New Data Association Technique for Target Tracking in Dense Clutter Environment Using Filtered Gate Structure

E. M.Saad elsayedmos@hotmail.com
Department of Electronics and Communication
Engineering Helwan University
Cairo, Egypt

El. Bardawiny bardawiny@hotmail.com
Department of Radar
MTC College
Cairo, Egypt

H.I.ALI hosam_ib_ali@yahoo.com
Department of Electronics and Communication
Engineering Helwan University
Cairo, Egypt

N. M. Shawky negmshawky@gmail.com
Department of Electronics and Communication
Engineering Helwan University
Cairo, Egypt

Abstract

Improving data association process by increasing the probability of detecting valid data points (measurements obtained from radar/sonar system) in the presence of noise for target tracking are discussed in this paper. We develop a novel algorithm by filtering gate for target tracking in dense clutter environment. This algorithm is less sensitive to false alarm (clutter) in gate size than conventional approaches as probabilistic data association filter (PDAF) which has data association algorithm that begin to fail due to the increase in the false alarm rate or low probability of target detection. This new selection filtered gate method combines a conventional threshold based algorithm with geometric metric measure based on one type of the filtering methods that depends on the idea of adaptive clutter suppression methods. An adaptive search based on the distance threshold measure is then used to detect valid filtered data point for target tracking. Simulation results demonstrate the effectiveness and better performance when compared to conventional algorithm.

Keywords: Target Tracking, Data Association, Probabilistic Data Association Algorithm, Kalman Filter.

1. INTRODUCTION

Real-world sensors often report more than one measurement that may be from a given target. These may be either measurements of the desired target or "clutter" measurements. Clutter refers to detections or returns from nearby objects, clouds, electromagnetic interference, acoustic anomalies, false alarms, etc. Data association algorithms allow the use of the Kalman filter (KF) structure for estimation in the presence of clutter. But data association can be the source of both track loss and computational complexity issues. Two most popular KF-based algorithms for single-target tracking in clutter using data association as probabilistic data association filter (PDAF) and nearest neighbor kalman filter (NNKF) used in comparative evaluation in simulation. Failing algorithm to track a target (track loss) may be from increasing the range of clutter density. Filtering methods for tracking targets in noise are well established [1], [2]. The way to separate signals from clutter in target tracking is to use a distance measure from the predicted target positions. Since it is computationally expensive to evaluate this for all measurements, we require a gating process in order to reduce the number of candidate measurements to be considered. The gating technique in tracking a maneuvering target in clutter is essential to make the subsequent algorithm efficient but it suffers from problems since the gate size itself determines the number of valid included measurements. If we choose a too small gate size, we can miss target-originated measurements on the other hand, if we choose a gate with too large size, we will obtain many unwanted non-target measurements, giving rise to increased computational complexity and decreased performance. To find a gate volume in which we regard measurements as valid is an important consideration. There have been many types of gating techniques studied. First of all, previous approaches have used constant parameters to determine the gate size [3]-[6]. Recently, adaptive and (locally) optimal approaches to estimate gate size have also been proposed under more restricted assumptions [7]-[14]. However, this estimation is often computationally intensive. Data association is responsible for deciding which of the existing multiple measurements in gate of the predicted position should update with a tracking target. Some data techniques use a unique to update a track; i.e. at most one observation is used to update a track. PDAF is An alternative approach to use all of the validated measurements with different weights (probabilities). Due to increase in the false alarm rate or low probability of target detection (target in dense clutter environment), most of the data association algorithms begin to fail. We propose here an algorithm which is less sensitivity to false alarm targets in the gate region size than PDA and NNKF algorithms. This proposed algorithm reduces the number of candidate measurements in the gate by a filtering method that compares the measurement in the gate at the prediction step with the current measurement in the same gate at the update step and then avoids any measurement in the current gate less than the threshold value due to comparison. This is called filtering gate method which is similar to an idea taken from adaptive clutter suppression filtering methods used in radar signal processing [15,16]. The filtering gate algorithm is combined with PDA algorithm to apply the proposed algorithm in tracking targets in presence of various clutter densities. Simulation results showed better performance when compared to the conventional PDA algorithm.

