1}{\sigma(T_1)}$$

$$\sigma(T_1) = \int_0^{T_1} S dt = \int_0^{T_1} \frac{a}{s} dt = \frac{aT_1}{s}; \quad EC = \frac{sQ_1}{aT_1}$$

Total sales revenue in the order cycle can be found as

$$SR = s \left[\alpha(1-r) \int_0^{T_1} a dt \right]$$

$$\pi_1(r, T_1) = \frac{TP_1}{T_1}$$

Thus total profit per unit time of the system is

$$= \frac{1}{T_1} [SR - PC - HC - DC - EC - OC]$$

On integration and simplification of the relevant costs, the total profit per unit time becomes

$$\pi_1 = \frac{1}{T_1} \left[s\alpha(1-r)aT_1 - h \left[\frac{e^{\theta T_1} - 1}{\theta} - T_1 \right] \frac{a\alpha}{\theta} - \frac{\theta da\alpha}{\theta} \left[\frac{e^{\theta T_1} - 1}{\theta} - T_1 \right] - \frac{sQ_1}{aT_1} - cQ_1 - C_0 \right] \quad (4)$$

If the product starts to deteriorate as soon as it is received in the stock, then there is only one option we may give post deterioration discount. The post deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product, i.e. $s(1-r) - c > 0$. Applying this constraint on unit total profit function we have the following maximization problem.

Maximize $\pi_1(r, T_1)$

Subject to $r < 1 - \frac{c}{s}$

$\forall r, T_1 \geq 0$

$\pi_1 = F_1 + F_2h + F_3d$

(5)

where $F_1 = \frac{1}{T_1} \left[s\alpha(1-r)aT_1 - \frac{sQ_1}{aT_1} - cQ_1 - C_0 \right]$

(6)

$F_2 = -\frac{1}{T_1} \left[\frac{e^{\theta T_1} - 1}{\theta} - T_1 \right] \frac{a\alpha}{\theta}$

(7)

$F_3 = -\frac{1}{T_1} \left[a\alpha \left(\frac{e^{\theta T_1} - 1}{\theta} - T_1 \right) \right]$

(8)

4. FUZZY MODEL

We replace the holding cost and disposal cost by fuzzy numbers \tilde{h} and \tilde{d} respectively. By expressing \tilde{h} and \tilde{d} as the normal triangular fuzzy numbers (h_1, h_0, h_2) and (d_1, d_0, d_2) ,

where, $h_1 = h - \Delta_1$, $h_0 = h$, $h_2 = h + \Delta_2$, $d_1 = d - \Delta_3$, $d_0 = d$, $d_2 = d + \Delta_4$ such that

$0 < \Delta_1 < h$, $0 < \Delta_2$, $0 < \Delta_3 < d$, $0 < \Delta_4$, $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy disposal cost are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \frac{h-h_1}{h_0-h_1}, & h_1 \leq h \leq h_0 \\ \frac{h_2-h}{h_2-h_0}, & h_0 \leq h \leq h_2 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$$\mu_{\tilde{d}}(d) = \begin{cases} \frac{d-d_1}{d_0-d_1}, & d_1 \leq d \leq d_0 \\ \frac{d_2-d}{d_2-d_0}, & d_0 \leq d \leq d_2 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Then the centroid for \tilde{h} and \tilde{d} are given by

$$M_{\tilde{h}} = \frac{h_1 + h_0 + h_2}{3} = h + \frac{\Delta_2 - \Delta_1}{3} \quad \text{and} \quad M_{\tilde{d}} = \frac{d_1 + d_0 + d_2}{3} = d + \frac{\Delta_4 - \Delta_3}{3} \quad \text{respectively.}$$

For fixed values of r and T_1 , let

$$Z(h, d) = F_1(r, T_1) + F_2(r, T_1)h + F_3(r, T_1)d = y$$

$$h = \frac{y - F_1 - F_3d}{F_2}, \quad \frac{\Delta_2 - \Delta_1}{3} = \psi_1 \quad \text{and} \quad \frac{\Delta_4 - \Delta_3}{3} = \psi_2$$

