pComparison of Energy Detection Based Spectrum Sensing Methods Over Fading Channels in Cognitive Radio

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Abstract

With the advance of wireless communications, the problem of bandwidth scarcity has become more prominent. Cognitive radio technology has come out as a way to solve this problem by allowing the unlicensed users to use the licensed bands opportunistically. To sense the existence of licensed users, many spectrum sensing techniques have been devised. This paper presents the energy detection based spectrum sensing technique. In the present work, the comparison of ROC curves has been done for various wireless fading channels using squaring and cubing operation. The cubing operation shows an improvement of up to 0.6 times as compared to the squaring operation for AWGN channel. For Rayleigh channel, the improvement achieved is up to 0.4 times as we move from squaring to cubing operation in an energy detector.

Keywords: Cognitive Radio, Spectrum Sensing, Probability of Detection.

1. INTRODUCTION

The progressive growth of the wireless communications, has led to under-utilization of the spectrum. It has also been observed that major portion of the spectrum is rarely used as it is reserved only for the licensed users while other is heavily used. Federal Communications Commission (FCC) proposed the solution to this problem by allowing the unlicensed users to use the licensed bands opportunistically and named it as Cognitive Radio (CR) [1]. Cognitive Radio is a sophisticated wireless system that gathers the information about the surrounding environment and adapts its transmission parameters accordingly [2]. There are two main characteristics of Cognitive Radio: Cognitive Capability (i.e. gathering the information about the environment) and reconfigurability (i.e. adapting its transmission parameters according to the gathered information) [3]. Since the unlicensed secondary (users) are allowed to utilize a licensed band only when they do not cause interference to the licensed (primary) users, spectrum sensing is considered as one of the most important elements of the Cognitive Radio. Spectrum sensing aims at monitoring the usage and the characteristics of the covered spectral band(s) and is thus required by the secondary users both before and during the use of licensed spectral bands [1].

Energy detector based approach, also known as radiometry or periodogram, is one of the popular methods for spectrum sensing as it is of non-coherent type and has low implementation complexity. In addition, it is more generic as receivers do not require any prior knowledge about the primary user's signal [4]. In this method, the received signal's energy is measured and compared against a pre-defined threshold to determine the presence or absence of primary user's signal. Moreover, energy detector is widely used in ultra wideband (UWB) communications to borrow an idle channel from licensed user. Detection probability (P_d), False alarm probability (P_f) and missed detection probability ($P_m = 1 - P_d$) are the key measurement metrics that are used to analyze the performance of an energy detector. The performance of an energy detector is illustrated by the receiver operating characteristics (ROC) curve which is a plot of P_d versus P_f or P_m versus P_f [5].

This paper is organized as follows: Section 2 describes the performance analysis of energy detector. Section 2.1 illustrates test statistic for energy detection using squaring operation. Section 2.2 and 2.3 describe the expressions for probability of detection using squaring operation for AWGN and Rayleigh channel respectively. Simulation Results for squaring operation and cubing operation are presented in Section 2.4 followed by conclusions in section 2.5.

2. PERFORMANCE ANALYSIS OF ENERGY DETECTOR

Energy detector is composed of four main blocks [6]:
1) Pre-filter.
2) A/D Converter (Analog to Digital Converter).
3) Squaring Device.
4) Integrator.

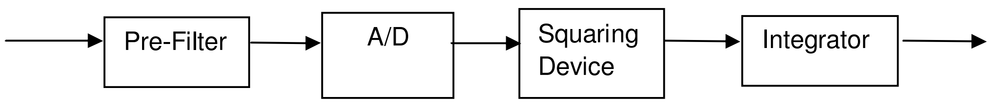

FIGURE 1: Block Diagram of Energy Detector [6].

The output that comes out of the integrator is energy of the filtered received signal over the time interval T and this output is considered as the test statistic to test the two hypotheses H_0 and H_1 [7].
H_0 : corresponds to the absence of the signal and presence of only noise.
H_1 : corresponds to the presence of both signal and noise.

2.1 Test Statistic Using Squaring Operation

Considering the following notations:

$x(t)$ is the transmitted signal waveform, $y(t)$ is the received signal waveform, $w_i(t)$ is in-phase noise component, $w_q(t)$ is quadrature phase component, B_N is noise bandwidth, N_0 is power-spectral density (two-sided), N is power spectral density (one-sided), T is the sampling interval, E_s is the signal energy, Λ is decision threshold.