2. BACKGROUND

State Space Model

In a dynamic state space model, the observed signals (observation/measurements) are associated with a state and measurement noise. Let the unobserved signal (hidden states) $\{x_t : t \in N\}, x_t \in X$ be modeled as a Markov process of initial distribution $p(x_0)$ and transition probability $p(x_t \mid x_{t-1})$. The observations $\{z_t : t \in N\}, z_t \in Z$ are assumed to be conditionally independent given the process $\{x_t : t \in N\}$ and of the marginal distribution $p(z_t \mid x_t)$.We have the following state and measurement space models at time *t*:

$$\begin{aligned} x_t &= A \quad x_{t-1} + w_t \\ z_t &= H \quad x_t + v_t \end{aligned} \tag{1}$$

where w_t and v_t are white Gaussian noise with zero mean and covariance *Q* and *R* respectively. *A* and *H* are matrices with appropriate sizes. The initial distribution is defined by $P \ (x_0) = N \ (x_0 \mid m_0, p_0)$ where the initial configuration defined by parameters m_0 and

p_0 is assumed to be known. In linear systems, the state space model is optimally addressed by the Kalman filter [14],[17]. The functioning of the Kalman filter consists of two recursive steps: prediction and update.

Filtered Gate Method

In the prediction step, Let $Z_{t-1} = \{z_{1,t-1}, z_{2,t-1}, \cdots z_{i,t-1}, \cdots z_{w_n,t-1}\}$ be a set of points in the 2-D Euclidean space at time t-1 where w_n is the number of points at time scan Δt and let $\hat{z}_t$ be a predicted position of the tracked target at time t. according to distance metric measure and gate size, let $\breve{z}_{t-1} = \{z_{1,t-1}, \dots, z_{j,t-1} \cdots z_{m,t-1}\}$ be a set of the candidate points detected in the gate of predicted position $\hat{z}_t$ whose elements are a subset from the set Z_{t-1} where j =1 to m (number of detected points in gate at time t-1) and $\breve{z}_{t-1}$ be a set of all valid points $z_{j,t-1}$ that satisfy the distance measure condition $\left| z_{i,t-1} - \hat{z}_t \right|$ < W where W is threshold value that determines the gate size and i =1 to w_n , j =1 to m, i. e $j = j+1$ after each valid point is detected up to m points. We consider each point $z_{j,t-1}$ in the gate is a center of very small square gate g_j its length is small δ where each value in the small gate g_j is approximately equal to $z_{j,t-1}$ i.e. $z_{j,t-1} \approx z_{j,t-1} - \delta/2$ to $z_{j,t-1} + \delta/2$.

In the updating step, let $Z_t = \{z_{1,t}, z_{2,t}, \dots, z_{w_k,t}\}$ be a set of points in the 2-D Euclidean space at time t where w_k is the number of points at time scan Δt. The candidate points detected in the same gate of predicted position $\hat{z}_t$ be a subset $\breve{z}_t = \{z_{1,t}, \dots, z_{k,t} \cdots z_{n,t}\}$ from the set Z_t where k =1 to n (number of detected points in gate at time t) and $\breve{z}_t$ be a set of all valid points $z_{k,t}$ that satisfy the distance measure condition $\left| z_{i,t} - \hat{z}_t \right|$ < W where i =1 to w_k , k=1 to n for k=k+1 after each valid point is detected. After receiving the measurement Z_t and detecting the valid measurements $\breve{z}_t$ in the gate , each point from $\breve{z}_t$ in the specified gate at time t is compared with the previous points $\breve{z}_{t-1}$ in the gate at time t-1 to detect the invalid points when $\left| z_{k,t} - z_{j,t-1} \right| < \delta$ and then exclude the point $z_{j,t-1}$ from the set $\breve{z}_{t-1}$ in the next iteration of comparison as shown in Fig. 1.

Finally, we obtain the reduced number of valid points in the gate while the other invalid points is not including in the data association process.