Let

By extension principle the membership function of the fuzzy profit function is given by

$$\begin{aligned} \mu_{\tilde{z}(h,d)}^{(y)} &= \text{Sup}_{(h,d) \in Z^{-1}(y)} \{ \mu_{\tilde{h}}(h) \vee \mu_{\tilde{d}}(d) \} \\ &= \text{Sup}_{d_1 \leq d \leq d_2} \left\{ \mu_{\tilde{h}} \left(\frac{y - F_1 - F_3d}{F_2} \right) \vee \mu_{\tilde{d}}(d) \right\} \end{aligned} \quad (11)$$

Now,

$$\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3d}{F_2} \right) = \begin{cases} \frac{y - F_1 - F_2h_1 - F_3d}{F_2(h_0 - h_1)}, & u_2 \leq d \leq u_1 \\ \frac{F_1 + F_2h_2 + F_3d - y}{F_2(h_2 - h_0)}, & u_3 \leq d \leq u_2 \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

where,

$$u_1 = \frac{y - F_1 - F_2h_1}{F_3}, \quad u_2 = \frac{y - F_1 - F_2h_0}{F_3} \quad \text{and} \quad u_3 = \frac{y - F_1 - F_2h_2}{F_3}$$

when $u_2 \leq d$ and $d \leq u_1$ then $y \leq F_1 + F_2h_0 + F_3d_0$ and $y \geq F_1 + F_2h_1 + F_3d_1$. It is clear that for every $y \in [F_1 + F_2h_1 + F_3d_1, F_1 + F_2h_0 + F_3d_0]$, $\mu_y(y) = PP'$. From the equations (9) and (12) the value of PP' may be found by solving the following equation:

$$\frac{d - d_1}{d_0 - d_1} = \frac{y - F_1 - F_2 h_1 - F_3 d}{F_2 (h_0 - h_1)}$$

$$d = \frac{(y - F_1 - F_2 h_1)(d_0 - d_1) + F_2 d_1 (h_0 - h_1)}{F_2 (h_0 - h_1) + F_3 (d_0 - d_1)}$$

or

$$PP' = \frac{d - d_1}{d_0 - d_1} = \frac{y - F_1 - F_2 h_1 - F_3 d}{F_2 (h_0 - h_1) + F_3 (d_0 - d_1)} = \mu_1(y)$$

Therefore, (say). (13)

When $u_3 \leq d$ and $d \leq u_2$ then $y \leq F_1 + F_2 h_2 + F_3 d_2$ and $y \geq F_1 + F_2 h_0 + F_3 d_0$. It is evident that for every $y \in [F_1 + F_2 h_0 + F_3 d_0, F_1 + F_2 h_2 + F_3 d_2]$, $\mu_{\tilde{y}}(y) = PP''$. From the equations (9) and (12), the value of PP'' may be found by solving the following equation:

$$\frac{d_2 - d}{d_2 - d_0} = \frac{F_1 + F_2 h_2 + F_3 d - y}{F_2 (h_2 - h_0)}$$

$$d = \frac{F_2 d_2 (h_2 - h_0) - (F_1 + F_2 h_2 - y)(d_2 - d_0)}{F_2 (h_2 - h_0) + F_3 (d_2 - d_0)}$$

or,

$$PP'' = \frac{d_2 - d}{d_2 - d_0} = \frac{F_1 + F_2 h_2 + F_3 d - y}{F_2 (h_2 - h_0) + F_3 (d_2 - d_0)} = \mu_2(y)$$

Therefore, (say). (14)

Thus the membership function for fuzzy total profit is given by

$$\mu_{\tilde{z}(\tilde{h}, \tilde{d})}(y) = \begin{cases} \mu_1(y); & F_1 + F_2 h_1 + F_3 d_1 \leq y \leq F_1 + F_2 h_0 + F_3 d_0 \\ \mu_2(y); & F_1 + F_2 h_0 + F_3 d_0 \leq y \leq F_1 + F_2 h_2 + F_3 d_2 \\ 0; & \text{otherwise} \end{cases} \quad (15)$$

$$P_1 = \int_{-\infty}^{\infty} \mu_{\tilde{z}(\tilde{h}, \tilde{d})}(y) dy \quad R_1 = \int_{-\infty}^{\infty} y \mu_{\tilde{z}(\tilde{h}, \tilde{d})}(y) dy$$

Now, let

Hence, the centroid for fuzzy total profit is given by

$$\tilde{\pi}_1 = M_{\tilde{TP}}(r, T_1) = \frac{R_1}{P_1}$$

$$= F_1(r, T_1) + F_2(r, T_1)h + F_3(r, T_1)d$$

$$+ \psi_1 F_2(r, T_1) + \psi_2 F_2(r, T_1)$$

$$M_{\tilde{TP}}(r, T_1) = F_1 + (h + \psi_1)F_2 + (d + \psi_2)F_3 \quad (16)$$

where, $F_1(r, T_1)$, $F_2(r, T_1)$ and $F_3(r, T_1)$ are given by equations (6), (7) and (8).

