

## On Channel Estimation of OFDM-BPSK and -QPSK over Nakagami-m Fading Channels

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### Abstract

This paper evaluates the performance of OFDM - BPSK & -QPSK based system with and without channel estimation over Nakagami-m fading channels. Nakagami-m variants are generated by decomposition of Nakagami random variable into orthogonal random variables with Gaussian distribution envelopes. Performance of OFDM system in Nakagami channel has been reported here. The results yield the optimum value of  $m$  based on BER and SNR. Using this optimum value of  $m$ , Channel estimation over flat fading has been reported here. It has been depicted clearly from simulated graphs that channel estimation has further reduced the BER. However, threshold value of  $m$  has played a vital role during channel estimation.

**Keywords**-OFDM, Fading distribution, Nakagami-m channel, Rayleigh fading, Channel estimation, Trained Symbol.

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### 1. INTRODUCTION

OFDM technique is a multi-carrier transmission technique, which is being recognized as an excellent method for high-speed bi-directional wireless data communication. The prime idea is that all queuing data in buffer are uniformly allocated on small sub-carriers. OFDM efficiently squeezes multiple modulated carriers tightly together reducing the required bandwidth but keeping the modulated signals orthogonal so that they do not interfere with each other. OFDM that is highly efficient technique shows favorable properties such as robustness to channel fading and inter symbol interference (ISI) and is more immune to noise. OFDM system is capable of mitigating a frequency selective fading channel to a set of parallel flat fading channels, which need relatively simple processes for channel equalization.

Rayleigh and Rician fading channels have already been deployed and studied in depth for OFDM systems. OFDM Rayleigh channel simulator for OFDM has been reported in [1]. Modification to existing model of simulator was proposed by considering the correlation between the sub channels of an OFDM system resulting into reduced computational complexity. BER performance in frequency selective Rician fading channel is studied in [2]. Estimation of OFDM system in Rayleigh faded channel is provided by many techniques, in [3], used the pilot symbol along with

the previously known channel coefficients for fast Rayleigh faded channels. In [4], Timing phase estimator for OFDM system in Rayleigh faded environment is proposed with low complexity. Nakagami-m fading distribution is another useful and important model to characterize the fading channel [5]. Kang et al. [6] modeled the OFDM- BPSK system with frequency selective fading channel. The work was further enhanced by Zheng et al. [7] by presenting asymptotic BER performance of OFDM system in frequency selective Nakagami-m channel. In [8], accurate error performance of OFDM systems was provided on basis of number of channel taps in Nakagami-m fading environment.

OFDM systems have gained an equivalent attention with flat fading environment. In [9], present the method of Channel estimation and Carrier frequency offset to design an OFDM receiver in flat fading environment. However, BER performance of OFDM system in flat fading channel using DBPSK modulation technique is studied by Lijun et al. [10]. Since, the frequency selective model of Nakagami-m channel is already presented in literature, so our motivation behind this paper is to study the performance of OFDM system using flat fading channel of Nakagami-m distribution and further to improve the BER by applying channel estimation.

This paper is organized as follows: In section 2, OFDM system model is described. In section 3, the mathematical model to generate the Nakagami-m fading channel is explained along with the OFDM transmitting signal. In section 4, channel estimation technique is discussed. The analysis of simulated results of performance of OFDM system without estimation is done in section 5, while results with estimation have been presented in section 6. Finally section 7, concludes the paper.

## 2. MODEL DESCRIPTION

A Complex base band OFDM signal with N subcarriers, is expressed as [11]:

$$s(t) = \sum_{k=0}^{N-1} D_k e^{j2\pi k f_o t} \quad 0 \leq t \leq T \quad (1)$$

For each OFDM symbol, the modulated data sequences are denoted by  $D(0), D(1), \dots, D(N-1)$ . Here,  $f_o$  denote the sub-carriers spacing and is set to  $f_o = 1/T$  the condition of orthogonality. After IFFT, the time-domain OFDM signal can be expressed as [11]:

$$S(n) = \frac{1}{N} \sum_{k=0}^{N-1} D_k e^{\frac{j2\pi k f_o n}{N}} \quad (2)$$

After IFFT, the modulated signal is up-converted to carrier frequency  $f_c$  and then the following signal is produced and transmitted through channel [11]:

$$x(t) = \text{Re} \left\{ \sum_{k=0}^{N-1} D_k e^{j2\pi k (f_o + f_c) t} \right\} \quad 0 \leq t \leq T \quad (3)$$

$x(t)$  represents the final OFDM signal in which sub-carriers shall undergo a flat fading channel.