The received signal $y(t)$ is filtered by a pre-filter which is a band-pass filter. The filtered signal is then passed through A/D converter i.e. converted to samples. Now, if noise $w(t)$ is a band-pass random process, its sample function can be written as [8, Eq. (5.4)]:

$$w(t) = w_i(t)\cos w_c t - w_q(t)\sin w_c t \qquad (1)$$

where w_c is angular frequency. If $w(t)$ is confined to a bandwidth of B_N and has a power-spectral density N_0, then $w_i(t)$ and $w_q(t)$ can be considered as two low-pass processes with bandwidth less than $B_N/2$ and power spectral density of each equal to $2N_0$. Now, if a sample function has Bandwidth B and duration T, then it can be described approximately by a set of sample values *2BT* or degrees of freedom will be 2BT . Thus, $w_i(t)$ and $w_q(t)$ each will have degrees of freedom, *d* equal to $B_N T$ [7]. Also, using approximation as in [9, Eq. (2.1-21)]**:**

$$\int_0^T w^2(t)dt = \frac{1}{2}\int_0^T [w_i^2(t) + w_q^2(t)]dt \qquad (2)$$

As $w_i(t)$ and $w_q(t)$ are considered as low-pass processes, therefore according to sampling theorem, $w_i(t)$ can be written as [10]:

$$w_i(t) = \sum_{k=-\infty}^{+\infty} c_{ik}\, sinc(B_N t - k) \qquad (3)$$

where $sincx = \frac{\sin\pi x}{\pi x}$ and $c_{ik} = w_i(\frac{k}{B_N})$ is a Gaussian random variable with zero mean and variance $\sigma_k{}^2 = 2N_0B_N, \forall k$.

Now, using the fact as in [7],

$$\int_{-\infty}^{\infty} sinc(B_N t - k)sinc(B_N t - m)dt = \begin{cases} \frac{1}{B_N}, & k = m \\ 0, & k \neq m \end{cases} \quad (4)$$

Therefore using (3) and (4), we obtain:

$$\int_{-\infty}^{\infty} w_i{}^2(t)dt = \frac{1}{B_N}\sum_{k=-\infty}^{+\infty} c_{ik}{}^2 \quad (5)$$

As $w_i(t)$ has $B_N T$ degrees of freedom over the interval $(0, T)$ [7], therefore

$$w_i(t) = \sum_{k=1}^{B_N T} c_{ik}\, sinc(B_N t - k), \qquad 0 < t < T \quad (6)$$

And the integral $\int_{-\infty}^{\infty} w_i{}^2(t)dt$ over the interval $(0, T)$ can be written as

$$\int_0^T w_i{}^2(t)\,dt = \frac{1}{B_N}\sum_{k=1}^{B_N T} c_{ik}{}^2 \quad (7)$$

Similarly,

$$\int_0^T w_q{}^2(t)dt = \frac{1}{B_N}\sum_{k=1}^{B_N T} c_{qk}{}^2 \quad (8)$$

Substituting $\frac{c_{ik}}{\sqrt{2B_N N_0}} = d_{ik}$ and $\frac{c_{qk}}{\sqrt{2B_N N_0}} = d_{qk}$ in (7) and (8), and using (2), we arrive at [7]:

$$\int_0^T w^2(t)dt = \left[\sum_{k=1}^{B_N T} d_{ik}{}^2 + \sum_{k=1}^{B_N T} d_{qk}{}^2\right]. N_0 \quad (9)$$

Similarly, considering transmitted signal $x(t)$ as a band-pass process [8], we have

$$\int_0^T x^2(t)\,dt = \left[\sum_{k=1}^{B_N T} b_{ik}{}^2 + \sum_{k=1}^{B_N T} b_{qk}{}^2\right]. N_0 \quad (10)$$

or,

$$\sum_{k=1}^{B_N T}(b_{ik}{}^2 + b_{qk}{}^2) = \frac{E_s}{N_0} \quad (11)$$

where $b_{ik} = \frac{x_i(\frac{k}{B_N})}{\sqrt{2B_N N_0}}$, $b_{qk} = \frac{x_q(\frac{k}{B_N})}{\sqrt{2B_N N_0}}$ and $E_s = \int_0^T x^2(t)\,dt$ is the signal energy.

The output of the integrator is $Y = \frac{1}{T}\int_0^T y^2(t)\,dt$.Test statistic can be Y or any quantity monotonic with Y. Taking Y' as the test statistic [7]:

$$Y' = \frac{1}{N_0}\int_0^T y^2(t)\,dt \quad (12)$$

Now, Under Hypothesis H_0, the received signal is only noise i.e. $y(t) = w(t)$, therefore using (9) test statistic Y' can be written as:

$$Y' = \sum_{k=1}^{B_N T}({d_{ik}}^2 + {d_{qk}}^2) \tag{13}$$

Thus, Test statistic Y'under H_0 is chi-square distributed [9] with $2B_N T$ degrees of freedom or $Y' \sim \chi^2_{2d}$ [11] .