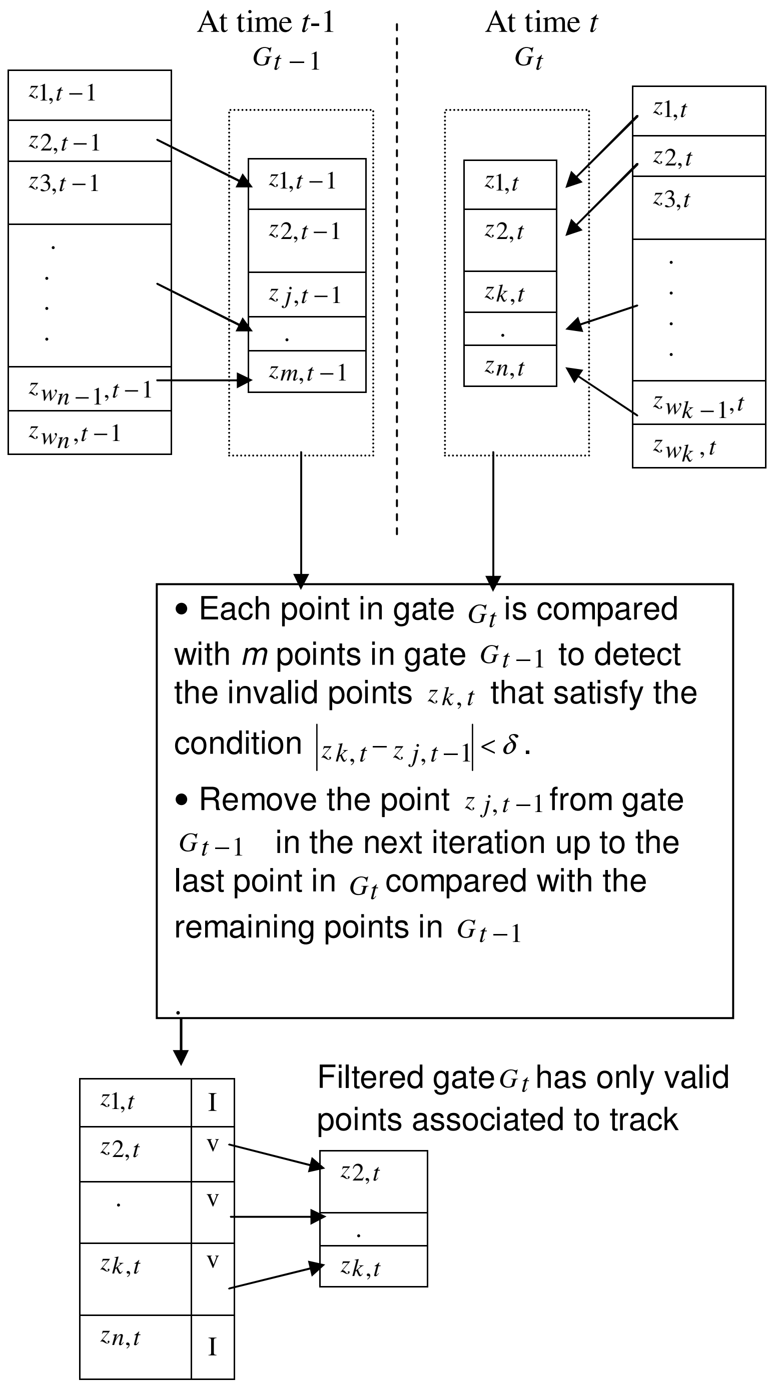


FIGURE 1: Filtered Gate scheme.

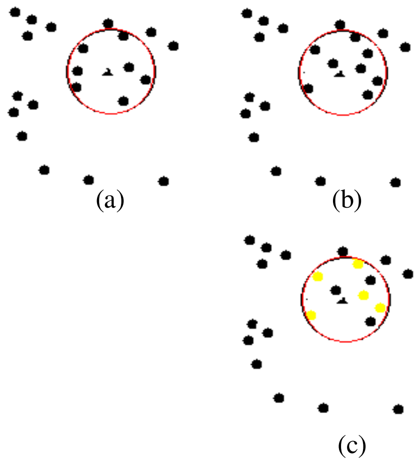


FIGURE 2: Gated measurements given an identical threshold for conventional PDA and filtered gate based PDA (FG-PDA): (a) measurements at previous scan. (b) Measurements at current scan. (c) Filtered gate based approach at the scan of (b).

3. INTEGRATION BETWEEN DATA ASSOCIATION AND FILTERED GATE

We propose an algorithm which depends on the history of observation for one scan and use a fixed threshold but operates similar to an adaptive estimator. In conventional data association approaches with a fixed threshold, all observations lying inside the reconstructed gate are considered in association. The gate may has a large number of observations due to heavy clutter, this leading to; increasing in association process since the probability of error to associate target-originated measurements my be increased. In our proposed algorithm a filtered gate structure is used to provide the possibility to decrease the number of observations in the gate by dividing the state of observations into valid and invalid that only the valid are considered in association. The proposed algorithm can be applied to all gate based approaches, including tracking and clustering. See Fig. 2. Fig. 2(a) and 2(b) show the candidates for association in both conventional probabilistic data association (PDA) and our proposed filtered gate based probabilistic data association (FG-PDA). Red circles represent the gated measurements. Our approach has measurements as well as ones inside the validated region but is divided into two states valid and invalid, yellow points represent invalid points as shown in Fig. 2(c).

4. IMPLEMENTATION OF PROBABILISTIC DATA ASSOCIATION FILTER USING FILTERED GATE METHOD

The Probabilistic Data Association (PDA) algorithm is used to calculate the probability that each validated measurements is attributable to the target in cluttered environments [1],[2],[4],[14],[18-19]. This algorithm is useful in tracking a single object (target) in clutter and referred to as the PDA filter (PDAF).