The post-deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product, i.e. $s(1-r)-c > 0$.

Applying this constraint on the unit total profit function in equation (17) we have the following maximization problem.

$$\text{Maximize } M_{\tilde{TP}_1}(r, T_1)$$

$$\text{Subject to, } r < 1 - \frac{c}{s} \quad (18)$$

$$\forall r, T_1 \geq 0$$

Our objective here is to determine the optimal values of r and T_1 to maximize the unit profit function. It is very difficult to derive the results analytically. Thus some numerical methods must be applied to derive the optimal values of r and T_1 , hence the unit profit function. There are several methods to

cope with constraint optimization problem numerically. But here we use penalty function method [7] and LINGO software to derive the optimal values of the decision variables.

a. Special Case

b. I Model for instant deterioration with no discount

In this case order level and unit profit function for model with constant deterioration and constant demand with no discount are obtained from (3) and (4) by substituting $r=0$ as

$$Q_2 = \frac{a}{\theta} (e^{\theta T_2} - 1) \tag{19}$$

From equation (4) total profit per unit time becomes

$$\begin{aligned} \pi_2(T_2) &= \frac{TP_2}{T_2} = \frac{1}{T_2} \left[saT_2 - (h + \theta d) \frac{a}{\theta} \left[\frac{e^{\theta T_2} - 1}{\theta} - T_2 \right] - \frac{sQ_2}{aT_2} - cQ_2 - C_0 \right] \\ &= F_4 + F_5h + F_6d \end{aligned} \tag{20}$$

where,

$$F_4 = \frac{1}{T_2} \left[saT_2 - \frac{sQ_2}{aT_2} - cQ_2 - C_0 \right] \tag{21}$$

$$F_5 = \frac{-a}{\theta T_2} \left[\frac{e^{\theta T_2} - 1}{\theta} - T_2 \right] \tag{22}$$

$$F_6 = -\frac{a}{\theta T_2} \left[\frac{e^{\theta T_2} - 1}{\theta} - T_2 \right] \tag{23}$$

Thus we have to determine T_2 from the fuzzy maximization problem

$$\begin{aligned} \text{maximize } M_{TC_2}(T_2) \\ \forall T_2 \geq 0 \end{aligned} \tag{24}$$

where, $M_{TC_2}(T_2) = F_4 + (h + \psi_1)F_5 + (d + \psi_2)F_6 = \tilde{\pi}_2$. (25)

5. MODEL ANALYSIS THEOREM

$$\text{For } n \neq 1, \tilde{\pi}_1 > \tilde{\pi}_2 \text{ if } r < \min \left\{ 1 - \frac{c}{s}, 1 - \frac{n \left(c + \frac{s}{aT_2} \right)}{s(n-1)} \right\}.$$

Proof:

The values of $\tilde{\pi}_1$ for fixed r are always less than optimal value of r . Thus it is sufficient to show that $\tilde{\pi}_1 > \tilde{\pi}_2$ for fixed r . Here, T_1 is the cycle length when post deterioration discount is applied on unit selling price to enhance the demand of decreased quality items. For the enhancement of demand the inventory depletion rate will be higher and consequently the cycle time will reduce. T_2 is the cycle length when no discount is applied on selling price. Obviously T_2 is greater than T_1 . Without loss of generality let both the profit function $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are positive.

$$\tilde{\pi}_1 - \tilde{\pi}_2 = \frac{\tilde{TP}_1}{T_1} - \frac{\tilde{TP}_2}{T_2} \geq \frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2}$$

$$\frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2} > 0 \qquad \frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2} > 0$$