Under Hypothesis H_1, received signal is the sum of signal and noise i.e. $y(t) = w(t) + x(t)$. Again considering $y(t)$ as a band-limited process [8], using equations (2-10), we arrive at [7]:

$$\int_0^T y^2(t)dt = \left[\sum_{k=1}^{B_N T}(d_{ik} + b_{ik})^2 + \sum_{k=1}^{B_N T}(d_{qk} + b_{qk})^2\right]. N_0 \tag{14}$$

Then, using (12) and (14), test statistic Y' can be written as:

$$Y' = \left[\sum_{k=1}^{B_N T}(d_{ik} + b_{ik})^2 + \sum_{k=1}^{B_N T}(d_{qk} + b_{qk})^2\right] \tag{15}$$

Thus, test statistic Y'under H_1 has a non-central chi-square distribution [9] with $2B_N T$ degrees of freedom and a non-centrality parameter λ given by $\frac{E_s}{N_0}$ [7]. Now, Defining Signal to Noise Ratio, γ in terms of non-centrality parameter as in [11]:

$$\gamma = \frac{E_s}{N} = \frac{E_s}{2N_0} = \frac{\lambda}{2} \tag{16}$$

Thus, test statistic Y' under H_1: $Y' \sim \chi^2_{2d}(\lambda)$ [11]. Also, probability density function of Y' can be expressed as [9, Eq. (2.3-21) & Eq. (2.3-29)]**:**

$$f_{Y'}(y) = \begin{cases} \frac{1}{2^d \Gamma(u)} y^{d-1} e^{-\left(\frac{y}{2}\right)}, & H_0 \\ \frac{1}{2}\left(\frac{y}{\lambda}\right)^{\frac{d-1}{2}} e^{-\left(\frac{\lambda+y}{2}\right)} I_{d-1}(\sqrt{\lambda y}), & H_1 \end{cases} \tag{17}$$

2.2 Probability Of Detection And False Alarm For AWGN Channel

Probability of detection P_d and false alarm P_f can be evaluated respectively by [11]:

$$P_d = P(Y' > \Lambda | H_1) \tag{18}$$

$$P_f = P(Y' > \Lambda | H_0) \tag{19}$$

where Λ is the decision threshold. Also, P_f can be written in terms of probability density function as:[12, Eq. (4-16) & Eq. (4-22)]

$$P_f = \int_\Lambda^\infty f_{Y'}(y)\,dy \tag{20}$$

Using (19),

$$P_f = \frac{1}{2^d \Gamma(d)} \int_\Lambda^\infty y^{d-1} e^{-\left(\frac{y}{2}\right)} dy \tag{21}$$

Dividing and multiplying the R.H.S. of above equation by 2^{d-1}, we get

$$P_f = \frac{1}{2\Gamma(d)} \int_\Lambda^\infty \left(\frac{y}{2}\right)^{d-1} e^{-\left(\frac{y}{2}\right)} dy \tag{22}$$

Substituting $\frac{y}{2} = t$, $\frac{dy}{2} = dt$ and changing the limits of integration to $(\frac{\Lambda}{2}, \infty)$, we get

$$P_f = \frac{1}{\Gamma(d)} \int_{\Lambda/2}^\infty (t)^{d-1} e^{-(t)}\,dt \tag{23}$$

or,

$$P_f = \frac{\Gamma(d, \Lambda/2)}{\Gamma(d)} \tag{24}$$

where $\Gamma(.)$ is the incomplete gamma function [13]. Now, Probability of detection can be written by making use of the cumulative distribution function [12, Eq. (4.22)].

$$P_d = 1 - F_{Y'}(\Lambda) \tag{25}$$

The cumulative distribution function (CDF) of Y' can be obtained (for an even number of degrees of freedom which is $2d$ in our case) as [9, Eq. (2.1-124)]:

$$F_{Y'}(y) = 1 - Q_d\left(\sqrt{\lambda}, \sqrt{y}\right) \tag{26}$$

Therefore, using (25) and (26), probability of detection P_d for AWGN channel is [11]:

$$P_d = Q_d\left(\sqrt{\lambda}, \sqrt{\Lambda}\right) \tag{27}$$

Using (16),

$$P_d = Q_d\left(\sqrt{2\gamma}, \sqrt{\Lambda}\right) \tag{28}$$

2.3 Probability Of Detection And False Alarm For Rayleigh Channel

Probability density function for Rayleigh channel is [12, Eq. (4-44)]:

$$f(\gamma) = \frac{1}{\bar{\gamma}} exp\left(\frac{-\gamma}{\bar{\gamma}}\right) \qquad ; \gamma \geq 0 \tag{29}$$