Notation for PDAF Approach

The PDAF calculates the associated probability of each element of the set of validated measurements at time *t*, denoted as $Z_t = \left\{z_t^i\right\}_{i=1:c_t}$ where z_t^i is the ith validated measurement and c_t is the number of measurements in the validation region at time *t*. under the Gaussian assumption for the prediction kernel $p\ (x_t \mid z_{1:t-1})$, the validation region is commonly taken to be the elliptical region

$$V_t = \left\{ Z : (z_t^i - H \ \overline{m}_t)^T S_t^{-1} (z_t^i - H \ \overline{m}_t) \ \le \gamma \right\} \quad (2)$$

Where γ is a given threshold and the covariance is defined by $S_t = H \ \overline{p}_t H^T + R$. We define the accumulation of validated measurements is $Z_{1:t} = \left\{ Z_j, \ \ for \ \ j \in \{1,....,t\} \right\}$.

Prediction step in PDAF Approach

We define the posterior distribution of x_t given the past sequence of observations $Z_{1:t-1}$ in the prediction step, i.e., $p \ (x_t | Z_{1:t-1})$. This process is equivalent to the prediction step of standard Kalman filter. The prediction distribution is defined by

$p \ (x_t | Z_{1:t-1}) = N \ (x_t; \overline{m}_t, \overline{p}_t)$,where

$\overline{m}_t = A \ \ m_{t-1}$ and $\overline{p}_t = A \ \ p_{t-1} A^T + Q$

Update Step in PDAF Approach

As mentioned in section 2.1, the hidden variables of the state space model are recursively estimated by the prediction and updating steps. The PDAF can be modeled as a state space model which can also be estimated using these recursive operations. First of all, the update step in PDAF approach is as

$$p \ (x_t | Z_{1:t}) = \sum_{i=0}^{c_t} N \ (x_t; m_t^i, p_t^i) \beta_{t,i} \quad (3)$$

where the $P \ (i | Z_{1:t}) = \beta_{t,i}$ is association probability and

$$p_t^i = \left[\overline{p}_t^{-1} + H^T R^{-1} H \right]^{-1} \text{ and } m_t^i = p_t^i \left[H^T R^{-1} z_t^i + \overline{p}_t^{-1} \overline{m}_t \right] \quad (4)$$

for $i \in \left\{ 0,....,c_t \right\}$. In addition, we have $m_t^0 = \overline{m}_t$ and $p_t^0 = \overline{p}_t$ for i = 0 where there is no target-originated measurement (i.e., z_t^0 = nil).

Estimating Conditional Probability in PDAF Approach

In order to obtain the filtering density, we require an estimate for the parameter $\beta_{t,i}$ for $i \in \left\{ 0,....,c_t \right\}$. Under the assumption of a poisson clutter model, the association probability $\beta_{t,i}$ can be estimated as [20-21]

$$\beta_{t,i} = P \ (i | Z_{1:t}) = \begin{cases} \dfrac{e_i}{b + \sum_{j=1}^{c_t} e_j}, & i = 1,.....c_t \\ \dfrac{b}{b + \sum_{j=1}^{c_t} e_j}, & i = 0 \end{cases} \quad (5)$$

where

$$
\begin{aligned}
e_i &= \exp\{D\} \\
b &= \lambda \left|2\pi S_t\right|^{1/2} \frac{1-p_D\, p_G}{p_D} \\
D &= -\frac{1}{2}(z_t^i - H\ \ \overline{m}_t)^{iT}\ S_t^{-1}(z_t^i - H\ \ \overline{m}_t)^i \\
&= -\frac{1}{2}(v_t)^{iT}\ S_t^{-1}(v_t)^i
\end{aligned}
\tag{6}
$$

and λ is a spatial density parameter. The functions p_D and p_G denote the probability of detection and Gaussian validation. The proposed filtering gate based PDAF is represented in algorithm 1. The algorithm is divided into four major parts: prediction, finding validated regions, estimating conditional probability and finally an update step. Since only finding validated regions component is fundamentally different from the conventional PDAF, we look at this in more detail.