It is sufficient to show that $\frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2}$ is an increasing function of r then our purpose will be served. Now differentiating it with respect to r we have,

$$\frac{\partial(\tilde{\pi}_1 - \tilde{\pi}_2)}{\partial r} = \frac{1}{T_2} \left[\left(s(n-1)(1-r) + n \frac{h + \theta d}{\theta} \right) \frac{aT_2}{(1-r)^{n+1}} + \left[-c - \frac{h + \theta d}{\theta} - \frac{s}{aT_2} \right] \frac{an(e^{\theta r_2} - 1)}{\theta(1-r)^{n+1}} \right]$$

Therefore, $\frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2} > 0 \Rightarrow \frac{\partial(\tilde{\pi}_1 - \tilde{\pi}_2)}{\partial r} > 0,$

i.e. if

$$\left[s(n-1)(1-r) + n \frac{h + \theta d}{\theta} \right] + \left[-c - \frac{h + \theta d}{\theta} - \frac{s}{aT_2} \right] \times \frac{n(e^{\theta r_2} - 1)}{\theta T_2} > 0 \tag{26}$$

Now, $\frac{(e^{\theta r_2} - 1)}{\theta T_2} > 1.$

we have,

$$s(n-1)(1-r) + n \left[-c - \frac{s}{aT_2} \right] > 0$$

i.e. $r < \frac{s(n-1) + n \left[-c - \frac{s}{aT_2} \right]}{s(n-1)}$

We have the restriction $r < 1 - \frac{c}{s}.$

Therefore, $\tilde{\pi}_1 > \tilde{\pi}_2$ if

$$r < \min \left\{ 1 - \frac{c}{s}, 1 - \frac{n \left(c + \frac{s}{aT_2} \right)}{s(n-1)} \right\} \tag{27}$$

Theorem indicates that for $n \neq 1$ post instant deterioration discount on unit selling price produces higher profit than that of instant deterioration with no discount on unit selling price in fuzzy environment, if the percentage of post deterioration discount on unit selling price is less than min

$$\left\{ 1 - \frac{c}{s}, 1 - \frac{n \left(c + \frac{s}{aT_2} \right)}{s(n-1)} \right\}$$

A simple managerial indication is that in pure inventory scenario if the product deteriorates after a certain time then it is always more profitable to apply only post deterioration discount on unit selling price and the amount of percentage discount must be less than the limit provided in equation (27) for the post deterioration discount.

6. NUMERICAL EXAMPLE

LINGO software is used to solve the aforesaid numerical example.

We redo the same example of [18] to see the optimal replenishment policy while considering the fuzzy holding cost, fuzzy disposal cost and entropy cost. The parameter values are $a=80$, $b=0.3$, $h=0.6$, $d=2.0$, $s=10.0$, $C_0=100.0$, $c=4.0$, $\theta=0.03$, $n=2.0$, $\Delta_1=0.1$, $\Delta_2=0.2$, $\Delta_3=0.5$, $\Delta_4=0.8$.

After 185 and 50 iterations in Table 2 we obtain the optimal replenishment policy for instant deterioration fuzzy entropic order quantity models with post deterioration discount and no discount respectively. The total profits for both the cases obtained here is at least 4.12% and 3.76% respectively less than that in [18] i.e. our CEOQ models. This is because we modified the model by introducing the hidden cost that is entropy cost where the optimal values for both the cases are 21.03623 and 20.28649 respectively. In Tables 3 and 4 we obtain the numerical results of different models like FEnOQ, FEOQ, CEnOQ and CEOQ for above two cases separately. The behaviour of the total profit to the lot size and the cycle length of post deterioration discounted model is shown in Figure 1.

TABLE-2: The Numerical Results of the Instant Deterioration Fuzzy Entropic Order Quantity (FEnOQ) Models (i=1,2)

| Model | Local optimal solution found at iteration | r | Ti | Qi | EC | π_i |
|--|---|--------|---------|----------|----------|----------|
| FEnOQ (Only post deterioration discount) | 185 | 0.0350 | 1.8221 | 160.8798 | 21.0362 | 354.1393 |
| FEnOQ (No discount) | 50 | - | 1.8814 | 154.8204 | 20.28649 | 353.6979 |
| % change | - | - | -3.1457 | 3.9136 | 3.6958 | 0.1248 |