## 3. CHANNEL MODEL

In this paper, the sub-channel spacing ( $f_o = 1/T$ ) is chosen so that the produced parallel fading sub-channels have flat fading characteristics. So, we have chosen Nakagami-m flat fading

channel with additive white Gaussian noise. In flat fading environment, the base-band signal at the input of receiver  $y(t)$  is as described as follows [11]:

$$y(t) = x(t) * r(t) + n(t) \tag{4}$$

where,  $x(t)$  denotes the base-band transmitted signal,  $r(t)$  is the Nakagami-m distributed channel envelope and  $n(t)$  is the additive white Gaussian noise with zero mean.

Nakagami- m fading distribution function is given by [5]

$$p_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mr^2}{\Omega}\right), r \geq 0 \tag{5}$$

Where,  $\Gamma(\cdot)$  is the Gamma function,  $\Omega = \overline{r^2}$  is the average power,  $m$  is fading parameter and  $r$  is Nakagami distribution envelope.

Since, Nakagami distribution encompasses Scattered, reflected and direct components of the original transmitted signal [12], it can be generated using the envelope of the both random signal processes  $r_{nlos}(t)$  for non line- of- sight envelope i.e. Rician and  $r_{los}(t)$  for line-of-sight i.e. Rayleigh as per the following expression[12]

$$r(t) = |r_{nlos}(t)| \exp(1 - m) + |r_{los}(t)| \cdot (1 - \exp(1 - m)) \tag{6}$$

So, this value of  $r(t)$  is used as envelope of Nakagami-m distributed channel.

#### 4. CHANNEL ESTIMATION

Channel estimation in frequency selective has different approach then compared with flat fading environment. A comparative study using Minimum Mean Squared Error (MMSE) and Least square (LS) estimator in frequency selective fading environment has been presented in [13]. The channel estimation based on comb type pilot arrangement is studied using different algorithms by bahai et. al.[14]. Semi-analytical method to evaluate BER of a quadrature phase shift keying (QPSK)-OFDM system in Nakagami,  $m < 1$  fading and additive noise where pilot-assisted linear channel estimation and channel equalization is described in [15]. A novel channel estimation scheme for OFDMA uplink packet transmissions over doubly selective channels was suggested in [16]. The proposed method uses irregular sampling techniques in order to allow flexible resource allocation and pilot arrangement. In flat fading environment, estimation of the channel using trained sequence of data has been studied and implemented in [17]. Channel phase was estimated during each coherence time. Then pilot data of some required percentage of data length (referred as training percentage in simulation) is inserted into the source data. It is used to estimate the random phase shift of the fading channel and train the decision to adjust the received signal with phase recover. So, finally phase estimation using training symbol is implemented in flat fading environment.

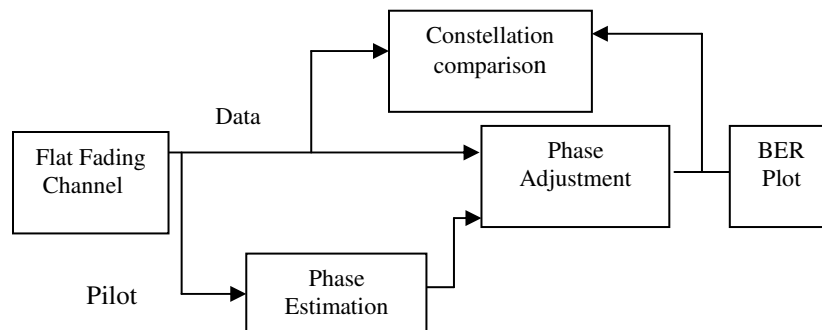


FIGURE 1: Simulation flow chart for Channel estimation [16].

In this paper, after simulating the OFDM system in Nakagami-m faded environment, OFDM system has been simulated with channel estimation scheme to obtain the improved results. The results obtained showed the significant variation in BER for with and without estimation curves.