Algorithm 1 PDAF using filtered gate

---

1. **for** t = 1 to T **do**
2. **Do prediction step**,

   $$x_{t\,|\,t-1} \sim P\ \ (x_t \,|\, Z_{1:t-1}) = N\ \ (x_t \,|\, \overline{m}_t\,, \overline{p}_t)$$

   where $\begin{Bmatrix} \overline{m}_t = A\ \ m_{t-1} \\ \overline{p}_t = A\ \ p_{t-1} A^T + Q \end{Bmatrix}$

3. **Finding validated region according to Algorithm 2**.
4. **Estimating conditional probability**, $\beta_{t,i}$ for

   for $i \in \left\{\ 0,\ldots.,c_t\ \right\}$,

   $$\beta_{t,i} = \begin{Bmatrix} \dfrac{e_i}{b + \sum\limits_{j=1}^{c_t} e_j}, & i = 1,\ldots..c_t \\ \dfrac{b}{b + \sum\limits_{j=1}^{c_t} e_j}, & i = 0 \end{Bmatrix}$$

   where

   $$e_i = \exp\left\{-\frac{1}{2}(v_t)^{iT}\ S_t^{-1}(v_t)^i\right\}$$

   $$b = \lambda \left|2\pi S_t\right|^{1/2} \frac{1-p_D\, p_G}{p_D}$$

5. **Do update step**,
6. Calculate the distribution of the missing observation $P\ \ (x_t \,|\, Z_{1:t-1})$ which is for *i* = 0,

   $m_t^0 = \overline{m}_t\,, \qquad p_t^0 = \overline{p}_t$

7. Calculate the distribution of the associated observation, $P\ (x_t \mid z_t^i, Z_{1:t-1}) = N\ (x_t; m_t^i, p_t^i)$ for $i = \left\{\ 1,....,c_t\ \right\}$

$$p_t^i = \left[\overline{p}_t^{-1} + H^T R^{-1} H\ \right]^{-1}$$
$$m_t^i = p_t^i \left[H^T R^{-1} z_t^i + \overline{p}_t^{-1} \overline{m}_t\right]$$

8. Calculate marginalized probability using Gaussian approximation, $P\ (x_t \mid Z_{1:t}) = N\ (x_t \mid m_t, p_t)$ *where* .

$$m_t = \sum_{i=0}^{c_t} \beta_{t,i} m_t^i$$
$$p_t = \sum_{i=0}^{c_t} \beta_{t,i} \left[p_t^i + (m_t^i - m_t)(m_t^i - m_t)^T\right] \tag{7}$$

9. end for

For finding the validated region, the filtered gate FG_PDAF after the prediction step checks the number of measurements z_{t-1}^i at time *t*-1 that lying inside the gate that determined by the same way of PDAF, then in the update step at time *t* also checks the number of measurements z_t^i that lying in the same gate. If any measurement in the current gate has approximately the same weight (position) to any measurement detected in the previous frame for the same gate within tolerance value with very small threshold δ as mentioned before, we consider this measurement be invalid in the gate and not taken in consideration to data association process.

Filtering the Validation Region to valid/invalid Observations

Intuitively, we find measurements in the gate with fixed size which are associated to the predicted position of the existing target before receiving new measurements. To update the predicted position, the new measurements in the gate is compared with the detected previous measurements in the same gate and avoid these new measurements which have approximately the same weight from data association process as described in algorithm 2.

Algorithm 2 Finding Validated Region of Filtered Gate based PDAF

---

1. **Find validated region for measurements at time *t*-1**:

$$Z_{t-1} = \{z_{t-1}^i\}, \quad i = 1,.....,m$$

By accepting only those measurements that lie inside the gate:

$$z_{t-1} = \left\{ z_{t-1} : (\ z_{t-1}^i - H\ \ \overline{m}_t)^T\ S_t^{-1} (\ z_{t-1}^i - H\ \ \overline{m}_t)\ \le \gamma \right\}$$

2. **Find validated region for measurements at time *t***:

$$Z_t = \{z_t^i\}, \quad i = 1,.....,n$$

By accepting only those measurements that lie inside the gate

$$z_t = \left\{ z_t : (z_t^i - H \ \overline{m}_t)^T S_t^{-1} (z_t^i - H \ \overline{m}_t) \ \le \gamma \right\}$$

where $\quad S_t = H \ \overline{p}_t H^T + R$

3. for $i = 1$ to n do

4. If $\left| z_t^i - z_{t-1}^j \right| < \delta \qquad j = 1,.....m$

   Set $z_t^i \ \ to \ \ I$,

   Remove z_{t-1}^j from the set Z_{t-1} , and set $m = m - 1$

5. Else

   Set $z_t^i \ \ to \ \ V$

6. End if
7. End for
8. **Obtain valid (V) measurements** c_t are included for data association process where the invalid (I) measurements c_f are excluded, i.e.:

$$Z_t \ \ be \ \ a \ \ set \ \ of \ \ all \ \ measurements \{z_t^i\} = V,$$

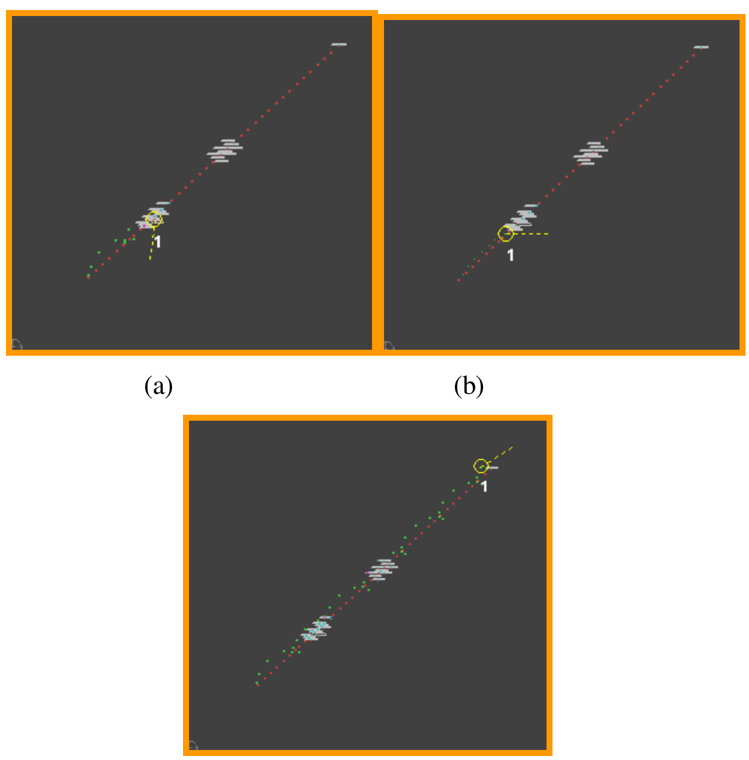
$$i = 1 \ \ to \ \ c_t \ where \ \ c_f = n - c_t$$

5. SIMULATION RESULTS

We used a synthetic dataset to highlight the performance of the proposed algorithm. The performance of the FG-PDAF is compared with a conventional PDAF and nearest neighbor kalman filter (NNKF) [22] . The synthetic data has a single track which continues from the first frame to the last frame. The mean and covariance for the initial distribution $p(x_0)$ is set to m_0 = [12, 15, 0, 0] and p_0 = diag ([400, 400, 100, 100]). The row and column sizes of the volume (V=$s_W \times s_H$). We initiate the other parameters as: τ $=148$,V=26x26 , λ= 0.001 , Δt = 4 , p_D =0.99, p_G =0.8, in addition, we also set the matrices of (1) as

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ Q = G \ \ G^T, \qquad R = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}, \qquad G = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}$$

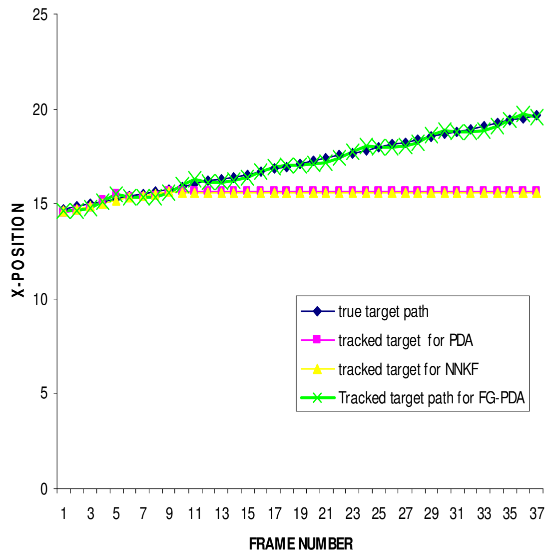
Given a fixed threshold ($\gamma = 10^{-5}$), we showed that the proposed FG-PDAF succeeded to track a target in dense clutter environment while the others conventional PDAF, NNKF failed to track a target as shown in Fig. 3. We obtain trajectories for X and Y components as shown in Fig. 4(a),(b). In this figure, the blue line represents the underlying truth target of the trajectory. Our proposed algorithm (green line) detects and associates the proper sequence of observation very well compared to PDAF (red line) and NNKF (yellow line).It is clear that the values obtained from using the proposal FG-PDA algorithm, is approximately attached to the values of the true target up to the processing of the last frame number while the values obtained from the conventional two algorithms(PDA and NNKF) is started to far from the true target values after 11 number of frames due to failing in tracking process with existing more numbers of false targets . We also compared error value and root mean square (RMSE) for different approaches as shown in Fig. 5,6. It is also noted that the absolute tracking error is very successful especially after 11 number of frame using the proposed algorithm that has far lower error, RMSE values than either PDAF or NNKF over frame numbers.
From results, Simulation have been achieved that the proposed algorithm improves the conventional PDA algorithm to be able to, continue tracking without losing the true target in heavy clutter environment, decreases the number of valid measurements region by avoiding the measurements that represent the false targets and thus, the performance of the data association process is increased.