TABLE-3: Comparison of Results for the different Post Deterioration Discount Models

| Model | Local optimal solution found at iteration | r | Ti | Qi | EC | π_1 |
|-------|---|--------|--------|----------|---------|----------|
| FEnOQ | 185 | 0.0350 | 1.8221 | 160.8798 | 21.0362 | 354.1393 |
| FEOQ | 193 | 0.0673 | 1.6063 | 151.3270 | - | 366.6226 |
| CEnOQ | 105 | 0.0392 | 1.8561 | 165.4009 | 21.1392 | 357.0641 |
| CEOQ | 196 | 0.0708 | 1.6367 | 155.4112 | - | 369.3739 |

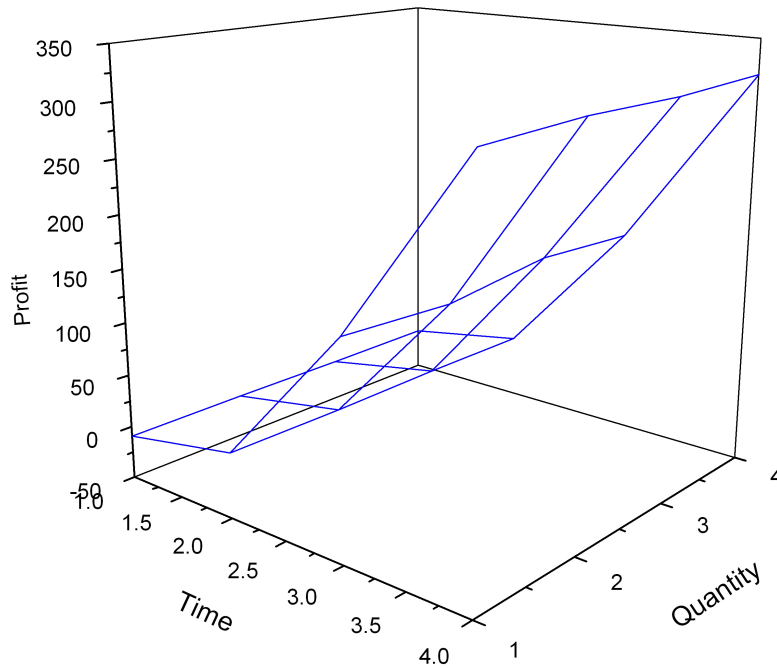


FIGURE1: The behaviour of the total profit to the lot size and the cycle length of post deterioration discounted model.

7. COMPARATIVE EVALUATION

Table 2 shows that 3.4% discount on post deterioration model is provided on unit selling price to earn 0.12% more profit than that with no discounted instant deterioration model. From Table 5 it indicates that the uncertainty and entropy cost are provided on the post deterioration discount model to lose 3.4%, 0.81% and 4.12% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. Similarly it shows that the no discounted deterioration model to lose 3.08%, 0.78% and 3.76% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. This paper investigates a computing schema for the EOQ in fuzzy sense. From Tables 3 and 4 it shows that the fuzzy and crisp results are very approximate, i.e. it permits better use of EOQ as compared to crisp space arising with the little change in holding cost and in disposal cost respectively. It indicates the consistency of the crisp case from the fuzzy sense.

TABLE-4: Comparison of Results for the different No Discounted Instant Deterioration Models

| Model | Local optimal solution found at iteration | T2 | Q2 | EC | π_2 |
|-------|---|--------|----------|----------|----------|
| FEnOQ | 50 | 1.8814 | 154.8207 | 20.28649 | 353.6979 |
| FEOQ | 32 | 1.7203 | 141.2193 | - | 364.9558 |
| CEnOQ | 48 | 1.9239 | 158.4196 | 20.2931 | 356.5054 |
| CEOQ | 34 | 1.7592 | 144.4996 | - | 367.5178 |

TABLE-5: Relative Error (RE) of Post Deterioration Discount and No Discounted Deterioration FEnOQ Models with the different Models

| | | | | |
|-------|------------|----------|------------|----------|
| FEnOQ | Q1 | 160.8798 | Q2 | 154.8207 |
| | π_1 | 354.1393 | π_2 | 353.6979 |
| FEOQ | Q11 | 151.3270 | Q21 | 141.2193 |
| | π_{11} | 366.6226 | π_{21} | 364.9558 |
| RE | % change | 6.3127 | % change | 9.6314 |
| | % change | -3.4050 | % change | -3.0847 |
| CEnOQ | Q12 | 165.4009 | Q22 | 158.4196 |
| | π_{12} | 357.0641 | π_{22} | 356.5054 |
| RE | % change | -2.7334 | % change | -2.2718 |
| | % change | -0.8191 | % change | -0.7878 |
| GEOQ | Q13 | 155.4112 | Q23 | 144.4996 |
| | π_{13} | 369.3739 | π_{23} | 367.5178 |
| RE | % change | 3.5188 | % change | -6.6665 |
| | % change | -4.1244 | % change | -3.7603 |