## 5. RESULTS WITHOUT ESTIMATION

To analyze the performance of OFDM-BPSK and -QPSK systems over Nakagami-m fading channel, we consider the total number of sub-carriers 400, the IFFT /FFT length is chosen to be 1024 by using Guard interval of length 256.

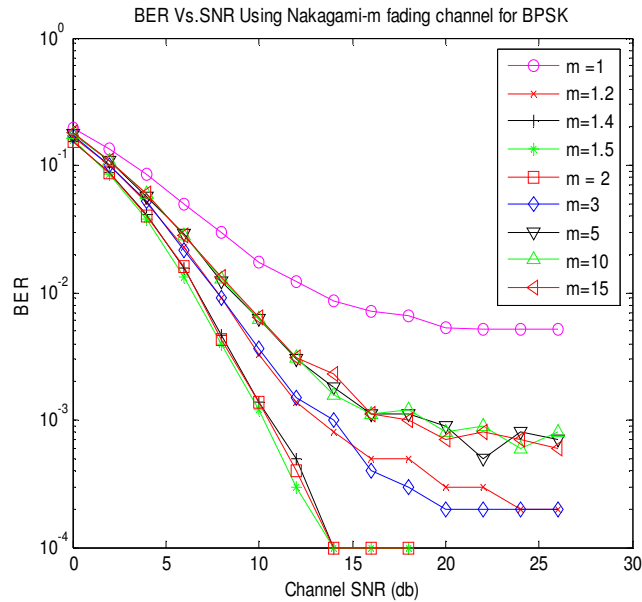
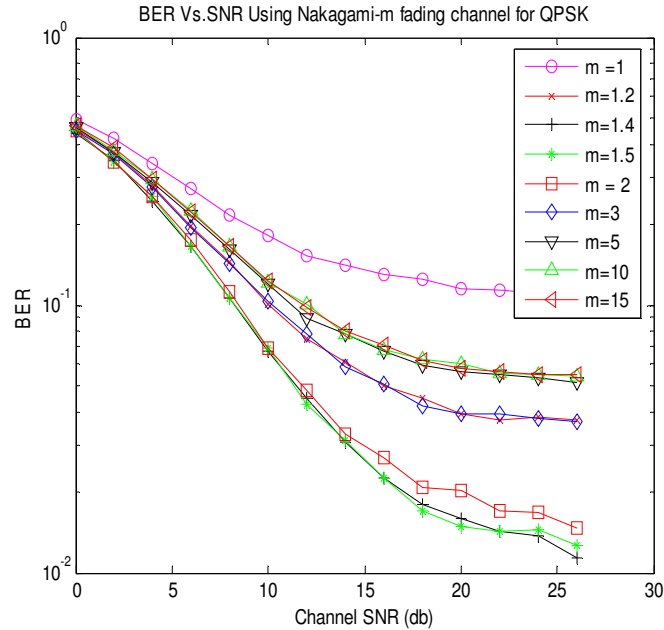


FIGURE 2: BER Vs. SNR for OFDM-BPSK system



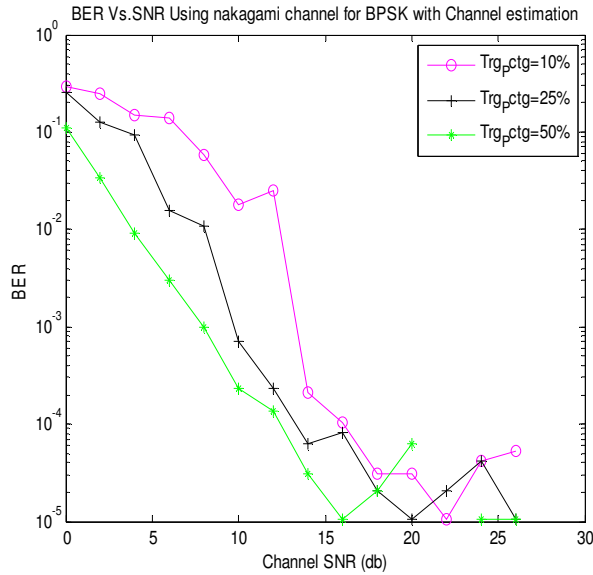
**FIGURE 3:** BER Vs. SNR for OFDM-QPSK System