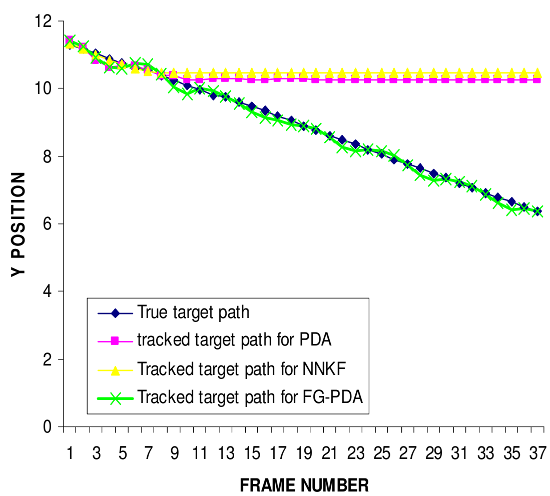
(a) (b)

(c)

FIGURE 3: The state of tracking a single target moving in heavy clutter using 3 approaches algorithm (a) PDAF failed to track (b) NNKF failed to track (c) FG-PDAF succeeded to track

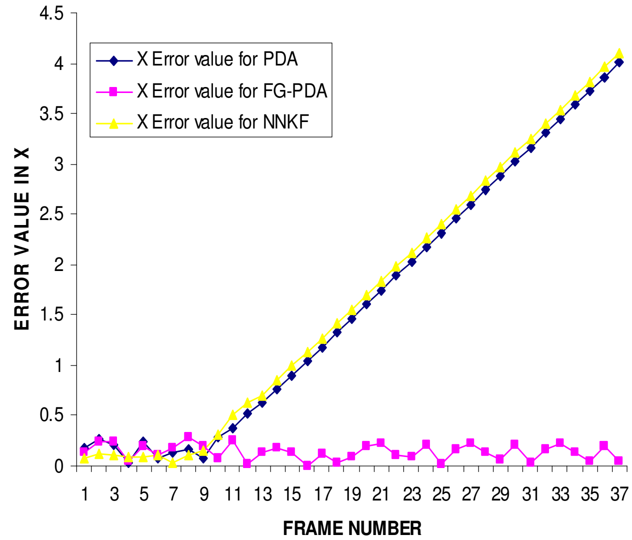


(a)

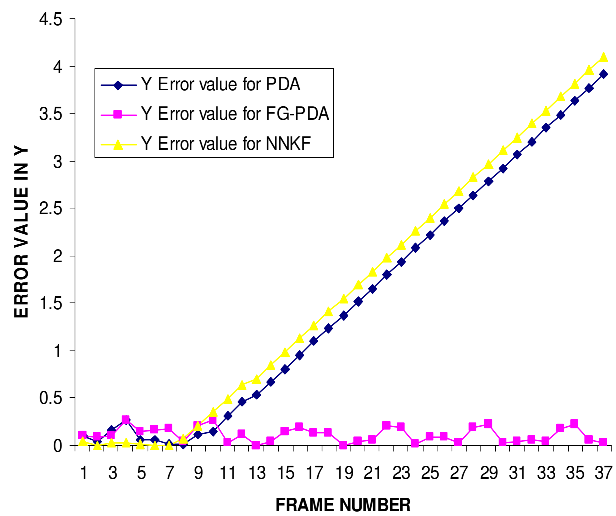


(b)

FIGURE 4: Trajectory for X and Y components for the 3 approaches algorithm used in tracking a target in dense clutter and the true target path (as presented in Fig. 3). (a) Trajectory for X (b) Trajectory for Y



(a)



(b)

FIGURE 5: Absolute error value for the 3 approaches algorithm in X- and Y-components where the error for FG-PDA is minimum compared to the increased error for PDA and NNKF due to failing in tracking (a) absolute error for X component (b) absolute error for Y component

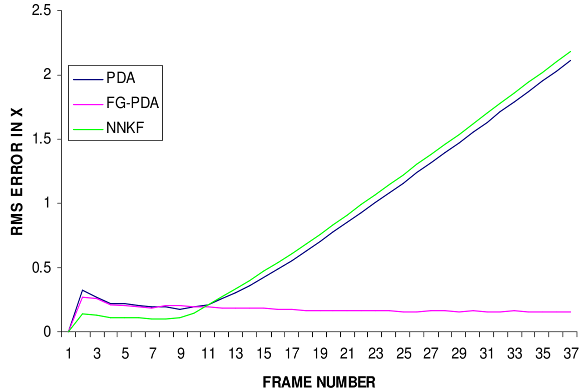


FIGURE 6: The root mean square error [RMSE] over frame number (each frame take 4 sec / one scan) for the 3 approaches algorithm and the RMSE is maintained minimum for the proposed FG-PDA and less sensitivity to dense clutter.