8. CRITICAL DISCUSSION

When human originated data like holding cost and disposal cost which are not precisely known but subjectively estimated or linguistically expressed is examined in this paper. The mathematical model is developed allowing post deterioration discount on unit selling price in fuzzy environment. It is found that, if the amount of discount is restricted below the limit provided in the model analysis, then the unit profit is higher. It is derived analytically that the post deterioration discount on unit selling price is to earn more revenue than the revenue earned for no discount model. The numerical example is presented to justify the claim of model analysis. Temporary price discount for perishable products to enhance inventory depletion rate for profit maximization is an area of interesting research. This paper introduces the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement of the process. This paper examines the idea by extending the analysis of [18] by introducing fuzzy approach and entropy cost to provide a firm its optimum discount rate, replenishment schedule, replenishment order quantity simultaneously in order to achieve its maximum profit.

Though lower amount of percentage discount on unit selling price in the form of post deterioration discount for larger time results in lower per unit sales revenue, still it is more profitable. Because the inventory depletion rate is much higher than for discount with enhanced demand resulting in lower amount inventory holding cost and deteriorated items. Thus it can be conjectured that it is always profitable to apply post deterioration discount on unit selling price to earn more profit. Thus the firm in this case can order more to get earn more profit.

These models can be considered in a situation in which the discount can be adjusted and number of price changes can be controlled. Extension of the proposed model to unequal time price changes and other applications will be a focus of our future work.

9. CONCLUSION

This paper provides an approach to extend the conventional system cost including fuzzy arithmetic approach for perishable items with instant deterioration for the discounted entropic order quantity model in the adequacy domain. To compute the optimal values of the policy parameters a simple and quite efficient policy model was designed. Theorem determines effectively the optimal discount rate r for post deterioration discount. Finally, in numerical experiments the solution from the instant deteriorated model evaluated and compared to the solutions of other different EnOQ and traditional EOQ policies.

However, we saw few performance differences among a set of different inventory policies in the existing literature. Although there are minor variations that do not appear significant in practical terms, at least when solving the single level, incapacitated version of the lot sizing problem. From our analysis it is demonstrated that the retailer's profit is highly influenced by offering post discount on selling price. The results of this study give managerial insights to decision maker developing an optimal replenishment decision for instant deteriorating product. Compensation mechanism should

also be included to induce collaboration between retailer and dealer in a meaningful supply chain. We conclude this paper by summarizing some of the managerial insights resulting from our work.

In general, for normal parameter values the relative payoff differences seem to be fairly small. The optimal solution of the suggested post deterioration discounted model has a higher total payoff as compared with no discounted model. Conventional wisdom suggests that workflow collaboration in a fuzzy entropic model in a varying deteriorating product in market place are promising mechanism and achieving a cost effective replenishment policy. Theoretically such extensions would require analytical paradigms that are considerably different from the one discussed in this paper, as well as additional assumptions to maintain tractability.

The approach proposed in the paper based on EnOQ model seems to be a pragmatic way to approximate the optimum payoff of the unknown group of parameters in inventory management problems. The assumptions underlying the approach are not strong and the information obtained seems worthwhile. Investigating optimal policies when demand are generated by other process and designing models that allow for several orders outstanding at a time, would also be challenging tasks for further developments. Its use may restrict the model's applicability in the real world. Future direction may be aimed at considering more general deterioration rate or demand rate. Uses of other demand side revenue boosting variables such as promotional efforts are potential areas of future research. There are numerous ways in which one could consider extending our model to encompass a wider variety of operating environments. The proposed paper reveals itself as a pragmatic alternative to other approaches based on constant demand function with very sound theoretical underpinnings but with few possibilities of actually being put into practice. The results indicate that this can become a good model and can be replicated by researchers in neighbourhood of its possible extensions. As regards future research, one other line of development would be to allow shortage and partial backlogging in the discounted model.

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