In this section, we have presented the simulation results using MATLAB™ by implementing two modulation formats for OFDM to get threshold value of fading parameter  $m$ . Figure 2 indicates the BER Versus SNR for OFDM-BPSK with different values of fading parameter  $m$ . It is well known, that at  $m = 1$ , Nakagami- $m$  fading corresponds to Rayleigh fading. So, the results for the same have been achieved through simulations. When value of  $m$  is increased, the BER starts reducing and value of  $10^{-4}$  is reported at  $m = 1.4, 1.5$  and  $2$ . Further, if we increase  $m$ , no reduction in BER has been reported rather it starts increasing. So threshold value of  $m$  is achieved to be  $1.4$ , to estimate the fading channel. This interesting fact about Nakagami- $m$  channel has also been reported by Zheng et al. [6]. We have further analyzed OFDM system using QPSK shown in Figure 3. Results obtained for OFDM-QPSK systems are similar in nature to that of OFDM-BPSK system. BER starts decreasing with increasing value of  $m$ . It is very well depicted from graph the threshold value of  $m = 1.4$  is achieved with BER value of  $10^{-2}$ . If value of  $m$  is further increased, BER starts increasing. Here, the value of  $m = 1$  gives the Rayleigh fading curve for OFDM systems.

Finally, BER performance of OFDM system in Nakagami channel degrades if we increase  $m$  beyond the certain threshold value [18].

## 6. RESULTS WITH ESTIMATION

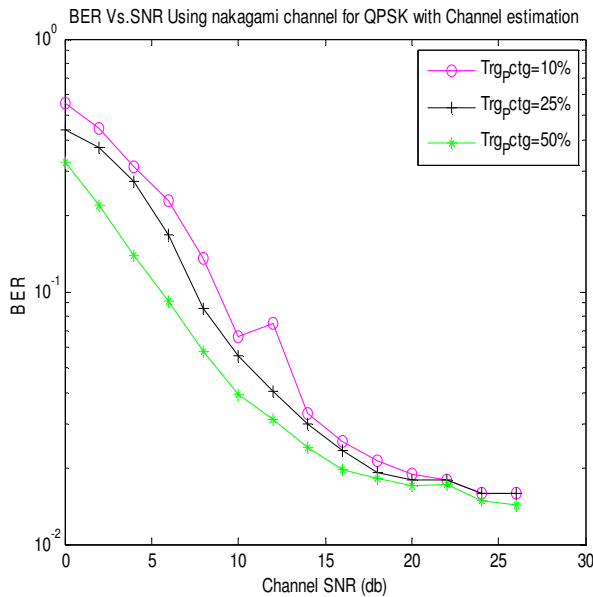
Trained symbols are added to source signal as discussed in section 4. The percentage of such symbol may be varied depending upon the system response to the trained sequence. We have analyzed the results for various percentage values of trained sequence. In this paper, improved results with channel estimation and OFDM implementation has been reported with threshold value of  $m = 1.4$  and varying value of training sequence over the range from 10% to 50%. Results for OFDM-BPSK and -QPSK have been indicated in Fig. 4 and 5 respectively.



**FIGURE 4:** BER Vs. SNR for OFDM-BPSK System

In depth analysis of these graphs show that by increasing the amount of trained symbols BER decreases. In Figure 4, BER value at SNR of 10db has decreased from 0.017885 to 0.00022 for training percentage of 10 to 50. The final value of BER in Figure 2 has been reported to be  $10^{-4}$ , whereas with estimation it has been reported to be  $10^{-5}$ , Hence, there is a significant improvement over BER in the modified implementation.

However, results obtained in Figure 5 are indicating curves as compared to those obtained in Figure 4 but the value of BER for OFDM- QPSK is higher than –BPSK system. Final value of BER in Figure 4 & Figure 5 has been reported to be  $10^{-2}$ . Channel estimation has same effect on this system, BER at SNR of 10db has decreased from 0.06587 to 0.03912 because of increased value of training symbol.



**FIGURE 5:** BER Vs. SNR for OFDM-QPSK System

## 7. CONCLUSIONS

In this paper, we have evaluated the performance of OFDM system using BPSK and QPSK with OFDM using Nakagami-m fading channel. Further the enhanced system performance has been implemented by phase estimation of channel. Threshold value of  $m$  has played a significant role in channel estimation as it provided the minimum value of BER.

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