6. CONCLUSION

We have showed that the probabilistic data association filter (PDAF) is improved by avoiding the false targets from the valid based measurement region using a filtering method in dense clutter environment. This approach can be used to overcome the clutter of gate based approaches in tracking. With even high threshold values for gate size, we can obtain smaller validated measurement regions with improving data association Process which have been shown to give targets the ability to continue tracking in dense clutter.

7. REEFRENCES

[1] Y. Bar-Shalom and W. D. Blair." *Multitarget Multisensor Tracking:Applications and Advances"*, volume III. Artech House, Norwood, MS, 2000.

[2] Y. Bar-Shalom and T. E. Fortmann. "*Tracking and Data Association"*. Academic Press, 1988.

[3] R. A. Singer, R. G. Sea, and K. B. Housewright." *Derivation and evaluation of improved tracking filters for use in dense multitarget environment*". IEEE Transactions on Information Theory, 20:423–432, 1974.

[4] Y. Bar-Shalom and E. Tse." *Tracking in a cluttered environment with probabilistic data-association*". Automatica, 1975.

[5] Y. Bar-Shalom. "*Tracking methods in a multitarget environment*". IEEE Transactions on Automated control, 23:618–626, 1978.

[6] D. B. Reid. "*An algorithm for tracking multiple targets*". IEEE Transactions on Automatic Control, 24:843–854, 1979.

[7] J. B. Collins and J. K. Uhlmann. "*Efficient gating in data association with multivariate Gaussian distribution states*". IEEE Transactions on Aerospace and Electronic Systems, 28(3):909–916, July 1992.

[8] S. S. Blackman and R. Popoli. "*Design and Analysis of Modern Tracking Systems"*. Artech House, 1999.

[9] F. J. Breidt and A. L. Carriquiry." *Highest density gates for target tracking*". IEEE Transactions on Aerospace and Electronic Systems,36(1):47–55, Jan. 2000.

[10] X. Wang, S. Challa, and R. Evans." *Gating techniques for maneuvering target tracking in clutter*". IEEE Transactions on Aerospace and Electronic Systems, 38(3):1087–1097, July 2002.

[11] Y. Kosuge and T. Matsuzaki. "*The optimum gate shape and threshold for target tracking*". In SICE Annual Conference, 2003.

[12] D. Musicki and M. R. Morelande. "*Gate Volume Estimation for Target Tracking*". In International Conference on Information Fusion, 2004.

[13] M. Wang, Q. Wan, and Z. You." *A gate size estimation algorithm for data association filters*". Science in China, 51(4):425–432, April 2008.

[14] Ji Won Yoon and Stephen J .Roberts *"Robust Measurement Validation in Target Tracking using Geometric Structure"* IEEE Signal Pocessing Letters,17(5):493-496,May 2010

[15] Simon Haykin. "*Radar Signal Processing*", *.* IEEE ASSP MAGAZINE*,* April 1985.

[16] G.Richard Curry. '*Radar System Performance Modeling*". Artich House, 2rd ed. Edition,2005.

[17] R. F. Stengel. "*Optimal Control and Estimation"*. Dover Publications, 1994.

[18] G. W. Pulford and R. J. Evans. "*Probabilistic data association for systems with multiple simultaneous measurements*". Automatica, 32(9):1311– 1316, Set. 1996.

[19] X. R. Li and Y. Bar-Shalom. "*Stability evaluation and track life of the PDAF for tracking in clutter*". IEEE Transactions on Automatic Control, 36(5):588–602, May 1991.

[20] T. Kirubarajan and Y. Bar-Shalom." *Probabilistic Data Association Techniques for Target Tracking in Clutter*". Proceedings of the IEEE, 92(3):536–557, Mar. 2004.

[21] Gad.A,Majdi. F and Farooq. M. *"A Comparison of Data Association Techniques for Target Tracking in Clutter"* Proceedings of information Fusion, of the Fifth international conference vol 2:1126-1133,Nov 2002.

[22] V P S Naidu. "*Data Association and Fusion Algorithms for Tacking in Presence of Measurement Loss*". IE(I) Journal -AS, Vol 86, pages 17–28, May 2005.