The Probability of detection for Rayleigh Channels is obtained by averaging their probability density function over probability of detection for AWGN Channel [11]:

$$P_{d,R} = \int_0^{\infty} P_d \, f(\gamma) d\gamma \tag{30}$$

where $P_{d,R}$ is the probability of detection for Rayleigh channel.

With (28) and (29), (30) becomes

$$P_{d,R} = \frac{1}{\bar{\gamma}} \int_0^{\infty} Q_d\left(\sqrt{2\gamma}, \sqrt{\Lambda}\right) exp\left(\frac{-\gamma}{\bar{\gamma}}\right) d\gamma \tag{31}$$

Now, substituting $\sqrt{\gamma} = x \,, \gamma = x^2, d\gamma = 2x dx$ in (31), we get

$$P_{d,R} = \frac{2}{\bar{\gamma}} \int_0^{\infty} x . Q_d\left(\sqrt{2}x, \sqrt{\Lambda}\right) exp\left(\frac{-x^2}{\bar{\gamma}}\right) dx \tag{32}$$

Considering the result [14]

$$\int_0^{\infty} dx . x . exp\left(\frac{-p^2x^2}{2}\right) Q_M(ax, b) = \frac{1}{p^2} . exp\left(-\frac{b^2}{2}\right) . \left\{\left(\frac{p^2+a^2}{a^2}\right)^{M-1} \left[exp\left(\frac{b^2}{2} . \frac{a^2}{p^2+a^2}\right) - \sum_{n=0}^{M-2} \frac{1}{n!}\left(\frac{b^2}{2} . \frac{a^2}{p^2+a^2}\right)^n\right] + \sum_{n=0}^{M-2} \frac{1}{n!}\left(\frac{b^2}{2}\right)^n\right\} \tag{33}$$

Comparing (32) and (33), $p^2 = \frac{2}{\bar{\gamma}}$, $a = \sqrt{2}$, $b = \sqrt{\Lambda}$, $M = d$

Thus, using (33), Probability of detection for Rayleigh channel can be expressed as [11]:

$$P_{d,R} = e^{(-\Lambda/2)} \sum_{n=0}^{d-2} \frac{1}{n!}\left(\frac{\Lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{d-1} \left[\exp\left(-\frac{\Lambda}{2(1+\bar{\gamma})}\right) - \exp\left(-\frac{\Lambda}{2}\right) \sum_{n=0}^{d-2} \frac{1}{n!}\left(\frac{\Lambda\bar{\gamma}}{2(1+\bar{\gamma})}\right)^n\right] \tag{34}$$

2.4 Simulation Results

The performance of energy detector is analysed using ROC (Receiver operating characteristics) curves for fading channels. Monte-Carlo method is used for simulation. It can be seen in the following figures that with increase in SNR (Signal to Noise Ratio), the performance of energy detection improves. FIGURE 2 and FIGURE 4 illustrates the ROC curves using squaring operation for AWGN and Rayleigh channel respectively. FIGURE 3 and FIGURE 5 depicts improvement in the performance of energy detector using cubing operation over AWGN and Rayleigh channel respectively. We assume time-bandwidth product=5.

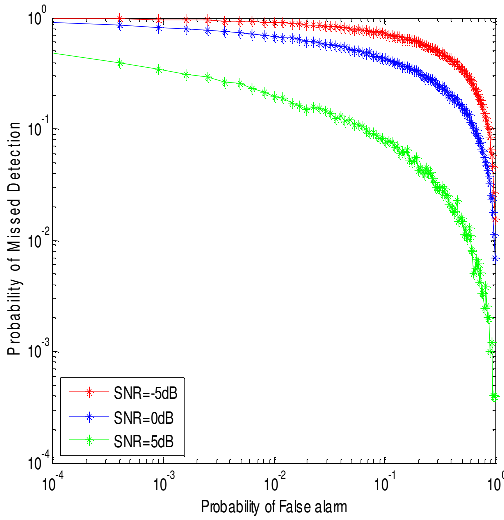

FIGURE 2: Complementary ROC Curves for AWGN using Squaring operation.

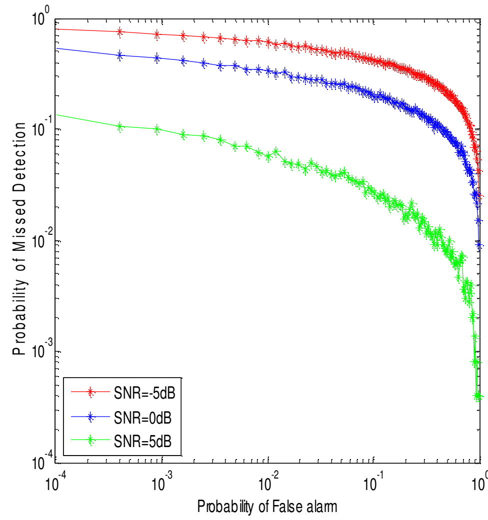

FIGURE 3: Complementary ROC Curves for AWGN using Cubing operation.

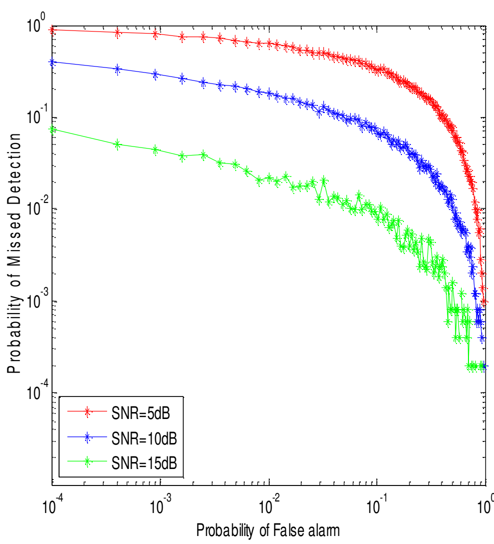

FIGURE 4: Complementary ROC for Rayleigh using Squaring operation.

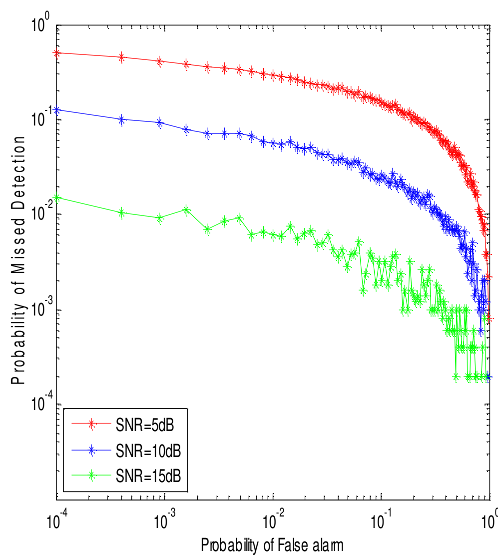

FIGURE 5: Complementary ROC for Rayleigh using Cubing operation.

The results obtained using cubing operation show an improvement of roughly one order of magnitude as compared to the energy detection method illustrated in [11]. The results obtained are quantified as shown in TABLE 1 and TABLE 2. These results illustrate improvement in probability of detection using cubing operation. This improvement has gone up to 0.4 times for Rayleigh Channel and 0.6 times for AWGN Channel. We assume time-bandwidth product=5 and Average SNR=5dB.

Probability of False Alarm	Probability of detection for AWGN Channel (Squaring Device)	Probability of detection for AWGN Channel (Cubing Device)	Improvement (times)
0.0001	0.5372	0.8656	0.6113
0.0441	0.8792	0.9594	0.0912
0.1681	0.9390	0.9758	0.0392
0.3721	0.9748	0.9862	0.0117
0.6561	0.9938	0.9952	0.0014

TABLE 1: Improvement using cubing operation for AWGN channel.

Probability of False Alarm	Probability of detection for Rayleigh Channel (Squaring Device)	Probability of detection for Rayleigh Channel (Cubing Device)	Improvement (times)
0.0001	0.6516	0.9746	0.4957
0.0441	0.4096	0.9982	0.0146
0.1681	0.9976	0.9996	0.0020
0.3721	0.9998	1.000	0.0002
0.6561	1.000	1.000	0

TABLE 2: Improvement using cubing operation for Rayleigh channel.

2.5 Conclusions

In the present work, the performance of energy detector is analysed. Closed form expressions for Probability of detection and false alarm over AWGN and Rayleigh channels are described. Using ROC Curves, it is shown that the Probability of detection is improved if cubing operation is used instead of squaring operation. Energy detection has the advantage of low implementation and computational complexities.

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